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FLUID MECHANICS

MECHANICAL ENGINEERING

Date of Test : 27/09/2024

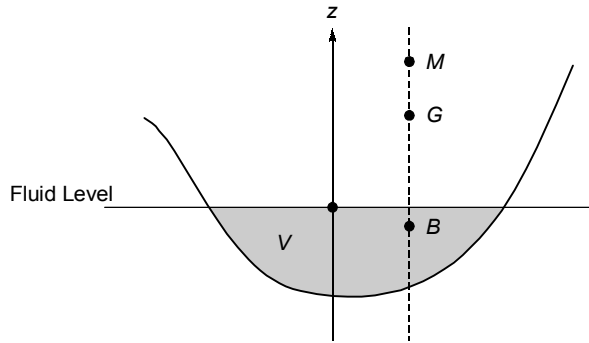
ANSWER KEY >

1. (d)	7. (b)	13. (c)	19. (b)	25. (b)
2. (b)	8. (b)	14. (d)	20. (d)	26. (c)
3. (a)	9. (b)	15. (c)	21. (b)	27. (b)
4. (c)	10. (d)	16. (b)	22. (a)	28. (c)
5. (c)	11. (a)	17. (d)	23. (a)	29. (a)
6. (c)	12. (d)	18. (a)	24. (b)	30. (a)

DETAILED EXPLANATIONS

1. (d)

Stable equilibrium condition



2. (b)

For geometrically similar model and prototype

$$\left(\frac{P}{N^3 D^5}\right)_{\text{model}} = \left(\frac{P}{N^3 D^5}\right)_{\text{prototype}}$$

Given,

$$N_m = N_p$$

⇒

$$\frac{P_m}{N_m^3 D_m^5} = \frac{P_p}{N_p^3 D_p^5}$$

$$\frac{P_m}{P_p} = \frac{N_m^3 D_m^5}{N_p^3 D_p^5}$$

$$\frac{P_m}{P_p} = \frac{2^3 N_p^3}{N_p^3} \times \frac{D_m^5}{16^5 D_m^5}$$

$$P_m = \frac{10 \times 10^6 \times 2^3}{16^5} W = 76.29 \text{ W}$$

3. (a)

$$P_A - H \times 9.81 \times 1 - 0.18 \times 9.81 \times 0.827 = P_B - 13.6 \times 9.81 \times (H + 0.53)$$

$$-H \times 9.81 - 1.4603 = 97 - 13.6 \times 9.81 \times H - 13.6 \times 9.81 \times 0.53$$

$$\Rightarrow H = 0.2245 \text{ m}$$

$$\therefore H = 22.45 \text{ cm}$$

4. (c)

$$\therefore \bar{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi d^2}{4}} \Rightarrow \frac{4Q}{\pi d^2} = \frac{4 \times 880 \times 10^{-9}}{\pi \times 0.50^2 \times 10^{-6}} = 4.48 \text{ m/s}$$

We know,
$$Q = \frac{\pi \Delta p D^4}{128 \mu L}$$

$$\Rightarrow \mu = \frac{\pi \Delta p D^4}{128 Q L} = \frac{\pi \times 10^6 \times (0.5)^4 \times 10^{-12}}{128 \times 880 \times 10^{-9} \times 1}$$

$$\mu = 1.74 \times 10^{-3} \text{ Ns/m}^2$$

5. (c)

$$Re_{\text{critical}} = 2000 = \frac{VD}{\nu} = \frac{\rho VD}{\mu}$$

$$2000 = \frac{950 \times V \times 0.15}{8 \times 10^{-2}}$$

$$V = 1.123 \text{ m/s}$$

$$\text{Head loss} = \frac{32\mu VL}{\rho g D^2} = \frac{32 \times 8 \times 10^{-2} \times 1.123 \times 300}{950 \times 9.81 \times 0.15^2}$$

$$= 4.113 \text{ m} \Rightarrow \text{Maximum difference in oil elevations.}$$

6. (c)

During cavitation, the vapour bubbles starts forming where the pressure of the liquid falls below the vapour pressure and sudden collapsing of these vapour bubbles in a region of higher pressure. When they collapse, a very high pressure is created. This causes pitting action on the surfaces over which they collapse. Hence during, cavitation and subsequent, pitting, pre-dominant forces are compressive forces.

7. (b)

Force on piston = Shear force due to oil viscosity

$$18 = \tau \cdot A$$

$$18 = \frac{\mu V}{h} \times \pi \times d \times L$$

$$\Rightarrow 18 = \frac{3 \times 0.1 \times V \times \pi \times 0.0795 \times 0.3}{0.025 \times 10^{-2}}$$

$$\Rightarrow V = 0.2001 \text{ m/s}$$

$$\Rightarrow V = 20 \text{ cm/s}$$

8. (b)

$$\text{Sensitivity} = \frac{1}{\sin \theta} = \frac{1}{\sin 30^\circ} = 2$$

9. (b)

Surface tension is due to cohesion between liquid particles at the surface.

10. (d)

$$P = \rho_{\text{Hg}} \times g \times H$$

$$6.8 \times 10^4 = 13.6 \times 10^3 \times 9.81 \times H$$

$$H_{\text{Hg}} = 0.5096 \text{ m}$$

$$H_{\text{water}} = \frac{13.6}{1} \times 0.5096$$

$$H_{\text{water}} = 6.931 \text{ m}$$

11. (a)

For the condition of verge of tipping, the centre of pressure must be at C.

$$\therefore \text{Height of C above B} = \frac{9.5}{3} = 3.16 \text{ m}$$

[∵ for a rectangular plane surface of height H, just completely inside fluid, the centre of pressure is at $\frac{H}{3}$ from base.]

12. (d)

$$\text{Volume of cube} = a^3$$

$$a^3 = 125 \times 10^{-3} \times 10^{-3} \text{ m}^3$$

$$\Rightarrow a = 5 \times 10^{-2}$$

$$\Rightarrow a = 0.05 \text{ m}$$

$$F = p \times A$$

$$P_{\text{bottom}} = p_{\text{atm}} + h_1 g \rho_{\text{oil}} + h_2 \rho_{\text{water}} g$$

$$= 101325 + 0.5 \times 0.8 \times 1000 \times 9.81 + 0.3 \times 1000 \times 9.81$$

$$P_{\text{bottom}} = 108192 \text{ Pa}$$

$$F = P_{\text{bottom}} \times A = 108192 \times 0.05^2 = 270.48 \text{ N}$$

$$T = \text{Upthrust} - W$$

$$= 125 \times 10^{-6} \times 1000 \times 9.81 - 125 \times 10^{-6} \times 0.77 \times 1000 \times 9.81$$

$$= 0.282 \text{ N}$$

13. (c)

$$u = \frac{\partial \psi}{\partial y}$$

$$u = 2x^2 + (x + t) 2y$$

∴ for face OB,

$$x \Rightarrow 0$$

$$u_{OB} = 2ty$$

Discharge through AB

$$\therefore Q_{AB} = \int_0^2 u_{OB} \cdot 5 dy = \int_0^2 2ty \cdot 5 dy$$

$$\text{At } t = 1$$

$$Q_{OB} = 20 \text{ units}$$

$$\therefore V = -\frac{\partial \psi}{\partial x} = -[4xy + y^2]$$

$$\text{At } y = 0$$

$$V = 0$$

$$\therefore Q_{AO} = 0$$

$$\begin{aligned} \therefore Q_{AB} &= Q_{OB} + Q_{OA} \\ &= 20 + 0 \\ &= 20 \text{ units} \end{aligned}$$

14. (d)

$$V_2 = \frac{Q}{A_2} = \frac{1.13 \times 10^{-6}}{\frac{\pi}{4} \times (0.0012)^2} \simeq 1 \text{ m/s}$$

$$\frac{P_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_f$$

$$h_f = z_1 - z_2 - \frac{\alpha_2 V_2^2}{2g}$$

$$\Rightarrow h_f = 0.6 - 0 - \frac{(2)(1)^2}{2 \times 9.81} = 0.5 \text{ m}$$

$$h_f = \frac{32 \mu V L}{\rho g D^2}$$

$$\Rightarrow 0.5 = \frac{32 \times \mu \times 0.3 \times 1}{9000 \times 0.0012^2}$$

$$\Rightarrow \mu = 6.75 \times 10^{-4} \text{ Pa-s}$$

15. (c)

$$\mu = 0.97 \text{ poise} = 0.097 \text{ Ns/m}^2$$

$$\rho = 0.9 \times 998 = 898.2 \text{ kg/m}^3$$

$$\dot{m} = \text{mass rate of flow} = \frac{100}{30} = 3.333 \text{ kg/s}$$

$$Q = \text{Volume rate of flow}$$

$$= \frac{3.333}{898.2}$$

$$\Rightarrow 3.711 \times 10^{-3} \text{ m}^3/\text{s} = 3.711 \text{ L/s}$$

$$\text{Area of flow} = A = \frac{\pi}{4} (0.1)^2 = 0.007854 \text{ m}^2.$$

$$\bar{U} = \frac{3.711 \times 10^{-3}}{0.007854} = 0.4725 \text{ m/s}$$

$$-\Delta P = \frac{32 \mu \bar{U} L}{D^2} = \frac{32 \times 0.097 \times 0.4725 \times 10.0}{(0.1)^2}$$

$$-\Delta P = 1467 \text{ Pa}$$

16. (b)

$$Re_L = \frac{UL}{\nu} = \frac{1.75 \times 5}{1.475 \times 10^{-5}}$$

$$Re_L = 5.932 \times 10^5$$

$$C_f = \frac{0.074}{Re_L^{1/5}} = \frac{0.074}{(5.932 \times 10^5)^{1/5}} = 5.183 \times 10^{-3}$$

Drag force on one side of the plate,

$$F_d = C_f \times \text{area} \times \frac{1}{2} \rho U^2$$

$$= 5.183 \times 10^{-3} \times (1.8 \times 5) \times 1.22 \times \frac{(1.75)^2}{2}$$

$$F_d = 0.0871 \text{ N}$$

17. (d)

∴ Continuity equation holds,

$$\therefore \frac{\pi}{4} \times (5)^2 \times 2 = \frac{\pi}{4} \times 3^2 \times x$$

$$x = 5.55 \text{ m/s}$$

Mars flow rate

$$\Rightarrow \dot{m} = \rho A_1 V_1 = 1000 \times \frac{\pi}{4} \times 0.05^2 \times 2 = 3.9269 \text{ kg/s}$$

Let f_x and f_y be the force in Right and vertically upward diversion respectively to hold the box in position.

∴ Now, $\Sigma f_x = 0$ [Box is stationary after applying force]

$$-\dot{m} \times V_1 \cos 65^\circ + f_x = -\dot{m} \times V_2 \cos 0^\circ$$

$$-3.9269 \times 2 \times \cos 65^\circ + f_x = -3.9269 \times 5.55 \times 1$$

$$f_x = -18.475 \text{ N.}$$

f_x must be in left as f_x comes out to be negative.

Similarly for vertical direction $\Sigma f_y = 0$

$$f_y - 3.9269 \times 2 \times \sin 65^\circ = 0$$

$$f_y = 7.11 \text{ N}$$

∴ It is towards vertically upward direction.

18. (a)

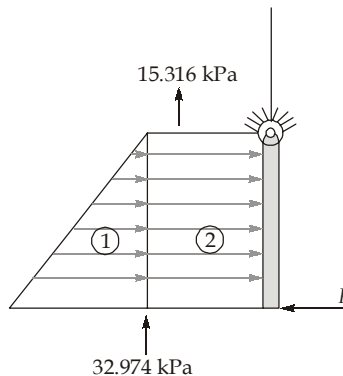
Gauge pressure at the upper end of the gate

$$= 80 + 0.9 \times 9.81 \times 4 - 100$$

$$= 15.316 \text{ kPa}$$

Gauge pressure at the lower end of the gate

$$= 80 + 0.9 \times 9.81 \times 6 - 100 = 32.974 \text{ kPa}$$



Using pressure diagram

For equilibrium,

Net moment about the hinge = 0

$$M_1 + M_2 = F \times 2$$

$$\frac{1}{2} \times (32.974 - 15.316) \times 2 \times 4 \times \left(\frac{2}{3} \times 2\right) + 15.316 \times 2 \times 4 \times \frac{2}{2} = F \times 2$$

On solving $F = 108.352 \text{ kN}$

19. (b)

$$H_m = H + \frac{fLV_d^2}{2gd} + 1.3 \times \frac{V_d^2}{2g} + \frac{V_d^2}{2g}$$

H_m = Net head required

$$H_m = 30 + \frac{4f'LV_d^2}{2gd} + 1.3 \times \frac{V_d^2}{2g} + \frac{V_d^2}{2g}$$

$$f' = 0.02 \quad A_d V_d = 50 \times 10^{-3} \text{ m}^3/\text{s}$$

$$d = 200 \text{ mm}$$

$$\frac{\pi}{4} \times 0.2^2 \times V_d = 50 \times 10^{-3}$$

$$V_d = 1.59 \text{ m/s}$$

$$\Rightarrow H_m = 30 + \frac{4 \times 0.02 \times 100 \times 1.59^2}{2 \times 9.81 \times 0.2} + \frac{1.3 \times 1.59^2}{2 \times 9.81} + \frac{1.59^2}{2 \times 9.81}$$

$$= 35.45 \text{ m}$$

$$\Rightarrow \text{Power} = \rho Q g H_m = 17388.225 \text{ W} = 17.388 \text{ kW}$$

20. (d)

τ_1 = shear stress at bottom

$$= \mu_1 \times \frac{V}{x}$$

$$\tau_2 = \text{shear stress at top} = \mu_2 \frac{V}{a-x}$$

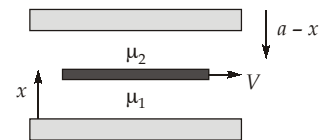
$$\text{drag force} = (\tau_1 + \tau_2) \times A = F_D$$

$$= F_D = A \times \left[\frac{\mu_1 V}{x} + \frac{\mu_2 V}{a-x} \right]$$

$$\frac{dF_D}{dx} = 0 = \frac{-\mu_1 V}{x^2} + \frac{\mu_2 V}{(a-x)^2} \Rightarrow \frac{\mu_1}{x^2} = \frac{\mu_2}{(a-x)^2}$$

$$a-x = \sqrt{\frac{\mu_2}{\mu_1}} x$$

$$\Rightarrow \frac{a\sqrt{\mu_1}}{\sqrt{\mu_1} + \sqrt{\mu_2}} = x$$



21. (b)

Momentum thickness,
$$\theta = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u}{U}\right) dy$$

Let,
$$z = \frac{y}{\delta} \quad (\text{For } y \rightarrow 0, z \rightarrow 0 \text{ and } y \rightarrow \delta, z \rightarrow 1)$$

$$\frac{u}{U} = 2z - z^2$$

$$= \int_0^1 (2z - z^2)(1 - 2z + z^2) \delta dz$$

$$= \delta \int_0^1 [(2z - z^2) - 2z(2z - z^2) + z^2(2z - z^2)] dz$$

$$= \delta \int_0^1 (2z - z^2 - 4z^2 + 2z^3 + 2z^3 - z^4) dz$$

$$= \delta \int_0^1 (2z - 5z^2 + 4z^3 - z^4) dz$$

$$\theta = \delta \left[z^2 - \frac{5z^3}{3} + z^4 - \frac{z^5}{5} \right]_0^1$$

$$= \delta \left(1 - \frac{5}{3} + 1 - \frac{1}{5} \right) = \delta \left(2 - \left(\frac{5}{3} + \frac{1}{5} \right) \right)$$

$$= \delta \left(2 - \frac{28}{15} \right) = \frac{2\delta}{15}$$

22. (a)

(1)
$$u = x^2 \cos y$$

$$v = -2x \sin y$$

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$= \frac{\partial u}{\partial x}(x^2 \cos y) + \frac{\partial}{\partial y}(-2x \sin y)$$

$$(2x)\cos y - (2x)\cos y = 0$$

hence it satisfy the continuity equation.

(2)
$$u = x + 2$$

$$v = 1 - y$$

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$= \frac{\partial}{\partial x}(x+2) + \frac{\partial}{\partial y}(1-y) = 1 - 1 = 0$$

hence, it satisfy the continuity equation.

$$(3) \quad u = xyt$$

$$v = x^3 - y^2 \frac{t}{2}$$

Continuity equation

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ &= \frac{\partial}{\partial x}(xyt) + \frac{\partial}{\partial y}\left(x^3 - \frac{y^2 t}{2}\right) = yt - yt = 0 \end{aligned}$$

hence, it satisfy continuity equation

$$(4) \quad u = \ln(x + y)$$

$$v = xy - \frac{y}{x}$$

Continuity equation

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ &= \frac{\partial}{\partial x} \ln(x + y) + \frac{\partial}{\partial y} \left(xy - \frac{y}{x} \right) = \frac{1}{x + y} + x - \frac{1}{x} \neq 0 \end{aligned}$$

Hence, it does not satisfy continuity equation.

23. (a)

$$\text{Pressure head} = \frac{\rho}{\rho g} = \frac{19.62 \times 10^3}{1000 \times 9.81 \times 0.8} = 2.5 \text{ m of oil}$$

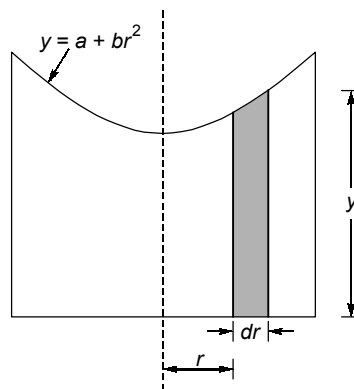
$$\text{Velocity head} = \frac{V^2}{2g} = \frac{Q^2}{2A^2 g} = \frac{(0.12)^2}{2 \times \left(\frac{\pi}{4} \times 0.25^2\right)^2 \times 9.81} = 0.3 \text{ m of oil}$$

$$\text{Datum head} = 2.7 \text{ m}$$

$$\text{Total head} = \text{Pressure head} + \text{Velocity head} + \text{Datum head}$$

$$= 2.5 + 0.3 + 2.7 = 5.5 \text{ m}$$

24. (b)



Elemental mass,

$$dm = (2\pi r) y (dr) \rho$$

total mass,

$$m = \int dm = \rho \int_0^R (2\pi r) dr \cdot y$$

$$= \rho \int_0^R (2\pi r)(a + br^2) dr = 2\pi\rho \left[\frac{ar^2}{2} + \frac{br^4}{4} \right]_0^R$$

$$m = 2\pi\rho \left[\frac{aR^2}{2} + \frac{bR^4}{4} \right]$$

$$m = \rho\pi R^2 \left[a + \frac{bR^2}{2} \right]$$

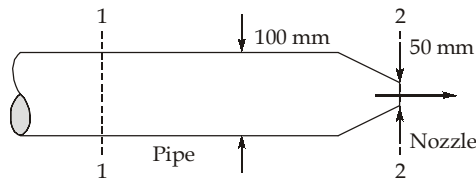
25. (b)

By using continuity equation,

$$V_1 A_1 = V_2 A_2$$

$$5 \times \frac{\pi}{4} (0.1)^2 = V_2 \times \frac{\pi}{4} (0.05)^2$$

$\Rightarrow V_2 = 20 \text{ m/s}$



From force balance in x-diagram

$$P_1 A_1 + R_x - P_2 A_2 = \rho Q (V_2 - V_1)$$

$$\Rightarrow R_x = \rho Q (V_2 - V_1) + P_2 A_2 - P_1 A_1$$

$$= 1000 \times \frac{\pi}{4} (0.1)^2 \times 5 (20 - 5) + 100 \times 10^3 \times \frac{\pi}{4} (0.05)^2 - 500 \times 10^3 \times \frac{\pi}{4} (0.1)^2$$

Force required to hold the nozzle, $R_x = -3141.6 \text{ N}$

26. (c)

From force balance at point of contact,

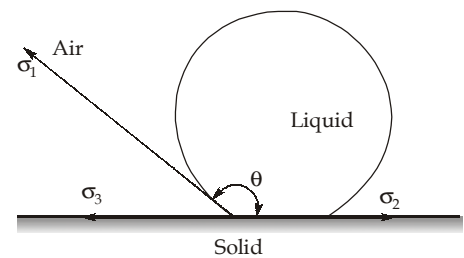
$$\sigma_1 \cos(180^\circ - \theta) + \sigma_3 = \sigma_2$$

or $\cos(180^\circ - \theta) = \frac{\sigma_2 - \sigma_3}{\sigma_1} = -\cos\theta$

$\therefore \sigma_1 = 0.0720 \text{ N/m}$ (liquid and air)
 $\sigma_2 = 0.0418 \text{ N/m}$ (liquid and solid)
 $\sigma_3 = 0.0008 \text{ N/m}$ (air and solid)

$$\cos\theta = \frac{0.0008 - 0.0418}{0.072} = -0.56944$$

$$\theta = 124.7^\circ$$



27. (b)

$$\therefore \text{Re} = \frac{16}{f} = \frac{16}{0.04} = 400$$

\therefore The flow is viscous.

The shear stress in case of viscous flow through a pipe is given by

$$\tau = \frac{-\partial p}{\partial x} \left(\frac{r}{2} \right)$$

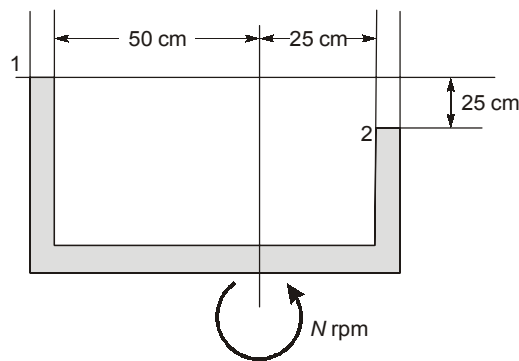
$\therefore \frac{\partial p}{\partial x}$ is constant across a section.

$$\therefore \tau \propto r$$

$$\therefore \frac{\tau}{r} = \frac{\tau_0}{R} = \frac{0.00981}{40} = \frac{\tau_0}{100}$$

$$\therefore \tau_0 = \frac{100 \times 0.00981}{40} = 0.0245 \text{ N/cm}^2$$

28. (c)



$$\frac{P_1^0}{\rho g} - \frac{V_1^2}{2g} + Z_1 = \frac{P_2^0}{\rho g} - \frac{V_2^2}{2g} + Z_2$$

$$Z_1 - Z_2 = \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$0.25 = \frac{1}{2g} \{r_1^2 \omega^2 - r_2^2 \omega^2\}$$

$$0.25 \times 2 \times 9.81 = \omega^2 \left\{ \left(\frac{50}{100} \right)^2 - \left(\frac{25}{100} \right)^2 \right\}$$

$$\omega = 5.115 \text{ rad/s}$$

$$N = \frac{60\omega}{2\pi} = \frac{60 \times 5.115}{2 \times \pi}$$

$$= 48.8 \text{ rpm}$$

29. (a)

$$Re_L \leq 2000$$

$$\Rightarrow \frac{\rho VD}{\mu} \leq 2000$$

$$\Rightarrow \frac{(0.92 \times 1000) \times \frac{Q}{A} \times (10 \times 10^{-2})}{0.9 \times 10^{-1}} \leq 2000$$

$$\Rightarrow Q \leq \frac{2000 \times 0.9 \times 10^{-1} \times A}{(0.92 \times 1000) \times (10 \times 10^{-2})}$$

$$\Rightarrow Q \leq 1.95652 \times \frac{\pi}{4} \times \left\{ \frac{10}{100} \right\}^2$$

$$\Rightarrow Q \leq 0.015366 \text{ m}^3/\text{s}$$

$$\Rightarrow Q \leq 15.36 \approx 15.4 \text{ L/s}$$

30. (a)

Given; $d = 3 \text{ m}$; $h_1 = 1.5 \text{ m}$; $h_2 = 4 \text{ m}$

$$\text{So, } \sin \theta = \frac{h_2 - h_1}{BC} = \frac{4 - 1.5}{3} = 0.833$$

$$\theta = 56.44^\circ$$

$$\text{Now, } h_{cp} = \bar{h} + \frac{I_G \sin^2 \theta}{A \bar{h}}$$

$$\bar{h} = \frac{d}{2} \sin \theta + h_1 = 2.75 \text{ m}$$

$$A = \frac{\pi d^2}{4}, I_G = \frac{\pi d^4}{64}$$

$$h_{cp} = 2.75 + \frac{\frac{\pi d^4}{64} \times (0.833)^2}{\frac{\pi d^2}{4} \times 2.75} = 2.75 + 0.1420 = 2.892 \text{ m}$$

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