CLASS TEST

S.No.:01SK_ABCDEFGHIJKL_30924

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DISCRETE MATHEMATICS

COMPUTER SCIENCE & IT

Date of Test : 30/09/2024

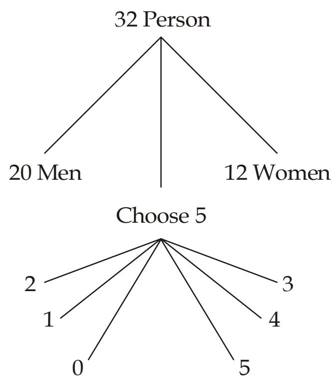
ANSWER KEY

1.	(b)	7.	(b)	13.	(a)	19.	(a)	25.	(c)
2.	(d)	8.	(a)	14.	(c)	20.	(b)	26.	(a)
3.	(b)	9.	(c)	15.	(d)	21.	(c)	27.	(c)
4.	(c)	10.	(d)	16.	(b)	22.	(c)	28.	(b)
5.	(c)	11.	(a)	17.	(a)	23.	(c)	29.	(c)
6.	(c)	12.	(a)	18.	(d)	24.	(c)	30.	(b)

DETAILED EXPLANATIONS

1. (b)

Given,



We must choose at least 3 women, so, we calculate 3 women, 4 women and 5 women and by addition rule add the results:

$$= {}^{12}C_3 \times {}^{20}C_2 + {}^{12}C_4 \times {}^{20}C_1 + {}^{12}C_5 \times {}^{20}C_0$$
$$= 220 \times 190 + 495 \times 20 + 792 \times 1 = 52492$$

2. (d)

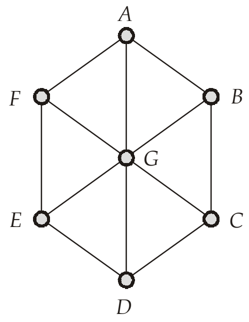
1. is valid by constructive dilemma.
2. is valid by destructive dilemma.
3. is valid by hypothetical syllogism.
All of the above are known rules of inference.

3. (b)

$$f(x) = x + 2 \qquad \text{Let } y = f(x)$$
$$y = x + 2 \qquad \Rightarrow x = f^{-1}(y)$$
$$\Rightarrow \quad x = y - 2$$
$$\Rightarrow \quad f^{-1}(y) = y - 2 \qquad \textbf{or } f^{-1}(x) = x - 2$$
$$g(3) = (1 + (3)^2)^{-1} = (1 + 9)^{-1} = \frac{1}{10}$$
$$f^{-1}g(3) = f^{-1}(g(3)) = g(3) - 2$$
$$\Rightarrow \quad f^{-1}g(3) = \frac{1}{10} - 2 = -1.9$$

4. (c)

Consider each options:
(a) Null graph of 6 vertices is 1-chromatic so it is correct.
(b) It is correct because tree with 2 or more vertices is always bichromatic.
(c) It is incorrect. Consider a wheel graph of 7 vertices.



The chromatic number of graph is 3.

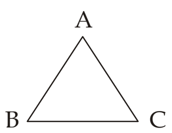
- Color 1 for G
- Color 2 for A, E, C
- Color 3 for F, B, D
- A wheel graph is 3-chromatic when n-vertices are odd and 4-chromatic when n-vertices is even.
- So here $n = 7$, $\left(\left\lfloor \frac{n}{2} \right\rfloor + 1\right) = \left(\left\lfloor \frac{7}{2} \right\rfloor + 1\right) = 4$ which is incorrect because only 3 colors are required to color the above wheel graph.

(d) This statement is correct because graph without odd length cycle having atleast 1 edge is bichromatic.

All other statements are true except option (c).

5. **(c)**

Considering each statements:

- If a graph is bipartite, then its two colourable. Because a bipartite graph can be represented as two groups of vertices such that vertices in same graph are not adjacent.
  Similarly, statement 2 is equivalent to statement 1.
- If a bipartite graph has a cycle, then it has to be of even length. Graph G is bipartite iff no odd length cycle.
- 

  This graph has a Hamiltonian circuit, but the cycle is of odd length and not bipartite.

$\therefore$ $3 \not\approx 4$

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  This graph is bipartite and 2 colorable but does not have Hamiltonian circuit.

So 1, 2 and 4 are equivalent statements.

6. **(c)**

$$a_n = -5_{a_{n-1}} + 6_{a_{n-2}}$$

$$a_n + 5_{a_{n-1}} - 6_{a_{n-2}} = 0$$

$$x^2 + 5x - 6 = 0$$

$$x^2 + 6x - x - 6 = 0$$
$$x(x + 6) - 1(x + 6) = 0$$
$$(x + 6)(x - 1) = 0$$
$$x = -6, \quad x = 1$$
$$\therefore \quad a_n = A(-6)^n + B \cdot (1)^n$$
$$a_n = A(-6)^n + B$$

7. (b)

Number of ways all vowels are together = $\dfrac{5! \times 4!}{2!\,2!} = 720$

Number of ways not all vowels are together = Total number of permutation – Number of ways all vowels are together

$$= \frac{8!}{2!\,2!} - 720$$

⇒ Number of not all vowels together = (10080 – 720) = 9360

8. (a)
Hamiltonian cycle for the above graph G is *abcdefa*.
Condition: Each node should be visited exactly once.

9. (c)
The theorem is every finite lattice is bounded but a bounded lattice may not be finite.

10. (d)
Complementary lattice may not have unique complement for every element.

11. (a)
$A \cup B \subseteq A \cap B$ holds true when $A = B$. It is true for empty as well as nonempty sets.
⇒ $|A| = |B|$ is true $|A| \geq 0$ ***eg.*** $A = B\ \{a, b\}$
Hence $A = \{\ \}$, $B = \{\ \}$ "always" is false.

12. (a)
***R* is reflexive:** Since $(a, b)\ R\ (a, b)$ for all elements (a, b) because $a = a$ and $b = b$ are always true.
***R* is symmetric:** Since $(a, b)\ R\ (c, d)$ and $a = c$ or $b = d$ which can be written as $c = a$ or $d = b$.
So, $(a, b)\ R\ (a, b)$ is true.
***R* is not antisymmetric:** Since $(1, 2)\ R\ (1, 3)$ and $1 = 1$ or $2 = 3$ true b/c $1 = 1$.
So $(1, 3)\ R\ (1, 2)$ but here $2 \neq 3$ so $(1, 2) \neq (1, 3)$.
So, only statement 1 and 2 are correct.

13. (a)
Total number of subset of 5 element = ${}^{25}C_5$

$$= \frac{25 \times 24 \times 23 \times 22 \times 21}{5 \times 4 \times 3 \times 2 \times 1} = 23 \times 22 \times 21 \times 5 = 53130$$

T be a 5 element subset contain no odd number = ${}^{12}C_5$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 792$$

So number of 5 element subset with atleast 1 odd number

$$T \subseteq S = {}^{25}C_5 - {}^{12}C_5$$
$$= 53130 - 792 = 52338$$

14. (c)

A number is relatively prime to 15 iff it is not divisible by 3 and not divisible by 5.

Set of integer from 1 to 1000 divisible by $3 = \left\lfloor \frac{1000}{3} \right\rfloor = 333.$

Set of integer from 1 to 1000 divisible by $5 = \left\lfloor \frac{1000}{5} \right\rfloor = 200.$

So, number of integer not relatively prime to 15 are

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= \left\lfloor \frac{1000}{3} \right\rfloor + \left\lfloor \frac{1000}{5} \right\rfloor - \left\lfloor \frac{1000}{15} \right\rfloor$$
$$= 333 + 200 - 66 = 467$$

So, number of integer relatively prime to 15 are

$$\overline{|A \cup B|} = 1000 - 467 = 533$$

15. (d)

"Not every satisfiable logic is Valid"

= Not (every satisfiable logic is Valid)

= Not ($\forall x$(satisfiable (x) $\Rightarrow$ Valid (x)) — option (a)

= Not ($\forall x$($\neg$satisfiable $\vee$ Valid (x)) — option (c)

= $\exists x$(satisfiable (x) $\wedge \neg$ Valid (x)) — option (b)

Statement (d) says every satisfiable logic is invalid.
So option (d) is not represent given statement.

16. (b)

- I is not D_{42} because the divisor 7 is missing. So, there is no way for I to be isomorphic to $(P\{a,b,c\}, \subseteq)$ as it needs to have 8 divisors but right now it has only 7.
- II is D_{66} a well known boolean algebra and has 8 vertices and its masses diagram will be isomorphic $(P(\{a,b,c\}), \subseteq)$.
- III is not isomorphic even though it looks like D_{70}, it is on the relation ≤, resulting in a chain, which won't be boolean algebra.

17. (a)

$T(n) - 9T(n-1) + 20T(n-2) = 0$
Let $a_n = T(n)$
$\Rightarrow a_n - 9a_{n-1} + 20a_{n-2} = 0$

$$t^2 - 9t + 20 = 0$$

$$t^2 + 5t - 4t + 20 = 0$$
$$t(t-5) - 4(t-5) = 0$$
$$(t-4)(t-5) = 0$$
$$t = 4, 5$$

Homogenous equation become

$$a_n = c_1 . 5^n + c_2 . 4^n \quad ...(1)$$

Put $n = 0$ in equation (1)

$$a_0 = c_1 . 5^0 + c_2 . 4^0$$
$$-3 = c_1 + c_2 \quad ...(2)$$

Put n = 1 in equation (1)

$$a_1 = c_1 . 5^1 + c_2 . 4^1$$
$$-10 = 5c_1 + 4c_2 \quad ...(3)$$

Solving equation (2) and (3) and get c_1 and c_2

$$(c_1 + c_2 = -3) \times 5$$
$$5c_1 + 4c_2 = -10$$

$$5c_1 + 5c_2 = -15$$
$$5c_1 + 4c_2 = -10$$
$$c_2 = -5 \quad \text{and} \quad c_1 = 2$$

Put value of c_1 and c_2 in eq. (1)

$$a_n = 2.5^n - 5.4^n$$

18. (d)

- S_1 is correct because connected graph has a Euler circuit if and only if it has number of odd degree vertices is 0.
- A connected graph has a Euler path if and only if it has number of odd degree vertices is either 0 or 2. Therefore a connected graph has Euler path but not euler circuit if and only it has exactly 2 vertices of odd degree therefore S_2 is correct.
- A complete graph of n-vertices contains $n - 1$ degree at each vertex which is greater than $\frac{n}{2}$ for all $n \geq 3$ therefore complete graph has a Hamiltonian circuit. So S_3 is correct
- C_6 is bipartite because any cycle graph with even number of vertices is bipartite. A complete graph with 4 vertices contains complete graph of 3 vertices which contains odd length cycle hence it is not bipartite. So statement S_4 is incorrect.

19. (a)

Maximum and minimum number of component given by:

$$n - K \leq e \leq \frac{(n-K+1)(n-K)}{2}$$

1.
$$n - K \leq e$$
$$n - e \leq K$$
$$10 - 6 \leq K \quad (\because \text{Minimum number of component})$$

2.
$$e \leq \frac{(n-K+1)(n-K)}{2}$$
$$6 \leq \frac{(10-K+1)(10-K)}{2}$$

$$2 \times 6 \leq (11 - K)(10 - K)$$
$$12 \leq (10 - K)(11 - K)$$
$$12 \leq K^2 + 110 - 21K$$
$$0 \leq K^2 + 98 - 21K$$
$$K^2 + 98 - 21K = 0$$
$$K = 14, 7$$

Maximum value of K is 7 because number of components never be larger than nodes.

20. **(b)**

Since bit are '0' and '1' form. The hamming distance relation on bit has a digraph which will be always an 5-cube where 5 is the number of bits.

- Chromatic number of n-cube = 2 (Since n-cube is always bipartite)
  So chromatic number of 5-cube = 2

i.e., '0' = One color
'1' = Second color

- Diameter of n-cube = n
  Diameter of 5 cube = 5

i.e., maximum length between any two vertex.

So ratio $\frac{2}{5} = \frac{X}{Y}$

$$Y - X = 5 - 2 = 3$$

21. **(c)**

Total number of terms = 6 + 1 = 7

So middle term is 4^{th} term.

$(x + y)^n$ has $(r + 1)^{th}$ term as ${}^nC_r\, x^{n-r}\, y^r$.

$[(3 + 1)^{th}$ term] 4^{th} term is

$$= {}^6C_3 \left(\frac{\sqrt{x}}{3}\right)^{6-3} \left(\frac{-3}{x\sqrt{y}}\right)^3$$

$$= {}^6C_3 \cdot \left(\frac{\left(\sqrt{x}\right)^3}{27}\right) \cdot \left(\frac{-27}{x^3 \cdot \left(\sqrt{y}\right)^3}\right)$$

$$= 20 \cdot \left(\frac{x^{3/2}}{27}\right) \cdot \left(\frac{-27}{x^3 \cdot (y)^{3/2}}\right)$$

$$= -20\left(\frac{x}{y}\right)^{\frac{3}{2}} \cdot \frac{1}{x^3}$$

22. **(c)**

I and IV are true. Let's see why IV is true first. We will treat set theory as boolean algebra here, and will demonstrate how to apply this approach.

Given, $S \subseteq R$, which in same as, $\Rightarrow S - R = \varphi$

The same can be written in boolean algebra as, $\Rightarrow S \wedge R' = 0$

Since we know that $\wedge$ is commutative, $\Rightarrow R' \wedge S = 0$

Now $R' \wedge S = 0$ is same as $R' \wedge (S')' = 0 \Rightarrow R' - S' = \varphi$

$\Rightarrow \quad R' \subseteq S'$

Therefore IV is correct.

In order to show that R^{-1} is a subset of S^{-1}, we just need to show that every element in R^{-1} belongs to S^{-1}. So let's assume that R is a subset of S. So if (a, b) is an element of R, then (a, b) belongs to S as well. As (a, b) belongs to R, (b, a) belongs to R^{-1}. Also, (a, b) belongs to S, (b, a) will belong to S^{-1}. Since we can show this presence of every element in R^{-1} in S^{-1}, we see that R^{-1} is a subset of S^{-1}. However II is clearly not true.

23. (c)

This problem corresponds to the number of non-negative integral solution to

$$x_1 + x_2 + x_3 = 10 \text{ with the conditions}$$
$$0 \le x_1 \le 10$$
$$0 \le x_2 \le 5$$
$$0 \le x_3 \le 3$$

Generating functions are required, since the variables have an upper constraint.

Generating function is

$(1 + x + x^2 \text{ .... } + x^{10})\ (1 + x + x^2 \text{ .... } + x^5)\ (1 + x \text{ .... } + x^3)$

$$= \left(\frac{1-x^{11}}{1-x}\right)\left(\frac{1-x^{6}}{1-x}\right)\left(\frac{1-x^{4}}{1-x}\right)$$

$$= \frac{(1-x^{11})(1-x^6)(1-x^4)}{(1-x)^3}$$

$$= (1-x^4-x^6+x^{10})\sum_{r=0}^{\infty} {}^{3-1+r}C_r \cdot x^r$$

$$= (1-x^4-x^6+x^{10})\sum_{r=0}^{\infty} {}^{r+2}C_r \cdot x^r$$

The coefficient of x^{10} in above generating function is

$${}^{12}C_{10} - {}^{8}C_{6} - {}^{6}C_{4} - {}^{2}C_{0} = 24$$

24. (c)

R^1 is nothing but R itself.

Now, $\quad R^2 = R \circ R$ i.e. composite of R with R

If $(a, b) \in R$, then $(a, c) \in R^2$ iff $(b, c) \in R$.

So, $\quad R^2 = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$

$\quad R^3 = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$

So, $\quad P = R^1 \cup R^2 \cup R^3$

$\quad = \{(1, 1), (2, 1), (3, 1), (4, 1), (4, 2)\}$

$\therefore \quad$ Cardinality $= 5$

25. (c)

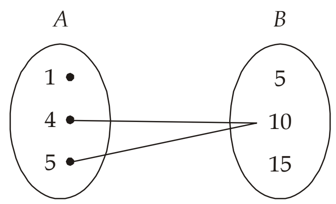
"f is one-one and onto".
Negation of this statement will be
"f is not one-one or not onto".
Now, according to statement R.

Let, $a_1 = 4 \in A$

$a_2 = 5 \in A$

$f(a_1) = f(a_2) = 10$

So,



Clearly this is condition of not one-one.
So, R is correct.
Now, Q is definition of onto so we have to take negation of this.
Therefore option (c) is correct answer.

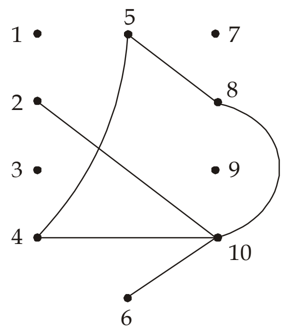
26. (a)

$(Z, +)$ is a group and $Z \subseteq Q$.
$(A, +)$ is not a group. Hence it is not a subgroup of $(Q, +)$.
$(B, +)$ is not a group. Hence it is not a subgroup of $(Q, +)$.

27. (c)

G is a planar graph. Every planar graph is 4 colorable. Every face is bordered by 3 edges.
So graph has possibilities of 3 or 4 colors.
k_3 colored with 3 and k_4 colored with 4 colors.

28. (b)



$$E = \{\{2, 10\}, \{4, 10\}, \{6, 10\}, \{8, 10\}, \{4, 5\}, \{5, 8\}\}$$

$\Rightarrow$ 6 edges are present in G.

29. **(c)**

$$4.r = 2.e \quad \text{[Planar graph]} \quad ...(i)$$
$$3.n = 2.e \quad \text{[Cubic graph]} \quad ...(ii)$$
$$n - e + r = 2 \quad ...(iii)$$

Substitute (i) and (ii) in (iii)

$$\Rightarrow \quad \frac{2e}{3} - e + \frac{2e}{4} = 2$$
$$\Rightarrow \quad 8e - 12e + 6e = 24$$
$$\Rightarrow \quad 14e - 12e = 24$$
$$\Rightarrow \quad 2e = 24$$
$$\Rightarrow \quad e = 12$$

30. **(b)**

'd' preceeds the set and $\{a, b, h\}$ suceeds the set.

$\therefore$ Lower bound = d

Upper bound = $\{a, b, h\}$

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