CLASS TEST

S.No.: 03SK_EC+EE_DEFG_071024

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ENGINEERING MATHEMATICS

EC + EE

Date of Test : 07/10/2024

ANSWER KEY

1. (a)	7. (c)	13. (a)	19. (a)	25. (c)
2. (b)	8. (d)	14. (b)	20. (c)	26. (b)
3. (b)	9. (b)	15. (a)	21. (a)	27. (a)
4. (c)	10. (a)	16. (b)	22. (b)	28. (d)
5. (a)	11. (a)	17. (c)	23. (d)	29. (b)
6. (b)	12. (d)	18. (d)	24. (d)	30. (a)

DETAILED EXPLANATIONS

1. (a)

$$\begin{bmatrix} 3 & 7.5 \\ -6 & 4.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -90 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ -6 & 4.5 & -90 \end{bmatrix}$$

$R_2 \leftarrow R_2 + 2R_1$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ 0 & 19.5 & -78 \end{bmatrix}$$

$$19.5y = -78$$

or $$y = -4$$

$$3x + 7.5y = 6$$

$$3x + 7.5(-4) = 6$$

$$3x = 36$$

$\Rightarrow$ $$x = 12$$

$\therefore$ $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$$

2. (b)

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(x-1) = 0$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(x^3-1) = 0$$

Also $$f(1) = 0$$

Thus $$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$$

$\Rightarrow$ f is continuous at $x = 1$

And Lf′(1) = 2, Rf′(1) = 1

$\Rightarrow f$ is not differentiable at $x = 1$

3. (b)

Probability of first item being defective,

$$P_1 = \frac{15}{50}$$

Probability of second item being defective,

$$P_2 = \frac{14}{49}$$

Probability of third item being defective,

$$P_3 = \frac{13}{48}$$

Probability that all three are defective,

$$P = P_1P_2P_3 = \frac{15}{50}\times\frac{14}{49}\times\frac{13}{48} = \frac{13}{560}$$

4. (c)

The characteristic equation $[A - \lambda I] = 0$

$$\text{i.e.} \quad \begin{bmatrix} 4-\lambda & 6 \\ 2 & 8-\lambda \end{bmatrix} = 0$$

$$\text{or} \quad (4 - \lambda)(8 - \lambda) - 12 = 0$$

$$\text{or} \quad 32 - 8\lambda - 4\lambda + \lambda^2 - 12 = 0$$

$$\Rightarrow \quad \lambda^2 - 12\lambda + 20 = 0$$

$$\Rightarrow \quad \lambda^2 - 10\lambda - 2\lambda + 20 = 0$$

$$\Rightarrow \quad (\lambda - 10)(\lambda - 2) = 0$$

$$\Rightarrow \quad \lambda = 10, 2$$

Corresponding to $\lambda = 10$, we have

$$[A - \lambda I]x = \begin{bmatrix} -6 & 6 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which gives, $-6x + 6y = 0$

$\Rightarrow$ $x = y$

$2x - 2y = 0$

$\Rightarrow$ $x = y$

i.e. eigen vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Corresponding to $\lambda = 2$, we have

$$[A - \lambda I]x = \begin{bmatrix} 2 & 6 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which gives, $2x + 6y = 0$ i.e. eigen vector $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

5. (a)

Given, $x = b(2 - \cos\theta)$, $y = b(\sin\theta + \theta)$

$$\therefore \quad \frac{dx}{d\theta} = b\sin\theta,$$

$$\frac{dy}{d\theta} = b(\cos\theta + 1)$$

$$\frac{dx}{dy} = \frac{dx/d\theta}{dy/d\theta} = \frac{b\sin\theta}{b(\cos\theta + 1)}$$

$$= \frac{2b\sin\left(\frac{\theta}{2}\right).\cos\left(\frac{\theta}{2}\right)}{b \times 2\cos^2\left(\frac{\theta}{2}\right)} = \tan\left(\frac{\theta}{2}\right)$$

6. (b)

Inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} 2 & 3 \\ 6 & 8 \end{bmatrix}^{-1} = \frac{1}{16-18}\begin{bmatrix} 8 & -3 \\ -6 & 2 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} 8 & -3 \\ -6 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 3/2 \\ 3 & -1 \end{bmatrix}$$

7. (c)

$$e^{1/z} = 1 + \frac{1}{z} + \frac{1}{2!z^2} + \frac{1}{3!z^3} + \ldots\ldots\ldots.$$

So, $e^{1/z}$ has pole at $z = 0$ which lies inside 'C' so, using Cauchy's residue theorem,

$$\int_C e^{1/z} dz = 2\pi i \,[\Sigma \text{Res}(e^{1/z} \text{ at poles inside } C)]$$

$$\underset{z \to 0}{\text{Res}}\, e^{1/z} = 1$$

$$\int_C e^{1/z} dx = 2\pi i\,[1] = 2\pi i$$

8. (d)

$$y = \sqrt{a^x + y}$$

$$y^2 = a^x + y$$

$$2y\frac{dy}{dx} = a^x \ln a + \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{a^x \ln a}{(2y-1)}$$

9. (b)

$$P(T) = 0.5$$

Probability of getting tails exactly 6 times is

$$8C_6 (0.5)^6 (0.5)^2 = \frac{7}{64}$$

10. (a)

$$P(-1 \le x \le 1) = \int_{-1}^{1} (0.1)\,dx$$

$$= 2 \times \frac{1}{10} = \frac{1}{5}$$

11. (a)

Required probability is

$$P = \frac{4}{52} \times \frac{2}{51} + \frac{4}{52} \times \frac{2}{51} = \frac{4}{663}$$

12. (d)

$$D = -96 \text{ for the given matrix}$$

$$|A| = \begin{vmatrix} 4 & 12 & 0 \\ 8 & 24 & 16 \\ -4 & 0 & 8 \end{vmatrix} = 2^3 \begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix}$$

(Taking 2 common from each row)

$$\therefore \quad \text{Det}(A) = (2)^3 \times D$$

$$= 8 \times (-96) = -768$$

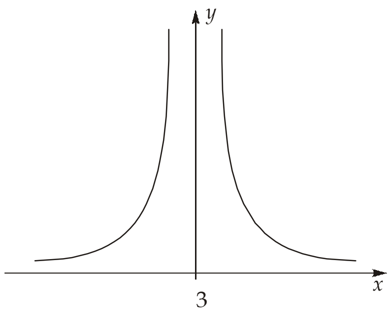
13. (a)

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)\,dx = \int_0^1 x^2\,dx + \int_1^2 (2-x)x\,dx$$

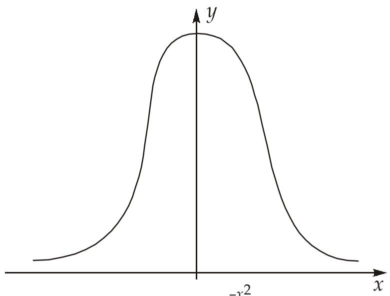
$$= \left.\frac{x^3}{3}\right|_0^1 + \left.\left(x^2 - \frac{x^3}{3}\right)\right|_1^2 = \frac{1}{3} + 4 - 1 - \frac{8-1}{3} = 1$$

14. (b)

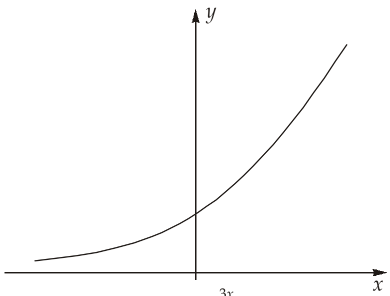
From the graphs below, we can see that only $2e^{-x^2}$ is strictly bounded.



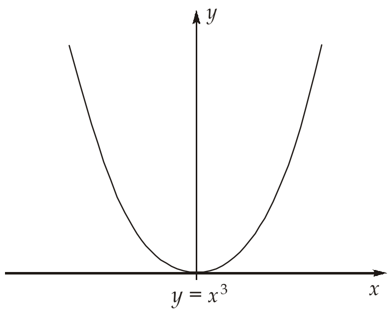
$y = \frac{3}{2x^2}$



$y = 2e^{-x^2}$



$y = e^{3x}$



$y = x^3$

15. (a)

Given that the partial differential equation is parabolic

$$\therefore \quad b^2 - 4ac = 0$$

$$b^2 - 4(4)(4) = 0$$

$$b^2 - 64 = 0$$

$$b^2 = 64$$

16. (b)

$$P[X > 1] = \int_1^{\infty} f(x)\,dx = \int_1^{\infty} e^{-2x}\,dx = \left.\frac{-e^{-2x}}{2}\right|_1^{\infty}$$

$$= -\left(\frac{e^{-2\infty}}{2} - \frac{e^{-2}}{2}\right) = \frac{e^{-2}}{2} = 0.067$$

17. (c)

$$\frac{dy}{dx} = 0.75y^2 \qquad (y = 1 \text{ at } x = 0)$$

Iterative equation by backward (implicit) Euler's method for above equation would be

$$y_{k+1} = y_k + h_f(x_{k+1}, y_{k+1})$$

$$y_{k+1} = y_k + h \times 0.75\, y^2_{k+1}$$

$\Rightarrow$ $0.75\, hy^2_{k+1} - y_{k+1} + y_k = 0$

Putting $k = 0$ in above equation

$$0.75hy_1^2 - y_1 + y_0 = 0$$

Since $y_0 = 1$ and $h = 1$

$$0.75y_1^2 - y_1 + 1 = 0$$

$\Rightarrow$
$$y_1 = \frac{1 \pm \sqrt{1^2 - 3}}{2 \times 0.75} = \frac{2}{3}(1 \pm i\sqrt{2})$$

18. (d)

$$\frac{dx}{dt} = 9x - 11y$$

$$\frac{dy}{dt} = 7x + 13y$$

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 & -11 \\ 7 & 13 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9x & -11y \\ 7x & +13y \end{bmatrix}$$

19. (a)

The volume of a solid generated by revolution about the x-axis, of the area bounded by curve $y = f(x)$, the x-axis and the ordinates $x = a$, $y = b$ is

Volume $\int_a^b \pi y^2 dx$

Here, $a = 2$, $b = 3$ and $y = 2\sqrt{x} \Rightarrow y^2 = 4x$

$\therefore$
$$\text{Volume} = \int_2^3 \pi 4x\,dx = 4\pi\left[\frac{x^2}{2}\right]_2^3 = 2\pi[x^2]_2^3 = 2\pi\,[9 - 4] = 10\pi$$

20. (c)

Given,

$$\text{Trace } A = 9$$
$$|A| = 24$$
$$\lambda_1 = 3$$
$$\lambda_1 + \lambda_2 + \lambda_3 = 9$$
$$\Rightarrow \quad 3 + \lambda_2 + \lambda_3 = 9$$
$$\Rightarrow \quad \lambda_2 + \lambda_3 = 6$$

21. (a)

$$\int_0^{\pi} \frac{dx}{c\left(\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right) + d\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)} = \int_0^{\pi} \frac{dx}{(c+d)\cos^2\frac{x}{2} + (c-d)\sin^2\frac{x}{2}}$$

$$= \int_0^{\pi} \frac{\sec^2\frac{x}{2}\,dx}{(c+d) + (c-d)\tan^2\frac{x}{2}} = \frac{1}{(c-d)}\int_0^{\pi} \frac{\sec^2\frac{x}{2}\,dx}{\frac{(c+d)}{(c-d)} + \tan^2\frac{x}{2}}$$

$$= \frac{2}{c-d}\sqrt{\frac{c-d}{c+d}}\left[\tan^{-1}\left\{\tan\frac{x}{2}\sqrt{\frac{c-d}{c+d}}\right\}\right]_0^{\pi}$$

$$= \frac{2}{c-d}\sqrt{\frac{c-d}{c+d}}\left[\tan^{-1}\infty - \tan^{-1}0\right]$$

$$= \frac{2}{\sqrt{(c-d)(c+d)}} \times \frac{\pi}{2} = \frac{\pi}{\sqrt{c^2 - d^2}}$$

22. (b)

Let
$$I = \int_0^{\infty} \frac{e^{-x}\sin bx}{x}\,dx$$

$$\frac{dI}{db} = \int_0^{\infty} \frac{\partial}{\partial b}\left(\frac{e^{-x}\sin bx}{x}\right)dx = \int_0^{\infty} \frac{e^{-x}x\cos bx}{x}\,dx$$

$$= \int_0^{\infty} e^{-x}\cos bx\,dx$$

We know that

$$\int e^{ax}\cos bx = \frac{e^{ax}}{a^2 + b^2}(a\cos bx + b\sin bx)$$

$$\int_0^{\infty} e^{-x}\cos bx\,dx = \left[\frac{e^{-x}}{1+b^2}[-\cos bx + b\sin bx]\right]_0^{\infty}$$

$$\frac{dI}{db} = \frac{1}{1+b^2}$$

Integrating both sides, $I = \tan^{-1} b$

23. (d)

$$y'' + 4y' + 4y = 0$$
$$(D^2 + 4D + 4)y = 0$$
$$\Rightarrow \quad (D + 2)(D + 2) = 0$$
$$\Rightarrow \quad D = -2, -2$$
$$\therefore \quad y = C_1 e^{-2x} + C_2 x e^{-2x}$$
$$y(0) = 0 \Rightarrow 0 = C_1$$
$$y(1) = 0 \Rightarrow 0 = C_1 + C_2$$
$$\Rightarrow \quad C_2 = 0$$
$$y = 0 \text{ is the solution}$$
$$\therefore \quad y(2) = 0$$

24. (d)

$$|A - \lambda I| = 0$$
$$\begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$
$$\begin{vmatrix} 3-\lambda & 1 \\ -2 & -\lambda \end{vmatrix} = 0$$
$$-3\lambda + \lambda^2 + 2 = 0$$
$$\lambda^2 - 3\lambda + 2 = 0$$
$$A^2 - 3A + 2 = 0$$
$$A - 3I + 2A^{-1} = 0$$

25. (c)

$$AX = \lambda X$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -6 \end{bmatrix} = (-6)\begin{bmatrix} 3 \\ -6 \end{bmatrix}$$
$$3a - 6b = -18 \quad \text{... (i)}$$
$$3c - 6d = 36 \quad \text{... (ii)}$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -3 \end{bmatrix} = (-3)\begin{bmatrix} 3 \\ -3 \end{bmatrix}$$
$$3a - 3b = -9 \quad \text{... (iii)}$$
$$3c - 3d = 9 \quad \text{... (iv)}$$

From equation (i) and (ii), $a = 0$ and $b = 3$.
From equation (ii) and (iv), $c = -6$ and $d = -9$.

$$\therefore \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -6 & -9 \end{bmatrix}$$

26. (b)

$$y = 7x^2 + 12x$$
$$\frac{dy}{dx} = 14x + 12$$
$$\left.\frac{dy}{dx}\right|_{x=1} = 26$$

$$\left.\frac{dy}{dx}\right|_{x=3} = 54$$

$\because$ x is defined in open interval $x = (1, 3)$

$\therefore$ $1 < x < 3$

$\therefore$ $26 < \frac{dy}{dx} < 54$

27. (a)

Since $\sum_{x=0}^{4} P(x) = 1$

x	0	1	2	3	4
$P(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{36}$	$\frac{5}{36}$
$xP(x)$	0	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{3}{36}$	$\frac{20}{36}$

$$c + 2c + 2c + c^2 + 5c^2 = 1$$

$$6c^2 + 5c - 1 = 0$$

$$c = \frac{1}{6}, -1$$

Since $P(x) \geq 0$, the possible value of

$$c = \frac{1}{6}$$

$$\text{Mean} = \sum_{x=0}^{4} xP(x) = 0 + \frac{2}{6} + \frac{4}{6} + \frac{3}{36} + \frac{20}{36}$$

$$= \frac{59}{36} = 1.638$$

$$\text{Variance} = \sigma^2 = E(x^2) - [E(x)]^2$$

$$= \left[0\left(\frac{1}{6}\right) + 1\left(\frac{2}{6}\right) + 4\left(\frac{2}{6}\right) + 9\left(\frac{1}{36}\right) + 16\left(\frac{5}{36}\right) - \left(\frac{59}{36}\right)^2\right]$$

$$= 1.45$$

28. (d)

Given differential equation is

$$x\frac{dy}{dx} + y = x^4$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{y}{x}\right) = x^3 \quad \text{... (i)}$$

Standard form of liebnitz linear equation is

$$\frac{dy}{dx} + Py = Q \quad \text{... (ii)}$$

where P and Q function of x only and solution is given by

$$y(IF) = \int Q(IF)dx + c$$

where, integrating factor (IF) = $e^{\int P dx}$

here in equation (ii),

$$P = \frac{1}{x}, \text{ and } Q = x^3$$

$$IF = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

Solution $y(x) = \int x^3 \cdot x dx + c = \frac{x^5}{5} + c$

Given condition, $y(2) = \frac{21}{5}$

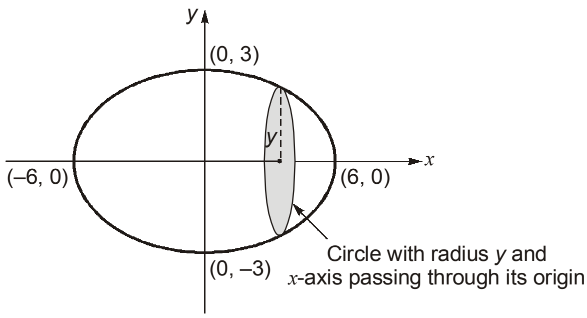
$\therefore$ $\frac{21}{5} \times 2 = \frac{2^5}{5} + c$

$\Rightarrow$ $c = \frac{42-32}{5} = 2$

$\therefore$ $yx = \frac{x^5}{5} + 2$

$\Rightarrow$ $y = \frac{x^4}{5} + \frac{2}{x}$

29. **(b)**



Volume generated $= \int_{-6}^{6} \pi y^2 dx = \int_{-6}^{6} \pi\left(\frac{36-x^2}{4}\right)dx$

$= \frac{\pi \times 2}{4}\int_{0}^{6}(36-x^2)dx = \frac{\pi}{2}\left[36x - \frac{x^3}{3}\right]_0^6 = 72\pi$

30. **(a)**

Given, $\frac{dy}{dx} + 3y = 0$ and $y(1) = 4$

$\frac{dy}{dx} = -3y$

$\int \frac{dy}{y} = \int -3dx$

$\Rightarrow$ $\ln y = -3x + c$

$\Rightarrow$ $y = e^{-3x}. e^{c} = c_1 e^{-3x}$

$y(1) = c_1 e^{-3} = 4$

$\Rightarrow$ $c_1 = \frac{4}{e^{-3}}$

So, $y = \frac{4}{e^{-3}} e^{-3x} = 4e^3 e^{-3x} = 80.34e^{-3x}$

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