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STRENGTH OF MATERIAL

CIVIL ENGINEERING

Date of Test : 24/09/2024

ANSWER KEY >

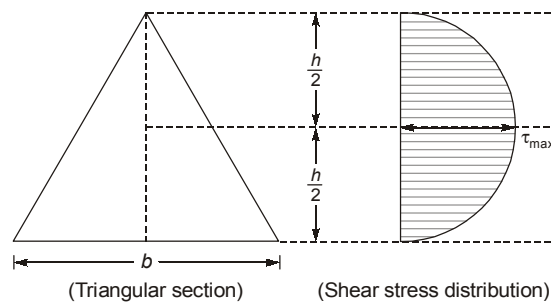
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|--------|---------|---------|---------|---------|
| 1. (b) | 7. (c) | 13. (b) | 19. (c) | 25. (b) |
| 2. (b) | 8. (d) | 14. (a) | 20. (b) | 26. (c) |
| 3. (d) | 9. (c) | 15. (b) | 21. (b) | 27. (b) |
| 4. (c) | 10. (c) | 16. (b) | 22. (a) | 28. (a) |
| 5. (c) | 11. (c) | 17. (d) | 23. (d) | 29. (a) |
| 6. (b) | 12. (d) | 18. (a) | 24. (c) | 30. (b) |

DETAILED EXPLANATIONS

1. (b)

$$\begin{aligned} \therefore \text{Stiffness, } k &\propto \frac{1}{\text{Number of coils } (n)} \\ \Rightarrow k_1 n_1 &= k_2 n_2 \\ \Rightarrow k_1 \times 25 &= k_2 \times 20 \\ \therefore k_2 &= 1.25 k_1 \end{aligned}$$

2. (b)



3. (d)

4. (c)

A couple anywhere in the beam will cause equal and opposite support reactions in the beam. So the SFD will be rectangular or uniform throughout the beam.

5. (c)

Theory of simple bending is only applicable to section of beam in which plane of loading is axis of symmetry \triangle and T have symmetry about loading axis (vertical axis) so theory of simple bending is applicable only to these sections.

6. (b)

$$R_A = \frac{100}{2} - \frac{25 \times 5/3}{10} = \frac{275}{6} \text{ kN}$$

\therefore Bending moment at D

$$\begin{aligned} M &= R_A \times 5 - \frac{10 \times (5)^2}{2} \\ &= \frac{275}{6} \times 5 - \frac{10 \times (5)^2}{2} = 104.167 \text{ kN-m} \end{aligned}$$

7. (c)

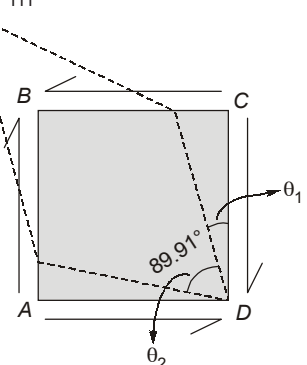
Shear strain,

$$\gamma = (\theta_1 + \theta_2)$$

$$= 90^\circ - (89.91^\circ) = 0.09^\circ = 0.09 \times \frac{\pi}{180}$$

$$\gamma = \frac{\pi}{2000}$$

($180^\circ = \pi$ radian, $1^\circ = \pi/180$ radian)



8. (d)

9. (c)

From the given stress tensor,

$$\begin{aligned} \epsilon_{xy} &= 0.002 \\ \text{Shear strain, } \phi_{xy} &= 2\epsilon_{xy} = 2 \times 0.002 = 0.004 \\ \text{Shear stress, } \tau_{xy} &= G \times \phi_{xy} \\ &= 90 \times 10^3 \times 0.004 \\ &= 360 \text{ MPa} \end{aligned}$$

10. (c)

$$e = \frac{b^2 h^2 t}{4I}$$

11. (c)

12. (d)

$$M = 3.5 \text{ kNm}; T = 5 \text{ kNm}; d = 80 \text{ mm}$$

$$\begin{aligned} \sigma_1 &= \frac{16}{\pi d^3} \left[M + \sqrt{M^2 + T^2} \right] \times 10^6 = \frac{16}{\pi(80)^3} \left[3.5 + \sqrt{(3.5)^2 + 5^2} \right] \times 10^6 \\ &= 95.5 \text{ N/mm}^2 \end{aligned}$$

$$\sigma_2 = \frac{16}{\pi d^3} \left[M - \sqrt{M^2 + T^2} \right] \times 10^6 = -25.89 \text{ N/mm}^2$$

$$\text{Maximum strain, } \epsilon = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} = \frac{1}{E} [95.5 + 0.28 \times 25.89] = \frac{102.75}{E}$$

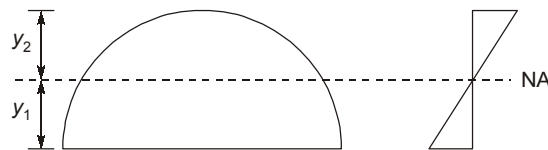
If 'σ' be the stress producing the same maximum strain then,

$$\frac{\sigma}{E} = \frac{102.75}{E}$$

⇒

$$\sigma = 102.75 \text{ N/mm}^2$$

13. (b)



$$y_1 = \frac{4r}{3\pi}$$

$$y_2 = r - \frac{4r}{3\pi}$$

$$\frac{\sigma_T}{y_2} = \frac{\sigma_C}{y_1}$$

⇒

$$\frac{\sigma_T}{\sigma_C} = \frac{y_2}{y_1} = \frac{\left[r - \frac{4r}{3\pi} \right]}{\left(\frac{4r}{3\pi} \right)} = \frac{3\pi}{4} - 1 = 1.36$$

14. (a)

For the triangular portion, let the load per unit metre is w .

$$\therefore \frac{1}{2} \times w \times 2 = 60$$

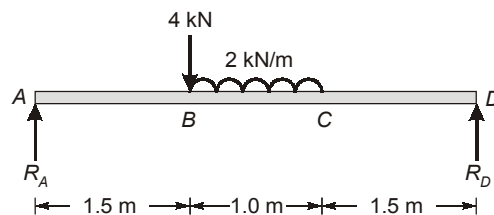
$$\Rightarrow w = 60 \text{ kN/m}$$

$$\therefore \sum M_A = 0$$

$$\Rightarrow 20 \times 5 - R_D \times 4 + 60 \times \left(1 + 2 \times \frac{2}{3}\right) = 0$$

$$\Rightarrow R_D = 60 \text{ kN}$$

15. (b)



$$\sum F_y = 0$$

$$\Rightarrow R_A + R_D = 4 + 2 \times 1$$

$$\Rightarrow R_A + R_D = 6 \quad \dots(i)$$

$$\sum M_D = 0$$

$$\Rightarrow R_A \times 4 - 4 \times 2.5 - 2 \times 1 \times 2 = 0$$

$$\Rightarrow R_A = 3.5 \text{ kN}$$

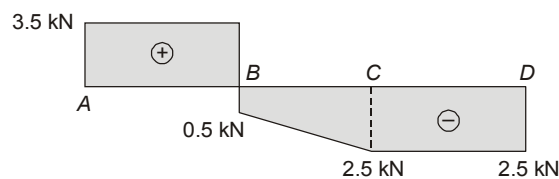
$$\text{and } R_D = 6 - 3.5 = 2.5 \text{ kN}$$

Shear force diagram:

$$SF_A = +R_A = 3.5 \text{ kN}$$

$$SF_B = +3.5 - 4.0 = -0.5 \text{ kN}$$

$$SF_C = -0.5 - (2 \times 1) = -2.5 \text{ kN}$$



We know that the maximum bending moment will occur at B , where the shear force changes sign i.e. the distance of point B from $A = 1.5 \text{ m}$.

16. (b)

The yielding is peculiar to structural steel and other materials do not possess well defined yield point. Yield strength is defined as the lowest stress at which extension of test piece increases without further increase in load.

17. (d)

At point A slope of BMD $\neq 0$ and BMD should be parabolic.

So from elimination, option (d) is correct.

18. (a)

As per Beltrami and Haigh,

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \leq \sigma_y^2$$

As per J.J. Guest theory,

$$(\sigma_1 - \sigma_2) = \sigma_y$$

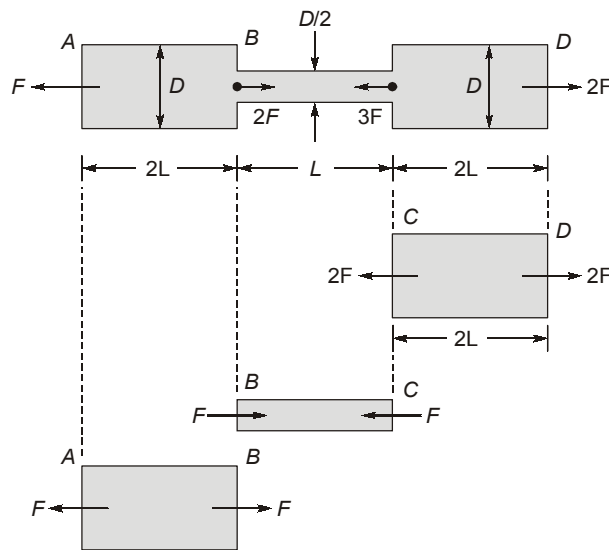
As per Von mises and Hencky theory,

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \leq \sigma_y^2$$

As per St. Venant theory,

$$\sigma_1 - \mu\sigma_2 \leq \sigma_y$$

19. (c)



$$\sigma_{AB} : \sigma_{BC} : \sigma_{CD} = \frac{F}{\frac{\pi}{4}D^2} : \frac{F}{\frac{\pi}{4}\left(\frac{D}{2}\right)^2} : \frac{2F}{\frac{\pi}{4}D^2} = 1 : 4 : 2$$

20. (b)

Given, $L = 2\text{m}$, $P = 120\text{ N}$, $L = 2000\text{ mm}$

Central deflection = 4 mm

$$\therefore \frac{PL^3}{48EI} = 4\text{ mm}$$

$$\therefore \frac{(120\text{ N})(2000\text{ mm}) \times L^2 (\text{mm}^2)}{48E (\text{N/mm}^2) \cdot I (\text{mm}^4)} = 4$$

$$\frac{EI}{l^2} = \frac{120 \times 2000}{48 \times 4} = 1250\text{ N}$$

Now, as per Euler,

$$P_e = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2} \quad (l_e = L)$$

$$= \pi^2 \times 1250\text{ N} = \frac{\pi^2 \times 1250}{1000}\text{ kN}$$

$$P_e = 12.34\text{ kN}$$

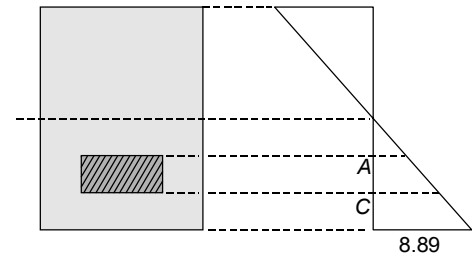
21. (b)

$$\sigma_{\max} = \frac{M}{I} y_{\max}$$

$$= \frac{20 \times 10^6 \times 150}{\left(\frac{150 \times (300)^3}{12} \right)} = 8.89 \text{ MPa}$$

$$\sigma_A = \frac{8.89}{150} \times 50 = 2.96 \text{ MPa}$$

$$\sigma_C = \frac{8.89}{150} \times 100 = 5.93 \text{ MPa}$$



$$\text{Tensile force in hatched area} = \frac{1}{2} (2.96 + 5.93) \times 50 \times 50 = 11.107 \text{ kN}$$

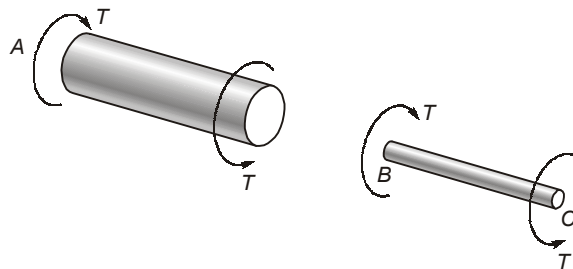
22. (a)

As we know that, (angle of twist)

$$\theta = \frac{TL}{GJ}$$

J for a circular bar of diameter 'd' is $\frac{\pi d^4}{32}$

The total angle of twist, θ_{total} is equal to the sum of the angles of twist for the two different sections. Torques is same for both sections.



$$\theta_{AC} = \theta_{AB} + \theta_{BC} = \frac{T \cdot (2L)}{GJ_1} + \frac{T \cdot L}{GJ_1}$$

$$0.0225 = \frac{T \cdot (2L)}{G \frac{\pi(2)^4}{32}} + \frac{T \cdot L}{G \frac{\pi(1)^4}{32}}$$

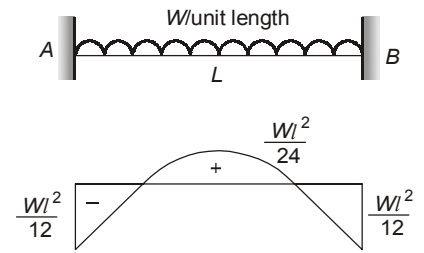
$$0.0225 = \frac{T \cdot (2L)}{G\pi} (32 + 4)$$

$$T = \frac{0.0225}{36} \times \frac{G\pi}{L} = 6.25 \times 10^{-4} \frac{G\pi}{L}$$

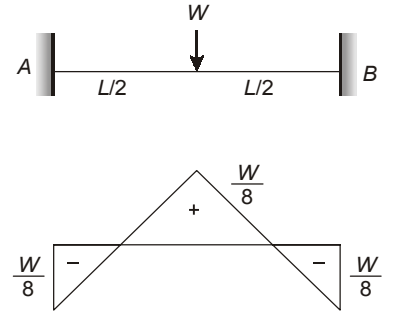
$$T = 0.000625 \frac{G\pi}{L}$$

23. (d)

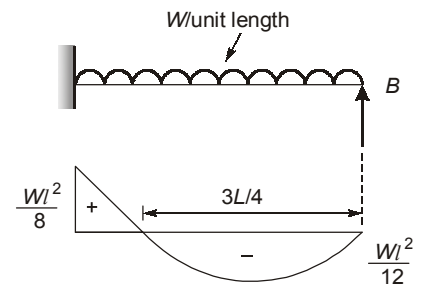
(a) Support moment = $\frac{\omega l^2}{12}$ (hogging)



(b) Support moment = $\frac{WL}{8}$ (hogging)



(c) Moment at fixed end = $\frac{WL^2}{8}$ (sagging)



(d) Support moment = $\frac{Wa^2}{2}$ (hogging)



24. (c)

$$E_s = 200 \text{ kN/mm}^2$$

$$E_c = 105 \text{ kN/mm}^2$$

$$D_c = \text{diameter of copper bar} = 100 \text{ mm}$$

$$D_s = \text{diameter of steel bar} = 50 \text{ mm}$$

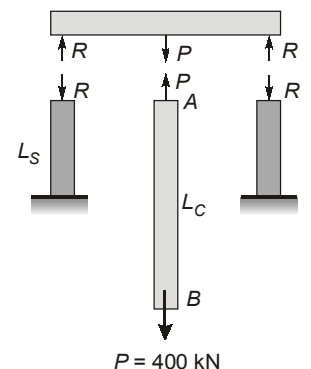
$$R = \frac{P}{2} = 200 \text{ kN}$$

∴ Vertical displacement of point B

$$= \frac{P.L_c}{A_c.E_c} + \frac{(P/2).L_s}{A_s.E_s}$$

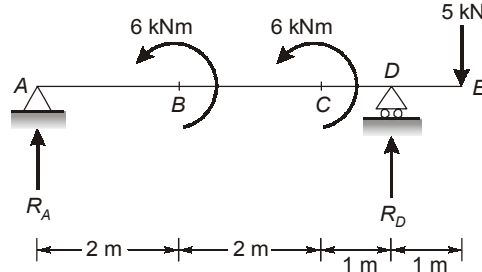
$$= \frac{400 \times 10^3 \text{ (N)} \times 8.8 \times 1000 \text{ (mm)}}{\frac{\pi}{4} \cdot (100)^2 \text{ mm}^2 \times 105 \times 10^3 \frac{\text{N}}{\text{mm}^2}} + \frac{200 \times 10^3 \text{ (N)} \times 0.8 \times 1000 \text{ (mm)}}{\frac{\pi}{4} \cdot (50)^2 \text{ mm}^2 \times 200 \times 10^3 \frac{\text{N}}{\text{mm}^2}}$$

$$= 4.675 \text{ mm}$$



25. (b)

Horizontal load at J produces a couple of 6 kNm (anticlockwise) and a thrust of 6 kN at A (\rightarrow), load of 6 kN at H produces 6 kNm couple (anticlockwise) and a thrust of 6 kN at A (\leftarrow). Therefore, net thrust at A becomes zero.



For support reactions, take moments about A,

$$\sum M_A = 0$$

$$\Rightarrow 6 + 6 + 5R_D - 5 \times 6 = 0$$

$$\therefore R_D = 3.6 \text{ kN}$$

$$\Rightarrow R_A = 5 - 3.6 = 1.4 \text{ kN}$$

Bending moment diagram:

$$M_A = 0 \text{ kNm}$$

$$M_B = R_A \times 2 = 1.4 \times 2 = 2.8 \text{ kNm}$$

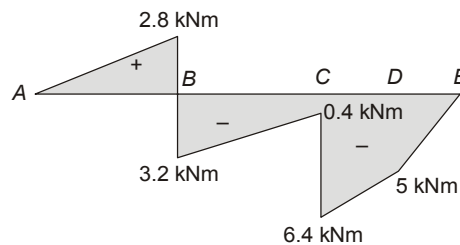
$$M'_B = 2.8 - 6 = -3.2 \text{ kNm}$$

$$M_C = 1.4 \times 4 - 6 = -0.4 \text{ kNm}$$

$$M'_C = -0.4 - 6 = -6.4 \text{ kNm}$$

$$M_D = 1.4 \times 5 - 6 - 6 = -5 \text{ kNm}$$

$$M_E = 0 \text{ kNm}$$



26. (c)

From the given Mohr's circle:

Maximum principal strain, $\epsilon_1 = +180 \mu$

Minimum principal strain, $\epsilon_2 = -80 \mu$

Radius of Mohr's circle of strain is engineering's stress, shear stress is twice of engineering stress.

Maximum shear strain = $2 \times$ Radius of Mohr's circle

$$= 2 \times \frac{180 - (-80)}{2} = 260 \mu$$

Normal strain on the plain of maximum shear strain

= Center of Mohr circle

$$= \frac{180 - 80}{2} = 50 \mu$$

27. (b)

$$\sigma_1 = \frac{f_1 + f_2}{2} + \sqrt{\left(\frac{f_1 - f_2}{2}\right)^2 + q^2}$$

$$\sigma_2 = \frac{f_1 + f_2}{2} - \sqrt{\left(\frac{f_1 - f_2}{2}\right)^2 + q^2}$$

Principal stresses are of opposite nature

∴

$$\sigma_1 \cdot \sigma_2 < 0$$

⇒

$$\left[\frac{f_1 + f_2}{2} + \sqrt{\left(\frac{f_1 - f_2}{2}\right)^2 + q^2} \right] \left[\frac{f_1 + f_2}{2} - \sqrt{\left(\frac{f_1 - f_2}{2}\right)^2 + q^2} \right] < 0$$

⇒

$$\left(\frac{f_1 + f_2}{2}\right)^2 - \left[\left(\frac{f_1 - f_2}{2}\right)^2 + q^2\right] < 0$$

⇒

$$\left(\frac{f_1 + f_2}{2}\right)^2 - \left(\frac{f_1 - f_2}{2}\right)^2 - q^2 < 0$$

⇒

$$\frac{4f_1 f_2}{4} - q^2 < 0$$

∴

$$f_1 f_2 < q^2$$

28. (a)

Let F be the force between the beam and the spring.

$$\text{Deflection of spring, } \delta = \frac{F}{K}$$

Upward deflection of beam due to F ,

$$\delta_1 = \frac{FL^3}{3EI}$$

Downward deflection of beam at B due to w ,

$$\delta_2 = \frac{wL^4}{8EI}$$

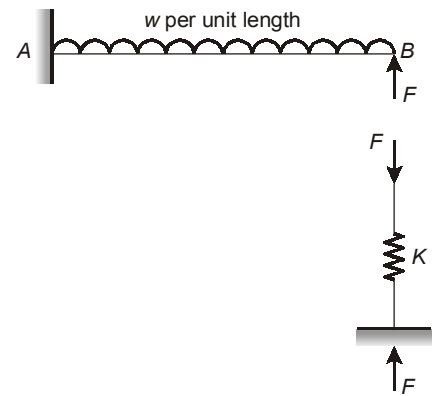
Now,

$$\delta_2 - \delta_1 = \delta$$

$$\frac{wL^4}{8EI} - \frac{FL^3}{3EI} = \frac{F}{K}$$

∴

$$F = \frac{\frac{wL^4}{8EI}}{\left(\frac{1}{K} + \frac{L^3}{3EI}\right)} = \frac{\frac{3}{8}wL}{1 + \frac{3EI}{KL^3}}$$



29. (a)

$$\sigma_1 = \frac{E}{1 - \mu^2} (\epsilon_1 + \mu \epsilon_2)$$

and

$$\sigma_2 = \frac{E}{1 - \mu^2} (\epsilon_2 + \mu \epsilon_1)$$

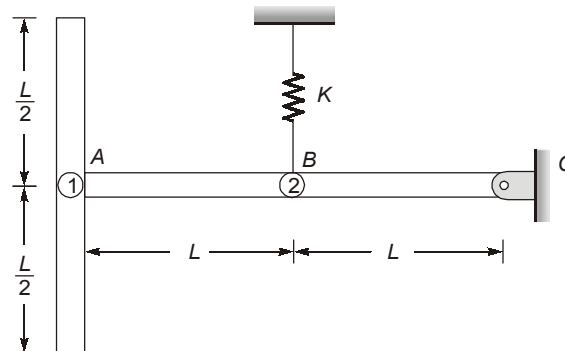
$$\sigma_1 = \frac{2 \times 10^5}{1 - 0.3^2} (0.00152 + 0.3 \times 0.00081) = 387.47 \text{ N/mm}^2$$

and

$$\sigma_2 = \frac{2 \times 10^5}{1 - 0.3^2} (0.00081 + 0.3 \times 0.00152) = 278.24 \text{ N/mm}^2$$

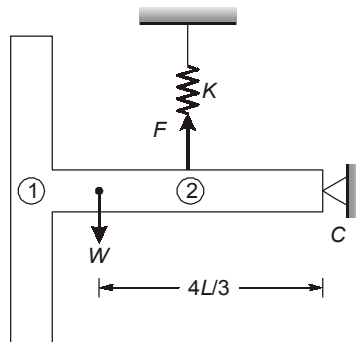
$$\tau_{\max} = \left| \frac{\sigma_1 - \sigma_2}{2} \right| = \left| \frac{387.47 - 278.24}{2} \right| = 54.61 \text{ N/mm}^2$$

30. (b)



Center of gravity of T-shape is at \bar{x} distance from C, calculated as

$$\begin{aligned} \bar{x} &= \frac{A_1 \bar{x}_1 + A_2 \bar{x}_2}{A_1 + A_2} \\ &= \frac{\left(\frac{L}{2} + \frac{L}{2}\right) \times 2L + 2L \times L}{L + 2L} = \frac{4}{3}L \end{aligned}$$



Taking moment about C

$$\Rightarrow W \times \frac{4}{3}L = F \times L$$

$$\Rightarrow F = Kx = \frac{4}{3}W$$

$$\Rightarrow x = \frac{4W}{3K}$$

