CLASS TEST

Sl. : 03IGCE_DEF_07102024

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FLUID MECHANICS

CIVIL ENGINEERING

Date of Test : 07/10/2024

ANSWER KEY

1.	(d)	7.	(a)	13.	(c)	19.	(b)	25.	(b)
2.	(a)	8.	(b)	14.	(b)	20.	(b)	26.	(b)
3.	(c)	9.	(d)	15.	(a)	21.	(d)	27.	(c)
4.	(b)	10.	(c)	16.	(a)	22.	(c)	28.	(a)
5.	(b)	11.	(d)	17.	(c)	23.	(c)	29.	(b)
6.	(d)	12.	(a)	18.	(b)	24.	(b)	30.	(c)

DETAILED EXPLANATIONS

1. (d)

Kinetic energy correction factor is given by

$$\alpha = \frac{1}{AV^3}\int u^3 dA$$

2. (a)

Given, $\rho = 981 \text{ kg/m}^3$

and $\tau = 0.2452 \text{ N/m}^2$

Velocity gradient, $\frac{du}{dy} = 0.2 \text{ s}^{-1}$

Now, using the equation

$$\tau = \mu\frac{du}{dy}$$

$$\Rightarrow \quad 0.2452 = \mu \times 0.2$$

$$\Rightarrow \quad \mu = \frac{0.2452}{0.2} = 1.226 \text{ Ns/m}^2$$

Kinematic viscosity is given by

$$\nu = \frac{\mu}{\rho} = \frac{1.226}{981} = 0.125\times10^{-2} \text{ m}^2/\text{sec}$$

$$= 12.5 \text{ cm}^2/\text{sec}$$

$$= 12.5 \text{ stokes}$$

3. (c)

Continuity equation must be satisfied

$$\therefore \quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}(ax+by)+\frac{\partial}{\partial y}(dx+cy) = 0$$

$$\Rightarrow \quad a + c = 0$$

4. (b)

5. (b)

For parallel pipes, head loss through the pipe is equal,

$$h_{f_1} = h_{f_2}$$

$$\Rightarrow \quad \frac{f_1L_1V_1^2}{2gd_1} = \frac{f_2L_2V_2^2}{2gd_2}$$

$$\Rightarrow \frac{500\times(0.5)}{0.3\times800}\times2\times9.81\times0.35 = V_2^2 \qquad \left(\frac{V_1^2}{2g} = 0.5 \text{ m}\right)$$

$$\Rightarrow \qquad V_2 = 2.674 \text{ m/s}$$

Discharge through pipe 2,

$$Q = A_2V_2$$

$$= \frac{\pi}{4}(0.35)^2 \times 2.476 = 0.2573 \text{ m}^3/\text{s}$$

6. (d)

7. (a)

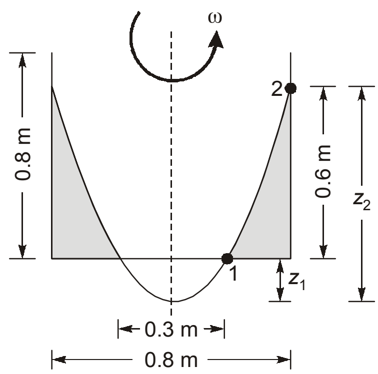
$$P_A - H \times 9.81 \times 1 - 0.18 \times 9.81 \times 0.827 = P_B - 13.6 \times 9.81 \times (H + 0.53)$$

$$-H \times 9.81 - 1.4603 = 97 - 13.6 \times 9.81 \times H - 13.6 \times 9.81 \times 0.53$$

$$\Rightarrow \qquad H = 0.2245 \text{ m}$$

$$\therefore \qquad H = 22.45 \text{ cm}$$

8. (b)



$$z_2 - z_1 = \frac{\omega^2}{2g}\left[R_2^2 - R_1^2\right]$$

$$0.6 = \frac{\omega^2}{2g}\left[0.4^2 - 0.15^2\right]$$

$$\omega = 9.253 \text{ rad/sec}$$

$$\frac{2\pi N}{60} = 9.253$$

$$N = 88.36 \text{ rpm}$$

9. (d)

In Navier's-Stoke equation viscous force term is considered and in all the above mentioned flow viscous force can't be neglected.

10. (c)

$K = 2.1 \times 10^9$ Pa; $E = 2.1 \times 10^{11}$ Pa

$\rho = 1000$ kg/m^3; $D = 400$ mm

$t = 4$ mm

Velocity of propagation of water hammer pressure

$$= \sqrt{\frac{K/\rho}{1+\frac{KD}{Et}}} = \sqrt{\frac{2.1\times\frac{10^9}{1000}}{1+1}} = 1024.7 \text{ m/s}$$

11. (d)

In steady uniform flow

Shear friction velocity, $V_* = \sqrt{\frac{\tau_0}{\rho}}$ ...(i)

Also, $V_* = \sqrt{\frac{f}{8}}\times V_{avg}$ ...(ii)

From eq. (i) and (ii)

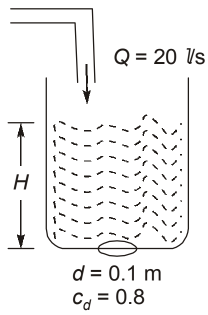
$$\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{f}{8}}\times V_{avg}$$

Squaring both sides, $\frac{\tau_0}{\rho} = \frac{f}{8}\times V_{avg}^2$

$\Rightarrow$ $\tau_0 = \frac{0.024}{8}\times 2\times 2\times 1000$

$= 12 \text{ N/m}^2$

12. (a)



Assume H is the level of water in the tank in steady condition.
For steady water level in the tank
Discharge through orifice

= Water enters in the tank

$$c_d\, a \cdot \sqrt{2gH} = 20 \times 10^{-3}$$

$$0.8\times\frac{\pi}{4}(0.1)^2\sqrt{2gH} = 0.02$$

$$H = 0.52 \text{ m}$$

13. (c)

Given: $u = xe^{-kt}$ and $v = y$

Equation of a streamline is given by

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\Rightarrow \quad \frac{dx}{xe^{-kt}} = \frac{dy}{y} \qquad \left[\text{Let } e^{-kt} = \lambda\right]$$

$$\Rightarrow \quad \frac{dx}{xk} = \frac{dy}{y}$$

$$\therefore \quad \ln x = k \ln y + \ln d$$

where 'd' is a constant.

$$\Rightarrow \quad \ln x = \ln y^{\lambda} + \ln d$$

$$\Rightarrow \quad x = dy^{\lambda}$$

$$\Rightarrow \quad y^{\lambda} = \frac{1}{d}x$$

$$\Rightarrow \quad y = \left(\frac{1}{d}\right)^{1/\lambda} x^{1/\lambda} = Cx^{1/e^{-kt}} \qquad \left[\text{where } \left(\frac{1}{d}\right)^{1/\lambda} = C\right]$$

$$\Rightarrow \quad y = Cx^{e^{kt}}$$

14. (b)

Given data:

Relative density of glass sphere = 2.7

Diameter of glass sphere = 1 mm

Velocity of sphere = 1.25 cm/s

Density of oil = 920 kg/m^3

$$\text{Reynold's number} = \frac{\rho v d}{\mu}$$

Let us assume that Stokes' law is valid then,

$$V = \frac{1}{18}D^2\left(\frac{\gamma_s - \gamma_f}{\mu}\right)$$

$$\frac{1.25}{100} = \frac{1}{18} \times \frac{\left(10^{-3}\right)^2 (2.7 \times 1000 \times 9.81 - 920 \times 9.81)}{\mu}$$

$$\mu = 0.0776 \text{ Pa.S}$$

$$\text{Reynold's number} = \frac{\rho VD}{\mu} = \frac{920 \times \frac{1.25}{100} \times 10^{-3}}{0.0776} = 0.148 < 1$$

Hence Stokes law is valid.

Therefore dynamic viscosity, μ = 0.0776 Pa.S

15. (a)

Distance from the centre where average velocity equal is to local velocity = $\frac{R}{\sqrt{2}}$

$\therefore$ Distance from boundary of pipe = $R - \frac{R}{\sqrt{2}} = 150\left(1 - \frac{1}{\sqrt{2}}\right)$ mm = 4.39 cm

16. (a)

Discharge ratio, $Q_r = \frac{Q_m}{Q_p} = L_{r_h}\left(L_{r_v}\right)^{3/2}$

$= \frac{1}{625} \times \left(\frac{1}{36}\right)^{3/2} = \frac{1}{135000}$

$\therefore$ $Q_p = \frac{Q_m}{Q_\gamma} = 0.025 \times 135000$

$= 3375\ \text{m}^3/\text{s}$

17. (c)

Reynold number,

$$\text{Re}_x = \frac{u_\infty x}{\nu} = \frac{2 \times 1}{1.5 \times 10^{-5}} = 1.33 \times 10^5$$

$$\delta = \frac{4.64x}{\sqrt{\text{Re}_x}} = \frac{4.64 \times 1}{\sqrt{1.33 \times 10^5}} = 0.0127$$

Now, $\frac{du}{dy} = u_\infty\left[\frac{3}{2}\cdot\frac{1}{\delta} - \frac{3}{2}\left(\frac{y^2}{\delta^3}\right)\right]$

$$\left.\frac{du}{dy}\right|_{y=0} = \frac{3u_\infty}{2\delta}$$

Now, shear stress,

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0} = \mu \times \frac{3u_\infty}{2\delta}$$

$$= \frac{3u_\infty \times \nu \times \rho}{2\delta} \qquad \because \mu = \nu\rho$$

$$= \frac{3 \times 2 \times 1.5 \times 10^{-5} \times 1.23}{2 \times 0.0127}$$

$= 4.36 \times 10^{-3}\ \text{N/m}^2$

18. (b)

Buoyancy force acts through center of gravity of displaced liquid.

A large metacentric height in a vessel improves stability and makes time period of oscillation shorter.

19. (b)

20. (b)

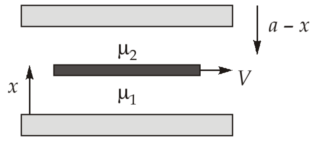
$$\bar{u} = \frac{R^2}{8\mu}\left(\frac{\Delta P}{L}\right)$$

$$\frac{32\mu\bar{u}}{D^2} = \frac{\Delta P}{L}$$

$$\Rightarrow \quad \frac{128\mu Q}{\pi D^4} = \frac{\Delta P}{L}$$

$$= 2535 \text{ Pa}$$

21. (d)



$$\tau_1 = \text{shear stress at bottom}$$

$$= \mu_1 \times \frac{V}{x}$$

$$\tau_2 = \text{shear stress at top} = \mu_2 \frac{V}{a-x}$$

$$\text{drag force} = (\tau_1 + \tau_2) \times A = F_D$$

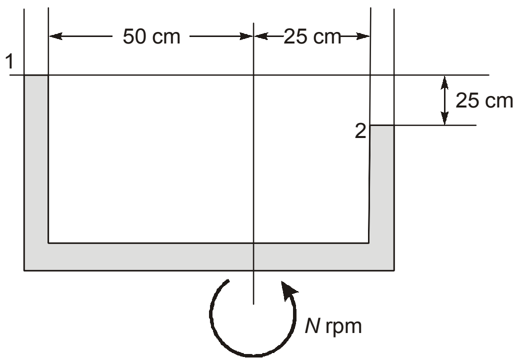
$$= F_D = A \times \left[\frac{\mu_1 V}{x} + \frac{\mu_2 V}{a-x}\right]$$

$$\frac{dF_D}{dx} = 0 = \frac{-\mu_1 V}{x^2} + \frac{\mu_2 V}{(a-x)^2} \Rightarrow \frac{\mu_1}{x^2} = \frac{\mu_2}{(a-x)^2}$$

$$a - x = \sqrt{\frac{\mu_2}{\mu_1}}x$$

$$\Rightarrow \quad \frac{a\sqrt{\mu_1}}{\sqrt{\mu_1} + \sqrt{\mu_2}} = x$$

22. (c)



$$\frac{\overset{0}{\cancel{P_1}}}{\cancel{\rho g}} - \frac{V_1^2}{2g} + Z_1 = \frac{\overset{0}{\cancel{P_2}}}{\cancel{\rho g}} - \frac{V_2^2}{2g} + Z_2$$

$$Z_1 - Z_2 = \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$0.25 = \frac{1}{2g}\left\{r_1^2\omega^2 - r_2^2\omega^2\right\}$$

$$0.25 \times 2 \times 9.81 = \omega^2 \left\{ \left(\frac{50}{100}\right)^2 - \left(\frac{25}{100}\right)^2 \right\}$$

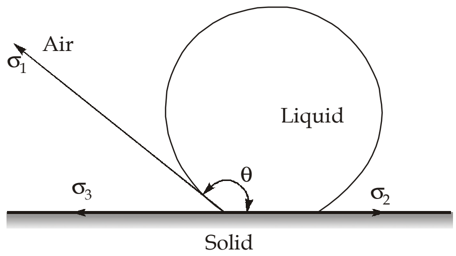
$$\omega = 5.115 \text{ rad/s}$$

$$N = \frac{60\omega}{2\pi} = \frac{60 \times 5.515}{2 \times \pi}$$

$$= 48.8 \text{ rpm}$$

23. (c)

From force balance at point of contact,



$$\sigma_1 \cos(180 - \theta) + \sigma_3 = \sigma_2$$

or $$\cos(180 - \theta) = \frac{\sigma_2 - \sigma_3}{\sigma_1} = -\cos\theta$$

∴ σ_1 = 0.0720 N/m (liquid and air)

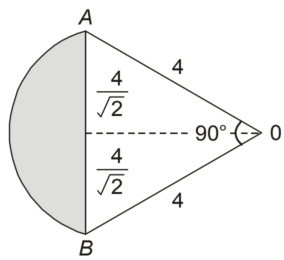
σ_2 = 0.0418 N/m (liquid and solid)

σ_3 = 0.0008 N/m (air and solid)

$$\cos\theta = \frac{0.0008 - 0.0418}{0.072} = -0.56944$$

$$\theta = 124.7°$$

24. (b)



$$\text{Area of shaded region} = \frac{\pi \times 4^2}{4} - \frac{1}{2} \times \frac{8}{\sqrt{2}} \times \frac{4}{\sqrt{2}}$$

$$= 4\pi - 8 = 4.566 \text{ m}^2$$

Horizontal force, $F_H = \rho g A \bar{x}$

$$= 1000 \times 9.81 \times \frac{8}{\sqrt{2}} \times \frac{4}{\sqrt{2}}$$

$$F_H = 156.96 \times 10^3 \text{ N}$$

Vertical force, $F_V = 4.566 \times 9.81 \times 10^3$

$$= 44.79246 \times 10^3 \text{ N}$$

∴ $$\frac{F_H}{F_V} = \frac{156.96}{44.9246} = 3.504$$

25. (b)

Hagen–Poiseuille's equation,

$$\Delta p = \frac{32\mu VL}{d^2}$$

where $$V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$$

$$\therefore \quad \Delta p = \frac{128\pi QL}{\pi d^4}$$

$$\Delta p \propto \frac{1}{d^4}$$

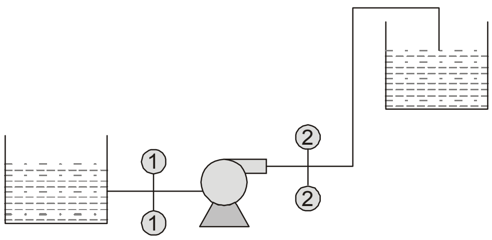
$$\Delta p \times d^4 = C$$

$$\Delta p_1 \, d_1^4 = \Delta p_2 \, d_2^4$$

$$\Delta p_1 \times d_1^4 = \Delta p_2 \times (2d_1)^4$$

or $$\Delta p_2 = \frac{\Delta p_1}{16}$$

26. (b)



$$S = 0.8$$

$$\therefore \quad \rho = 0.8 \times 1000 = 800 \text{ kg/m}^3$$

$$W_p = 50 \text{ J/kg}$$

$$V_1 = 1 \text{ m/s}$$

$$V_2 = 7 \text{ m/s}$$

Applying Bernoulli's equation between sections (1)–(1) and (2)–(2)

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 + W_P = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

where $$z_1 = z_2$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + W_P = \frac{p_2}{\rho} + \frac{V_2^2}{2}$$

$$\frac{p_2 - p_1}{\rho} = \frac{V_1^2 - V_2^2}{2} + W_P$$

$$p_2 - p_1 = \rho\left[\frac{V_1^2 - V_2^2}{2} + W_P\right]$$

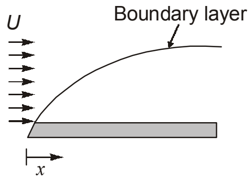
$$= 800\left[\frac{(1)^2 - (7)^2}{2} + 50\right] = 20800 \text{ N/m}^2 = 20.8 \text{ kN/m}^2$$

27. (c)

For laminar flow over flat plate,

$$\delta = \frac{5x}{\sqrt{Re}}$$

where $$Re = \frac{Ux}{\nu}$$



$\therefore$ $$\delta = \frac{5x}{\left(\frac{Ux}{\nu}\right)^{1/2}}$$

$$\delta = \frac{5\sqrt{x}}{\sqrt{U/\nu}}$$

$$\delta \propto x^{1/2}$$

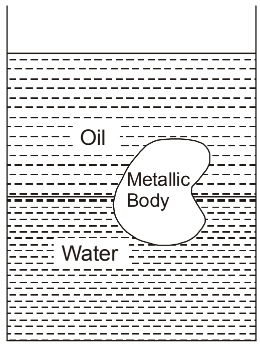
28. (a)

Let V = Volume of metallic body

45% of volume is in oil

i.e., V_{oil} = 45% of V = $0.45V$

and $V_{water} = 0.55V$



For equilibrium condition,

Net buoyant force = $(F_B)_w + (F_B)_{oil}$

= Weight of body

$$M_b g = \rho_w V_w g + \rho_{oil} V_{oil} g$$

$$M_b = \rho_w V_w + \rho_{oil} V_{oil}$$

$$\rho_b V = \rho_w \times 0.55V + \rho_{oil} \times 0.45V$$

or $$\rho_b = \rho_w \times 0.55 + \rho_{oil} \times 0.45$$

$$= 1000 \times 0.55 + 700 \times 0.45$$

$$= 550 + 315 = 865 \text{ kg/m}^3$$

29. (b)

For 2D-flow velocity field is:

$$\vec{V} = \frac{x}{x^2+y^2}\hat{i} + \frac{y}{x^2+y^2}\hat{j}$$

so $$u = \frac{x}{x^2+y^2} \text{ and } v = \frac{y}{x^2+y^2}$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= \frac{x}{x^2+y^2}\frac{\partial}{\partial x}\left[\frac{x}{x^2+y^2}\right] + \frac{y}{x^2+y^2}\frac{\partial}{\partial y}\left[\frac{x}{x^2+y^2}\right]$$

$$= \frac{x}{x^2+y^2}\left[\frac{x(-2x)}{(x^2+y^2)^2} + \frac{1}{x^2+y^2}\right] + \frac{y}{x^2+y^2}\left[\frac{-x(2y)}{(x^2+y^2)^2}\right]$$

$$= \frac{-2x^3}{(x^2+y^2)^3} + \frac{x}{(x^2+y^2)^2} - \frac{2xy^2}{(x^2+y^2)^3}$$

$$= \frac{-2x^3 + x(x^2+y^2) - 2xy^2}{(x^2+y^2)^3}$$

$$= \frac{-2x^3 + x^3 + xy^2 - 2xy^2}{(x^2+y^2)^3} = \frac{-x^3 - xy^2}{(x^2+y^2)^3}$$

$$= \frac{-x(x^2+y^2)}{(x^2+y^2)^3}$$

$$a_x = \frac{x}{(x^2+y^2)^2}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= \frac{x}{x^2+y^2}\frac{\partial}{\partial x}\left[\frac{y}{x^2+y^2}\right] + \frac{y}{x^2+y^2}\frac{\partial}{\partial y}\left[\frac{y}{x^2+y^2}\right]$$

$$= \frac{x}{x^2+y^2}\frac{(-2xy)}{(x^2+y^2)^2} + \frac{y}{x^2+y^2}\times\left[\frac{-2y^2}{(x^2+y^2)^2} + \frac{1}{x^2+y^2}\right]$$

$$= \frac{-2x^2y}{(x^2+y^2)^3} - \frac{2y^3}{(x^2+y^2)^3} + \frac{y}{(x^2+y^2)^2}$$

$$= \frac{-2x^2y - 2y^3 + y(x^2+y^2)}{(x^2+y^2)^3} = \frac{-x^2y - y^3}{(x^2+y^2)^3} = \frac{-y(x^2+y^2)}{(x^2+y^2)^3}$$

$$a_y = \frac{-y}{(x^2+y^2)^2}$$

30. (c)

Given velocity field,

$$\vec{V} = (-x^2 + 3y)\hat{i} + (2xy)\hat{j}$$

where $u = -x^2 + 3y$ and $v = 2xy$

The acceleration components along x and y-axis.

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

and $$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= (-x^2 + 3y) \times (-2x) + 2xy \times 3$$

$$= 2x^3 - 6xy + 6xy = 2x^3$$

and $$a_y = (-x^2 + 3y) \times 2y + 2xy \times 2x$$

$$= -2yx^2 + 6y^2 + 4x^2y = 2yx^2 + 6y^2$$

At point (1, –1),

$$a_x = 2$$

and $$a_y = 2 \times (-1) \times 1 + 6 \times (-1)^2$$

$$= -2 + 6 = 4$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = 2i + 4\hat{j}$$

Resultant acceleration,

$$a = \sqrt{4+16} = \sqrt{20}$$

$$= \sqrt{4 \times 5} = 2\sqrt{5}$$

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