CLASS TEST S.No.: 03SKCS_FGHIJKL_181124

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Theory of Computation

COMPUTER SCIENCE & IT

Date of Test : 18/11/2024

ANSWER KEY

1.	(d)	7.	(a)	13.	(d)	19.	(a)	25.	(b)
2.	(d)	8.	(d)	14.	(1)	20.	(a)	26.	(c)
3.	(a)	9.	(c)	15.	(b)	21.	(d)	27.	(c)
4.	(c)	10.	(b)	16.	(b)	22.	(d)	28.	(a)
5.	(d)	11.	(d)	17.	(c)	23.	(d)	29.	(a)
6.	(d)	12.	(a)	18.	(c)	24.	(c)	30.	(a)

DETAILED EXPLANATIONS

1. (d)

$$\left.\begin{array}{l} A \rightarrow aAb \\ A \rightarrow \in \end{array}\right\} \text{ generates } \{a^n b^n : n \geq 0\}$$

Now, $S \rightarrow Ab$

$\rightarrow a^n b^n b$

$L = \{a^n b^{n+1} : n \geq 0\}$

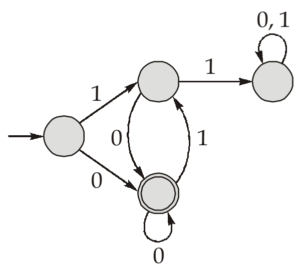
So, option (d) is the correct answer.

2. (d)

$L = \{x \in \{0, 1\}^* \mid x \text{ ends with 0 and not contains 2 consecutive 1's}\}$

R.E. $= (0 + 10)^* (0 + 10)$

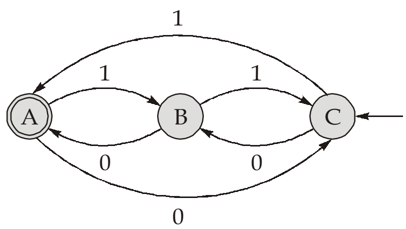
$= (0 + 10)^+$



So, option (d) is correct.

3. (a)

The finite automata obtained by F_1 is



Automata F_2 is same as F_1. So $L(F_1) = L(F_2)$.

4. (c)

L can be written as $\{w \# (0 + 1)^* w^R (0 + 1)^*\}$

The context-free grammar that generate L is

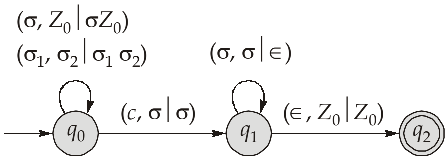
$S \rightarrow TX$

$T \rightarrow 0T0 \mid 1T1 \mid \#X$

$X \rightarrow 0X \mid 1X \mid \in$

5. (d)

The transition diagram of the PDA is as shown below. In the figure σ, σ_1 and σ_2 represent a or b.



PDA accepting $\{wcw^R | w \in (a, b)^* \text{ and } |w| \geq 1\}$.

6. (d)

7. (a)

- Context free languages are not closed under complementation and intersection.
  Hence option (b) and (c) is false.
- DPDA is less powerful than PDA. Hence there is CFL language which can not be accepted by DPDA.
  Hence option (d) is false.

8. (d)

Recursively enumerable problems are not closed under complementation. Set difference $A - B$ can be written as $A \cap B'$ where B' denotes complementation of B.
So we can say that recursively enumerable problems are not closed under set difference.
Hence all other options are correct.

9. (c)

$$L = \{a^i b^j c^k | i > k, 0 \leq j < 3, k \geq 0\}$$

The context-free grammar for the given language can be
$S \rightarrow aS$
$S \rightarrow aSc$
$S \rightarrow a$
$S \rightarrow ab$
$S \rightarrow abb$

10. (b)

$$\text{Prefix }(L) = \{\in, b, ba, bab, baba\}$$
$$\text{Suffix }(L) = \{\in, a, ba, aba, baba\}$$
$$A = \{\in, b, ba, bab, baba\} \cap \{\in, a, ba, aba, baba\}$$
$$A = \{\in, ba, baba\}$$

There are 3 strings present in language A.

11. (d)

Consider $L_1 = \{a^n b^m \mid n = m\}$
and $L_2 = \{a^n b^m \mid n \neq m\}$
then $L_1 \cap L_2 = \phi$
and $L_1 \cup L_2 = a^* b^*$
Since there exits a DFA for $L_1 \cup L_2$
Hence option (b) is false.
Option (c) is also false since for both L_1 and L_2 there does not exits any DFA.
Now, In order to verify option (a)

Consider $L_1 = a^* b^*$

$L_2 = \{a^n b^m \mid n = m\}$

then, $L_1 \cap L_2 = \{a^n b^m \mid n = m\}$

and $L_1 \cup L_2 = a^* b^*$

Clearly, no DFA exists for $L_1 \cap L_2$

Here option (a) is also false.

Therefore, (d) is the only choice.

12. (a)

In formal language theory, a context - free grammar, G, is said to be in chomsky normal form if all of its production rule are of the form

$$A \to BC$$

or $$A \to a$$

Hence, only G_1 is in CNF.

13. (d)

$$L(M) = \{w \mid n_a(w) = 0 \text{ or } n_a(w) \geq 4\}$$

$$\overline{L(M)} = L(\bar{M}) = \{w \mid 1 \leq n_a(w) \leq 3\}$$

So, option (d) is correct.

14. (1)

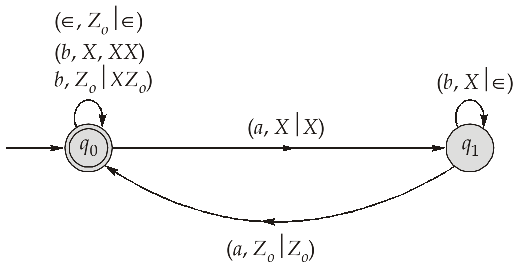
The given Mealy machine computes XNOR.

So, $\alpha \odot \beta = \overline{\alpha \oplus \beta} = \overline{0 \oplus 0} = 1$

15. (b)

The Turing Machine T accepts the regular language corresponding to the regular expression $aa^* + bb^*$.

16. (b)



$$L = \{(b^n a b^n a)^m \mid m, n \geq 0\}$$

17. (c)

L_1 : is DCFL, push all 0's and 1's in the stack the for every 0 of the string, start popping from the stack.

L_2 : is DCFL, for every 0 in the string push two 0's in the stack, for every '1', pop a '0' from the stack, then skip operation will be applied on all 0's.

L_3 : $0^l 1^{2l} 0^n$, this is not even a CFG. Due to three level dependency, it can't be solved using single stack.

L_4 : Here we need to compare each 2 with 0 and each 3 with 1. However, in both the cases top of stack contains 1's and 2's respectively. So, can't be solved using single stack.

18. (c)

$$L = \{a^m\, b^n\, c^k \mid m, n, k > 0 \text{ and } m = k\}$$

Here, a's are replaced by x and c's are replaced by y in every scan from $q_0 \to q_1 \to q_2 \to q_0$.
To reach final state, atleast one b should appear and atleast one y (y represents c hence a also must appear) should appear.

$\therefore$ $L = a^i\, b^j\, c^i \mid i, j > 0\}$ is accepted by TM

So option (c) is correct.

19. (a)

Since M is a TM that halts on all input, so $L(M)$ is decidable. So, $L(M) \neq L'$. Since decidable language cannot be equal to some undecidable language.

So, $L = \phi$

Hence decidable and recursive.

20. (a)

Regular language are closed under Half (L).

21. (d)

$L_1 \cap L_2$ will generate CSL since $L_1 \cap L_2$ is $\{a^n b^n c^n \mid n \geq 0\}$.

22. (d)

Checking CFL is equivalence, equality and subset problems for CFL are undecidable.

23. (d)

$\boldsymbol{L_1}$ **:** $\{a^n\, b^m\, a^k \mid k = mn\}$ is not CFL, since can not implement it with single stack.

$\boldsymbol{L_2}$ **:** $\{a^{m+n}\, b^{n+m}\, c^m \mid n, m \geq 1\}$ is non-CFL since here more than 1 comparison present i.e., $\{a^m\, a^n\, b^n\, b^m\, c^m\}$. Hence cannot be implement by single stack.

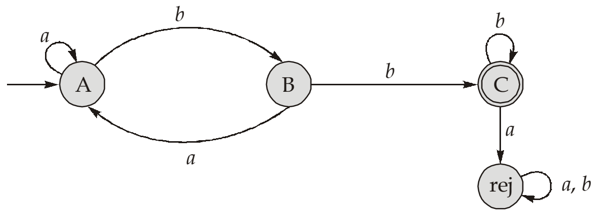
$\boldsymbol{L_3}$ **:** $\{a^n\, b^n\, c^m \mid m < n \text{ and } m, n \geq 1\}$ is non-CFL since more than 1 comparison are present simultaneously i.e. after comparison of $n = n$, we left with on c^m and we cannot compare $m < n$ or not.

24. (c)

L_1 is recursive, since M_0 is halts on every inputs. So, when we run M.
Then If M_0 halt and accept the string then M accepts the string.
Else if M_0 halt and reject the string then M rejects the string.
So, M is always halt i.e., recursive language.
L_2 is recursive enumerable, since M halts only for strings that accepted by TM M_0.
So M is semidecidable.
Hence L_2 is recursively enumerable language.
Both (a) and (b) are true.

25. (b)

The DFA for the regular expression $a^*b\ (aa^*b)^*bb^*$ will be,



26. (c)

$$L_1.L_2 = \text{(Regular) . (CSL)}$$

L_2 is $a^n\ b^n\ c^n$, but L_1 can be any regular language.

Case 1: If $L_1 = \phi$,

$\Rightarrow$ $L_1.L_2 = \phi.\ \{a^n\ b^n\ c^n\} = \phi$ is regular, which is also CSL.

Case 2: If $L_1 = \{\varepsilon\}$

$\Rightarrow$ $L_1.L_2 = \{\varepsilon\}.\ \{a^n\ b^n\ c^n\} = \{a^n\ b^n\ c^n\}$ is CSL.

$L_1.L_2$ is always CSL but it may or may not be regular.

27. (c)

- Languages accepted by push down automata are not closed under complementation.
- Turing decidable languages are closed under union and Kleen star operation.
- Recursive enumerable languages are not closed under complementation.

28. (a)

$$L_2 = \{a^nb^n\},\ L_1 = \{a^*b^*\}$$
$$L = (a^*b^*) \cup ((a+b)^* - \{a^nb^n\})$$
$$= a^*b^* + \{a^mb^n \mid m \neq n\} + (a+b)^*\ ba\ (a+b)^*$$
$$= (a+b)^* + \{a^mb^n \mid m \neq n\}$$
$$= (a+b)^*$$

29. (a)

The above PDA represents the language,

$$L = \{a^mb^nc^k \mid \text{if } (m \text{ is even}) \text{ then } n = k\}$$

30. (a)

$$Y = A \cup L_1 \cup L_2 \cup \ldots \cup L_n \cup B$$
$$Y = \Sigma^* \cup L_1 \cup L_2 \cup \ldots \cup L_n \cup \phi \qquad (\because \text{ union with } \Sigma^* = \Sigma^*)$$
$$Y = \Sigma^*$$

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