CLASS TEST

S.No.: 01SKCE_AB_30012025

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Kolkata

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

ENGINEERING MATHEMATICS

CIVIL ENGINEERING

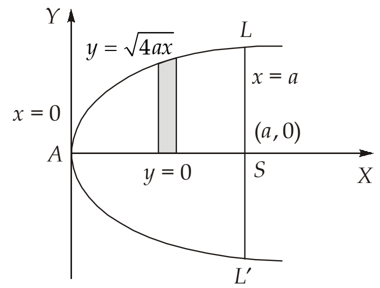
Date of Test : 30/01/2025

ANSWER KEY

1.	(d)	7.	(c)	13.	(c)	19.	(d)	25.	(b)
2.	(a)	8.	(c)	14.	(b)	20.	(c)	26.	(b)
3.	(a)	9.	(b)	15.	(a)	21.	(d)	27.	(c)
4.	(a)	10.	(c)	16.	(c)	22.	(a)	28.	(c)
5.	(a)	11.	(c)	17.	(b)	23.	(c)	29.	(d)
6.	(b)	12.	(d)	18.	(c)	24.	(c)	30.	(a)

DETAILED EXPLANATIONS

1. (d)



$$\text{Required area} = 2 \times (\text{area ASL})$$

$$= 2\int_0^a \int_0^{2\sqrt{ax}} dy\,dx = 2\int_0^a 2\sqrt{ax}\,dx$$

$$= 4\sqrt{a}\left(\frac{x^{3/2}}{3/2}\right)_0^a = \frac{8a^2}{3}$$

2. (a)

$$\vec{a}\times(\vec{b}\times\vec{c}) = (a \cdot c)b - (a \cdot b)c$$

3. (a)

$$P(F) = 0.3$$
$$P(G) = 0.4$$
$$P(F \cap G) = 0.2$$

Now, $P(F \cup G) = P(F) + P(G) - P(F \cap G)$

$$= 0.3 + 0.4 - 0.2 = 0.5$$

$\therefore$ $P(F \cup G) = 0.5$

4. (a)

The auxilary equation is

$$2\lambda^2 + 8 = 0$$
$$\lambda = \pm 2\,i$$

Therefore the complimentary function,

$$y(t) = c_1 \cos 2t + c_2 \sin 2t$$
$$y'(t) = -2c_1 \sin 2t + 2c_2 \cos 2t$$

But we have,

$$y(0) = 0,$$
$$y'(0) = 10$$

Therefore, we get

$$c_1 = 0 \text{ and } c_2 = 5$$

Therefore the solution is

$$y(t) = 5 \sin 2t$$

So, $y(1) = 5 \sin 2$

5. (a)

The valid equality for an orthogonal matrix Q would be

$$Q^T = Q^{-1}$$

© Copyright: MADE EASY

6. (b)

In Simpson's $\frac{1}{3}$rd rule the curve is replaced by a second degree polynomial i.e. parabola.

7. (c)

Given functions are

$$f(x\ ,\ y) = x^3 - 3xy^2$$
$$g(x,\ y) = 3x^2\ y - y^2$$

So, $$\frac{\partial f}{\partial x} = 3x^2 - 3y^2$$

and $$\frac{\partial g}{\partial y} = 3x^2 - 2y$$

$\therefore$ $$\frac{\partial f}{\partial y} = -6\ xy$$

$$\frac{\partial g}{\partial x} = 6\ xy$$

$\therefore$ $$\frac{\partial f}{\partial y} = -\frac{\partial g}{\partial x}$$

8. (c)

Given function, $$f(x) = (K^2 - 4)x^2 + 6x^3 + 8x^4$$
$$f'(x) = 32x^3 + 18x^2 + 2(K^2 - 4)x$$
$$f''(x) = 96x^2 + 36x + 2(K^2 - 4)$$

For maxima, $$f''(0) = 2(K^2 - 4) < 0$$

$\Rightarrow$ $$K^2 - 4 < 0$$

$\Rightarrow$ $$K^2 < 4$$

$\Rightarrow$ $$-2 < K < 2$$

9. (b)

Method will fail when,

$$f'(x_n) = 0$$

Hence, $$2(x - 1) + 1 = 0$$
$$2x - 2 + 1 = 0$$
$$2x - 1 = 0$$
$$x = \frac{1}{2} = 0.5$$

Alternate Solution:

By Newton-Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Here, $f(x) = (x-1)^2 + x - 3$

$f'(x) = 2(x-1) + 1$

Therefore with various x_n, we get

x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
0	–2	–1	–2
0.5	–2.25	0	–∞
1	–2	1	3
3	4	5	2.2

with the initial guess of 0.5, the next iteration value is –∞.

10. (c)

$$\lim_{x\to 0}\left(\frac{e^{5x}-1}{x}\right)^2 = \left(\lim_{x\to 0}\frac{e^{5x}-1}{x}\right)^2$$

Applying L's hospital rule

$$\left(\lim_{x\to 0}\frac{5e^{5x}}{1}\right)^2 = 25$$

11. (c)

$$\int \vec{F}\cdot\vec{dl} = \int_1^2 2x\,dx = \left[x^2\right]_1^2 = 3$$

12. (d)

Angle between the vectors is given by

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = -\frac{3}{4}+\frac{1}{4} = -\frac{1}{2}$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \quad \theta = 120°$$

13. (c)

$$\text{Maximum of } \frac{e^{\sin x}}{e^{\cos x}} = e^{\sin x - \cos x} = e^{\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)}$$

Maximum occurs when $\sin\left(x-\frac{\pi}{4}\right)$ is maximum, i.e., 1

$$\therefore \quad \text{Maximum value} = e^{\sqrt{2}}$$

14. (b)

The characteristic equation of A is

© Copyright: MADE EASY

$$\begin{vmatrix} 1-\lambda & 1 \\ 2 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow$ $\lambda^2 - 4\lambda - 5 = 0$ ...(i)

By Cayley-Hamilton theorem, A must satisfy its characteristic equation (i), so that,

$A^2 - 4A - 5I = 0$ ...(ii)

Multiplying (ii) by A^{-1}, we get

$$A - 4I - 5A^{-1} = 0$$

$$A^{-1} = \frac{1}{5}(A-4I) = \frac{1}{5}\left\{\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

$$= \frac{1}{5}\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$$

Alternate Solution:

$$A^{-1} = \frac{1}{3-8}\begin{bmatrix} 3 & -4 \\ -2 & 1 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$$

$\lambda^5 - 4\lambda^4 - 7\lambda^3 + 11\lambda^2 - \lambda - 10I$ by the polynomial $\lambda^2 - 4\lambda - 5$, we obtain

$$\lambda^5 - 4\lambda^4 - 7\lambda^3 - \lambda - 10I = (\lambda^2 - 4\lambda - 5)(\lambda^3 - 2\lambda + 3) + \lambda + 5$$

$$= \lambda + 5$$

Hence,

$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I = A + 5$

Which is a linear polynomial in A.

15. (a)

In the equation $Mdx + Ndy = 0$

Theorem-1:

If $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N}$ be a function of x only (say $f(x)$), then $e^{\int f(x)dx}$ is an integrating factor.

Theorem-2:

If $\dfrac{\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}}{M}$ be a function of y only (say $f(y)$), then $e^{\int f(y)dy}$ is an integrating factor.

16. (c)

$$x = (N)^{1/N}$$

$$x^N = N$$

$$x^N - N = 0$$

$$f(x) = x^N - N$$

$$f'(x) = N \cdot x^{N-1}$$

Newton-Raphson Iteration,

$$x_{K+1} = x_K - \frac{f(x_K)}{f'(x_K)} = x_K - \frac{x_K^N - N}{N x_K^{N-1}}$$

$$= \frac{N x_K^N - x_K^N + N}{N x_K^{N-1}}$$

$$x_{K+1} = \left(\frac{N-1}{N}\right) x_K + x_K^{1-N}$$

17. (b)

Probability of defective item, $p = \frac{1}{10}$,

Probability of non defective item,

$$q = \frac{9}{10}$$

$$n = 10, \quad r = 2$$

$$p(r) = {}^nC_r \, p^r q^{n-r}$$

$$p(2) = {}^{10}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{10-2} = \frac{10 \times 9}{1 \times 2} \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^8$$

$$= 0.1937$$

18. (c)

$$x^2 + y^2 = 1, \qquad x + y + z = 3,$$

$$z = 0$$

$$\text{Required volume} = \iiint dx\,dy\,dz$$

$$= \iiint dx\,dy\,[z]_0^{3-x-y}$$

$$= \iiint (3 - x - y)\,dx\,dy$$

On putting x = r cos θ, $y = r$ sin θ, $dx\ dy = r\ d\theta\ dr$, we get

$$= \iint (3 - r\cos\theta - r\sin\theta)\, r\, d\theta\, dr$$

$$= \iint (3 - r\cos\theta - r\sin\theta)\, r\, d\theta\, dr$$

we get,

$$= \int_0^{2\pi} d\theta \int_0^{L} (3r - r^2\cos\theta - r^2\sin\theta)\, dr$$

© Copyright: MADE EASY

$$= \int_0^{2\pi} d\theta \left(\frac{3r^2}{2} - \frac{r^3}{3}\cos\theta - \frac{r^3}{3}\sin\theta\right)_0^1$$

$$= \int_0^{2\pi} \left(\frac{3}{2} - \frac{1}{3}\cos\theta - \frac{1}{3}\sin\theta\right) d\theta$$

$$= \left[\frac{3}{2}\theta - \frac{1}{3}\sin\theta + \frac{1}{3}\cos\theta\right]_0^{2\pi}$$

$$= 3\pi - \frac{1}{3}\sin 2\pi + \frac{1}{3}\cos 2\pi - \frac{1}{3}$$

$$= 3\pi$$

19. (d)

$$\left(\frac{1+i}{1-i}\right)^n = 1$$

$$\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n = 1$$

$$\left(\frac{1-1+2i}{1+1}\right)^n = 1$$

$$(i)^n = 1$$

$n = 4$ is the smallest positive integer.

20. (c)

We have, $$(2xy + x^2)\frac{dy}{dx} = 3y^2 + 2xy$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3y^2 + 2xy}{2xy + x^2}$$

Put, $$y = Vx$$

$$\frac{dy}{dx} = V + x\frac{dV}{dx}$$

On substitution, the given equation becomes

$$V + x\frac{dV}{dx} = \frac{3V^2x^2 + 2Vx^2}{2Vx^2 + x^2}$$

$$V + x\frac{dV}{dx} = \frac{3V^2 + 2V}{2V + 1}$$

$$x\frac{dV}{dx} = \frac{V^2 + V}{2V + 1}$$

$$\Rightarrow \quad \int\left(\frac{2V+1}{V^2+V}\right) dV = \int \frac{dx}{x}$$

$$\Rightarrow \qquad \ln(V^2 + V) = \log_e x + \ln c$$

$$\Rightarrow \qquad \frac{y^2}{x^2} + \frac{y}{x} = cx$$

$$\Rightarrow \qquad y^2 + xy = cx^3$$

21. (d)

Given, $\frac{d^2y}{dx^2} - 4\left(\frac{dy}{dx}\right) + 3y = 0$

A.E. is $m^2 - 4m + 3 = 0$

$$(m-3)(m-1) = 0$$

$$m = 1, 3$$

C.F. is $y = Ae^x + Be^{3x}$ i.e., $p = 1, q = 3$

So, $p + q = 1 + 3$

$$= 4$$

22. (a)

Given, $AX = B$

$$\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$$

For infinite solution,

$$\rho(A) = \rho(A/B) < n$$

$$\rho[A/B] = \begin{bmatrix} 3 & -2 & 1 & b \\ 5 & -8 & 9 & 3 \\ 2 & 1 & a & -1 \end{bmatrix}$$

$R_3 \leftrightarrow R_1$

$$= \begin{bmatrix} 2 & 1 & a & -1 \\ 3 & -2 & 1 & b \\ 5 & -8 & 9 & 3 \end{bmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - 2R_1$

$$= \begin{bmatrix} 2 & 1 & a & -1 \\ 1 & -3 & 1-a & b+1 \\ 1 & -10 & 9-2a & 5 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$= \begin{bmatrix} 2 & 1 & a & -1 \\ 1 & -3 & 1-a & b+1 \\ 0 & -7 & 8-a & 4-b \end{bmatrix}$$

$R_2 \to R_2 - R_1/2$

© Copyright: MADE EASY

$$= \begin{bmatrix} 2 & 1 & a & -1 \\ 0 & \frac{-7}{2} & 1-\frac{3a}{2} & b+\frac{3}{2} \\ 0 & -7 & 8-a & 4-b \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$= \begin{bmatrix} 2 & 1 & a & -1 \\ 0 & \frac{-7}{2} & 1-\frac{3a}{2} & b+\frac{3}{2} \\ 0 & 0 & 6+2a & 1-3b \end{bmatrix}$$

For infinite solution,

$$6 + 2a = 0 \Rightarrow a = -3$$

$$1 - 3b = 0 \Rightarrow b = \frac{1}{3}$$

23. (c)

Let, $r = y - z\ ,\ s = z - x,\ t = x - y$

$$u = u(r, s, t)$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\cdot\frac{dr}{du} + \frac{\partial u}{\partial s}\cdot\frac{ds}{dx} + \frac{\partial u}{\partial t}\cdot\frac{dt}{dx}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial z}(0) + \frac{\partial u}{\partial z}(-1) + \frac{\partial u}{\partial t}(1)$$

$$= \frac{\partial u}{\partial t} - \frac{\partial u}{\partial s} \quad \text{...(i)}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\cdot\frac{dr}{dy} + \frac{\partial u}{\partial s}\cdot\frac{ds}{dy} + \frac{\partial u}{\partial t}\cdot\frac{dt}{dy}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}(1) + \frac{\partial u}{\partial z}(0) + \frac{\partial u}{\partial t}(-1)$$

$$= \frac{\partial y}{\partial r} - \frac{\partial u}{\partial t} \quad \text{...(ii)}$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}\cdot\frac{dr}{dz} + \frac{\partial u}{\partial s}\cdot\frac{ds}{\partial z} + \frac{\partial u}{\partial t}\cdot\frac{dt}{dz}$$

$$\frac{\partial y}{\partial z} = \frac{\partial u}{\partial r}(-1) + \frac{\partial u}{\partial s}(1) + \frac{\partial u}{\partial t}(0)$$

$$= \frac{\partial u}{\partial s} - \frac{\partial u}{\partial r} \quad \text{...(iii)}$$

By adding equation (i), (ii) and (iii),

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

24. (c)

Given, $\lim\limits_{n\to\infty} \dfrac{an^2 + bn + c}{pn^2 + q}$

$$= \lim_{n\to\infty} \frac{n^2\left(a + \frac{b}{n} + \frac{c}{n^2}\right)}{n^2(p + q/n^2)}$$

$$= \lim_{n\to\infty} \frac{a+0+0}{p+0} = \frac{a}{p}$$

25. (b)

Given, $f(x) = x^3 - 9x^2 + 24x + 10$

$f'(x) = 3x^2 - 18x + 24 = 0$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

$x = 2, 4$

$f''(x) = 6x - 18 = 6(x - 3)$

$f''(2) = -6 \Rightarrow x = 2$ is local maxima

$f''(4) = 6 \Rightarrow x = 2$ is local minima

Global maximum value of $f(x) = \max[f(1), f(2), f(6)]$

$= \max[26, 30, 46]$

$= 46$

26. (b)

Given, $$I = \int_0^{\pi/2} \sin^4 x \cos^6 x \, dx$$

$$= \frac{(4-1)(4-3)\cdot(6-1)(6-3)(6-5)}{(4+6)(4+6-2)(4+6-4)(4+6-6)(4+6-8)} \times \frac{\pi}{2}$$

$$= \frac{3\cdot 1\cdot 5\cdot 3\cdot 1}{10\cdot 8\cdot 6\cdot 4\cdot 2} \times \frac{\pi}{2}$$

$$= \frac{45\pi}{7860} = \frac{3\pi}{512}$$

27. (c)

$s = x^2 + y^2 + z^2 = 4$

$x^2 + y^2 + z^2 - 4 = 0$

grad (s) $\nabla(s)\big|_{(1,1,-1)} = 2\hat{i} + 2\hat{j} - 2\hat{k}$

Maximum value of directional derivative = $|\nabla(s)|$

$$= \sqrt{2^2 + 2^2 + 2^2} = 2\sqrt{3}$$

© Copyright: MADE EASY

28. (c)

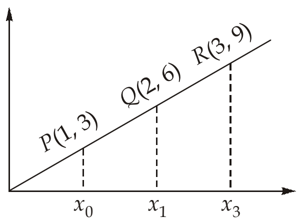
$$I = \oint_c (yzdx + zxdy + xydz)$$

$$\oint_c \vec{F}\cdot dr = \iint_R (\nabla \times \vec{F})\cdot \hat{n}\, ds$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix} = \hat{i}(x-x) - \hat{j}(y-y) + \hat{k}(z-z) = 0$$

$$I = 0$$

29. (d)



x	1	2	3
y	3	6	9

$$h = \frac{b-a}{n} = \frac{3-1}{2} = 1$$

Value of integral using trapezoidal rule

$$I = \frac{h}{2}\left[(y_0 + y_2) + 2y_1\right] = \frac{1}{2}[(3+9) + 2\times 6]$$

$$I = 12$$

Value of integral using Simpson's 1/3 method

$$I = \frac{h}{3}\times\left[(y_0 + y_2) + 4y_1\right] = \frac{1}{3}\times\left[(3+9) + 4\times 6\right]$$

$$= \frac{1}{3}[36] = 12$$

Difference = 12 – 12 = 0

30. (a)

$$I = \int_2^4 x^4 dx, \qquad h = \frac{b-a}{n}$$

By trapezoidal rule, $h = \frac{4-2}{2} = 1$

x	2	3	4
y	16	81	256

$$\int_2^4 x^4 dx = \frac{h}{2}\left[(y_0 + y_2) + 2y_1\right]$$

$$= \frac{1}{2}\left[(16 + 256) + 2\times 81\right] = 217$$

■■■■