CLASS TEST

S.No.: 01SK_EC+EE_AB_300125

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ENGINEERING MATHEMATICS

EC + EE

Date of Test : 30/01/2025

ANSWER KEY

1. (d)	7. (a)	13. (b)	19. (c)	25. (d)
2. (a)	8. (a)	14. (a)	20. (d)	26. (a)
3. (a)	9. (c)	15. (b)	21. (c)	27. (c)
4. (a)	10. (b)	16. (b)	22. (b)	28. (b)
5. (a)	11. (c)	17. (b)	23. (a)	29. (b)
6. (b)	12. (b)	18. (d)	24. (c)	30. (d)

DETAILED EXPLANATIONS

1. (d)

$$(A^{\theta})^{\theta} = \overline{\left[\{(\bar{A})'\}'\right]} = \left[\bar{\bar{A}}\right] = A$$

$$(A + B)^{\theta} = (\overline{A+B})' = (\bar{A}+\bar{B})' = (\bar{A})' + (\bar{B})' = A^{\theta} + B^{\theta}$$

$$(AB)^{\theta} = (\overline{AB})' = (\bar{A}\cdot\bar{B})' = (\bar{B})'\cdot(\bar{A})' = B^{\theta}\cdot A^{\theta}$$

2. (a)

$$f(x) = 3x(x-2)$$
$$= 3x^2 - 6x$$
$$f'(x) = 6x - 6$$
$$f''(x) = 6 > 0$$

since $f''(x) > 0$, the function has a minimum.

$$f'(x) = 0$$
$$6x - 6 = 0$$
$$x = 1$$

Therefore at $x = 1$, the function has a minimum.

3. (a)

Let the eigen values are α and β.

From the given matrix

$$\text{Trace} = \alpha + \beta = 0$$
$$\text{Determinant} = \alpha\beta = +1$$

$\therefore$ $\alpha = i$ or $-i$

$\beta = -i$ or i correspondingly.

So the eigen values of the given matrix are i and $-i$.

4. (a)

For the equation: $\frac{dy}{dx} + Py = Q$ the integrating factor is $e^{\int P dx}$.

For this given equation after dividing by $\cos^2 x$

$$P = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\text{Integrating factor} = e^{\int \sec^2 x\, dx} = e^{\tan x}$$

5. (a)

$$F = xy$$

$\therefore$ $dF = x\,dy + y\,dx$

6. (b)

Given,
$$\begin{vmatrix} 1 & 3 & 2 \\ 4 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 1(3-1) - 3(12-2) + 2(4-2)$$
$$= 2 - 30 + 4$$
$$= -24$$

7. (a)

The event R has 5 possibilities that is showing {1, 2, 3, 4, 5} i.e. $P(R) = \frac{5}{6}$.

The event S has 3 possibilities, that is showing of {1, 3, 5} i.e.

$$P(S) = \frac{3}{6} = \frac{1}{2}$$
$$P(R \cap S) = \frac{3}{6} = \frac{1}{2}$$

We know by definition for any event A, B

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$P\left(\frac{R}{S}\right) = \frac{P(R \cap S)}{P(S)} = \frac{1/2}{1/2} = 1$$

Where as,
$$P\left(\frac{S}{R}\right) = \frac{P(R \cap S)}{P(R)} = \frac{3/6}{5/6} = \frac{3}{5}$$

8. (a)

Probability of 'Ace' to be drawn = $\frac{{}^{4}C_1}{{}^{52}C_1}$

Probability of two successive 'Aces' drawn

$$= \frac{{}^{4}C_1}{{}^{52}C_1} \times \frac{{}^{4}C_1}{{}^{52}C_1} = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

9. (c)

A : The number 5 appears at least once.

B : The sum of the numbers appearing is 8.

The set of elementary events are therefore

$$A = \{(5, 1), (5, 2), (5,3), (5, 4)\ (5, 5), (5, 6), (1, 5), (2, 5), (3, 5), (4, 5), (6, 5)\}$$
$$B = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$$

and
$$A \cap B = \{(5, 3), (3, 5)\}$$

The total outcomes of throwing two dice is 6 × 6 = 36

Hence,
$$P(A) = \frac{11}{36},$$

$$P(B) = \frac{5}{36},$$

$$P(A \cap B) = \frac{2}{36}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}$$

10. (b)

It is upper triangular matrix (UTM) and we know that for UTM the eigen values are its diagonal eleements.

Alternate Solution:

The characteristics equation is

$$[A - \lambda I] = 0$$

i.e., $$\begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix} = 0$$

$$(3 - \lambda)(2 - \lambda)(5 - \lambda) = 0$$

Thus the eigen values of A are 2, 3, 5.

11. (c)

Given,
$$px + qy + rz = 0$$
$$px + ry + pz = 0$$
$$rx + py + qz = 0$$

Let, $$A = \begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix}$$

The system is $AX = 0$

This is a homogenous system, such a system has nontrivial solution if $|A| = 0$.

So, $$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

$$p(qr - p^2) - q(q^2 - pr) + r(pq - r^2) = 0$$

$$p^3 + q^3 + r^3 - 3\,pqr = 0$$

$$\Rightarrow (p + q + r)(p^2 + q^2 + r^2 - pq - qr - rp) = 0$$

$p = q = r$ satisfies the above equation.

12. (b)

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 0$$

$$A_x = \lambda y x^2 + yz$$

$$\frac{\partial A_x}{\partial x} = 2\lambda\, yx$$

$$A_y = x(y^2 - z^2)$$

$$\frac{\partial A_y}{\partial y} = 2xy$$

$$A_z = 2xy(z - xy)$$

$$\frac{\partial A_z}{\partial z} = 2xy$$

Thus, $2\lambda xy + 2xy + 2xy = 0$

$\Rightarrow$ $\lambda = -2$

13. (b)

For $n = 0$,

$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^n$$

$$\frac{dy}{dx} = -1$$

$\Rightarrow$ $y = -x + C$ (represents family of straight line)

$n = -1$, $\frac{dy}{dx} = -\frac{y}{x}$

$$\frac{dy}{y} = -\frac{dx}{x}$$

$$\int \frac{1}{y} dy = -\int \frac{1}{x} dx$$

$$\ln y = -\ln x + \ln C$$

$$\ln (yx) = \ln C$$

$xy = C$ (represents rectangular hyperbola)

$n = 1$, $\frac{dy}{dx} = -\frac{x}{y}$

$$\int y\, dy = -\int x\, dx$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + C$$

$x^2 + y^2 = 2C$ (represents family of circles)

14. (a)

Order is index of a derivative present in a partial differential equation (i.e. maximum)

Here, $\frac{\partial^2 u}{\partial x^2}$ has highest index = 2

$$\Rightarrow \quad \text{order} = 2$$

15. (b)

Poles of the integrand are given by putting the denominator equal to zero,

$$z^2 - 1 = 0,$$

$$\Rightarrow \quad z = \pm 1$$

The circle with centre $z = 1$ and radius unity encloses a simple pole at $z = 1$

By Cauchy integral formula,

$$\int_C \frac{3z^2 + z}{z^2 - 1} dz = 2\pi i \left[\frac{3z^2 + z}{z + 1}\right]_{z=1} = 2\pi i \left(\frac{3+1}{1+1}\right) = 4\pi i$$

16. (b)

The given equation may be rewritten as

$$\frac{dy}{dx} = \left(\frac{y}{x}\sec^2\frac{y}{x} - \tan\frac{y}{x}\right)\cos^2\frac{y}{x}$$

which is a homogeneous equation,

Putting $y = Vx$ becomes

$$V + x\frac{dV}{dx} = (V\sec^2 V - \tan V)\cos^2 V$$

or
$$x\frac{dV}{dx} = V - \tan V \cos^2 V - V$$

Separating the variables,

$$\frac{\sec^2 V}{\tan V} dV = \frac{-dx}{x}$$

Integrating both sides,

$$\ln \tan V = -\ln x + \ln C$$

$$x \tan V = C$$

or
$$x \tan\frac{y}{x} = C$$

17. (b)

Given, $(D^3 + 2D^2 - D - 2)y = e^x$

P.I.
$$y = \frac{e^x}{D^3 + 2D^2 - D - 2}$$

$\because$ at $D = 1$,

$$D^3 + 2D^2 - D - 2 = 0$$

$$\therefore \qquad y = x \cdot \frac{1}{f'(D)} \cdot e^x = \frac{x e^x}{3D^2 + 4D - 1} = \frac{x e^x}{3 + 4 - 1}$$

$$\text{P.I.} = \left(\frac{xe^x}{6}\right)$$

18. (d)

Given equation is Couchey-Euler differential equation

$$x^2 \frac{d^2 y}{dx^x} - x\frac{dy}{dx} + y = (\ln x)^2$$

$$(\theta\,(\theta - 1) - \theta + 1)y = z^2 \qquad \left\{\begin{aligned} \theta &= \frac{d}{dz} \\ z &= \ln x \end{aligned}\right.$$

$$(\theta^2 - 2\theta + 1)y = z^2$$

$$\text{P.I.} = \frac{z^2}{D^2 - 2D + 1} = \frac{z^2}{(D-1)^2} = (1 - D)^{-2}\,(z^2)$$

$$= (1 + 2D + 3D^2 \ldots)\,(z^2) \qquad \left\{\because (1-x)^{-n} = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots\right.$$

$$= z^2 + 2(2z) + 3(2)$$

$$= z^2 + 4z + 6$$

$$\text{P.I.} = (\ln x)^2 + 4(\ln x) + 6$$

19. (c)

Given, characteristics equation of A is $p^3 + ap^2 + bp + c = 0$

Product of $p_1\, p_2\, p_3 = -c = \text{Det (A)}$

$$\Rightarrow \qquad c = -\text{Det (A)}$$

$$\therefore \qquad |A| = \begin{vmatrix} 3 & 3 & 3 \\ 2 & 1 & 1 \\ 1 & 5 & 6 \end{vmatrix}$$

$$= 3(6 - 5) - 3(12 - 1) + 3(10 - 1)$$

$$= 3(1) - 3(11) + 3(9)$$

$$= 3 - 33 + 27$$

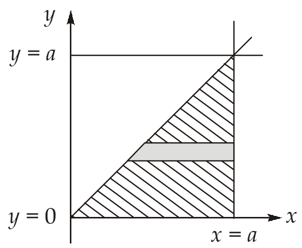
$$= -3$$

Hence, $\qquad c = -(-3) = 3$

20. (d)

By changing limit, y can also be written as

$$I = \int_{x=0}^{x=a} \int_{y=0}^{y=x} \frac{x}{x^2 + y^2}\, dy\, dx$$

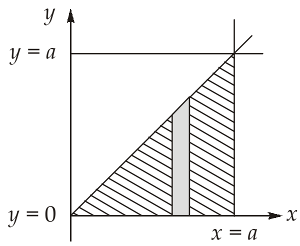


$$I = \int_0^a \left[x \times \frac{1}{x}\tan^{-1}\left(\frac{y}{x}\right)\right]_{y=0}^{y=x} dx$$

$$I = \int_0^a \left[\tan^{-1}\left(\frac{x}{x}\right) - \tan^{-1}(0)\right] dx$$

$$= \frac{\pi}{4}\int_0^a dx$$

$$I = \frac{\pi a}{4}$$



21. (c)

Given, $\vec{V} = xy^2\hat{i} + 2yx^2z\hat{j} - 3yz^2\hat{k}$

$$\text{curl } \vec{V} = \nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2 & 2yx^2z & -3yz^2 \end{vmatrix}$$

$$\text{curl } \vec{V} = \hat{i}(-3z^2 - 2yx^2) - \hat{j}(0-0) + \hat{k}(4xyz - 2xy)$$

$$\text{curl } \vec{V} = \hat{i}(-3z^2 - 2yx^2) + \hat{k}(4xyz - 2xy)$$

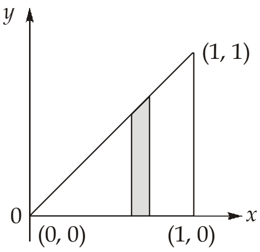
$$(\text{curl } \vec{V})_{(1,\,-1,\,1)} = \hat{i}(-3+2) + \hat{k}(-4+2)$$

$$= -\hat{i} - 2\hat{k}$$

$$= -(\hat{i} + 2\hat{k})$$

22. (b)

Given, $I = \int_c (x^2y\,dx + x^2dy)$



∵ By green theorem, we have

$$\int_c (Mdx + Ndy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

$$\int \left(x^2y\,dx + x^2dy\right) = \int_{x=0}^{x=1}\int_{y=0}^{y=x} (2x - x^2)dy\,dx$$

$$= \int_0^1 (2x - x^2)[y]_0^x\,dx$$

$$= \int_0^1 (2x - x^2)\cdot x\,dx$$

$$= \int_0^1 (2x^2 - x^3)dx$$

$$= \left.\frac{2x^3}{3} - \frac{x^4}{4}\right|_0^1$$

$$= \frac{2}{3} - \frac{1}{4}$$

$$= \frac{8-3}{12} = \frac{5}{12}$$

23. (a)

$\because$ $$\text{div}(\vec{F}) = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(F_1\hat{i} + F_2\hat{j} + F_3\hat{k}\right)$$

$$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$= 2 + (-1) + 2$$

$$= 3$$

24. (c)

Given, $v(x, y) = e^x \sin y$

then $\dfrac{\partial v}{\partial x} = e^x \sin y$

$$\frac{\partial v}{\partial y} = e^x \cos y$$

$\because$ $u = u(x, y)$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$$ [For function to be analytic $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = \dfrac{-\partial v}{\partial x}$]

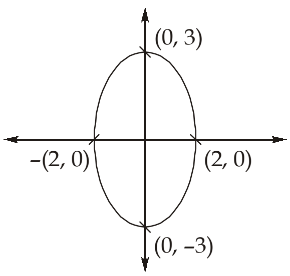
$$= \frac{\partial v}{\partial y}dx + \left(-\frac{\partial v}{\partial x}\right)dy$$

$$\int \partial u = \int e^x \cos y\, dx + \int (-e^x \sin y)dy$$ (In second integral only the term free from x will be integrated)

$$u = e^x \cos y + c$$

25. (d)

Given, $$I = \int_C \frac{4z^2 + z + 5}{z-4}dz$$



Pole $z - 4 = 0$, $z = 4$

$\because$ Point (4, 0) lies outside the ellipse

Hence, Integral $I = 0$

26. (a)

Given $u(r, \theta) = -r^3 \sin 3\theta$

$\because$ $f(z) = u(r, \theta) + iv(r, \theta)$

$\therefore$ $f'(z) = u_r + iv_r$

$$f'(z) = \left(\frac{\partial u}{\partial r}\right) + i\left[\frac{-1}{r}\left(\frac{\partial u}{\partial \theta}\right)\right]$$

For analytic function, $\left\{\left(\frac{\partial u}{\partial r}\right) = \frac{1}{r}\left(\frac{\partial V}{\partial \theta}\right) \text{ and } \left(\frac{\partial u}{\partial \theta}\right) = -r\left(\frac{\partial v}{\partial r}\right)\right.$

$$u = -r^3 \sin 3\theta$$

$$\frac{\partial u}{\partial r} = -3r^2 \sin 3\theta$$

$$\frac{\partial u}{\partial \theta} = -3r^3 \cos 3\theta$$

$$f'(z) = -3r^2 \sin 3\theta + i\left[-\frac{1}{r}(-3r^3 \cos 3\theta)\right]$$

$$f'(z) = -3r^2 \sin 3\theta + i\,(3r^2 \cos 3\theta)$$

Put $r = z$ and $\theta = 0$,

$$f'(z) = i(3z^2)$$

$$f(z) = iz^3 + c$$

27. (c)

Bag-1	**Bag-2**
4 - White	3 - White
2 - Black	3 - Black

$$P(1) = \frac{1}{2} = \text{Probability of selecting bag-1}$$

$$P(2) = \frac{1}{2} = \text{Probability of selecting bag-2}$$

$$P(w) = P\left(\frac{w}{1}\right)P(1) + P\left(\frac{w}{2}\right)P(2) = \frac{4}{6}\times\frac{1}{2}+\frac{3}{6}\times\frac{1}{2}$$

$$P(w) = \frac{7}{12}$$

28. (b)

$$P(H) = \frac{1}{2};$$

$$P(T) = \frac{1}{2}$$

Probability of getting 4 heads in 6 tosses

$$= {}^nC_r\, p^r\, q^{n-r} = {}^6C_4\, p^4 q^2$$

$$= \frac{6!}{4!2!}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2 = \frac{6\times 5}{1\times 2}\times\left(\frac{1}{2}\right)^6 = \frac{15}{64}$$

29. (b)

Given, $X = y_1 + y_2$

$\mu_1 = 2, \qquad \sigma_1 = 2$

$\mu_2 = 4, \qquad \sigma_2 = 2$

$\because$ y_1 and y_2 are two independent random variable

$$X = y_1 + y_2$$

$$\mu(X) = \mu(y_1 + y_2)$$

$$\mu(X) = \mu(y_1) + \mu(y_2)$$

$$\mu(X) = 2 + 4$$

$$\mu(X) = 6$$

$$\text{var }(X) = \text{var }(y_1 + y_2)$$

$$= \text{var }(y_1) + \text{var }(y_2) + 2\text{ cov}(y_1 y_2)$$

$\because$ Y_1 and Y_2 are independent event

$\therefore$ $$\text{var }(X) = \text{var }(y_1) + \text{var }(y_2)$$

$$\text{var }(X) = 4 + 4$$

$$\text{var }(X) = 8$$

30. (d)

Probability of A hitting the target $= \dfrac{3}{5}$

Probability of B hitting the target $= \dfrac{2}{5}$

Probability of C hitting the target $= \dfrac{3}{4}$

Probability of at least two shots hitting the target

$$= P(2) + P(3)$$

Probability that 2 shots hit the target,

$$P(2) = P(A)P(B)P(\bar{C}) + P(A)P(\bar{B})P(C) + P(\bar{A})P(\bar{B})P(C)$$

$$= \frac{3}{5}\times\frac{2}{5}\times\left(1-\frac{3}{4}\right)+\frac{3}{5}\times\frac{3}{4}\times\left(1-\frac{2}{5}\right)+\frac{2}{5}\times\frac{3}{4}\left(1-\frac{3}{5}\right)$$

$$= \frac{6}{25}\times\frac{1}{4}+\frac{9}{20}\times\frac{3}{5}+\frac{6}{20}\times\frac{2}{5} = \frac{45}{100} = \frac{9}{20}$$

Probability that 3 times hit the target,

$$P(3) = \frac{3}{5}\times\frac{2}{5}\times\frac{3}{4} = \frac{18}{100}$$

Hence,required probability $= P(2) + P(3)$

$$= \frac{9}{20}+\frac{18}{100} = \frac{63}{100}$$

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