



**MADE EASY**

Leading Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Kolkata

Web: [www.madeeasy.in](http://www.madeeasy.in) | E-mail: [info@madeeasy.in](mailto:info@madeeasy.in) | Ph: 011-45124612

# CONTROL SYSTEM

EC-EE

Date of Test : 05/02/2025

## ANSWER KEY >

- |        |         |         |         |         |
|--------|---------|---------|---------|---------|
| 1. (b) | 7. (d)  | 13. (a) | 19. (a) | 25. (b) |
| 2. (a) | 8. (b)  | 14. (a) | 20. (b) | 26. (c) |
| 3. (a) | 9. (b)  | 15. (d) | 21. (a) | 27. (c) |
| 4. (b) | 10. (d) | 16. (a) | 22. (b) | 28. (a) |
| 5. (c) | 11. (c) | 17. (b) | 23. (b) | 29. (c) |
| 6. (a) | 12. (d) | 18. (d) | 24. (d) | 30. (b) |

## DETAILED EXPLANATIONS

1. (b)

Where,

$$N = P - 2$$

$$N = 2 \text{ (Anti clock wise)}$$

$$P = \text{Open loop poles on right hand side of } s\text{-plane.}$$

$$Z = \text{Closed loop poles on right hand side of } s\text{-plane.}$$

$$2 = 2 - Z$$

$$Z = 0$$

As closed loop poles on right hand side of  $s$ -plane are zero.  
System is stable.

2. (a)

Open loop transfer function

$$= \frac{Ks + b}{s^2 + as + b - Ks - b} = \frac{Ks + b}{s^2 + (a - K)s}$$

$$K_v = \lim_{s \rightarrow 0} s \cdot \frac{(Ks + b)}{s(s + a - K)} = \frac{b}{a - K}$$

$$\therefore e_{ss} = \frac{1}{\left(\frac{b}{a - K}\right)} = \frac{a - K}{b}$$

3. (a)

$$c(t) = 12.5 e^{-6t} \sin 8t$$

$$c(s) = \frac{12.5(8)}{(s + 6)^2 + 8^2} = \frac{100}{s^2 + 12s + 100}$$

$$\omega_n^2 = 100$$

$$\omega_n = 10 \text{ r/s}$$

$$2\xi\omega_n = 12$$

$$\xi = \frac{12}{2(10)} = 0.6$$

4. (b)

$$G(s)H(s) = \frac{K}{s(s + 3 - j\sqrt{3})(s + 3 + j\sqrt{3})} = \frac{K}{s(s^2 + 6s + 12)}$$

5. (c)

$$G(s) = \frac{2(1 - sT)}{s(s + 2)}$$

Characteristic equation for the system is

$$1 + G(s) = 0$$

$$\Rightarrow s^2 + 2s + 2 - 2sT = 0$$

$$\Rightarrow s^2 + (2 - 2T)s + 2 = 0$$

By Routh array analysis,

$$\begin{array}{c|cc} s^2 & 1 & 2 \\ s^1 & 2-2T & 0 \\ s^0 & 2 & \end{array}$$

For system to be stable, if  $2 - 2T > 0$ ; i.e.  $T < 1$ .

6. (a)

Characteristic equation is

$$1 + \frac{K(s+2)}{s(s+10)(s-1)} = 0$$

System has roots farther to the left of -1.

Substitute  $(s - 1)$  in place of 's' we get

$$1 + \frac{K(s-1+2)}{s(s-1)(s-1+10)(s-2)} = 0$$

$$1 + \frac{K(s+1)}{(s-1)(s+9)(s-2)} = 0$$

$$(s-1)(s+9)(s-2) + K(s+1) = 0$$

$$(s-1)(s^2+7s-18) - s^2-7s+18 + Ks + K = 0$$

$$s^3 + 7s^2 - 18s - s^2 - 7s + 18 + Ks + K = 0$$

$$s^3 + 6s^2 - 25s + 18 + Ks + K = 0$$

$$s^3 + 6s^2 + (K-25)s + (18+K) = 0$$

From RH criterion

$$\begin{array}{c|cc} s^3 & 1 & K-25 \\ s^2 & 6 & 18+K \\ s^1 & \frac{6(K-25)-(18+K)}{6} & \\ s^0 & 18+K & \end{array}$$

$$18 + K > 0$$

$$K > -18$$

and  $6(K-25) - 18 - K > 0$

$$5K > 168$$

$$K > \frac{168}{5}$$

$\therefore$  range of K is  $K > 33.6$

7. (d)

The gain margin is the inverse of the intersection of the root loci plot to the imaginary axis and if it does not intersect then the gain margin will be infinite.

8. (b)

From the given equation,

We can get open loop transfer function of the system

$$G(s) = \frac{K(s+1)}{s(s^2 + 5s + 2)}$$

$$P = 3 ; Z = 1$$

∴ Number of asymptotes,  $P - Z = 2$

$$\therefore \text{Angle of asymptotes} = \frac{(2q+1)180^\circ}{P-Z}$$

$$\therefore P - Z = 2 \Rightarrow q = 0, 1$$

∴ Angle of asymptotes are  $90^\circ$  and  $270^\circ$ .

9. (b)

$$\text{Transfer function} = \frac{-R_2 \left[ \frac{R_1 C_1 s + 1}{R_2 C_2 s + 1} \right]}{R_1}$$

10. (d)

$$\frac{C(s)}{R(s)} = \frac{5 \times 4 \times 2 \times 10(1+1) + 10 \times 10 \times 2 \times (1+4)}{1+4+4+1+4+4}$$

$$\frac{C(s)}{R(s)} = \frac{400 \times 2 + 200 \times 5}{18} = 100$$

11. (c)

Forward paths:

$$F_1 = G_1 G_2 G_3 G_4 ; \Delta_1 = 1$$

$$F_2 = G_1 G_6 G_4 ; \Delta_2 = 1$$

$$F_3 = G_1 G_7 ; \Delta_3 = 1 - G_5$$

Individual loops:

$$L_1 = G_2 H_1 ; L_2 = G_5$$

Two non-touching loops,

$$L_{12} = G_2 H_1 (G_5)$$

$$\Delta = 1 - G_2 H_1 - G_5 + G_2 H_1 G_5$$

$$\therefore \frac{X_5}{X_1} = \frac{G_1 G_2 G_3 G_4 + G_1 G_4 G_6 + G_1 G_7 (1 - G_5)}{1 - G_2 H_1 + G_2 G_5 H_1 - G_5}$$

12. (d)

Characteristics equation is

$$s^5 - 2s^4 - 2s^3 + 4s^2 + s - 2 = 0$$

$$\begin{array}{l|ccc} s^5 & 1 & -2 & 1 \\ s^4 & -2 & 4 & -2 \\ s^3 & 0(-1) & 0(1) & 0 \\ s^2 & 2 & -2 & 0 \\ s^1 & 0(4) & 0 & 0 \\ s^0 & -2 & & \end{array}$$

Auxiliary equation-1 is,

$$-2s^4 + 4s^2 - 2 = 0$$

$$\frac{d}{ds}(-2s^4 + 4s^2 - 2) = 0$$

$$-8s^3 + 8s = 0$$

$$-s^3 + s = 0$$

Auxiliary equation-2 is,

$$2s^2 - 2 = 0$$

$$\frac{d}{ds}(2s^2 - 2) = 0$$

$$4s = 0$$

3 sign changes in the first column.

Therefore 3 poles are in RHP out of which 2 are symmetric.

13. (a)

$$1 + KG = 0$$

$$s^3 + 6s^2 + 12s + Ks + 8 - 4K = 0$$

$$(s^3 + 6s^2 + 12s + 8) + K(s - 4) = 0$$

$$1 + \frac{K(s-4)}{s^3 + 6s^2 + 12s + 8} = 0$$

∴ Open loop transfer function,

$$G(s) = \frac{(s-4)}{s^3 + 6s^2 + 12s + 8}$$

|       |                          |    |
|-------|--------------------------|----|
| $s^3$ | 1                        | 12 |
| $s^2$ | 6                        | 8  |
| $s^1$ | $\frac{72-8}{6} = 10.66$ |    |
| $s^0$ | 8                        |    |

There is no sign change in first column of RH table.

∴ Open loop system is stable.

14. (a)

$$\text{Transfer function} = C[sI - A]^{-1} B + D$$

$$= \frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+3 & 1 \\ 0 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{s(s+3)} = \frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+3 \\ 0 \end{bmatrix}}{s(s+3)} = \frac{1}{s}$$

15. (d)

$$\frac{s+3}{s+6} = \frac{3}{6} \left[ \frac{1 + \frac{1}{3}s}{1 + \frac{1}{6}s} \right] = \frac{\alpha(1+Ts)}{(1+\alpha Ts)}$$

$$\alpha = \frac{1}{2}; \quad T = \frac{1}{3}$$

$$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right) = \sin^{-1}\left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right) = \sin^{-1}\left(\frac{1}{3}\right)$$

$$\phi_m = 19.47^\circ$$

$$\omega_m = \frac{1}{T\sqrt{\alpha}} = \frac{1}{\frac{1}{3}\sqrt{\frac{1}{2}}} = 3\sqrt{2}$$

$$\omega_m = 3\sqrt{2} \text{ rad/sec}$$

16. (a)

Characteristics equation,

$$1 + G(s) = 0$$

$$1 + \frac{(K_1s + K_2s^2 + K_3)}{s^2(s-1)} = 0$$

$$s^3 + (K_2 - 1)s^2 + K_1s + K_3 = 0$$

From the given closed loop poles,

$$(s+1)(s+1)(s+2) = 0$$

$$(s^2 + 2s + 1)(s+2) = 0$$

$$s^3 + 2s^2 + 2s^2 + 4s + s + 2 = 0$$

$$s^3 + 4s^2 + 5s + 2 = 0$$

$$K_2 - 1 = 4$$

$$K_2 = 5; \quad K_1 = 5; \quad K_3 = 2$$

$\therefore K_1, K_2, K_3$  are 5, 5, 2

$\therefore K_1 + K_2 + K_3 = 5 + 5 + 2 = 12$

17. (b)

$$G(s) = \frac{K}{s(s+2)(s+10)}$$

$$\angle G(j\omega) = -90 - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

$$-180 = -90 - \tan^{-1}\left(\frac{\omega_{pc}}{2}\right) - \tan^{-1}\left(\frac{\omega_{pc}}{10}\right)$$

$$\tan^{-1}\frac{\omega_{pc}}{2} + \tan^{-1}\left(\frac{\omega_{pc}}{10}\right) = 90$$

$$\frac{\frac{\omega_{pc}}{2} + \frac{\omega_{pc}}{10}}{1 - \frac{\omega_{pc}^2}{20}} = \infty$$

$$\omega_{pc}^2 = 20$$

$$\omega_{pc} = \sqrt{20}$$

$$|G(j\omega)| = \frac{K}{\omega\sqrt{\omega^2 + 2^2}\sqrt{\omega^2 + 10^2}}$$

$$\text{Gain margin} = 20\log\left|\frac{1}{G(j\omega)}\right|$$

$$G = 20\log_{10}\frac{\sqrt{20}\sqrt{20+4}\sqrt{20+100}}{K}$$

$$20\log_{10}2 = 20\log_{10}\frac{\sqrt{20}\sqrt{20+4}\sqrt{20+100}}{K}$$

$$K = 120$$

18. (d)

$$K_v = \lim_{s \rightarrow 0} s \cdot G(s)$$

$$K_v = \lim_{s \rightarrow 0} s \cdot \frac{K(s+10)(s+15)}{s(s+3)(s+7)(s+20)}$$

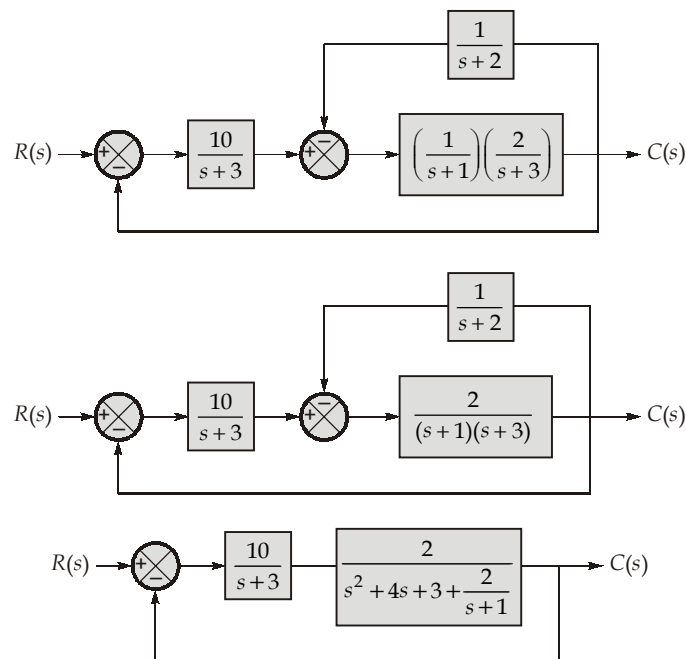
$$K_v = \frac{K(10)(15)}{(3)(7)(20)} = \frac{K(5)}{14}$$

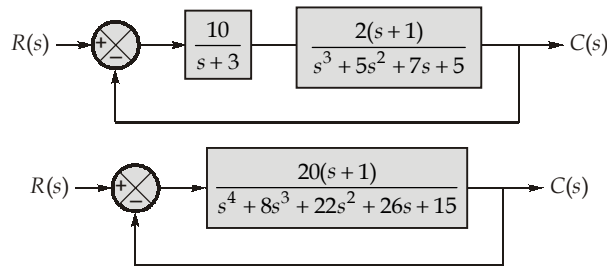
$$e_{ss} = \frac{25}{K_v} = \frac{25}{K(5)} \times 14$$

$$0.1 = \frac{5 \times 14}{K}$$

$$K = \frac{5 \times 14}{0.1} = 700$$

19. (a)





No. of poles at  $s = 0$  in open loop transfer function is zero,  
 $\therefore$  Type = 0

20. (b)

$$|G(j\omega)| = \frac{2}{\omega^2 + 1}$$

$$\frac{2}{1 + \omega_{gc}^2} = 1$$

$$2 = 1 + \omega_{gc}^2$$

$$\omega_{gc} = 1 \text{ rad/sec}$$

$$\angle G(j\omega)|_{\omega=\omega_{gc}} = -2 \tan^{-1} \omega_{gc} = -90^\circ$$

$$PM = 180 - 90^\circ = 90^\circ$$

21. (a)

$$\%M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$$

$$= e^{-0.5\pi/\sqrt{1-0.5^2}} \times 100$$

$$\%M_p = 16.3\%$$

$$\%M_p = \frac{P-3}{3} \times 100 = 16.3$$

$$\frac{P-3}{3} = 0.163$$

$$P = 3(1.163)$$

$$P = 3.489$$

22. (b)

Characteristic equation of the closed loop system,

$$s^3 + as^2 + 2s + 1 + Ks + K = 0$$

$$s^3 + as^2 + (K+2)s + (K+1) = 0$$

By using Routh Hurwitz table,

|       |                          |       |
|-------|--------------------------|-------|
| $s^3$ | 1                        | $K+2$ |
| $s^2$ | $a$                      | $K+1$ |
| $s^1$ | $\frac{a(K+2)-(K+1)}{a}$ |       |
| $s^0$ | $K+1$                    |       |

If ' $s^1$ ' term is equal to zero, then row of zeros occurs



$$\therefore \frac{a(K+2)-(K+1)}{a} = 0$$

$$\therefore a = \frac{1+K}{2+K}$$

and also,  $K > -1$

From the given options, if  $K = 2$  then

$$a = \frac{1+2}{2+2} = \frac{3}{4} = 0.75$$

Option (b) satisfy the condition.

23. (b)

Checking angle criterion of root locus

$$\angle G(s)H(s)|_{s=s_1} = -4\tan^{-1}(-2) \neq 180^\circ$$

$$\angle G(s)H(s)|_{s=s_2} = -4\tan^{-1}(1) = -180^\circ$$

$\angle G(s)H(s)|_{s=s_2}$  satisfies angle criterion.

So it lies on root locus.

24. (d)

The output is given as a function of time.

The final value of the output is,

$$\lim_{t \rightarrow \infty} c(t) = 1$$

Hence,  $t_d$  (at 50% of final value) is the solution,

$$\text{i.e.} \quad 0.5 = 1 - e^{-t_d}$$

$$e^{-t_d} = 0.5$$

$$t_d = \ln 2 \quad (\text{or}) \quad 0.693 \text{ sec}$$

25. (b)

$$\text{Given,} \quad K_r = 2 \Rightarrow \lim_{s \rightarrow 0} sG(s) = \lim_{s \rightarrow 0} s \frac{5s+9}{s(5s^2+4s+3K_t)}$$

$$2 = \frac{9}{3K_t}$$

$$\therefore K_t = 1.5$$

$$\therefore G(s) = \frac{5s+9}{s(5s^2+4s+4.5)}$$

$$\text{Also step error coefficient, } K_s = \lim_{s \rightarrow 0} \frac{5s+9}{s(5s^2+4s+4.5)} = \infty$$

$$\therefore \text{Steady state error, } e_{ss} = \frac{1}{1+K_s} = \frac{1}{\infty} = 0$$

26. (c)

$$\text{Given,} \quad h(t) = e^{-t} + e^{-2t}$$

$$H(s) = \frac{1}{s+1} + \frac{1}{s+2}$$

But 
$$H(s) = \frac{C(s)}{R(s)} = \frac{1}{s+1} + \frac{1}{s+2}$$

Given, 
$$R(s) = \frac{1}{s}$$

$$C(s) = R(s) \cdot H(s) = \frac{1}{s} \left[ \frac{1}{s+1} + \frac{1}{s+2} \right]$$

$$C(s) = \frac{1}{s(s+1)} + \frac{1}{s(s+2)} = \left( \frac{1}{s} - \frac{1}{s+1} \right) + \frac{1}{2} \left( \frac{1}{s} - \frac{1}{s+2} \right)$$

$\therefore$  
$$c(t) = L^{-1} [C(s)]$$
  

$$c(t) = (1.5 - e^{-t} - 0.5e^{-2t}) u(t)$$

27. (c)

Given, 
$$K_i = \frac{0.1}{s}$$
  

$$I_o = 40\%$$
  

$$e = \text{Error (constant)} = 20\%$$

For an integral controller

$$(I_{\text{out}} - I_o) = \int_0^t K_i e dt$$

When the error does not vary with time, the equation becomes

$$(I_{\text{out}} - I_o) = K_i e t$$

$\therefore$  
$$I_{\text{out}} = K_i e t + I_o$$
  
 At  $t = 2$  sec, 
$$I_{\text{out}} = 0.1 \times 20 \times 2 + 40 = 44\%$$

28. (a)

Given, 
$$c(t) = 1 + 0.5e^{-30t} - 1.5e^{-10t}$$

Impulse response, 
$$\frac{dc(t)}{dt} = 0 - 15e^{-30t} + 15e^{-10t}$$

Then the transfer function,

$$\text{TF} = L [\text{Impulse response}]$$

$$\text{TF} = \frac{-15}{s+30} + \frac{15}{s+10}$$

$$\text{TF} = \frac{-15}{s+30} + \frac{15}{s+10} = \frac{-15s - 150 + 15s + 450}{(s+10)(s+30)}$$

Therefore natural frequency,  $\omega_n = \sqrt{300} = 17.32 \text{ rad}$

29. (c)

$$sI - A = \begin{bmatrix} s+4 & 1 \\ -3 & s+1 \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} s+1 & -1 \\ 3 & s+4 \end{bmatrix} \frac{1}{(s+4)(s+1)+3}$$

Transfer function,  $G(s) = C(sI - A)^{-1}B = [1 \ 0] \frac{\begin{bmatrix} s+1 & -1 \\ 3 & s+4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}{(s+4)(s+1)+3}$

$$G(s) = \frac{s}{s^2 + 5s + 7}$$

$$2\xi\sqrt{7} = 5 \quad \{\because 2\xi\omega_n = 5\}$$

$$\Rightarrow \xi = \frac{5}{2\sqrt{7}} = 0.944$$

30. (b)

The transfer function, TF =  $C[sI - A]^{-1}B + D$

$$C = [0 \ 0 \ 1] ; B = \begin{bmatrix} 6 \\ 4 \\ 5 \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 0 & -3 \\ 1 & 0 & -4 \\ 0 & 1 & 5 \end{bmatrix} = \begin{bmatrix} s & 0 & 3 \\ -1 & s & 4 \\ 0 & -1 & s-5 \end{bmatrix}$$

$$\text{TF} = [0 \ 0 \ 1] \begin{bmatrix} s & 0 & 3 \\ -1 & s & 4 \\ 0 & -1 & s-5 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ 4 \\ 5 \end{bmatrix}$$

On solving,  $\text{TF} = \frac{5s^2 + 4s + 6}{s^3 + 5s^2 + 4s + 3}$

$$C(\infty) = \lim_{s \rightarrow 0} s[\text{TF}] \frac{1}{s} = \frac{6}{3} = 2$$

