CLASS TEST S.No.: 02_SK_ECEE_J_31052025

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SIGNAL & SYSTEM

EC-EE

Date of Test : 31/05/2025

ANSWER KEY

1.	(b)	7.	(d)	13.	(b)	19.	(d)	25.	(c)
2.	(b)	8.	(a)	14.	(b)	20.	(c)	26.	(a)
3.	(d)	9.	(d)	15.	(c)	21.	(a)	27.	(c)
4.	(c)	10.	(a)	16.	(b)	22.	(b)	28.	(c)
5.	(c)	11.	(d)	17.	(c)	23.	(d)	29.	(a)
6.	(c)	12.	(a)	18.	(c)	24.	(b)	30.	(c)

DETAILED EXPLANATIONS

1. (b)

Energy over one period, $E_{\text{period}} = \int_0^{T_0} \left|e^{j\omega_0 t}\right|^2 dt$

$$= \int_0^{T_0} 1 \cdot dt = T_0$$

Average power over one period,

$$P_{\text{period}} = \frac{1}{T_0} \times E_{\text{period}}$$

$$= \frac{1}{T_0} \times T_0 = 1$$

2. (b)

3. (d)

Let $X(\omega)$ and $Y(\omega)$ be the Fourier transform of $x(t)$ and $y(t)$ respectively. Then,

$$X(\omega) = \frac{1}{2 + j\omega}$$

$$Y(\omega) = \frac{1}{1 + j\omega}$$

Frequency response, $H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2 + j\omega}{1 + j\omega}$

$$= \frac{1 + 1 + j\omega}{1 + j\omega}$$

$$H(\omega) = 1 + \frac{1}{1 + j\omega}$$

Taking inverse Fourier transform of $H(\omega)$ yields the impulse response $h(t)$.

$$h(t) = \delta(t) + e^{-t}u(t)$$

4. (c)

Given that, $\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{d}{dt}x(t) - 2x(t)$

Taking Laplace transform of the above equation,

$$s^2Y(s) + 2sY(s) + Y(s) = sX(s) - 2X(s)$$

$$Y(s)[s^2 + 2s + 1] = X(s)[s - 2]$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s - 2}{s^2 + 2s + 1}$$

5. (c)

Signal will be uniquely reconstructed if sampling frequency,

$$f_s \geq f_{Ny}$$
$$f_s \geq (2 \times f_{max})$$
$$f_{max} \leq \frac{f_s}{2} = \frac{350}{2} = 175 \text{ Hz}$$

For sin (1000t),

$\Rightarrow$
$$f_{max} = \frac{1000}{2\pi} = 159.15 \text{ Hz}$$

6. (c)

If, $f(t) \xrightleftharpoons{\text{F.T.}} F(j\omega)$

Then, $f^*(t) \xrightleftharpoons{\text{F.T.}} F^*(-j\omega)$ ...using conjugate property

For any real signal,

$$f(t) = f^*(t)$$

$\downarrow\uparrow$ F.T.

$$F(j\omega) = F^*(-j\omega)$$

7. (d)

As we know that ;

$a^n \cos\omega_o n \, u(n)$ has ROC $|z|>|a|$

$$\therefore \quad \left(-\frac{2}{3}\right)^n \cos\left(\frac{2\pi n}{8}\right)u(n) \longrightarrow |z|>\frac{2}{3}$$

Also,
$$x[-n] \xleftrightarrow{zT} X[z^{-1}] \; ; \; \text{ROC} = \frac{1}{R}$$

$$\therefore \quad \left(-\frac{2}{3}\right)^{-n} \cos\left(-\frac{2\pi n}{8}\right)u(-n) = \left(-\frac{3}{2}\right)^n \cos\left(\frac{2\pi n}{8}\right)u(-n) \text{ will have ROC : } |z|<\frac{3}{2}.$$

8. (a)

$$x(n) = \frac{3}{n+\delta(n)}u(n-1)$$

For $n > 0$, we get

$$x(n) = \frac{3}{n}u(n-1) \qquad [\because \; \delta(n) = 0 \; ; \text{ for } n > 0]$$

$\Rightarrow$
$$nx(n) = 3u(n-1)$$

Take z-transform on both sides,

$$-\frac{zdX(z)}{dz} = \frac{3z^{-1}}{1-z^{-1}}$$

$$\frac{dX(z)}{dz} = -3z^{-1}\frac{z^{-1}}{1-z^{-1}}$$

Integrate on both sides,

$$\int dX(z) = -3\int\frac{z^{-2}}{1-z^{-1}}dz = -3\int\frac{1}{z(z-1)}dz$$

$$= -3\int\left(\frac{1}{z-1}-\frac{1}{z}\right)dz = 3\ln\left(\frac{z}{z-1}\right)$$

9. (d)

$$x(t) = 1 + 2\cos(20\pi t) + 3\sin(30\pi t)$$

The maximum frequency present in the given signal is,

$$f_{max} = \frac{30\pi}{2\pi}\text{ Hz}$$

$\therefore$ Nyquist rate is given by,

$$f_s = 2f_{max} = \frac{30\pi}{2\pi} \times 2 = 30\text{ Hz}$$

The maximum allowable time interval between the sample values is given by,

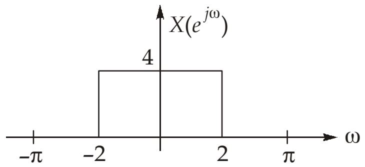
$$T_s = \frac{1}{f_s} = \frac{1}{30} = 33.33\text{ msec}$$

The minimum number of sample values required to be stored in order to produce 2s of this waveform is given by,

$$s = \frac{2s}{33.33\text{ ms}} = 60$$

10. (a)

$$\frac{4\sin 2n}{\pi n} \Rightarrow$$



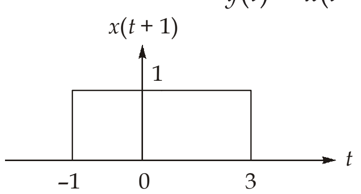
Passeval's energy theorem,

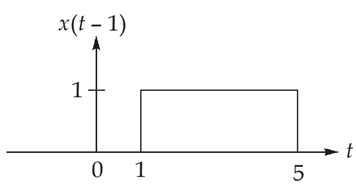
$$\text{Energy of } x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|X(e^{j\omega})\right|^2 d\omega$$

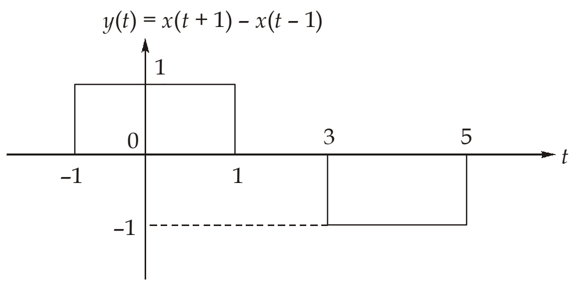
$$= \frac{1}{2\pi}\int_{-2}^{2} 4^2\, dw = \frac{16 \times 4}{2\pi} = \frac{32}{\pi}$$

11. (d)

$$y(t) = x(t) * h(t)$$
$$= x(t) * [(\delta(t+1) - \delta(t-1)]$$
$$= x(t) * \delta(t+1) - x(t) * \delta(t-1)$$
$$y(t) = x(t+1) - x(t-1)$$







12. (a)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

Putting $t = 0$ in above expression we get,

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{0}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)d\omega$$

$$\therefore \quad \int_{-\infty}^{\infty} X(\omega)d\omega = 2\pi x(0) = 2\pi\cdot 1 = 2\pi$$

13. (b)

The given waveform has half wave symmetry

i.e., $$x(t) = -x\left(t \pm \frac{T}{2}\right)$$

$\therefore$, a_k will be zero for even integer values of k.

14. (b)

Case-I: Consider the output $y(t)$ at $t = -\pi$

$$y(t)|_{t=-\pi} = x[\sin(-\pi)] = x(0)$$

In this case, present output depends on future input.

$\Rightarrow$ The system is non-causal.

- Checking linearity:

  Let $x_1(t)$ and $x_2(t)$ be the two arbitrary inputs.

$$x_1(t) \rightarrow y_1(t) = x_1[\sin(t)]$$
$$x_2(t) \rightarrow y_2(t) = x_2[\sin(t)]$$
$$x_3(t) \rightarrow y_3(t) = x_3[\sin(t)]$$

  Let $x_3(t)$ is a linear combination of $x_1(t)$ and $x_2(t)$.

$$x_3(t) = ax_1(t) + bx_2(t) \qquad \text{...(i)}$$

  where a and b are arbitrary scalars.

$$y_3(t) = x_3[\sin(t)]$$
$$= ax_1[\sin(t)] + bx_2[\sin(t)] \qquad \text{[using (i)]}$$
$$= ay_1(t) + by_2(t)$$

  $\therefore$ The system is linear.

15. (c)

$$\int_{t_1}^{t_2} x(t)\delta(t-t_0) = \begin{cases} x(t_0), & t_1 < t_0 < t_2 \\ 0, & \text{otherwise} \end{cases}$$

$$I = \int_0^2 (3t^2+4)\delta(t-1) = 3(1)^2 + 4 = 7$$

16. (b)

$$x(t) = 10\sin(12\pi t) + 4\cos(18\pi t)$$

This is combination of two signals having fundamental time period T_1 and T_2.

$$T_1 = \frac{2\pi}{12\pi} = \frac{1}{6}$$

$$T_2 = \frac{2\pi}{18\pi} = \frac{1}{9}$$

$$\text{Total time period, } T = \frac{\text{LCM of numerator of } (T_1, T_2)}{\text{HCF of denominator of } (T_1, T_2)} = \frac{1}{3}$$

$$\therefore \quad \text{Fundamental frequency} = \frac{1}{T} = 3\text{ Hz}$$

17. (c)

$$\sin\left[\frac{2\pi}{4}n\right]u(n) \overset{zT}{\longleftrightarrow} \frac{z\sin\frac{\pi}{2}}{z^2 - 2z\cos\frac{\pi}{2} + 1} = \frac{z}{z^2+1}$$

Using time-shifting property,

$$\sin\left[\frac{\pi}{2}(n-1)\right]u(n-1) \overset{zT}{\longleftrightarrow} \frac{1}{z^2+1}$$

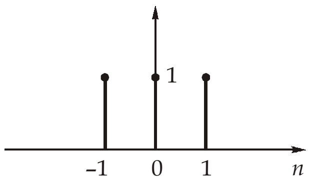
As we know that ;

$$nx(n) \overset{zT}{\longleftrightarrow} -z\frac{dX(z)}{dz}$$

$$\therefore \quad n\sin\left[\frac{\pi}{2}(n-1)\right]u(n-1) \overset{zT}{\longleftrightarrow} -z\frac{d}{dz}\left(\frac{1}{z^2+1}\right) = \frac{2z^2}{(z^2+1)^2}$$

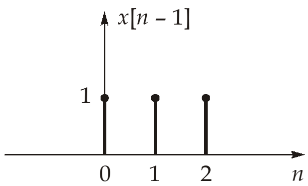
18. (c)

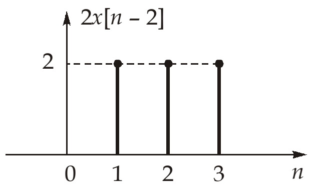
$$x[n] = \text{rect}_2[2n]$$

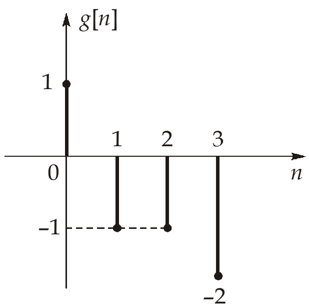


$$\therefore \quad g[n] = x[n] \otimes \{\delta[n-1] - 2\delta[n-2]\}$$

$$= x[n-1] - 2x[n-2]$$

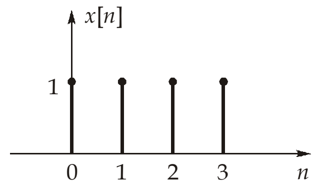




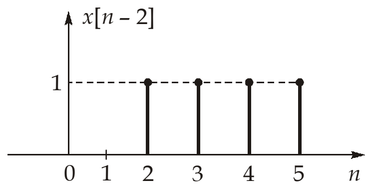


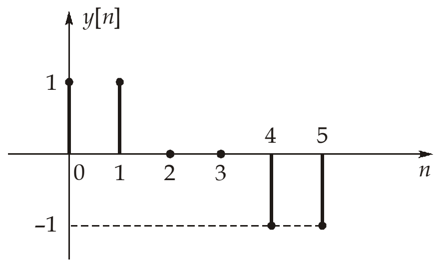
⇒ $g[2] = -1$ and $g[3] = -2$

19. (d)



$$y[n] = x[n] \otimes (\delta[n] - \delta[n-2])$$
$$y[n] = x[n] - x[n-2]$$





∴ Number of non-zero samples = 4.

20. (c)

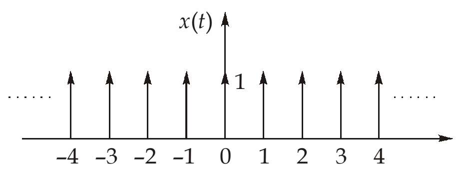
$$\text{rect}_3[n] \xleftrightarrow{\text{DTFT}} \frac{\sin\left[\left(3+\frac{1}{2}\right)2\pi f\right]}{\sin \pi f} = \frac{\sin 7\pi f}{\sin \pi f}$$

⇒ $$2\text{rect}_3[n-2] \xleftrightarrow{\text{DTFT}} \frac{2\sin 7\pi f}{\sin \pi f} e^{-j4\pi f} = Y(f)$$

$$Y(f)\Big|_{f=\frac{1}{8}} = \frac{2\sin\frac{7\pi}{8}}{\sin\frac{\pi}{8}} e^{-j\frac{4\pi}{8}} = -2j$$

21. (a)

Input $x(t)$



$$T_0 = 1 \text{ sec},$$

$$\omega_0 = \frac{2\pi}{1} = 2\pi \text{ rad/sec}$$

Fourier series coefficient of $x(t)$,

$$C_n = \frac{1}{T_0} \int_{<T_0>} x(t)\, e^{-jn\omega_0 t} \cdot dt$$

$$= \frac{1}{1} \int_{-1/2}^{1/2} \delta(t) \cdot e^{-jn\omega_0 t} dt$$

$$\left[\because \delta(t)\cdot x(t) = \delta(t)\cdot x(0)\right]$$

$$C_n = \frac{e^{-jn\omega_0 \times 0}}{1}\int_{-1/2}^{1/2} \delta(t)\cdot dt = 1$$

Now,
$$h(t) = e^{-4t}\,u(t)$$
$$\downarrow\uparrow \text{ F.T.}$$
$$H(\omega) = \frac{1}{4+j\omega}$$

Let the Fourier series coefficient of output $y(t)$ is C_n

There is a relation between C_n and C_n' and given by

$$C_n' = C_n \cdot H(n\,\omega_0)$$

$$C_n' = 1\times\frac{1}{4+j2n\pi} = \frac{1}{4+j2n\pi}$$

Therefore F.S.C of output $y(t)$ is

$$C_n' = \frac{1}{4+j2n\pi}$$

22. (b)

Overall impulse response of interconnected LTI system is

$$h(n) = h_1(n) * [h_2(n) + h_3(n) * h_4(n)]$$
$$= h_1(n) * h_2(n) + h_1(n) * h_3(n) * h_4(n)$$
$$= \left[\frac{1}{2}\delta(n)+\frac{1}{4}\delta(n-1)+\frac{1}{2}\delta(n-2)\right] * u(n+2) + \left[\frac{1}{2}\delta(n)+\frac{1}{4}\delta(n-1)+\frac{1}{2}\delta(n-2)\right]$$
$$* \; u(n+2) * \delta(n-2)$$
$$= \frac{1}{2}u(n+2)+\frac{1}{4}u(n+1)+\frac{1}{2}u(n)+\frac{1}{2}u(n)+\frac{1}{4}u(n-1)+\frac{1}{2}u(n-2)$$

$$h(n) = \left[\frac{1}{2},\frac{3}{4},\underset{\uparrow}{\frac{7}{4}},2,\frac{5}{2},\frac{5}{2},....\right],\quad \sum_{n=-\infty}^{0} h(n) = \frac{1}{2}+\frac{3}{4}+\frac{7}{4} = 3$$

23. (d)

$$x_2(t) = 2[u(t+1) - u(t)] + u(t) - u(t-1)$$
$$x_2(t) = 2[x_1(t+1)] + x_1(t)$$
$$x_2(t) = 2x_1(t+1) + x_1(t)$$

Since given system is LTI,

$$x_1(t) \rightarrow \boxed{\text{LTI system}} \rightarrow y_1(t)$$

$$2x_1(t+1) \rightarrow \boxed{\text{LTI system}} \rightarrow y_1'(t) = 2y_1(t+1)$$

Therefore,

$$2x_1(t+1)+x_1(t) \rightarrow \boxed{\text{LTI system}} \rightarrow y_2(t) = 2y_1(t+1)+y_1(t)$$

24. (b)

We know that, $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

Replace t by $t + 2$,

$$x(t+2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega(t+2)}d\omega$$

Now put, $t = 0$

$$x(2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j2\omega}d\omega$$

Therefore, $$\int_{-\infty}^{\infty} X(\omega)e^{j2\omega}d\omega = 2\pi\, x(2) = 2\pi \times (-1) = -2\pi$$

25. (c)

Given, $$y(n) = \beta\, x(n-1) - \alpha y(n-2)$$

$\downarrow\uparrow$ Z.T.

$$Y(z) = \beta\, z^{-1}\, x(z) - \alpha z^{-2}\, Y(z)$$

$$\frac{Y(z)}{X(z)} = \frac{\beta z^{-1}}{1+\alpha z^{-2}}$$

Given solution is causal and stable therefore $h(n) = 0;\ n < 0$

Therefore, $$\sum_{n=0}^{\infty} h(n)z^{-n} = \frac{\beta z^{-1}}{1+\alpha z^{-2}}$$

Put, $z = 1$

$$\sum_{n=0}^{\infty} h(n) = \frac{\beta}{1+\alpha}$$

$$-1 = \frac{\beta}{1+\alpha}$$

$$\alpha = -\beta - 1$$

26. (a)

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

$$= X_1 e^{jn\omega_0 t} + X_{-1}e^{-jn\omega_0 t} + X_3 e^{j3\omega_0 t} + X_{-3}e^{-j3\omega_0 t}$$

$$= 2e^{j\frac{\pi}{4}t} + 2e^{-j\frac{\pi}{4}t} + 4je^{j\frac{3\pi}{4}t} - 4je^{-j\frac{3\pi}{4}t}$$

$$= 4\left(\frac{e^{j\frac{\pi}{4}t} + e^{-j\frac{\pi}{4}t}}{2}\right) - 8\left(\frac{e^{j\frac{3\pi}{4}t} - e^{-j\frac{3\pi}{4}t}}{2j}\right)$$

$$= 4\cos\frac{\pi}{4}t - 8\sin\frac{3\pi}{4}t$$

$$= 4\cos\left(\frac{\pi}{4}t\right) + 8\cos\left(\frac{3\pi}{4}t + \frac{\pi}{2}\right)$$

27. (c)

$$\frac{dy(t)}{dt}+10y(t) = \int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau - x(t)$$

$$\frac{dy(t)}{dt}+10y(t) = [x(t) * z(t)] - x(t)$$

Take fourier transform of above equation

$$j\omega\, Y(\omega) + 10Y(\omega) = X(\omega)\cdot Z(\omega) - X(\omega)$$

$$Z(\omega) = \frac{1}{1+j\omega}+3,$$

$$\therefore \quad Y(\omega)(10 + j\omega) = X(\omega)\left[\frac{1}{1+j\omega}+3-1\right]$$

$$\frac{Y(\omega)}{X(\omega)} = \frac{3+2j\omega}{(1+j\omega)(10+j\omega)}$$

$$H(\omega) = \frac{3+2j\omega}{(1+j\omega)(10+j\omega)}$$

28. (c)

$$X(z) = \ln\left(\frac{\alpha}{\alpha-z^{-1}}\right);\quad \text{ROC}: |z| > \frac{1}{\alpha}$$

$$= -\ln(1 - (\alpha z)^{-1})$$

Now expanding it by Taylor series, we get,

$$X(z) = \left[(\alpha z)^{-1}+\frac{(\alpha z)^{-2}}{2}+\frac{(\alpha z)^{-3}}{3}+\ldots\right]$$

$$= \sum_{K=1}^{\infty}\frac{[(\alpha z)^{-1}]^K}{K}$$

$$X(z) = \sum_{K=1}^{\infty}\frac{\alpha^{-K}}{K}\cdot z^{-K}$$

Taking the inverse z-transform, we get,

$$x[n] = \sum_{K=1}^{\infty}\frac{\alpha^{-K}}{K}\delta(n-K) \qquad [\because\ \delta[n-K] \longleftrightarrow z^{-K}]$$

$$\therefore \quad x[n] = \left(\frac{\alpha^{-n}}{n}\right)u[n-1]$$

29. (a)

$$L[x(t)] = X(s) = \int_{-\infty}^{\infty} t \cdot e^{-2|t|} e^{-st} dt$$

$$= \int_{-\infty}^{0} t \cdot e^{-(s-2)t} dt + \int_{0}^{\infty} t \cdot e^{-(s+2)t} dt$$

The first integral converges if Re(s) < 2, and second converges if Re{s} > –2 therefore

ROC ; $-2 < \text{Re}(s) < 2$

$\therefore$

$$X(s) = \left\{\left[t \cdot \frac{e^{-(s-2)t}}{(s-2)}\right]_{-\infty}^{0} - \int_{\infty}^{0} \frac{e^{-(s-2)t}}{-(s-2)}\right\} + \left\{\left[t \cdot \frac{e^{-(s+2)t}}{-(s+2)}\right]_{0}^{\infty} - \int_{0}^{\infty} \frac{e^{-(s+2)t}}{-(s+2)} dt\right\}$$

$$= \frac{-1}{(s-2)^2} + \frac{1}{(s+2)^2}, \quad \text{ROC} ; -2 < \text{Re}(s) < 2$$

30. (c)

$$4y(n) = \alpha y(n-2) - 3x(n) + \beta x(n-1)$$

Taking z-transform,

$$4Y(z) = \alpha Y(z) z^{-2} - 3X(z) + \beta X(z) z^{-1}$$

$$Y(z)[4 - \alpha z^{-2}] = X(z)\,[\beta z^{-1} - 3]$$

$$\frac{Y(z)}{X(z)} = \left(\frac{\beta z^{-1} - 3}{4 - \alpha z^{-2}}\right)$$

For system to be stable, β can be of any value.

Poles of the system are given by

$$4z^2 - \alpha = 0$$

$$z = \sqrt{\frac{\alpha}{4}}$$

For system to be stable poles must lie inside the unit circle

$$|z| = \sqrt{\frac{\alpha}{4}} < 1$$

$\Rightarrow$ $$|\alpha| < 4$$

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