CLASS TEST

S.No.: 01SKEC_ABCDEFGHI_020625

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COMMUNICATIONS

ELECTRONICS ENGINEERING

Date of Test : 02/06/2025

ANSWER KEY

1.	(b)	7.	(a)	13.	(c)	19.	(a)	25.	(d)
2.	(c)	8.	(b)	14.	(c)	20.	(0)	26.	(a)
3.	(b)	9.	(c)	15.	(d)	21.	(a)	27.	(a)
4.	(c)	10.	(d)	16.	(c)	22.	(c)	28.	(c)
5.	(a)	11.	(b)	17.	(a)	23.	(c)	29.	(b)
6.	(c)	12.	(c)	18.	(a)	24.	(b)	30.	(b)

Detailed Explanations

1. (b)

$$\text{Carrier power} = (A_{rms})^2 = (50)^2 = 2.5\text{ kW}$$

$$\therefore \quad \text{Total power} = P_c\left(1+\frac{\mu^2}{2}\right) = 2.5\left(1+\frac{(0.6)^2}{2}\right) = 2.95\text{ kW}$$

2. (c)

$$\theta(t) = 10^8\pi t + 5\sin(4\pi \times 10^6 t)$$

$$\left.\frac{d\theta(t)}{dt}\right|_{t=0} = 10^8\pi + 5 \times 4\pi \times 10^6$$

$$f_i = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = \frac{1}{2\pi}[10^8\pi + 5\times 4\pi\times 10^6]$$

$$f_i = \frac{1}{2\pi}[10^8\pi + 20\pi\times 10^6] = 5\times 10^7 + 1\times 10^7$$

$$= 6 \times 10^7 = 60\text{ MHz}$$

3. (b)

$$s(t) = \frac{\mu A_c}{2}\cos[2\pi(f_c+f_m)t] + \frac{\mu A_c}{2}\cos[2\pi(f_c-f_m)t] + A_c A_m\cos(2\pi f_c t)$$

Thus, $\mu = 0.5$

4. (c)

The amplitude of uniformly distributed is equal to $\frac{1}{K}$.

$$\therefore \quad E[X^{K-1}] = \int_{-\infty}^{\infty} X^{K-1} f_x(x)dx$$

$$= \frac{1}{K}\int_0^k x^{K-1}dx = \frac{1}{K}\left|\frac{x^K}{K}\right|_0^K = (K)^{K-2}$$

5. (a)

Since, $Y = 2x$

Thus, $\frac{dx}{dy} = \frac{1}{2}$

$$f_Y(y) = \frac{dx}{dy} f_X(x) = \frac{dx}{dy} f_X\left(\frac{y}{2}\right)$$

$$\therefore \quad K = \frac{1}{2}$$

6. (c)

$$H = P\log_2\frac{1}{P} + (1-P)\log_2\frac{1}{1-P}$$

$$\frac{dH}{dp} = 0$$

Which gives as $P = \frac{1}{2}$

7. **(a)**

$$P = E[X^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega)d\omega = \frac{1}{2\pi}[5+1.5+1.5] = \frac{8}{2\pi} = 1.27$$

8. **(b)**

9. **(c)**

$$T_b = \frac{1}{0.1(10^6)} = 10^{-5}\text{ s}$$

$$f_b = 75\text{ kHz} = 75 \times 10^3\text{ Hz}$$

Now,
$$\alpha = 2f_b T_b - 1 = 1.5 - 1$$
$$= 0.5$$

10. **(d)**

$$(\text{SNR}) \propto (2^n)^2$$

$\therefore$
$$\frac{(\text{SNR})_2}{(\text{SNR})_1} = \frac{(2^{n+1})^2}{(2^n)^2} = 4$$

11. **(b)**

$$f_I = f_c + 2(IF)$$
$$IF = 10\text{ MHz}$$
$$\left(\frac{C_{max}}{C_{min}}\right) = \left(\frac{f_{Lo_2}}{f_{Lo_1}}\right)^2$$

In order to avoid image frequency

$$f_{Lo_1} = f_{o_1} + IF = 88 + 10 = 98\text{ MHz}$$
$$f_{Lo_2} = f_{o_2} + IF = 108 + 10 = 118\text{ MHz}$$

$\therefore$
$$\frac{C_{max}}{C_{min}} = \left(\frac{118}{98}\right)^2 = 1.449 : 1$$

12. **(c)**

$$y(t) = 4x(t) + 10x^2(t)$$

$\therefore$
$$y(t) = 4[m(t) + \cos(\omega_c t)] + 10[m(t) + \cos(\omega_c t)]^2$$
$$= 4m(t) + 4\cos(\omega_c t) + 10m^2(t) + \frac{10}{2} + \frac{10}{2}\cos(2\omega_c t) + 20m(t)\cos(\omega_c t)$$

$\therefore$
$$y(t) = 4\cos(\omega_c t) + 20m(t)\cos(\omega_c t) = 4[1 + 5m(t)]\cos(\omega_c t)$$

Now,
$$\max\{|m(t)|\} = A_m$$

$\therefore$
$$\mu = \max\{5|m(t)|\}$$
$$\mu = 5A_m$$
$$0.8 = 5A_m$$
$$A_c = 0.16$$

13. (c)

$$s(t) = x(t) + y(t) \text{ which is transmitted}$$

$$\therefore \quad s(t) = A_c\cos(\omega_c t) + m(t)\cos(\omega_c t) - \hat{m}(t)\sin(\omega_c t)$$

$$= [A_c + m(t)]\cos(\omega_c t) - \hat{m}(t)\sin(\omega_c t)$$

The output of the envelope detector is represented as $|s(t)|$ which is equal to

$$|s(t)| = \sqrt{[A_c + m(t)]^2 + [\hat{m}(t)]^2}$$

$$|s(t)| = \sqrt{A_c^2 + 2A_c m(t) + m^2(t) + [\hat{m}(t)]^2}$$

$\therefore$ A_c is very large

$$|s(t)| \approx \sqrt{A_c^2 + 2A_c m(t)}$$

$$|s(t)| \approx A_c\left[1 + \frac{2}{A_c}m(t)\right]^{1/2} \approx A_c\left[1 + \frac{m(t)}{A_c}\right]$$

$$|s(t)| \approx A_c + m(t)$$

14. (c)

$$s_o(t) = \alpha s_i^2(t) = \alpha A^2\cos^2(\theta) \qquad \text{where } \theta = \omega_c t + \beta\sin\omega_m t$$

$$= \frac{\alpha A^2}{2}[1 + \cos 2\theta] = \frac{\alpha A^2}{2} + \frac{\alpha A^2}{2}\cos 2\theta$$

$\because$ The output $s_o(t)$ is passed through a BPF, thus

$$y(t) = \frac{\alpha A^2}{2}\cos 2\theta$$

$$\therefore \quad y(t) = \frac{\alpha A^2}{2}\cos 2(\omega_c t + \beta\sin\omega_m t)$$

15. (d)

$$Y(t) = X(t) - X(t + \tau)$$

and $$\overline{X^2(t)} = R_X(0)$$

Also, we have to obtain the mean value of the random process $Y(t)$ as

$$E[Y(t)] = 0$$

$$\sigma_x^2 = E[Y^2(t)] - E[Y(t)]^2 = E\left[\{X(t) - X(t+\tau)\}^2\right]$$

$$= E[X^2(t)] - 2E[X(t)\cdot X(t+\tau)] + E[X^2(t+\tau)]$$

$$= R_X(0) - 2R_X(\tau) + R_X(0) = 2R_X(0) - 2R_X(\tau)$$

$$= 2[R_X(0) - R_X(\tau)]$$

16. (c)

$$X(t) = 6e^{At}$$

$$R_X(t_1, t_2) = E[X(t_1)X(t_2)] = E[6e^{At_1}\, 6e^{At_2}]$$

$$= 36\left[\frac{1}{2}\int_0^2 e^{A(t_1+t_2)}dA\right] = 18\left[\frac{e^{A(t_1+t_2)}}{t_1+t_2}\right]_0^2 = \frac{18}{t_1+t_2}[e^{2(t_1+t_2)} - 1]$$

17. (a)

The procedure are identical, let $\in = e$,

The above value should be non-negative, this will happen only for $\left(\frac{T-t_o}{T}\right)$ and also since x can have only two values A and 0, thus

Here, $$F_X(x|\varepsilon = e) = P\{X \le x|\varepsilon = e\}$$

$$= \left[\frac{(T-t_o)}{T}u(x) + \frac{t_o}{T}u(x-A)\right]$$

Because 'x' can have only value of zero and A.

Thus, $$f_X(x|\varepsilon = e) = \left[\frac{(T-t_o)}{T}\right]\delta(x) + \frac{t_o}{T}\delta(x)$$

$$f_X(x) = \int_{-\infty}^{\infty} f_X(x|\varepsilon = e) f_\varepsilon(e)de$$

$$= \left[\frac{(T-t_o)}{T}\right]\delta(x) + \left(\frac{t_o}{T}\right)\delta(x-A)$$

18. (a)

Given, $y = \frac{1}{2}(x+|x|)$

when $x > 0$, $$y = \frac{1}{2}(x+|x|) = \frac{1}{2}(x+x) = \frac{2x}{2} = x,\ y>0$$

and $y = x$, $\quad y > 0$

$$\therefore \quad F_Y(y) = P[X \le y|X \ge 0]$$

$$= \frac{P[X \le y, X \ge 0]}{P(X \ge 0)} = \frac{P(X \le y, X \ge 0)}{1-P(X<0)}$$

$$= \frac{P(0 \le X \le y)}{1-P(X \le 0)} = \frac{F_X(y)-F_X(0)}{1-F_X(0)}$$

$$\therefore \quad f_Y(y) = \frac{d}{dy}F_Y(y) = \frac{f_X(y)}{1-F_X(0)}$$

19. (a)

$$\int_{-\infty}^{\infty} f_X(x)dx = 1$$

$$\int_0^{\infty} Cxe^{-x}dx = 1$$

$$C\left[x\left(\frac{e^{-x}}{-1}\right) - 1\left(\frac{e^{-x}}{1}\right)\right]_0^{\infty} = 1$$

$$C(0+1) = 1$$

$$\therefore \quad C = 1$$

20. (0)

$$P(X,Y) = \begin{bmatrix} 0.25 & 0.25 \\ 0.15 & 0.15 \\ 0.1 & 0.1 \end{bmatrix}$$

$$P(x_1) = 0.25 + 0.25 = 0.5$$
$$P(x_2) = 0.15 + 0.15 = 0.3$$
$$P(x_3) = 0.1 + 0.1 = 0.2$$

and
$$P(y_1) = 0.5$$
$$P(y_2) = 0.5$$

$$P(X) = P(x_1)\log_2\frac{1}{P(x_1)} + P(x_2)\log_2\frac{1}{P(x_2)} + P(x_3)\log_2\frac{1}{P(x_3)}$$

$$= 0.5\log_2\frac{1}{0.5} + 0.3\log_2\frac{1}{0.3} + 0.2\log_2\frac{1}{0.2}$$

$$= 1.485 \text{ bit/symbol}$$

and,
$$H(Y) = P(y_1)\log_2\frac{1}{P(y_1)} + P(y_2)\log_2\frac{1}{P(y_2)}$$

$$= 0.5\log_2\frac{1}{0.5} + 0.5\log_2\frac{1}{0.5} = 1 \text{ bit/symbol}$$

$$H(X,Y) = 0.25\log_2\frac{1}{0.25} + 0.25\log_2\frac{1}{0.25} + 0.15\log\frac{1}{0.15} + 0.15\log\frac{1}{0.15} + 0.1\log_2\frac{1}{0.1} + 0.1\log\frac{1}{0.1}$$

$$= 2.485 \text{ bits/symbol}$$

$$I(X,Y) = H(X) + H(Y) - H(X,Y)$$
$$= 1.485 + 1 - 2.485 = 0 \text{ bits/symbol}$$

21. (a)

$$f_s = 10 f_N = 10 \times 20 = 200 \text{ kHz}$$

$$\delta f_s \geq \text{Max}\left|\frac{dm(t)}{dt}\right|$$

$$\delta \times 200 \text{ kHz} \geq 2\pi f_m A_m$$

$$\delta \geq \frac{2\pi \times (10\times 10^3) \times \left(\frac{1}{2}\right)}{200 \text{ kHz}}$$

$$\delta \geq 0.157 \text{ Volts}$$

22. (c)

According to MAP criterion

$$\frac{f_X(r|s_1)}{f_X(r|s_o)} \underset{H_1}{\overset{H_0}{\lessgtr}} \frac{P_o}{P_1}$$

Now,
$$f_X(r \mid s_1) = \frac{1}{\sqrt{\pi N_o}}\exp -\frac{(x-\mu)^2}{N_o}$$

and
$$f_X(r \mid s_1) = \frac{1}{\sqrt{\pi N_o}}\exp -\frac{(x-1)^2}{N_o}$$

$$f_X(r \mid s_0) = \frac{1}{\sqrt{\pi N_o}} \exp - \frac{(x+1)^2}{N_o}$$

$\therefore$ ACC to MAP criterion

$$\frac{\exp - \dfrac{(t_o - 1)^2}{N_o}}{\exp - \dfrac{(t_o + 1)}{2}} = \frac{P_0}{P_1} = 2$$

$$4t_o = 2\ln 2$$

$$t_o = \frac{1}{2}\ln 2 = 0.346$$

23. **(c)**

$$P_e = 1 - P_c$$

Now, since we are using ML criterion, and the input symbols are equally likely, then we can directly choose the output based on the maximum value of the transmission probabilities.

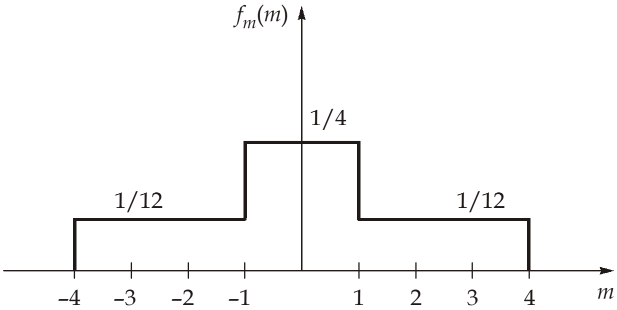
$$P_c = P(y_1)\, P(y_1 \mid x_1) + P(x_3) \cdot P(y_2 \mid x_3) + P(x_1) \cdot P(y_3 \mid x_1)$$

$$= \frac{1}{3}[0.5 + 0.5 + 0.4] = \frac{7}{15}$$

$\therefore$ $$P_e = 1 - \frac{7}{15} = \frac{8}{15} = 0.533$$

24. **(b)**

To create an optimum quantize of 3-bits all the message symbols should be equiprobable thus, the area under each quantized value must be same



Thus, for a 3-bit quantizer we need 8 levels. From the graph we can observe that the quantization level can be chosen as $\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}$ and $\pm\frac{7}{2}$.

Thus, signal power, $$\sigma_m^2 = \int_{-4}^{4} m^2 f_m(m)\,dm$$

$$= 2\left[\left[\frac{1}{4}\frac{m^3}{3}\right]_0^1 + \left[\frac{1}{12}\cdot\frac{m^3}{3}\right]_1^4\right] = \frac{11}{3} \text{ Watts}$$

Quantized noise power,

$$\sigma_q^2 = 2\left[\int_0^1 \left(m - \frac{1}{2}\right)^2 \times \frac{1}{4}dm + \int_1^2 \left(m - \frac{3}{2}\right)^2 \times \frac{1}{12}dm + \int_2^3 \left(m - \frac{5}{2}\right)^2 \times \frac{1}{12}dm + \int_3^4 \left(m - \frac{7}{2}\right)^2 \times \frac{1}{12}dm\right]$$

$$= 2\left[\frac{1}{4}\int_{-1/2}^{1/2} \lambda^2 d\lambda + \frac{1}{12}\int_{-1/2}^{1/2} \lambda^2 d\lambda + \frac{1}{12}\int_{-1/2}^{1/2} \lambda^2 d\lambda + \frac{1}{2}\int_{-1/2}^{1/2} \lambda^2 d\lambda + \frac{1}{12}\int_{-1/2}^{1/2} \lambda^2 d\lambda\right]$$

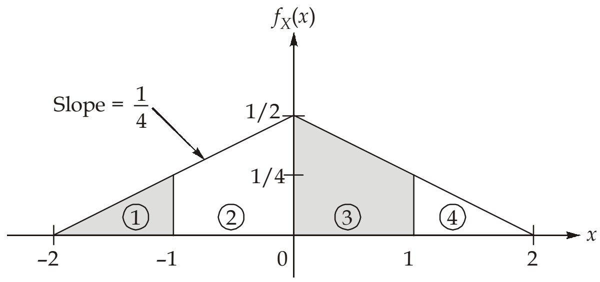
$$= \int_{-1/2}^{1/2} \lambda^2 d\lambda = 2\int_{0}^{1/2} \lambda^2 d\lambda = \frac{1}{12}$$

$$(\text{SNR})_9 = \frac{\sigma_m^2}{\sigma_n^2} = \frac{11/3}{1/12} = 44$$

$$\therefore \quad (\text{SNR})_{9\,(\text{dB})} = 10\log_{10}(44) = 16.43 \text{ dB}$$

25. (d)

Considering the given PDF with quantization regions, as follows:



The quantized samples exist at the output of the quantizer with the probabilities as follows:

$$P(y_1) = \text{Area of region} - 1 = \frac{1}{2}\times 1\times\frac{1}{4} = \frac{1}{8}$$

$$P(y_2) = \text{Area of region} - 2 = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$$

$$P(y_3) = P(y_2) = \frac{3}{8}$$

$$P(y_4) = P(y_2) = \frac{1}{8}$$

Entropy at the quantizer output is,

$$H(Y) = \frac{2}{8}\log_2(8) + \frac{6}{8}\log_2\left(\frac{8}{3}\right) \text{ bits/sample}$$

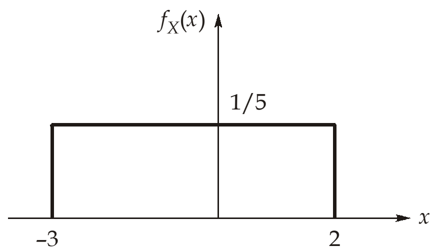
$$= \log_2(8) - \frac{6}{8}\log_2(3) = 3 - \frac{3}{4}\log_2(3) \text{ bits/sample}$$

$$= 1.81 \text{ bits/sample}$$

26. (a)

$$N_o = E[(X - x_q)^2] = \int_{-\infty}^{\infty} (x - x_q)^2 f_X(x)\, dx$$

Now, since the input is in the range of [-3 to 2] the PDF of the input signal can be drawn as



$$\therefore \quad N_o = \int_{-3}^{0} \frac{1}{5}(x+1)^2\,dx + \int_{0}^{2} \frac{1}{5}(x-2)^2\,dx$$

$$= \frac{1}{5}\frac{(x+1)^3}{3}\Bigg|_{-3}^{0} + \frac{1}{5}\frac{(x-2)^3}{3}\Bigg|_{0}^{2}$$

$$= \frac{1}{15}(1+8) + \frac{1}{15}(+8) = \frac{17}{15}\text{ W} = 1.13\text{ W}$$

27. (a)

$$G = \begin{bmatrix} 1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & | & 1 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad H = \begin{bmatrix} 1 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & | & 0 & 1 & 0 \\ 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

Hence option 'A'.

28. (c)

$$(SNR)_0 = \frac{3P}{M_{\max}^2}\cdot 2^{2n}$$

$$P = 30\text{ mW} = 30 \times 10^{-3}\text{ W}$$

$$M_{\max} = 3.8\text{ V}$$

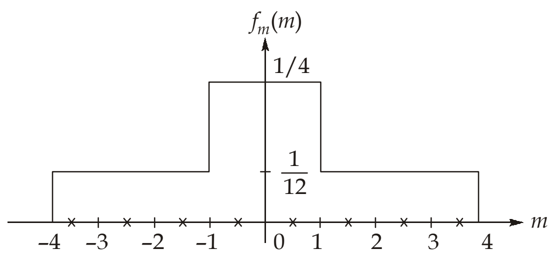
$$10\log(SNR)_0 = 10\log\left[\frac{3P\cdot 2^{2n}}{M_{\max}^2}\right]$$

$$20 = 10\log\left[\frac{3\times 30\times 10^{-3}\times 2^{2n}}{(3.8)^2}\right]$$

$$\therefore \quad n = 7$$

29. (b)

3-bit quantizer means 8-levels.



The eight levels can be chosen as the midpoint of the amplitude level for optimum quantization. Now, the signal power is equal to

$$\sigma_m^2 = \int_{-4}^{4} m^2 f_m(m)\,dm = 2\int_{0}^{4} m^2 f_m(m)\,dm$$

$$= 2\left[\left(\frac{m^3}{12}\right)_0^1 + \left(\frac{m^3}{36}\right)_1^4\right] = \frac{11}{3}\text{W}$$

Now, the quantization noise is equal to

$$\sigma_q^2 = 2\left[\frac{1}{4}\int_0^1\left(m-\frac{1}{2}\right)^2 dm + \frac{1}{12}\int_1^2\left(m-\frac{3}{2}\right)^2 dm + \frac{1}{12}\int_2^3\left(m-\frac{5}{2}\right)^2 dm + \frac{1}{12}\int_3^4\left(m-\frac{7}{2}\right)^2 dm\right]$$

$$\sigma_q^2 = \frac{2}{3}\left[\frac{1}{4}\left(m-\frac{1}{2}\right)^3\Bigg|_0^1 + \frac{1}{12}\left(m-\frac{3}{2}\right)^3\Bigg|_1^2 + \frac{1}{12}\left(m-\frac{5}{2}\right)^3\Bigg|_2^3 + \frac{1}{12}\left(m-\frac{7}{2}\right)^3\Bigg|_3^4\right]$$

$$\sigma_q^2 = \frac{1}{12}\text{W}$$

$$\therefore \quad (\text{SNR})_q = \frac{\sigma_m^2}{\sigma_q^2} = \frac{11/3}{1/12} = 44$$

$$\therefore \quad (\text{SNR}_q)_{\text{dB}} = 16.43 \text{ dB}$$

30. (b)

$$\therefore \quad (\text{BW})_{\text{QPSK}} = \frac{R_b}{\log_2(4)}$$

$$\Rightarrow \quad (\text{BW})_{\text{QPSK}} = 50 \text{ kHz}$$

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