CLASS TEST

S.No.: 01SKCS_ABCDEFGH_030625

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Theory of Computation

COMPUTER SCIENCE & IT

Date of Test : 03/06/2025

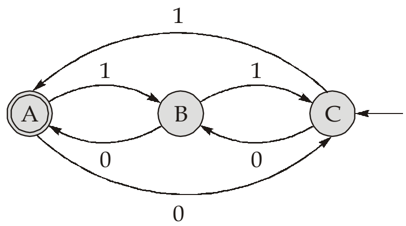
ANSWER KEY

1.	(a)	7.	(d)	13.	(a)	19.	(a)	25.	(d)
2.	(d)	8.	(b)	14.	(a)	20.	(d)	26.	(d)
3.	(c)	9.	(a)	15.	(a)	21.	(a)	27.	(b)
4.	(a)	10.	(c)	16.	(d)	22.	(c)	28.	(d)
5.	(a)	11.	(a)	17.	(c)	23.	(a)	29.	(c)
6.	(d)	12.	(b)	18.	(c)	24.	(a)	30.	(b)

DETAILED EXPLANATIONS

1. (a)

The finite automata obtained by F_1 is



Automata F_2 is same as F_1. So $L(F_1) = L(F_2)$.

2. (d)

Regular expression for the language that does not end with ab is

(a + b)* (aa + ba + bb) + a + b + ∈

Option (a) can not generate ∈, a, b.

Hence it is not correct regular expression.

3. (c)

- S_1 is correct and S_2 is incorrect.
- S_1 can be written as $(000)^n$ where $n \geq 1$. Regular grammar for S_1 is $S \rightarrow S000/000$. Hence S_1 is regular.
- S_2 can be written as $(00)^{(x+y)}$ where $x \geq 1$ and $n \geq 1$. S_2 can be further reduced to $(00)^x$ where $x \geq 2$. Regular grammar for S_2 is $S \rightarrow S00/0000$. Hence S_2 is also regular language.

4. (a)

- Context free languages are not closed under complementation and intersection.
  Hence option (b) and (c) is false.
- DPDA is less powerful than PDA. Hence there is CFL language which can not be accepted by DPDA.
  Hence option (d) is false.

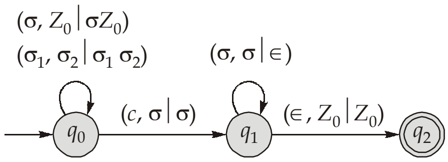
5. (a)

6. (d)

Checking CFL is equivalence, equality and subset problems for CFL are undecidable.

7. (d)

The transition diagram of the PDA is as shown below. In the figure σ, σ_1 and σ_2 represent a or b.



PDA accepting $\{wcw^R \mid w \in (a, b)^* \text{ and } |w| \geq 1\}$.

8. (b)

Above DFA is accepting the language 'L' which have even number of characters with length ≥ 2.

Minimized DFA will be,

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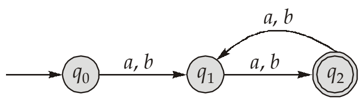
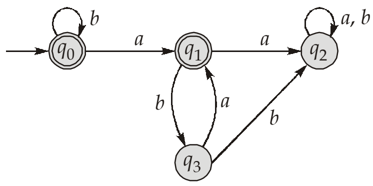


$\therefore$ Number of states are 3.

9. (a)



$$q_0 = b^* \quad \text{...(0)}$$
$$q_1 = q_0a + q_3a \quad \text{...(1)}$$
$$q_2 = q_1b \quad \text{...(2)}$$

Put equation (0) and (2) in (1)

$$q_1 = b^*a + q_1\,ba$$
$$q_1 = b^*a\,(ba)^*$$

Since two final states

So, final expression is = $b^* + b^*a\,(ba)^*$

10. (c)

- L_1 is CFL but not regular as there is infinite comparison between 0 and 1. It is CFL because only 1 comparison and k is not comparing with others, it is simply greater than 0 which can be done by DCFL as well.
- L_2 is DCFL but not regular.
- L_3 DCFL but not regular same concept as explained for L_1.
- L_4 is regular since the given comparisons is finite number. Hence we can make finite automata for the given L_4.

Hence all four are CFL. Since it is asking CFL but not regular so 3 is the correct answer.

11. (a)

$$L_1 = \{a^n 0 b^n \mid n \geq 1\}$$

All a's are push until 0 is not encounter. When 0 is come skip it and after the 0 for every b pop the a's present in stack.

So, language is DCFL.

$$L_2 = \{0^p\,(ab)^* \mid p \text{ is prime numbers}\}$$

Since we can not create PDA for language 0^p where p is prime numbers. By using pumping lemma we can prove that above language is not CFL.

$$L_3 = \{w\,0\,w \mid w \in (0,b,a)^*\}$$

String matching is done in CSL.

12. (b)

I. $\{a^m\, b^n\, c^p\, d^q \mid m + p = n + q\}$ is CFL because we can rearrange the equation as $m - n + p - q = 0$ which can be done by push, pop, push and pop respectively and check if stack is empty or not.

II. $\{a^m\, b^n\, c^p\, d^q \mid mn = p + q\}$ is not a CFL, since mn involves multiplying number of a's and number of b's which cannot be done by a PDA.

III. $\{a^m\, b^n\, c^p\, d^q \mid m = n \text{ and } p = q\}$ is CFL since one comparison at a time can be done by PDA.

IV. $\{a^m\, b^n\, c^p\, d^q \mid m = n = p \text{ and } p = q\}$ is not CFL since $m = n = p$ is a double comparison which can not be done by PDA.

13. (a)

(a) $\overline{L}_3 \cup L_4 = \overline{\text{REC}} \cup \text{RE} = \text{RE}$

(b) $L_1 \cdot L_2 = \text{Reg} \cdot \text{CFL} = \text{CFL}$

(c) $L_1^* \cap L_2 = (\text{Reg})^* \cap \text{CFL} = \text{Reg} \cap \text{CFL} = \text{CFL}$

(d) $\overline{L}_2 \cup L_3 = \overline{\text{CFL}} \cup \text{REC} = \text{CSL} \cup \text{REC} = \text{REC}$

So, only (a) correct.

14. (a)

At state q_2, since this for $10^n\, 1^n$, so we need to push all 0's in the stack, hence it becomes $(0, 0 \mid 00)$. Next, at state q_3 since this is for $110^n\, 1^{2n}$, so every single 0, we push two 0's, hence $0, z \mid 00z$, then for add 0's $0, 0 \mid 000$, then for c, now all 0's have been pushed in q_3 now to check the comparison between 1 and 0's need to be pop.

Hence at $c = (1, 0 \mid \in)$.

15. (a)

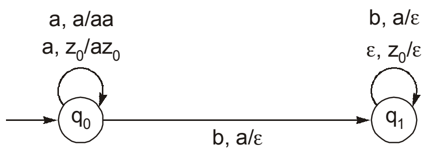
- Languages accepted by push down automata may closed under complementation. *e.g.* regular languages which are CFL also, closed under complement.
- Turing decidable languages are closed under union and Kleen star operation.
- Recursive enumerable languages are closed under intersection.

16. (d)

Multi tape TM ≅ multi tape multi-head TM ≅ Multi track TM.

$\Rightarrow$ All given TM's are equally expressive.

17. (c)



It accepts $L = \{a^n b^n \mid n \geq 1\}$ and $L = L_2$.

Only L_2 is the subset of L.

18. (c)

Given PDA is NPDA, hence two comparison with or is possible. If you observe the automata, at q_2 for every input 'a' there is two transition on epsilon which proves it is NPDA. The upper branch comparing a with b and lower branch comparing a with 'c' and both leads to the final states. L accepts epsilon as well. Hence $L = \{a^i\, b^j\, c^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } i = k\}$.

19. (a)

The above PDA represents the language,

$$L = \{a^m b^n c^k \mid \text{if } (m \text{ is even}) \text{ then } n = k\}$$

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20. (d)

- Finiteness property of a CFG is decidable, which can be decidable with the help of variable dependency graph.
- Push-down automata need not be always deterministic. In fact power of non-deterministic PDA is greater than the deterministic.
- Deterministic CFL are closed under complement, hence recursive too.
- DCFL is not closed under union.

21. (a)

$\mathbf{L_1}$ **:** $\{a^p b^p b^q a^q \mid p, q > 0\} = \{a^p b^{p+q} a^q \mid p, q > 0\} = \{a^p b^t a^q \mid t = p + q\}$

Here push, pop are clear i.e. when to push and when to pop and contain only 1 comparison i.e. $t = p + q$ so it is DCFL.

$\mathbf{L_2}$ **:** $\{a^p b^q a^q b^p \mid p, q > 0\}$

L_2 is DCFL.

Considering the statements,

$\mathbf{S_1}$ **:** $(\text{DCFL})^C$ = DCFL, so S_1 is true.

$\mathbf{S_2}$ **:** $(\text{DCFL})^R = (\text{CFL})^R$ = CFL (by closer property). Every CFL is CSL so S_2 is false.

$\mathbf{S_3}$ **:** $L_1 \cap L_2 = \{a^p b^p b^q a^q\} \cap \{a^p b^q a^q b^p\} = \phi \Rightarrow$ Regular. So S_3 is false.

$\mathbf{S_4}$ **:** Both L_1 and L_2 is DCFL and hence CSL too. So S_4 is true.

22. (c)

Traversing the states of the turing machine, it can be seen that for every 'a' as the input, it is accepting 3 b's. For every 'a' machine writes 'X' on the tape, then take right moves till it reaches 'b'. For every 3 b's it writes symbol Y.

Hence accepting the language $L = \{a^m b^n \mid 3m = n; m, n \geq 0\}$.

23. (a)

The language $\{0^p 1^{2p}\}$ is context-free language, hence it is recursive also. Since $L(M) \leq_p$ REC, so $L(M)$ also recursive, now given input (i.e. recursive language) to turing machine and finding it is accept or not is non-trival property so it is undecidable by Rice's theorem.

24. (a)

- L_1 is regular , since we can create DFA for given language.
- L_2 is CFL, since their is a comparison between number of a and number of b in strings i.e., difference is less than equal to 10.
- L_3 is regular, since by making $w = \epsilon$ and $c = (a+b)^*$ we get language $(a+b)^*$ which contain every string belongs to $wcww^r$.

25. (d)

By pumping lemma, we can never say that a language is regular or CFL. It can only be used to prove that a certain language is not regular or not CFL.

Since pumping lemma isn't satisfied for regular, hence we can say it is not regular, but since the lemma is satisfied for context-free, we can't say that the language is CFL.

26. (d)

The PDA drawn above is describing the language:

$$L = \{a^n b^{n+m} c^m \mid n \geq 0, m \geq 1\}$$

Out of the above five string for the blank spaces only the string 'bbcc' i.e., 'aabbbbbccc'.

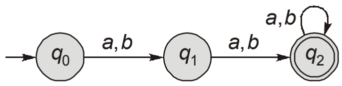
27. (b)

So, $|w_1| \geq 0$, R.E. $= (a+b)^*$

$|x| \geq 0$, R.E. $= (a+b)^*$

$|w_2| > 1 = |w_2| \geq 2$, R.E. $= (a+b)\,(a+b)\,(a+b)^*$

$$L = \{w_1 x w_2 | w, x \in \{a, b\}^*, |w_1| \geq 0, |w_2| > 1 \text{ and } |x| \geq 0\}$$
$$= (a+b)^*\,(a+b)^*\,(a+b)^2\,(a+b)^*$$
$$= (a+b)^*\,(a+b)^*\,(a+b)^*\,(a+b)^2$$
$$= (a+b)^*\,(a+b)^2 = (a+b)^2\,(a+b)^*$$



28. (d)

- **S_1:** Subset of infinite language may be finite or infinite. So **false**.
- **S_2:** Subset of regular language can be any language. So **false**. (Since regular language is not closed under subset operation)
- **S_3:** Subset of CFL can be any language (Since CFL is not closed under subset operation) So, **false**.

(**Note:** No language is clsoed under subset and superset operation).

29. (c)

$L_1 = \{\in, 012, 001122, 000111222, \ldots\}$

$L_2 = \{\in, 001122, 000011112222, \ldots\}$

$L_3 = \{\in, 0, 1, 2, 01, 02, \ldots, 012, \ldots\}$

So, $L_1 \subseteq L_3$ and $L_2 \subseteq L_1$ is **true**.

30. (b)

- Let A be the set of real numbers between [–3, –2] and B be the set of all natural numbers.

  $A \cap B = \phi$

- Let A be $A_{TM} = \{\langle M, w\rangle$ M is a TM and M accepts 'w'$\}$

  $B = \Sigma^*$

Here $A \subseteq B$ but A is not reducible to B.

■■■■

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