CLASS TEST

S.No.:02DLSKME_ABCDEFGHI_61125

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# THEORY OF MACHINES

## MECHANICAL ENGINEERING

Date of Test : 06/11/2025

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (b) | 7. | (d) | 13. | (b) | 19. | (d) | 25. | (a) |
| 2. | (b) | 8. | (d) | 14. | (c) | 20. | (c) | 26. | (d) |
| 3. | (b) | 9. | (d) | 15. | (a) | 21. | (b) | 27. | (d) |
| 4. | (c) | 10. | (b) | 16. | (d) | 22. | (d) | 28. | (d) |
| 5. | (a) | 11. | (c) | 17. | (c) | 23. | (c) | 29. | (d) |
| 6. | (c) | 12. | (c) | 18. | (b) | 24. | (c) | 30. | (c) |

## DETAILED EXPLANATIONS

**1. (b)**

**2. (b)**

$s$ = 10 cm, $l$ = 50 cm, $p$ = 30 cm, $q$ = 40 cm

for Grashoff's chain rule to be satisfied,

$$s + l \le p + q$$
$$10 + 50 \le 30 + 40$$
$$60 < 70$$

Chain rule satisfied.

here coupler '$AB$' which is a shortest link can rotate completely relative to fixed link OC. Because there is atleast one revolving link if the sum of the lengths of the largest and shortest link is less than the sum of the lengths of the other two links.

**3. (b)**

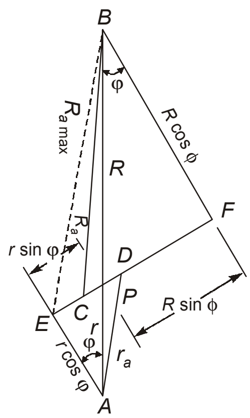
$$\eta_G \times N_i T_i + N_o T_o = 0$$
$$0.93 \times 1000 \times 1440 + (-840) T_o = 0$$
$$T_o = 1594.29 \text{ Nm}$$
$$\therefore \quad T_i + T_o + T_h = 0$$
$$1000 + 1594.3 + T_h = 0$$
$$T_h = -2594.3 \text{ Nm}$$

(–) sign mean torque should be applied to opposite of input torque or rotation.

**4. (c)**

**5. (a)**

**6. (c)**




Maximum path of contact = $(r + R)\sin\phi$

$$r = \frac{100}{2} = 50 \text{ mm}$$
$$\frac{R}{r} = G$$
$$\frac{R}{r} = 5$$
$$\Rightarrow \quad \frac{R}{50} = 5$$
$$R = 250 \text{ mm}$$

$\therefore$ According to the question,

$$(r + R)\sin\phi = 117.2 \text{ mm}$$
$$\text{from above } \phi = 22.995$$
$$\therefore \quad \phi = 23^\circ$$

**7. (d)**

For involute teeth profile, $\phi$ remains constant.

**8.** **(d)**

**9.** **(d)**

For damper not to be used,

$$r > \sqrt{2}$$

$$\therefore \quad \frac{\omega}{\omega_n} > \sqrt{2}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\Rightarrow \quad \omega_n = \sqrt{\frac{6.25\times1000}{10}} = 25$$

$$\therefore \quad \omega > \left(25\sqrt{2}\right)$$

$$\omega > 35.3553$$

For (d), 36, damper not be to used.

**10.** **(b)**

1.1079 rad/s on comparing it with,

$$m\ddot{x}+c\dot{x}+kx = 0$$

$$m = 20$$

$$c = 60$$

$$k = 25$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{20}} = 1.118$$

$$\xi = \frac{c}{2\sqrt{mk}}$$

$$\xi = \frac{6}{2\sqrt{20\times25}}$$

$$\xi = 0.1341$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\therefore \quad \omega_d = 1.118\sqrt{1-0.1341^2}$$

$$\therefore \quad \omega_d = 1.1079 \text{ rad/s} \simeq 1.11 \text{ rad/s}$$

**11.** **(c)**

$$T_{\text{mean}} = \frac{1}{\pi}\int_0^{\pi} Td\theta = \frac{1}{\pi}\int_0^{\pi}(1000+400\sin 2\theta-500\cos 2\theta)\times.d\theta$$

$$T_{\text{mean}} = 1000 \text{ Nm}$$

At any instant, $\Delta T = T - T_{\text{mean}}$

$$= (1000 + 400 \sin 2\,\theta - 500 \cos 2\,\theta) - 1000$$

$$= 400 \sin 2\,\theta - 500 \cos 2\,\theta$$

$\therefore$ $\Delta T$ is zero, when

$$400 \sin 2\,\theta - 500 \cos 2\,\theta = 0$$

$$\tan 2\theta = \frac{5}{4}$$

$$2\theta = 51.34° \qquad \text{or} \qquad 231.34°$$

$$e_{max} = \int_{25.67}^{115.67} (400\sin 2\theta - 500\cos 2\theta) \times d\theta$$

$$e_{max} = 640.312 \text{ Nm}$$

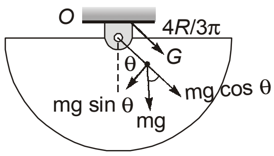
$$e_{max} = Ik_s\left[\frac{\omega_{max}+\omega_{min}}{2}\right]^2$$

$$640.312 = 500 \times .35^2 \times k \times 10\pi \times 10\pi$$

$$k = 0.01059$$

$$\therefore \quad k\% = 1.059\ \%$$

**12. (c)**




Displacing the semi-circular disk by small angle θ,

∴ We have,

$$\text{External torque} = \text{Inertial torque}$$

$$(mg\sin\theta)\times\frac{4R}{3\pi} = -I\ddot{\theta}$$

∴ For small θ, $\sin\theta \approx \theta$

$$mg\theta\times\frac{4R}{3\pi} = -I\ddot{\theta}$$

$$\therefore \quad I\ddot{\theta} + mg\times\frac{4R}{3\pi}\theta = 0$$

$$I = \frac{mR^2}{2} \qquad \left[\because \text{ Polar moment of inertia of disc} = \frac{MR^2}{2}\right]$$

$$\frac{mR^2}{2}\ddot{\theta} + mg\frac{4R}{3\pi}\theta = 0$$

$$\therefore \quad \ddot{\theta} + \left(\frac{mg4R}{3\pi} \div \frac{mR^2}{2}\right)\theta = 0$$

∴ Comparing with $\ddot{\theta} + \omega_n^2\theta = 0$

$$\therefore \quad \omega_n = \sqrt{\frac{mg4R\times 2}{3\pi\times mR^2}}$$

$$\Rightarrow \quad \omega_n = \sqrt{\frac{8g}{3\pi R}}$$

**13. (b)**

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} \qquad k_1 = f_1^2(2\pi)^2 \times m$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} \qquad k_2 = f_2^2(2\pi)^2 \times m$$

Since system is in series, so

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

$$f^2 \times (2\pi)^2 \times m = \frac{f_1^2 \times (2\pi)^2 \times m \times f_2^2 \times (2\pi)^2 \times m}{f_1^2 \times (2\pi)^2 \times m + f_2^2 \times (2\pi)^2 \times m}$$

$$f = \frac{f_1 \times f_2}{\sqrt{f_1^2 + f_2^2}}$$

**14. (c)**

$$\omega_1 = \frac{2\pi \times 460}{60} = 48.171 \text{ rad/sec}$$

$$\omega_2 = \frac{2\pi \times 435}{60} = 45.55 \text{ rad/sec}$$

$$\text{Energy released} = \frac{1}{2}I\left(\omega_1^2 - \omega_2^2\right) = \frac{1}{2} \times m \times k^2(48.171^2 - 45.55^2)$$

$$= \frac{1}{2} \times \frac{2 \times 1000}{9.81} \times 1.2 \times 1.2 \times 245.642$$

$$= 36 \text{ kJ}$$

**15. (a)**

$$m = 60 \text{ kg}$$

$$s = 40 \text{ kN/m} = 40 \times 10^3 \text{ N/m}$$

Damping = 20% of critical value

$$\omega_d = \omega_n\left(\sqrt{1-\xi^2}\right)$$

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{40 \times 10^3}{60}} = 25.8198 \text{ rad/s}$$

$$\xi = \frac{C}{C_C} = 0.2$$

$$\omega_d = 25.8198\sqrt{1-0.2^2}$$

$$= 25.2982 \text{ rad/s}$$

$$f_n = 4.02 \text{ Hz}$$

**16. (d)**

By equilibrium method

$$I\ddot{\theta} + kx\left(\frac{L}{2}\right) = 0$$

$$I\ddot{\theta}+\frac{kL}{2}\left(\theta\times\frac{L}{2}\right) = 0$$

$$I\ddot{\theta}+\frac{kL^2}{4}\theta = 0$$

$$\frac{mL^2}{12}\ddot{\theta}+\frac{kL^2}{4}\theta = 0$$

$$\omega_n = \sqrt{\frac{kL^2}{4}\times\frac{12}{mL^2}} = \sqrt{\frac{3k}{m}} = \sqrt{\frac{3\times 300}{1}}$$

$$\omega_n = 30 \text{ rad/s}$$

**17. (c)**

$$m = 3 \text{ kg}$$
$$F_0 = 40 \text{ N}$$
$$A = 15 \text{ mm}$$
$$T = 0.24 \text{ sec}$$

$$A = \frac{F_0/s}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\text{Resonance } \frac{\omega}{\omega_n} = 1$$

$$A = \frac{F_0/s}{2\xi} \quad \text{...(1)}$$

$$\omega_n = \frac{2\pi}{T} = \frac{2\pi}{0.24} = 26.1799 \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{s}{m}}$$

$$s = 2056.1675 \text{ N/m}$$

Putting these values in equation (1),

$$0.015 = \frac{40}{2056.1675\times 2\times\xi}$$

$$\xi = \frac{40\times 1000}{15\times 2056.1675\times 2}$$

$$\xi = 0.6484$$

$$\xi = \frac{C}{C_C}\frac{\text{(Damping coefficient)}}{\text{(Critical damping coefficient)}}$$

$$C = 0.6484\times 2\sqrt{2056.1675\times 3}$$

$$C = 101.859 \text{ Ns/m}$$

**18. (b)**

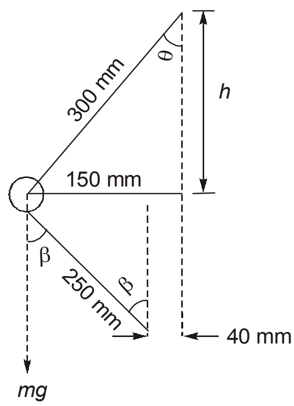
$$m = 96 \text{ kg}$$
$$G = 85 \times 10^9 \text{ N/m}^2$$
$$k = 0.4 \text{ m}$$
$$d = 0.04 \text{ m}$$
$$I = mk^2 = 96 \times 0.04^2 = 15.36 \text{ kg.m}^2$$
$$J = \frac{\pi}{32}d^4 = \frac{\pi}{32}\times(0.04)^4$$
$$= 2.5132 \times 10^{-7} \text{ m}^4$$

Total torsional stiffness of shaft,

$$q = q_1 + q_2$$
$$q = \frac{GJ}{L_1} + \frac{GJ}{L_2} = 85\times10^9\times2.5132\times10^{-7}\left[\frac{1}{1}+\frac{1}{0.8}\right]$$
$$= 48066.3676 \text{ Nm}$$
$$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$$
$$fn = \frac{1}{2\pi}\sqrt{\frac{48066.3676}{15.36}}$$
$$= 8.9031 \text{ Hz}$$
$$= 8.90 \text{ Hz}$$

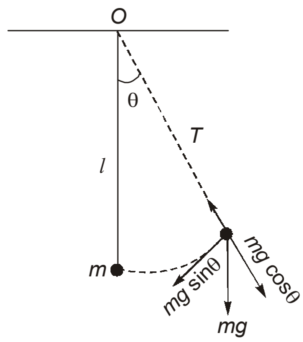
**19. (d)**

$$h = \sqrt{0.3^2 - 0.15^2} = 0.2598 \text{ m}$$
$$\sin\beta = \frac{150-40}{250}$$
$$\beta = 26.1038°$$
$$\sin\theta = \frac{150}{300}$$
$$\theta = 30°$$
$$k = \frac{\tan\beta}{\tan\theta} = \frac{\tan 26.1038}{\tan 30°} = 0.8486$$




$$\text{Power} = \left[m + \frac{M}{2}(1+k)\right]gh\left(\frac{4c^2}{1+2c}\right)$$
$$= [6+20(1+0.8486)]9.81\times0.2598\left(\frac{4\times0.01^2}{1+2\times0.01}\right)$$
$$= 0.04341 \text{ Nmm}$$
$$= 43.41 \text{ Nm}$$

**20. (c)**




$$T = -mgl\sin\theta$$
$$T = -mgl\theta \qquad [\because \sin\theta \simeq \theta \text{ as } \theta \text{ is very small}]$$
$$T = I\alpha = -mgl\theta$$
$$I = ml^2$$
$$\alpha = \frac{-g\theta}{l} \qquad \text{...(i)}$$

and we know $\alpha = \ddot{\theta} = -\omega_n^2\theta$ ...(ii)

Comparing (i) and (ii)

$$\omega_n = \sqrt{\frac{g}{l}}$$

So, $\omega_n \propto \sqrt{g}$

$$g_{\text{Moon}} = \frac{g_{\text{Earth}}}{6}$$
$$(\omega_n)_{\text{Moon}} = \sqrt{\frac{g_{\text{Moon}}}{l}}$$
$$(\omega_n)_{\text{Moon}} = \frac{1}{\sqrt{6}}\sqrt{\frac{g_{\text{Earth}}}{l}} = \frac{1}{\sqrt{6}}(\omega_n)_{\text{Earth}}$$
$$(\omega_n)_{\text{Moon}} = 0.4082[\omega_n]_{\text{Earth}}$$

**21. (b)**

Absolute acceleration of $B$ will be vector addition of centripetal acceleration, tangential acceleration due to rotation of link $OA$ and Coriolis acceleration due to sliding motion of block $B$.

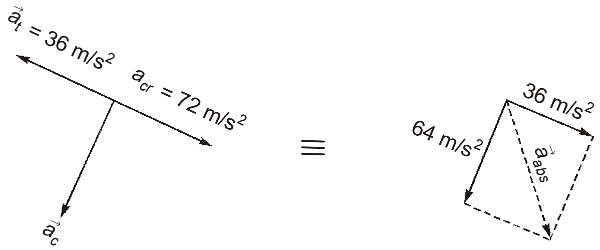
$$\vec{a}_c = \omega^2 r \text{ (towards centre)}$$
$$= 4^2 \times 4 = 64 \text{ m/s}^2$$
$$\vec{a}_t = r\alpha \text{ (tangential in direction of } \alpha)$$
$$= 4 \times 9 = 36 \text{ m/s}^2$$

Coriolis acceleration, $\vec{a}_{cr} = 2\, V_{\text{sliding}}\, \omega$

$$= 2 \times 9 \times 4 = 72 \text{ m/s}^2$$

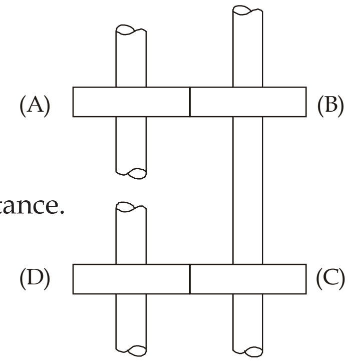
Direction of Coriolis acceleration $\rightarrow$ Rotating direction of sliding velocity which is inwards through 90° in the direction of the angular velocity which is counter clockwise.

Therefore direction of Coriolis acceleration is opposite and collinear to the tangential acceleration.




$$|\vec{a}_{abs}| = \sqrt{64^2 + 36^2} = 73.43 \text{ m/s}^2$$

**22. (d)**




Since $A$ and $C$ are driving gears

Let $\quad T_A = 18$ and $T_C = 24$

All the gears have same module so, to equate the center distance

$$T_A + T_B = T_C + T_C$$

Now, $\quad T_B = 36$ and $T_D = 30$ for the equal center distance.

Now,
$$\frac{N_D}{N_A} = \frac{N_D}{N_C} \times \frac{N_B}{N_A} = \frac{T_C}{T_D} \times \frac{T_A}{T_B}$$

$$\frac{200}{N_A} = \frac{24}{30} \times \frac{18}{36}$$

$$N_A = 500 \text{ rpm} = 52.36 \text{ rad/s}$$

**23. (c)**

$$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) = \ln\left(\frac{90}{70}\right)$$

$$\delta = 0.2513$$

and
$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

put
$$\zeta^2 = X$$

$$\delta = \frac{2\pi\sqrt{X}}{\sqrt{1-X}}$$

$$\delta^2 = \frac{4\pi^2 X}{1-X}$$

$$\Rightarrow \quad 0.2513^2(1 - X) = 4\pi^2 X$$

$$0.063 - 0.063X = 4\pi^2 X$$

$$0.063 = 39.54X$$

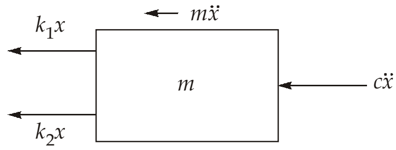
$$\Rightarrow \quad X = 1.59 \times 10^{-3}$$

$$\xi = \sqrt{X}$$

$$\xi = 0.0399 \simeq 0.040$$

**24. (c)**

As per given system after disturbing system by $x$.




Equation of motion $m\ddot{x}+c\dot{x}+(k_1+k_2)x = 0$

$$\ddot{x}+\frac{c}{m}\dot{x}+\frac{(k_1+k_2)x}{m} = 0$$

$$2\xi\omega_n = \frac{c}{m}$$

$$\omega_n = \sqrt{\frac{k_1+k_2}{m}}$$

For critically damped, $\xi = 1$

$$c = c_c = 2m\omega_n$$

$$c_c = 2\sqrt{m(k_1+k_2)}$$

**25. (a)**

$$R = 100 \text{ mm}, r = 45 \text{ mm}, \phi = 20°$$

$\therefore$ $$R_b = R\cos\phi = 100\cos 20° = 93.97 \text{ mm}$$

$$r_b = r\cos\phi = 45\cos 20° = 42.29 \text{ mm}$$

$\therefore$ Base circle radius is property of gear geometry so it does not change with center distance:

$$(R_b + r_b) = (R' + r')\cos\phi'$$

where $R' + r'$ is new center distance,

Previous center distance $= R + r = 145$ mm

New center distance $= 1.03 \times 145 = 149.35$ mm

$$\frac{93.97+42.29}{149.35} = \cos\phi'$$

$$\phi' = 24.17°$$

**26. (d)**

When piston is at 40% of stroke.

$$\text{Stroke} = 40 \text{ cm}; r = 20 \text{ cm}; l = 90 \text{ cm}$$

$$n = \frac{l}{r} = 4.5$$

$$x = r\left[(1-\cos\theta)+\left(n-\sqrt{n^2-\sin^2\theta}\right)\right]$$

$$0.4 \times 40 = 20\left[(1-\cos\theta)+\left(n-\sqrt{n^2-\sin^2\theta}\right)\right]$$

$$0.8 = 1 - \cos\theta + 4.5 - \sqrt{4.5^2-\sin^2\theta}$$

$$(\cos\theta - 4.7) = -\sqrt{4.5^2-\sin^2\theta}$$

$$\cos^2\theta + 4.7^2 - 2 \times 4.7 \times \cos\theta = 4.5^2 - \sin^2\theta$$

$$1 + 4.7^2 - 4.5^2 = 2 \times 4.7 \times \cos\theta$$

$$\cos\theta = 0.302$$

$$\theta = 72.41°$$

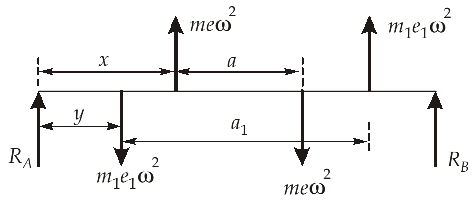
**27. (d)**

$$\Delta E = 3000 \text{ Nm}$$

rim contributes only 90% of MOI

$$k = 90\%$$
$$C_s = 0.2$$
$$\omega = \frac{2\pi\times 250}{60} = 26.1799 \text{ rad/s}^2$$
$$k \times \Delta E = I\omega^2 C_s$$
$$0.9 \times 3000 = I \times 26.1799^2 \times 0.2$$
$$I_r = 19.6968 \text{ kg-m}^2$$
$$m_r = \frac{I_r}{R^2} = \frac{19.6968}{(0.45)^2} = 97.268 \text{ kg}$$
$$m_r = 2\pi R \times b \times t \times \rho$$
$$97.268 = 2\pi \times 0.45 \times b \times t \times 7000$$
$$b \times t = 4.9145 \times 10^{-3} \text{ m}^2$$

**28.** **(d)**




from equilibrium equation

$$R_A + R_B + m_1e_1\omega^2 + me\omega^2 = m_1e_1\omega^2 + me\omega^2$$
$$R_A + R_B = 0$$

By taking moment about support '$A$',

$$m_1e_1\omega^2 \times y + me\omega^2(x + a) = me\omega^2 x + m_1e_1\omega^2(y + a_1) + R_B \times l$$
$$m_1e_1\omega^2 y + me\omega^2 x + me\omega^2 a = me\omega^2 x + m_1e_1\omega^2 y + m_1e_1\omega^2 a_1 + R_B \times l$$
$$R_B l = (me\omega^2 a - m_1e_1\omega^2 a_1)$$

if $$me\omega^2 a = m_1e_1\omega^2 a_1$$
$$mea = m_1e_1a_1 \text{ then } R_B = 0 \Rightarrow R_A = 0$$

**29.** **(d)**

$$\text{magnification factor} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

for peak amplitude

$$\frac{d(MF)}{dr} = 0 = \frac{-\frac{1}{2}\left[2(1-r^2)(-2r) + 4\xi^2 2r\right]}{\left[(1-r^2)^2 + (2\xi r)^2\right]^{3/2}} = 0$$

$$\Rightarrow \quad 2(1 - r^2)(2r) - 4\xi^2(2r) = 0$$
$$2\xi^2 = 1 - r^2$$
$$\Rightarrow \quad r = \sqrt{1 - 2\xi^2}$$

Given: $$\xi = 0.30$$
$$r = \sqrt{1 - 2\times(0.3)^2}$$
$$r = 0.9055$$

**30.** **(c)**

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