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STRENGTH OF MATERIAL

MECHANICAL ENGINEERING

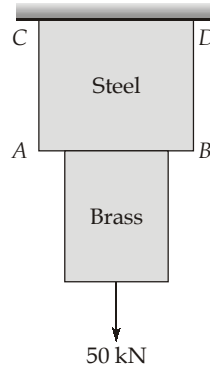
Date of Test : 28/11/2025

ANSWER KEY >

1. (d)	7. (b)	13. (b)	19. (d)	25. (a)
2. (d)	8. (d)	14. (b)	20. (a)	26. (d)
3. (d)	9. (c)	15. (d)	21. (c)	27. (c)
4. (d)	10. (d)	16. (c)	22. (a)	28. (b)
5. (b)	11. (b)	17. (b)	23. (d)	29. (c)
6. (a)	12. (b)	18. (b)	24. (d)	30. (d)

DETAILED EXPLANATIONS

1. (d)



$$(\sigma_{\max})_{\text{brass}} = (\sigma)_{AB}$$

$$(\sigma_{\max})_{\text{brass}} = \frac{50 \times 10^3 + 8500 \times 9.81 \times 50 \times 10^{-4} \times 5}{50 \times 10^{-4}} = 10.42 \text{ MPa}$$

$$\begin{aligned} (\sigma_{\max})_{\text{steel}} &= \frac{50 \times 10^3 + 8500 \times 9.81 \times 50 \times 10^{-4} \times 5 + 7800 \times 9.81 \times 10 \times 60 \times 10^{-4}}{60 \times 10^{-4}} \\ &= 9.44 \text{ MPa} \end{aligned}$$

2. (d)

The resultant couple acting on the beam due to end loadings is

$$= \frac{M}{2} - \frac{M}{3} = \frac{M}{6} \text{ (anticlockwise)}$$

It must be in equilibrium by another couple generated by the vertical reactions at B and C.

$$\therefore R \times l = \frac{M}{6}$$

$$\Rightarrow R = \frac{M}{6l}$$

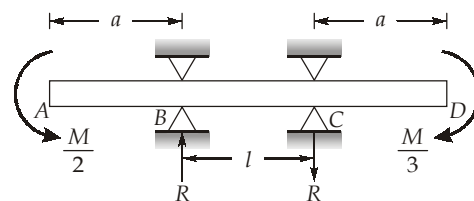
Shear force between B and C

$$= R = \frac{M}{6l} = \text{Constant}$$

Bending moment between B and C

$$= -\frac{M}{2} + \frac{M}{6l}x \quad (x \text{ is taken from } B)$$

$\therefore$ Bending moment is linearly varying between the supports.

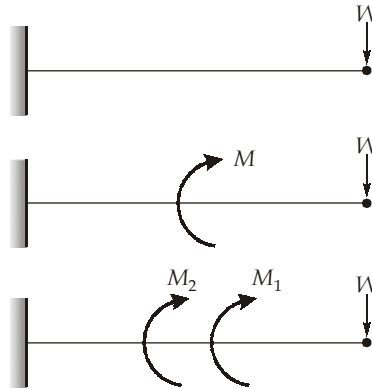


3. (d)

The shear centre for any transverse section of a beam is the point of intersection of the bending axis and the plane of the transverse section.

4. (d)

For the given SFD, infinite number of loading diagrams will be possible. e.g.



Similarly, infinite number of concentrated moments are possible and all will have the same SFD.

5. (b)

Given: $D = 10$ cm, $d = 1$ cm,

$$\text{Spring index (c)} = \frac{D}{d} = 10$$

Wahl's correction factor (k_w) includes both direct shear and curvature effect

$$\Rightarrow k_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 10 - 1}{4 \times 10 - 4} + \frac{0.615}{10} = 1.1448$$

6. (a)

$$\begin{aligned} \tan 2\theta_{p_1} &= \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} \\ \theta_{p_1} &= \frac{1}{2} \tan^{-1} \left(\frac{2 \times (-5)}{8 - 5} \right) \\ &= \frac{1}{2} \tan^{-1} \left(\frac{-10}{3} \right) = \frac{1}{2} \times -73.3 = -36.65^\circ \end{aligned}$$

$$\text{and } \theta_{p_2} = \theta_{p_1} + 90 = -36.65 + 90 = 53.349^\circ$$

7. (b)

We know that,

$$E = \frac{9KG}{3K + G}$$

$$\Rightarrow 3KE + EG = 9KG$$

$$\Rightarrow 9KG - 3KE = EG$$

$$\Rightarrow K(9G - 3E) = GE$$

$$\begin{aligned} \Rightarrow K &= \frac{GE}{9G - 3E} = \frac{120 \times 42}{9 \times 42 - 3 \times 120} \\ &= 280 \text{ GPa} \end{aligned}$$

8. (d)

A beam is said to be of uniform strength, when bending stress is same at every section, along its longitudinal axis.

9. (c)

As we know that, for a beam

$$\theta_{B/A} = \frac{[\text{Area under BMD from A to B}]}{EI}$$

For beam fixed at both ends

$$\theta_A = \theta_B = 0$$

$$\text{So, } \theta_{B/A} = 0 = \frac{[\text{Area under BMD}]}{EI} \quad [EI \neq 0]$$

⇒ So, sum of area under BMD will be always equal to zero between the two fixed ends.

10. (d)

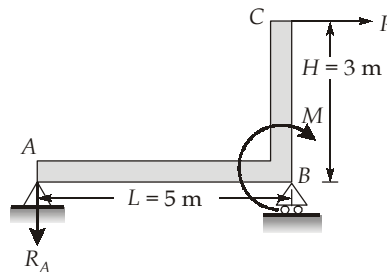
Effective equivalent length,

$$l_e = \frac{L}{3}$$

As,

$$P_e = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{(L/3)^2} = \frac{9\pi^2 EI}{L^2}$$

11. (b)



Here, maximum rotation of joint B is asked but concentrated moment at B is not present. So, assume a dummy moment M at B.

Consider equilibrium of bracket:

$$P \times H + M = R_A \times L$$

$$\therefore R_A = \left(\frac{PH + M}{L} \right)$$

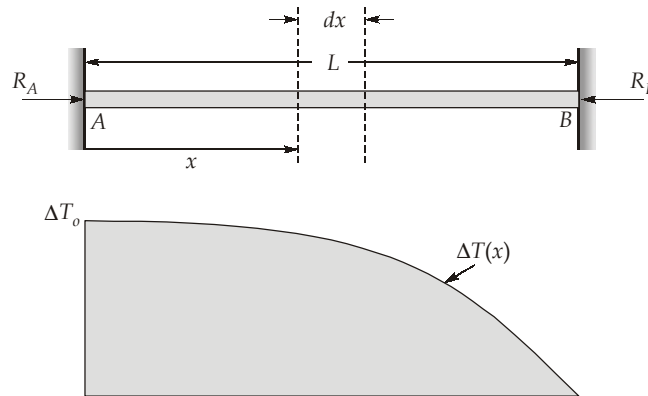
The total strain energy due to bending of the bracket:

$$\begin{aligned} \int_0^H \frac{(Px)^2}{2EI} dx + \int_0^L \left[\frac{\left(\frac{PH + M}{L} \right)x}{2EI} \right]^2 dx &= \frac{P^2 H^3}{6EI} + \int_0^L \left(\frac{P^2 H^2 x^2 + M^2 x^2 + 2PHMx^2}{2EIL^2} \right) dx \\ &= \frac{P^2 H^3}{6EI} + \frac{P^2 H^2 L^3}{6EIL^2} + \frac{M^2 L^3}{6EIL^2} + \frac{2PHML^3}{6EIL^2} \end{aligned}$$

Differentiating with respect of M and putting,
 $M = 0$, we get,

$$\begin{aligned}\theta_B &= \frac{2PHL}{6EI} = \frac{PHL}{3EI} = \frac{20 \times 10^3 \times 3 \times 5}{3 \times 5 \times 10^6} \\ &= 0.02 \text{ radian} = 1.146^\circ\end{aligned}$$

12. (b)



Here the temperature is varying non-uniformly so take a small element of length dx at a distance of x from A.

Expansion due to temperature variation:

$$\begin{aligned}(d\delta)_T &= \alpha(\Delta T)dx \\ &= \alpha\Delta T_0 \left(1 - \frac{x^2}{L^2}\right) dx\end{aligned}$$

$$\therefore \delta_T = \int_0^L \alpha\Delta T_0 \left(1 - \frac{x^2}{L^2}\right) dx = \alpha\Delta T_0 \left(L - \frac{L}{3}\right) = \frac{2}{3}\alpha\Delta T_0 L$$

This expansion is restricted by the supports.

So, by equation of compatibility,

$$\begin{aligned}\delta_T + (\delta L)_{\text{reactions}} &= 0 \\ \Rightarrow \frac{2}{3}\alpha\Delta T_0 L + \frac{RL}{AE} &= 0 \quad (\because R_A = R_B = R)\end{aligned}$$

$$\Rightarrow R = -\frac{2}{3}\alpha\Delta T_0 AE$$

$$\Rightarrow R = \frac{2}{3}EA\alpha\Delta T_0 \text{ (Compressive)}$$

$$\therefore \text{Thermal stress, } \sigma_T = \frac{R}{A} = \frac{2}{3}E\alpha\Delta T_0$$

13. (b)

The radius of curvature (R_C) of the bent wire is the distance from the centre of the drum to the neutral axis of the cross-section of the wire.

$$\therefore R_C = R + \frac{d}{2}$$

From bending equation we have,

$$\frac{M}{I} = \frac{E}{R_C}$$

$$\therefore M = \frac{EI}{R_C} = \frac{EI}{R + \frac{d}{2}} = \frac{2EI}{2R + d}$$

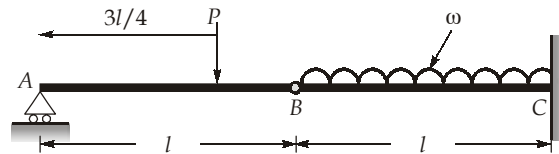
$$\text{or, } M = \frac{2E \times \frac{\pi}{64} \times d^4}{2R + d} = \frac{\pi E d^4}{32(2R + d)} \quad \text{Ans.}$$

Maximum bending stress,

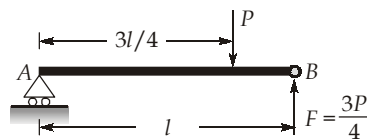
$$(\sigma_b)_{\max} = \frac{M}{Z}; \quad Z = \text{Section modulus}$$

$$\therefore (\sigma_b)_{\max} = \frac{\pi E d^4}{32(2R + d)} \times \frac{32}{\pi d^3} = \frac{Ed}{2R + d} \quad \text{Ans.}$$

14. (b)



For AB:

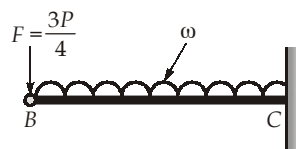


Taking moment about 'A'

$$F \times l = P \times \frac{3l}{4}$$

$$\therefore F = \frac{3P}{4}$$

For BC:

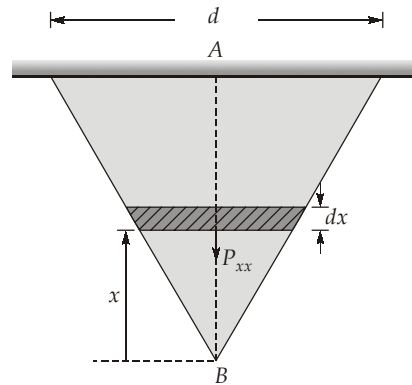


This is a cantilever subjected to one concentrated point load at the end and a uniformly distributed load 'w' throughout.

$$\text{so, } \delta_B = \frac{Fl^3}{3EI} + \frac{wl^4}{8EI}$$

$$\delta_B = \frac{\left(\frac{3P}{4}\right)L^3}{3EI} + \frac{wl^4}{8EI} = \frac{PL^3}{4EI} + \frac{wl^4}{8EI}$$

15. (d)



$$d_{xx} = \frac{dx}{L}$$

$$A_{xx} = \frac{\pi}{4} \times (d_{xx})^2 = \frac{\pi}{4} \times \left(\frac{dx}{L}\right)^2 = \frac{Ax^2}{L^2}$$

[Let the area at the fixed end, $A = \frac{\pi}{4}d^2$]

$$\therefore P_{xx} = \gamma \cdot \frac{A_{xx}x}{3} = \frac{\gamma Ax^3}{3L^2}$$

So, axial loading diagram is cubic parabola.

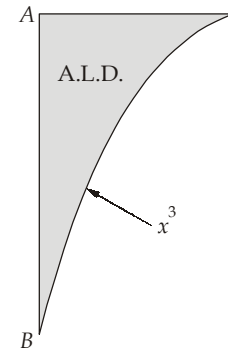
$$(\sigma_{\text{axial}})_{xx} = \frac{P_{xx}}{A_{xx}} = \frac{\gamma x}{3}$$

$$(\sigma_{\text{max}})_{\text{axial}} = \sigma_A = \frac{\gamma L}{3}$$

$$\therefore (\delta L)_{\text{conical bar}} = \int_0^L (\delta \text{strip}) = \int_0^L \frac{P_{xx}}{A_{xx}E_{xx}} dx$$

$$\therefore (\delta L)_{\text{conical bar}} = \int_0^L \frac{\gamma A_{xx}x}{3} \times \frac{1}{A_{xx}E} dx$$

$$= \int_0^L \frac{\gamma x}{3E} dx = \frac{\gamma L^2}{6E}$$



Under self weight, stress is independent of area so where the load is maximum, stress will be maximum.

16. (c)

From the given figure, for x face:

$$\sigma_x = 20 \text{ MPa}, \sigma_y = 30 \text{ MPa}, \tau_{xy} = 40 \text{ MPa}, \theta = -30^\circ \text{ (A.C.W.)}$$

$$\therefore \sigma'_x = \left(\frac{20+30}{2}\right) + \left(\frac{20-30}{2}\right) \cos(-60^\circ) + 40 \times \sin(-60^\circ)$$

$$= -12.14 \text{ MPa}$$

For y face:

$$\sigma'_y = \left(\frac{30+20}{2} \right) + \left(\frac{30-20}{2} \right) \cos(-60^\circ) - 40 \times \sin(-60^\circ)$$

$$= 62.14 \text{ MPa}$$

$$\tau_{x'y'} = - \left(\frac{20-30}{2} \right) \sin(-60^\circ) + 40 \times \cos(-60^\circ)$$

$$= 15.67 \text{ MPa}$$

17. (b)

Maximum shear stress in the shaft:

The net torque acting on the shaft is

$$\frac{1}{2} \times t \times L = \frac{tL}{2}$$

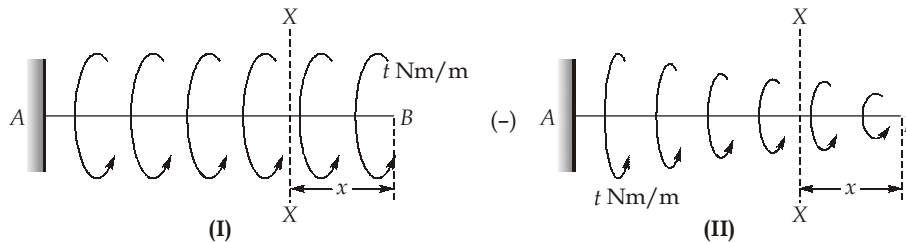
So, the reaction torque at 'A' = $\frac{tL}{2}$ in opposite direction.

$$\therefore \text{Maximum torque} = \frac{tL}{2} \text{ (at A)}$$

$\therefore$ Maximum shear stress in the shaft

$$\tau_{\max} = \frac{16T}{\pi d^3} = \frac{16 \times tL}{2 \times \pi d^3} = \frac{8tL}{\pi d^3}$$

Angle of twist of the shaft:



Consider a section x-x on the both shafts of length 'dx'.

$$(d\theta)_I = \frac{T_{xx} dx}{GJ} = \frac{32(tx) dx}{G \times \pi d^4} \quad [\because (T_{xx})_I = tx]$$

$$\therefore \theta_I = \int_0^L \frac{32tx dx}{G\pi d^4} = \frac{16tL^2}{\pi G d^4}$$

$$\begin{aligned} (d\theta)_{II} &= \frac{T_{xx} dx}{GJ} = \frac{\left(\frac{tx^2}{2L} \right) dx}{GJ} \quad [\because (T_{xx})_{II} = \frac{tx^2}{2L}] \\ &= \frac{tx^2 dx}{2L \times G \times \frac{\pi}{32} d^4} \end{aligned}$$

$$\therefore \theta_{II} = \int_0^L \frac{16tx^2 dx}{LG\pi d^4} = \frac{16tL^2}{3G\pi d^4}$$

$\therefore$ Angle of twist of the shaft,

$$\theta_{BA} = \theta_I - \theta_{II}$$

$$\theta_{BA} = \frac{16tL^2}{\pi Gd^4} - \frac{16}{3} \times \frac{tL^2}{G\pi d^4} = \left(16 - \frac{16}{3}\right) \left(\frac{tL^2}{G\pi d^4}\right)$$

$$\theta_{BA} = \frac{32tL^2}{3\pi Gd^4}$$

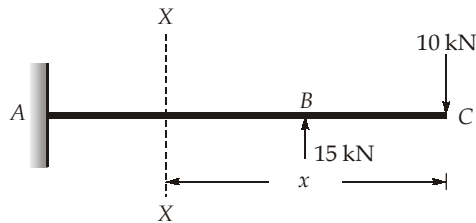
18. (b)



The deflection of point B is zero so,

$$\frac{10 \times (0.75)^3}{3EI} + \frac{2.5 \times (0.75)^2}{2EI} - \frac{R_B \times (0.75)^3}{3EI} = 0$$

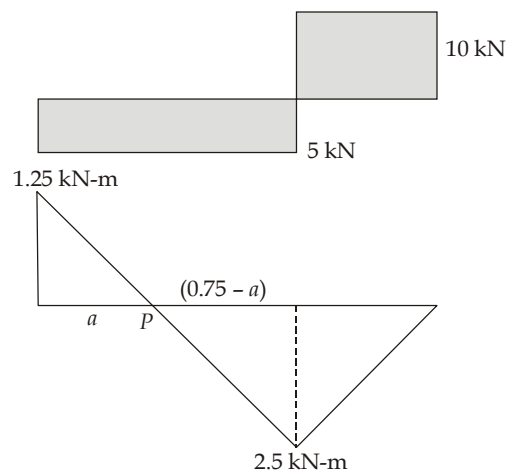
$$\Rightarrow R_B = 15 \text{ kN}$$



Net moment at A:

$$M_A = 15 \times 0.75 - 10 \times 1$$

$$= 1.25 \text{ kNm}$$



$$\therefore \frac{1.25}{a} = \frac{2.5}{(0.75 - a)}$$

$$\Rightarrow a = 0.25 \text{ m}$$

$\therefore$ The point of contraflexure will lie at 0.25 m from fixed end.

19. (d)

Given: $P = 13 \text{ kN}$, $C = 5$, $G = 80 \text{ GPa}$, $U = 2.1 \times 10^6 \text{ Nmm}$, $n = 8$

Spring is subjected an axial pull, so the wire will undergo twisting. Therefore energy stored in the

spring is $(U) = \frac{T^2 L}{2GJ}$

$$T = P \frac{D}{2}; L = \pi D n; J = \frac{\pi d^4}{32}$$

$$\Rightarrow U = \frac{P^2 D^2 \times \pi D n \times 32}{4 \times G \times \pi d^4} = \frac{4P^2 D^3 n}{G d^4}$$

$$\Rightarrow U = \frac{4P^2 C^3 n}{G d}$$

For safe design, $\frac{4P^2 C^3 n}{G d} \leq 2.1 \times 10^6 \text{ N-mm}$

$$\frac{4 \times (13000)^2 \times (5)^3 \times 8}{80 \times 10^3 \times d} \leq 21 \times 10^6$$

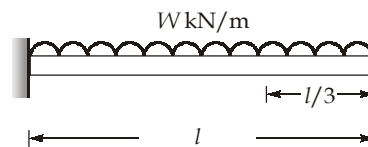
$$d \geq 4.023 \text{ mm}$$

$$\Rightarrow d = 5 \text{ mm}$$

$$\frac{D}{d} = 5$$

$$D = 20.115 \simeq 21 \text{ mm}$$

20. (a)



Maximum bending will be at fixed end,

i.e. $M = (Wl) \times \frac{l}{2} = \frac{Wl^2}{2}$

Bending moment at $\frac{l}{3}$ distance from free end

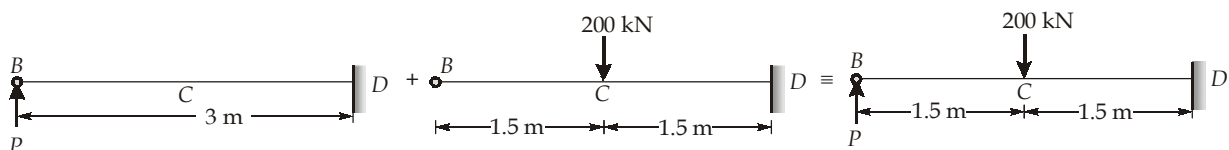
$$\Rightarrow M' = \left(\frac{Wl}{3} \right) \times \left(\frac{l}{6} \right) = \frac{Wl^2}{18}$$

$$M' = \frac{2}{9} \times \frac{Wl^2}{2} = \frac{M}{9} = 0.1111 = 11.11\% M$$

21. (c)

As given that member AB is rigid, so $(\delta_B)_{\text{net}} = 0$.

Let 'P' be the reaction at 'B' due to resistance of AB member.

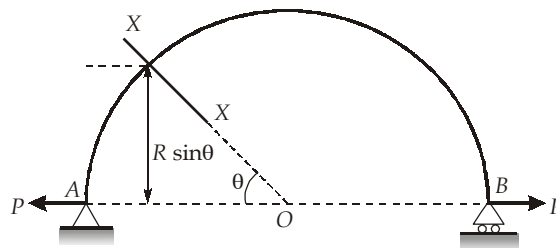


$$\begin{aligned}
 \text{So, } \frac{P_1 L^3}{3EI} &= \frac{P_2 (L/2)^3}{3EI} + \frac{P(L/2)^2}{2EI} \times \frac{L}{2} \\
 \Rightarrow \frac{P_1 L^3}{3EI} &= \frac{PL^3}{EI} \left[\frac{1}{24} + \frac{1}{16} \right] \\
 \Rightarrow \frac{P_1 L^3}{3EI} &= \frac{5}{48} \frac{PL^3}{EI} = \frac{5}{48} \times 200 \times \left(\frac{L^3}{EI} \right) \\
 \Rightarrow P_1 &= 62.5 \text{ kN}
 \end{aligned}$$

22. (a)

- Maximum deflection of beam is $\frac{5WL^4}{384EI}$.

23. (d)



Consider a section X-X at any angle θ with horizontal.

The bending moment at section X-X is given as,

$$M_x = PR \sin \theta$$

Strain energy stored in arch,

$$\begin{aligned}
 U &= \int \frac{M_x^2 dx}{2EI} = \int_0^\pi \frac{(PR \sin \theta)^2 (R d\theta)}{2EI} \\
 &= \frac{P^2 R^3}{2EI} \times 2 \int_0^{\pi/2} \sin^2 \theta d\theta = \frac{P^2 R^3}{EI} \times \frac{\pi}{4}
 \end{aligned}$$

24. (d)

Actual column is having end constraints as fixed and free but it was considered as fixed and hinged column.

$$(L_{eq})_{actual} = 2L$$

$$\Rightarrow (P_e)_{actual} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{4L^2} \quad \dots (i)$$

$$\begin{aligned}
 (P_e)_{assumed} &= \frac{\pi^2 EI}{(L/\sqrt{2})^2} \quad \because (L_{eq}) = \frac{L}{\sqrt{2}} \\
 &= \frac{2\pi^2 EI}{L^2}
 \end{aligned}$$

$$\% \text{error} = \frac{\text{Error}}{(P_e)_{Actual}} = \frac{\frac{\pi^2 EI}{L^2} \left[2 - \frac{1}{4} \right]}{\frac{\pi^2 EI}{L^2} \times \frac{1}{4}} = 700\%$$

Comment : The boundary value big impact on critical load calculation.

25. (a)
Cylindrical shell,

$$\sigma_h = \frac{PD}{2t}$$

$$\sigma_L = \frac{PD}{4t}$$

$$\text{Volumetric strain} = \frac{PD}{4tE}(5 - 4\mu)$$

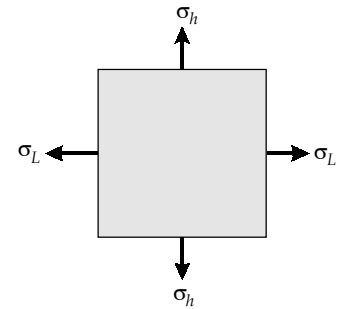
$$= \frac{\sigma_h}{2E}(5 - 4\mu)$$

$$\epsilon_V = \frac{\Delta V}{V} = \frac{5000}{\frac{\pi}{4} \times 300^2 \times 500}$$

$$= \frac{\sigma_h \times (5 - 4 \times 0.24)}{2 \times 200 \times 10^3}$$

$\Rightarrow$

$$\sigma_h = 14 \text{ MPa}$$



26. (d)

$$\epsilon_A = \epsilon_{0^\circ} = \epsilon_x = 500 \times 10^{-6}$$

$$\epsilon_C = \epsilon_{90^\circ} = \epsilon_y = -100 \times 10^{-6}$$

$$(\epsilon_n)_\theta = \left(\frac{\epsilon_x + \epsilon_y}{2} \right) + \left(\frac{\epsilon_x - \epsilon_y}{2} \right) \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$(\epsilon_n)_{\theta=45^\circ} = \epsilon_B = \left[\left(\frac{500 - 100}{2} \right) + \left(\frac{500 + 100}{2} \right) \cos 90^\circ + \frac{\gamma_{xy}}{2} \sin 90^\circ \right] \times 10^{-6}$$

$$= 450 \times 10^{-6}$$

$$\gamma_{xy} = 500 \times 10^{-6}$$

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2} \right)^2 + \left(\frac{\gamma_{xy}}{2} \right)^2}$$

$$= \left(200 \pm \sqrt{(300)^2 + (250)^2} \right) \times 10^{-6}$$

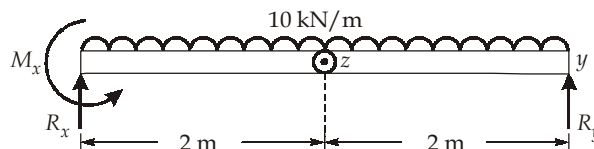
$$= (200 \pm 390.5) \times 10^{-6}$$

$$= 590.5 \times 10^{-6}, -190.5 \times 10^{-6}$$

$$\epsilon_1 = 590.5 \times 10^{-6}$$

27. (c)

Shear force at section/support x will be equal to the vertical reaction at x



$$R_x + R_y = 10 \times 4 = 40 \text{ kN}$$

$$(M_z) = 0 \quad (\text{From right side})$$

$$R_y \times 2 = 10 \times 2 \times 1$$

$$R_y = 10 \text{ kN}$$

$$R_x = 40 \text{ kN} - 10 \text{ kN} = 30 \text{ kN}$$

28. (b)

Induced maximum normal stress due to bending,

$$\begin{aligned}
 M_{\max} &= \text{Wind pressure} \times \text{Area of sign board} \times \text{Distance} \\
 &= 7.5 \times 10^3 \times 3 \times 1 \times 6.0 \text{ Nm} \\
 &= 135 \times 1000 \text{ Nm}
 \end{aligned}$$

$$\text{Section modulus} = \left(\frac{0.25^4 - d^4}{12 \times 0.125} \right)$$

$$\sigma_{\max} = \frac{M_{\max}}{Z}$$

$$\Rightarrow \frac{10^3 \times 135 \times 12 \times 0.125}{(0.25^4 - d^4)} \leq 240 \times 10^6$$

$$0.25^4 - d^4 \geq 8.4375 \times 10^{-4}$$

$$d^4 \geq 3.0625 \times 10^{-3}$$

$$d \geq 235.244$$

$$d = 236 \text{ mm}$$

29. (c)

Strain at $\delta = 6.5 \text{ mm}$

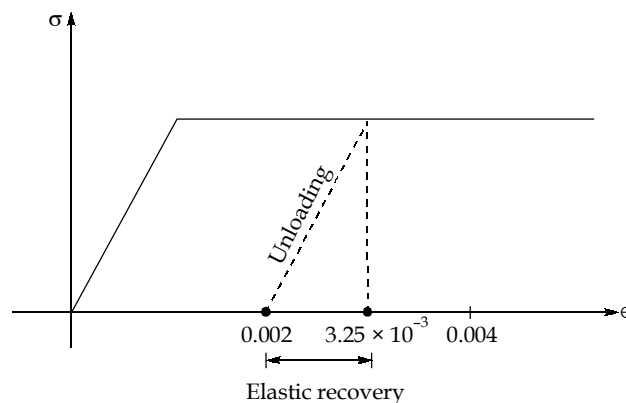
$$\epsilon = \frac{\delta}{L} = \frac{6.5}{2000} = 3.25 \times 10^{-3}$$

After removal of load there will be elastic recovery.

$$\epsilon_{\text{er}} = \frac{\sigma_y}{E} = \frac{250}{200 \times 10^3} = 1.25 \times 10^{-3}$$

$$\begin{aligned}
 \text{Plastic strain} &= \epsilon - \epsilon_{\text{er}} = 3.25 \times 10^{-3} - 1.25 \times 10^{-3} \\
 &= 2 \times 10^{-3}
 \end{aligned}$$

$$\text{Change in final length} = 2 \times 10^{-3} \times 2000 = 4 \text{ mm}$$



30. (d)

Torsional moment acting on AB is $T = P \times b$

$$\text{Moment of inertia, } J = \frac{\pi}{32} \times d^4$$

$$\text{Angle of twist, } \theta = \frac{TL}{GJ} = \frac{(P \times b) \times a}{G \times \frac{\pi}{32} d^4}$$

$$\text{Deflection of } c = b \times \theta = \frac{32Pb^2a}{\pi Gd^4}$$

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