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# CONTROL SYSTEMS

EC | EE

**Date of Test : 22/12/2021****ANSWER KEY ➤**

- |        |         |         |         |         |
|--------|---------|---------|---------|---------|
| 1. (b) | 7. (d)  | 13. (c) | 19. (d) | 25. (a) |
| 2. (d) | 8. (b)  | 14. (c) | 20. (b) | 26. (d) |
| 3. (b) | 9. (d)  | 15. (a) | 21. (b) | 27. (b) |
| 4. (d) | 10. (a) | 16. (b) | 22. (b) | 28. (b) |
| 5. (b) | 11. (a) | 17. (c) | 23. (b) | 29. (b) |
| 6. (c) | 12. (d) | 18. (c) | 24. (c) | 30. (a) |

## Detailed Explanations

1. (b)

As the three blocks are connected in cascade the overall transfer function is given by the multiplication of individual blocks.

$$\begin{aligned} \therefore x_1 \times x_2 \times x_3 &= \frac{(s+1)}{s(s+2)(s+3)} \\ \frac{1}{s(s+2)} \times \frac{(s+2)}{(s+3)} \times x_3 &= \frac{(s+1)}{s(s+2)(s+3)} \\ x_3 &= \frac{(s+1)}{(s+2)} \end{aligned}$$

2. (d)

At  $\omega = 0$ , the plot for system I, started from  $-270^\circ$  hence it represents a type 3 system.

At  $\omega = 0$ , the plot for system II has slope of 0 dB/dec and therefore it is a type 0 system.

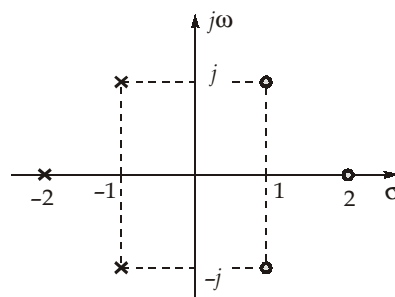
3. (b)

The Routh's table can be formed as

$s^3$	4	4
$s^2$	$3(s^2)$	$3(0)$
$s^1$	$(0)6$	$(0)$
$s^0$	$\frac{18-0}{6} = 3$	

as there is no sign change in the first column of Routh array thus, there will be no pole lie on the RHS of  $s$ -plane. Also the row of zero occurs that indicates the complex conjugate poles exists on  $j\omega$  axis.

4. (d)



5. (b)

Here, the encirclement to the critical point is 1 (in clock wise direction) and 2 (in counter clockwise direction).

$$\therefore N = -1 + 2 = 1$$

6. (c)

Closed loop transfer function  $\frac{C(s)}{R(s)} = \frac{K}{s^2 + 10s + K}$

Here,  $2\xi\omega_n = 10$   
 $\xi\omega_n = 5$

Therefore,  $\omega_n = \frac{5}{0.75}$

$\therefore K = \omega_n^2 = \left(\frac{5}{0.75}\right)^2 = 44.44$

7. (d)

$$T(s) = \frac{C(s)}{R(s)} = \frac{10(s+2)}{(s^2 + 4s + 8)}$$

$\therefore$  The step response  $C(s)$  is,

$$C(s) = \frac{10(s+2)}{(s^2 + 4s + 8)} \times R(s) = \frac{10(s+2)}{s(s^2 + 4s + 8)}$$

and the steady state response is,

$$\lim_{t \rightarrow \infty} c(t) = \lim_{s \rightarrow 0} sC(s) = \lim_{s \rightarrow 0} s \frac{10(s+2)}{s(s^2 + 4s + 8)} = \frac{10}{4} = 2.5$$

8. (b)

The roots of the characteristic equation are given by,

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{1-\xi^2}$$

Here,  $-\xi\omega_n = -3$

$$\Rightarrow \omega_n = \frac{3}{\xi} \quad \dots(i)$$

and  $\omega_n\sqrt{1-\xi^2} = 2 = \omega_d \quad \dots(ii)$

By putting the value of  $\omega_n$  in equation (ii), we get,

$$\frac{3}{\xi}\sqrt{1-\xi^2} = 2$$

$$\frac{9}{\xi^2} \times (1-\xi^2) = 4$$

or  $9(1-\xi^2) = 4\xi^2$

$$9 - 9\xi^2 - 4\xi^2 = 0$$

or  $13\xi^2 = 9$

or  $\xi = \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}}$

9. (d)

The steady state offset is given by,

$$e_{\text{off}} = \lim_{s \rightarrow 0} sC(s) \Big|_{R(s)=0}$$

When  $R(s) = 0$ ,

$$C(s) = D(s) - C(s) \left[ \frac{4K}{(2s+1)} \right]$$

$$C(s) = \frac{D(s)}{1 + \frac{4K}{(2s+1)}}$$

$$\therefore e_{\text{off}} = \lim_{s \rightarrow 0} \frac{s \cdot \frac{0.3}{s} (2s+1)}{(2s+1) + 4K}$$

or

$$e_{\text{off}} = \frac{0.3}{4K+1}$$

For  $e_{\text{off}} = 0$ ,

$$K = \infty$$

10. (a)

The characteristic equation is given by,

$$1 + G(s)H(s) = 0$$

Here,

$$G(s) = \frac{4}{s(s+0.2)} \text{ and } H(s) = (1+2s)$$

$$\therefore 1 + \frac{4(1+2s)}{s(s+0.2)} = 0$$

$$s^2 + 0.2s + 8s + 4 = 0$$

$$s^2 + 8.2s + 4 = 0$$

11. (a)

The transfer function of the above circuit is,

$$\frac{E_0(s)}{E_i(s)} = \frac{R_2 R_4 (R_1 C_1 s + 1)}{R_1 R_3 (R_2 C_2 s + 1)} = \frac{R_4 C_1}{R_3 C_2} \left( \frac{s + \frac{1}{R_1 C_1}}{s + \frac{1}{R_2 C_2}} \right)$$

$$= K \alpha \left( \frac{1 + sT}{1 + \alpha sT} \right) = K_c \left( \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \right)$$

Here,  $T = R_1 C_1$  and  $\alpha T = R_2 C_2$

If  $R_1 C_1 > R_2 C_2$  then  $\alpha < 1$ .

Thus, it represents a phase lead network.

12. (d)

The CE of the given system is,

$$1 + G(s) = 0$$

$$1 + \frac{2s^2 + as + 50}{(s+1)^2(s+2)} = 0$$

$$(s^2 + 2s + 1)(s + 2) + 2s^2 + as + 50 = 0$$

$$s^3 + 2s^2 + s + 2s^2 + 4s + 2 + 2s^2 + as + 50 = 0$$

$$s^3 + 6s^2 + (5 + a)s + 52 = 0$$

For system to be stable

$$(5 + a)6 > 52$$

$$30 + 6a > 52$$

$$6a > (52 - 30)$$

$$a > \frac{22}{6}$$

$$a > 3.667$$

13. (c)

As the system is said to be stable,  
Therefore, no open loop pole in the RHS.

$$\therefore P = 0$$

The intersection point  $\left(-\frac{4}{5}K, 0\right)$  and  $K > \frac{5}{4}$ .

$$\therefore \frac{4}{5}K > 1$$

$$\text{or } -\frac{4}{5}K < -1$$

That means the Nyquist plot encircles the critical point two times in the clockwise direction

$$\text{Hence, } N = -2$$

$$\Rightarrow N = P - Z$$

$$\Rightarrow -2 = -Z \text{ or } Z = 2$$

14. (c)

The closed loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{4s + 1}{4s^2 + 4s + 1} = \frac{1}{4} \times \frac{4s + 1}{\left(s + \frac{1}{2}\right)^2}$$

$$\text{For unit step input, } R(s) = \frac{1}{s}$$

$$C(s) = \frac{1}{4} \times \frac{4s + 1}{s \left(s + \frac{1}{2}\right)^2} \quad \dots (i)$$

15. (a)

1<sup>st</sup> line has slope of 12 dB/oct = 40 dB/dec, thus there is  $s^2$  term in the numerator.

At  $\omega = 0.5$  rad/sec, slope changes from +12 dB/oct to +6 dB/oct. Therefore, the term  $\left(1 + \frac{s}{0.5}\right)$

should be added to the denominator.

At  $\omega = 1$  rad/sec, slope changes from +6 dB/oct to 0 dB/oct, thus, a term  $(1 + s)$  should be added to the denominator.

At  $\omega = 5$  rad/sec, again the slope changes from 0 dB/oct to -6 dB/oct, thus, the term  $\left(1 + \frac{s}{5}\right)$

should be added to the denominator.

$\therefore$  The transfer function can be written as

$$T(s) = \frac{K(s^2)}{\left(1 + \frac{s}{0.5}\right)\left(1 + \frac{s}{5}\right)(1+s)}$$

Determining the  $K$  value, we get,

$$32 = 20\log K + 40\log \omega - 20\log \frac{\omega}{0.5} - 20\log \frac{\omega}{5} - 20\log \omega$$

$$32|_{\omega=5 \text{ rad/sec}} = 20\log K + 40\log 5 - 20\log \frac{5}{0.5} - 20\log 1 - 20\log 5$$

$$= 20\log K + 27.95 - 20 - 0 - 13.97$$

$$32 = 20\log K - 6.02$$

$$\log K = \frac{38.02}{20} = 1.901$$

$$K = 79.615$$

$$\therefore \text{The overall transfer function is, } T(s) = \frac{79.6s^2}{(2s+1)(s+1)(0.2s+1)}$$

16. (b)

The state equation can be written as.

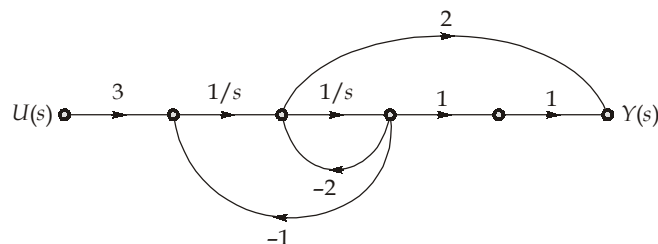
$$\dot{x}_1 = -2x_1 + x_2$$

$$\dot{x}_2 = -x_1 + 3u$$

and

$$y = x_1 + 2x_2$$

$\therefore$  The signal flow graph corresponding to the state equations is



17. (c)

For the given system put  $s = j\omega$

$$\text{we get, } G(j\omega)H(j\omega) = \frac{32}{j\omega(j\omega + \sqrt{6})^3}$$

$$|G(j\omega)H(j\omega)| = \frac{32}{\omega(\sqrt{\omega^2 + 6})^3}$$

$$\text{at } \omega = \sqrt{2} \text{ rad/sec,}$$

$$|G(\sqrt{2})H(\sqrt{2})| = \frac{32}{\sqrt{2}(\sqrt{2}+6)^3} = 1$$

$$\therefore |G(j\omega)H(j\omega)|_{\omega=\sqrt{2}\frac{\text{rad}}{\text{sec}}} = 1$$

Thus, the gain cross over frequency =  $\sqrt{2}$  rad/sec

$$\text{Also,} \quad -180 = -90 - 3 \tan^{-1} \frac{\omega_{pc}}{\sqrt{6}}$$

$$\tan^{-1} \frac{\omega_{pc}}{\sqrt{6}} = \frac{-90}{-3}$$

$$\Rightarrow \quad \frac{\omega_{pc}}{\sqrt{6}} = \tan 30^\circ$$

$$\therefore \quad \omega_{pc} = \sqrt{2} \text{ rad/sec}$$

$$\therefore \quad \omega_{gc} = \omega_{pc}$$

$$GM = 0 \text{ dB and PM} = 0^\circ$$

The system represents a marginally stable system.

18. (c)

$$\frac{C(s)}{R(s)} = \frac{1}{1+s\tau}$$

The response of the system,

$$C(s) = \frac{1}{1+s\tau} \times R(s)$$

$$C(s) = \frac{1}{1+s\tau} \times \frac{5}{s}$$

Taking inverse Laplace transform, we get,

$$c(t) = 5(1 - e^{-t/\tau}) u(t)$$

Now,

$$c(t) = 4.2 \quad \text{at } t = 0.35 \text{ msec}$$

By putting these values, we get,

$$4.2 = 5(1 - e^{-0.35/\tau})$$

$$0.16 = e^{-0.35/\tau}$$

or

$$\tau = 0.19 \text{ msec}$$

19. (d)

Given,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K+p}{s^2+qs+p}$$

For  $H(s) = 1$ ,

$$\frac{G(s)}{1+G(s)} = \frac{K+p}{s^2+qs+p}$$

or

$$G(s) [s^2 + qs + p] = (K+p) + G(K+p)$$

or

$$G(s) = \frac{K+p}{s^2+qs+p-K-p} = \frac{K+p}{s^2+qs-K}$$

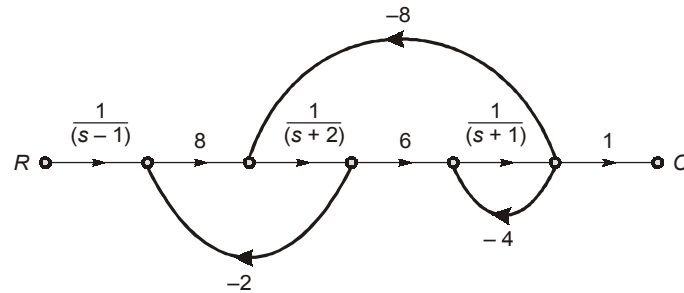
This is a type-0 system.

For a type-0 system, for unit ramp input,

$$e_{ss} = \infty$$

20. (b)

The signal flow graph of the given system can be drawn as,



The forward path,

$$F_1 = \frac{48}{(s-1)(s+2)(s+1)}$$

The feedback loops,

$$L_1 = -\frac{16}{s+2}$$

$$L_2 = -\frac{4}{(s+1)}$$

$$L_3 = -\frac{48}{(s+2)(s+1)}$$

The non-touching loop pair,

$$L_{1,2} = \frac{-(16) \times (-4)}{(s+2)(s+1)}$$

∴ The closed loop transfer function using Mason's gain formula is,

$$T(s) = \frac{\frac{48}{(s-1)(s+2)(s+1)}}{1 + \frac{16}{s+2} + \frac{4}{s+1} + \frac{48}{(s+1)(s+2)} + \frac{64}{(s+1)(s+2)}}$$

$$= \frac{\frac{48}{(s-1)}}{(s+1)(s+2) + 16(s+1) + 4(s+2) + 112}$$

$$T(s) = \frac{48}{(s-1)[s^2 + 3s + 2 + 16s + 16 + 4s + 8 + 112]}$$

$$= \frac{48}{s^3 + 23s^2 + 138s - s^2 - 23s - 138}$$

$$= \frac{48}{s^3 + 22s^2 + 115s - 138}$$

∴ The characteristic equation is,

$$s^3 + 22s^2 + 115s - 138 = 0$$

Using Routh's criterion,



$$\begin{array}{c|cc}
 s^3 & 1 & 115 \\
 s^2 & 22 & -138 \\
 s^1 & 121.27 & 0 \\
 s^0 & -138 & 
 \end{array}$$

∴ There is a sign change in the first column of Routh's tabular form, the given system is unstable.

21. (b)

Using the Routh's tabular form

$$\begin{array}{c|cccc}
 s^6 & 1 & 8 & 20 & 16 \\
 s^5 & 2 & 12 & 16 & 0 \\
 s^4 & 2(s^4) & 12(s^2) & 16(s^0) & \\
 s^3 & 8 & 24 & 0 & \\
 s^2 & 6 & 16 & 0 & \\
 s^1 & \frac{16}{6} & 0 & 0 & \\
 s^0 & 16 & & & 
 \end{array}$$

Since there is no sign change in the first column of the Routh array, the system does not have any pole in the RHS of s-plane. However the row of zeros occur which gives the auxiliary equation

$$\begin{aligned}
 A(s) &\Rightarrow 2s^4 + 12s^2 + 16 = 0 \\
 &\Rightarrow s^4 + 6s^2 + 8 = 0
 \end{aligned}$$

and the roots are given by,

$$s = \pm j\sqrt{2}, \pm j2$$

Hence the system is said to be marginally stable.

22. (b)

The open loop transfer function is given as,

$$G(s) H(s) = \frac{1}{s(10s - 1)}$$

Put,  $s = j\omega$

$$G(j\omega) H(j\omega) = \frac{1}{j\omega(10j\omega - 1)}$$

$$|G(j\omega) H(j\omega)| = \frac{1}{\omega\sqrt{100\omega^2 + 1}}$$

$$\angle G(j\omega) H(j\omega) = -90^\circ - 180^\circ + \tan^{-1}(10\omega) = -270^\circ + \tan^{-1}(10\omega)$$

For  $\omega = 0$ ,

$$|G(0) H(0)| = \infty$$

and

$$\angle G(0) H(0) = \left(-270^\circ + \tan^{-1}(10\omega)\right)\Big|_{\omega=0} = -270^\circ$$

For  $\omega = \infty$ ,

$$|G(\infty) H(\infty)| = 0$$

and

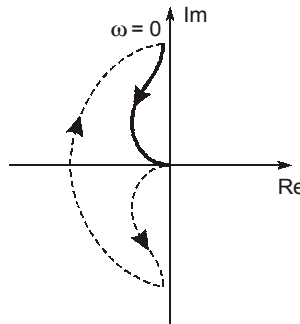
$$\angle G(\infty) H(\infty) = \left(-270^\circ + \tan^{-1}(10\omega)\right)\Big|_{\omega=\infty} = -180^\circ$$

There is an open loop pole at origin. To map this pole,

$$s = re^{j\theta} \Big|_{r \rightarrow 0, \theta \rightarrow -\frac{\pi}{2} \text{ to } \frac{\pi}{2}}$$

$$\begin{aligned} G(s)H(s) &= \frac{-1}{s(1-10s)} = \lim_{r \rightarrow 0} \frac{e^{j\pi}}{re^{j\theta}(1-10re^{j\theta})} \Big|_{\theta \rightarrow -\frac{\pi}{2} \text{ to } \frac{\pi}{2}} \\ &= \lim_{r \rightarrow 0} \frac{1}{r} e^{j(\pi-\theta)} \Big|_{\theta \rightarrow -\frac{\pi}{2} \text{ to } \frac{\pi}{2}} = \infty e^{j\theta} \Big|_{\theta \rightarrow \frac{3\pi}{2} \text{ to } \frac{\pi}{2}} \end{aligned}$$

Thus, the Nyquist plot can be drawn as,



23. (b)

The gain cross-over frequency  $\omega_{gc}$  can be calculated as,

$$|G(j\omega)|_{\omega=\omega_{gc}} = 1$$

Here,

$$G(j\omega) = \frac{32}{j\omega(j\omega + \sqrt{6})^3}$$

$$|G(j\omega)| = \frac{32}{\omega(\sqrt{\omega^2 + 6})^3} = 1$$

At  $\omega = \sqrt{2}$  rad/sec

$$|G(j\omega)|_{\omega=\sqrt{2} \text{ rad/sec}}$$

$$\frac{32}{\sqrt{2} \times \sqrt{8} \times \sqrt{8} \times \sqrt{8}} = 1$$

Thus,  $\omega = \sqrt{2}$  rad/sec is the gain cross-over frequency. Now, the phase cross-over frequency is calculated as

$$\angle G(j\omega)H(j\omega) = -180^\circ$$

Here,

$$\angle G(j\omega) = -90^\circ - 3 \tan^{-1} \frac{\omega}{\sqrt{6}}$$

or

$$\frac{\tan^{-1} \omega}{\sqrt{6}} = 30^\circ$$

or

$$\frac{\omega}{\sqrt{6}} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

or

$$\omega_{pc} = \sqrt{2} \text{ rad/sec}$$

$$\omega_{gc} = \omega_{pc} = \sqrt{2} \text{ rad/sec}$$

The given system represents a marginally stable system having GM = 0 dB and PM = 0°.

24. (c)

Given that, 
$$G(s) = \frac{25}{s(s+1)(s+5)}$$

Let the compensator, 
$$G_c(s) = \frac{(s+\omega_z)}{(s+\omega_p)}$$

The open loop transfer function of the compensated system can be given as,

$$L(s) = G(s) G_c(s) = \frac{25(s+\omega_z)}{s(s+1)(s+5)(s+\omega_p)}$$

The velocity error constant of the compensated system will be,

$$K_v = \lim_{s \rightarrow 0} sL(s) = \frac{25}{5} \left( \frac{\omega_z}{\omega_p} \right) = 5 \left( \frac{\omega_z}{\omega_p} \right)$$

Given that, 
$$e_{ss} = \frac{1}{K_v} < 0.05$$

So, 
$$K_v > \frac{1}{0.05} = 20$$

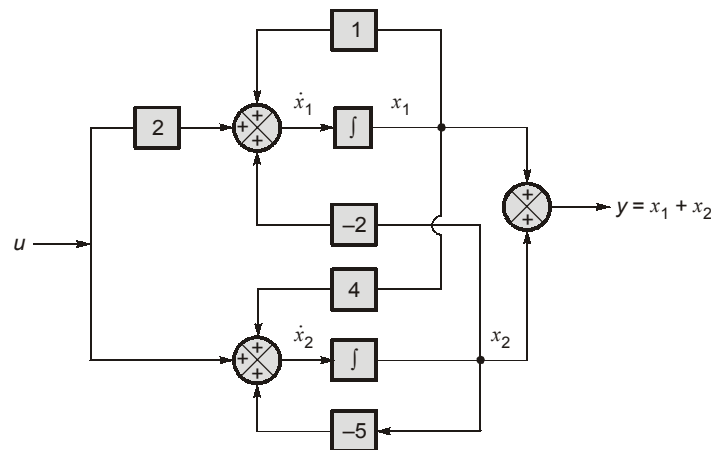
$$5 \left( \frac{\omega_z}{\omega_p} \right) > 20$$

$$\frac{\omega_z}{\omega_p} > 4$$

Only option (c) satisfies this.

25. (a)

Redrawing the given block diagram, we get,



As per the block diagram, state equations are,

$$\dot{x}_1 = x_1 - 2x_2 + 2u$$

$$\dot{x}_2 = 4x_1 - 5x_2 + u$$

and

$$y = x_1 + x_2$$

$\therefore$  State model,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u$$

$$y = [1 \ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

**Check for controllability:**

$$\begin{aligned} Q_c &= [B : AB] \\ &= \begin{bmatrix} 2 & : & \begin{pmatrix} 1 & -2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ 1 & : & \begin{pmatrix} 4 & -5 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{bmatrix} = \begin{bmatrix} 2 & : & (2-2) \\ 1 & : & (8-5) \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \\ |Q_c| &\neq 0 \Rightarrow \text{Controllable} \end{aligned}$$

**Check for observability:**

$$\begin{aligned} Q_o &= [C^T : A^T C^T] \\ &= \begin{bmatrix} 1 & : & \begin{pmatrix} 1 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ 1 & : & \begin{pmatrix} -2 & -5 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 1 & -7 \end{bmatrix} \\ |Q_o| &\neq 0 \Rightarrow \text{Observable} \end{aligned}$$

26. (d)

As per root locus transfer function

$$G(s) H(s) = \frac{k}{(s+4)(s^2+2s+2)}$$

$$\text{CE: } 1 + G(s) H(s) = 0$$

$$s(s^2+2s+2) + 4(s^2+2s+2) + k = 0$$

$$s^3 + 6s^2 + 10s + (8+k) = 0$$

$$s^3 \quad 1 \quad 10$$

$$s^2 \quad 6 \quad 8+k$$

$$s^1 \quad -\frac{(8+k)-60}{6} \quad 0$$

$$s^0 \quad 8+k \quad 0$$

$$\text{Row,} \quad s^1 = 0$$

$$\Rightarrow \quad 8+k = 60$$

$$k = 52$$

For calculation of intersection points,

$$6s^2 + (8+k) = 0$$

$$6s^2 + (60) = 0$$

$$s^2 = -10$$

$$s = \pm j\sqrt{10}$$

Thus points of intersection are,

$$s = \pm j\omega = \pm j\sqrt{10}$$

27. (b)

The gain margin of the system can be given as,

$$GM = 20 \log_{10} \frac{1}{|G(j\omega_{pc})|}$$

$\omega_{pc}$  is independent of the value of  $K$ .

So,  $GM = C - 20 \log_{10}(K)$

Where,  $C$  is a term independent of " $K$ ".

For  $K = 2$ ,  $GM = 32 \text{ dB}$

So,  $C = 32 + 20 \log_{10}(2)$

When  $GM = 25 \text{ dB}$ ,  $25 = C - 20 \log_{10}(K)$

$$25 = 32 + 20 \log_{10}(2) - 20 \log_{10}(K)$$

$$20 \log_{10}(K) = 7 + 20 \log_{10}(2) = 13.02$$

$$K = 10^{(13.02/20)} = 4.48$$

28. (b)

The maximum phase lead is given by,

$$\phi_m = \sin^{-1} \left( \frac{1-\alpha}{1+\alpha} \right)$$

For high pass filter/lead compensator,

$$\tau = R_1 C$$

and

$$\alpha = \frac{R_2}{R_1 + R_2} ; \alpha < 1$$

By putting the value of  $\alpha$  in the above relation, we get,

$$\begin{aligned} \phi_m &= \sin^{-1} \left( \frac{1 - \frac{R_2}{R_1 + R_2}}{1 + \frac{R_2}{R_1 + R_2}} \right) \\ &= \sin^{-1} \left( \frac{R_1 + R_2 - R_2}{R_1 + R_2 + R_2} \right) = \sin^{-1} \left( \frac{R_1}{R_1 + 2R_2} \right) \end{aligned}$$

29. (b)

The characteristic equation is given by,

$$|sI - A| = 0$$

$$\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 3 & 1 \\ 2 & 4 & 1 \\ -5 & -2 & -2 \end{bmatrix} = \begin{bmatrix} s & -3 & -1 \\ -2 & s-4 & -1 \\ 5 & 2 & s+2 \end{bmatrix}$$

$$\therefore \begin{vmatrix} s & -3 & -1 \\ -2 & s-4 & -1 \\ 5 & 2 & s+2 \end{vmatrix} = 0$$

$$s[(s-4)(s+2)+2] + 3[-2(s+2)+5] - 1[-4-5(s-4)] = 0$$

$$\Rightarrow s[s^2 - 2s - 8 + 2] + 3[-2s - 4 + 5] - [-4 - 5s + 20] = 0$$

$$\Rightarrow s[s^2 - 2s - 6] + 3[-2s + 1] - [-5s + 16] = 0$$

$$\Rightarrow s^3 - 2s^2 - 6s - 6s + 3 + 5s - 16 = 0$$

$$\Rightarrow s^3 - 2s^2 - 7s - 13 = 0$$

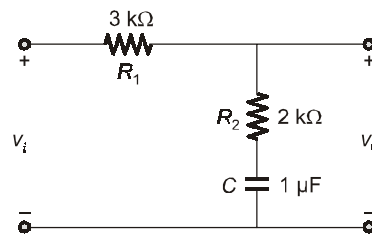
Using Routh's tabular form,

$$\begin{array}{c|cc} s^3 & 1 & -7 \\ s^2 & -2 & -13 \\ s^1 & -13.5 & 0 \\ s^0 & -13 & 0 \end{array}$$

Here, the total number of sign changes in the first column of Routh array is 1, therefore only one pole lie in the RHS of s-plane.

30. (a)

For the circuit shown,



$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{1}{sC}}{R_1 + R_2 + \frac{1}{sC}}$$

$$= \left( \frac{R_2}{R_1 + R_2} \right) \cdot \frac{\left( s + \frac{1}{R_2 C} \right)}{\left[ s + \left( \frac{R_2}{R_1 + R_2} \right) \frac{1}{R_2 C} \right]}$$

$\therefore$

$$\alpha = \frac{R_1 + R_2}{R_2} = \frac{3\text{ k}\Omega + 2\text{ k}\Omega}{2\text{ k}\Omega} = \frac{5}{2} = 2.50$$

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