CLASS TEST

S.No.: 02SK_ECEE_H+I_110624

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NETWORK THEORY

EC-EE

Date of Test : 11/06/2024

ANSWER KEY

1.	(a)	7.	(b)	13.	(b)	19.	(b)	25.	(b)
2.	(b)	8.	(a)	14.	(d)	20.	(c)	26.	(c)
3.	(c)	9.	(a)	15.	(c)	21.	(a)	27.	(d)
4.	(b)	10.	(c)	16.	(a)	22.	(c)	28.	(c)
5.	(d)	11.	(c)	17.	(a)	23.	(b)	29.	(d)
6.	(c)	12.	(b)	18.	(d)	24.	(d)	30.	(a)

DETAILED EXPLANATIONS

1. (a)

The average value of periodic signal can be calculated by considering one time period

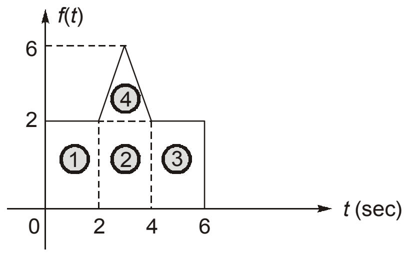
$$= \frac{\text{Total area under the graph for one period}}{T_0}$$

Total area under the graph for one period = Area 1 + Area 2 + Area 3 + Area 4

here Area 1 = Area 2 = Area 3 = Area 4 = 4

and T_0 = 8 sec

$$\text{Average value} = \frac{4+4+4+4}{8} = \frac{16}{8} = 2$$

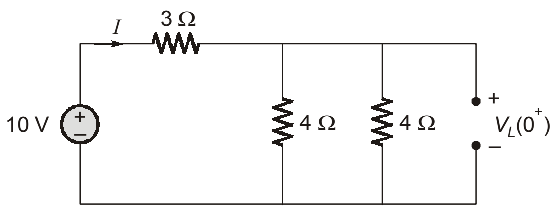


2. (b)

Before closing the switch, the circuit was not energized, therefore, current through inductor and voltage across capacitor are zero.

After closing the switch, at $t = 0^+$ inductor acts as open-circuit and capacitor acts as short-circuit.

Equivalent circuit at $t = 0^+$



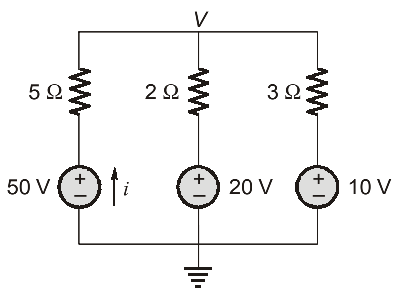
$$I = \frac{10}{3 + 4 || 4} = 2\text{ A}$$

$$V_L(0^+) = I \times (4 || 4)$$

$$= 2 \times 2 = 4\text{ V}$$

3. (c)

using source transformation



applying KCL at V

$$\frac{V-50}{5}+\frac{V-20}{2}+\frac{V-10}{3} = 0$$

$$\frac{6V-300+15V-300+10V-100}{30} = 0$$

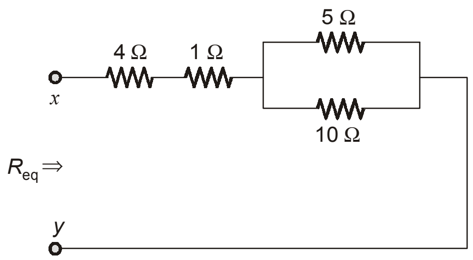
$$31V-700 = 0$$

or $$V = \frac{700}{31} = 22.58 \text{ V}$$

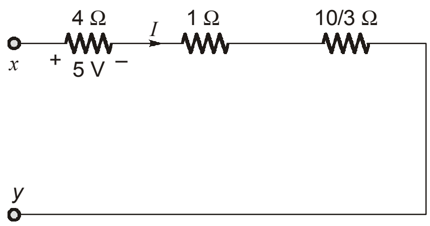
$$i = \frac{50-V}{5}$$

$$= \frac{50-22.58}{5} = 5.48 \text{ A}$$

4. (b)



(a)



(b)

$$R_{eq} = 4\ \Omega + 1\ \Omega + (5\,||\,10)\Omega$$

$$= 5\ \Omega + \left(\frac{5\times 10}{15}\right)\Omega$$

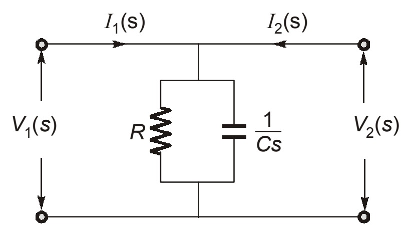
$$= 5+\frac{10}{3} = \frac{25}{3}\ \Omega$$

Also from (b) $$I = \frac{5}{4} \text{ A}$$ (in series circuit current remains same)

∴ $$V_{xy} = \frac{5}{4}\times\frac{25}{3} = 10.417 \text{ V}$$

5. (d)

Converting the network into s-domain



$$V_2(s) = I_1(s)\left(\frac{R\times\frac{1}{Cs}}{R+\frac{1}{Cs}}\right)$$

or $$\frac{V_2(s)}{I_1(s)} = \frac{R}{RCs+1} = \frac{1}{\left(s+\frac{1}{RC}\right)}$$

∴ $$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = \frac{1}{\left(s+\frac{1}{RC}\right)} = \frac{1}{1(s+1)}$$

$$Z_{21}(s) = \frac{1}{(s+1)}$$

6. (c)

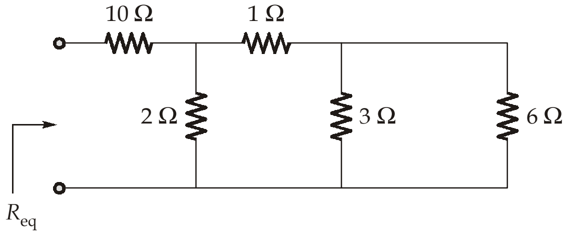
$$P = P_0 + P_1 + P_2$$

$$= 4^2 \times 20 + \left(\frac{5}{\sqrt{2}}\right)^2 \times 20 + \left(\frac{3}{\sqrt{2}}\right)^2 \times 20 \qquad \left[\because I_{rms} = \frac{I_{max}}{\sqrt{2}}\right]$$

$$= (16 + 12.5 + 4.5) \times 20 = 660 \text{ W}$$

7. (b)

The resistance $6\,\Omega \| 3\,\Omega$ and $12\,\Omega \| 4\,\Omega$ also 1 Ω is in series with 5 Ω, thus, the circuit can be redrawn as



$$\therefore \quad R_{eq} = 10\,\Omega + 2\,\Omega \| (1 + 3\,\Omega \| 6\,\Omega)$$

$$R_{eq} = 11.2\,\Omega$$

8. (a)

$$Z_L = j\omega L = j\Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j(1)\,(0.05)} = -j20\,\Omega$$

$$\therefore \quad Z_{eq} = j + 2 \| (-j20) = 1.98 + j0.802\,\Omega$$

and $Z_L(5j) = 5j\Omega$

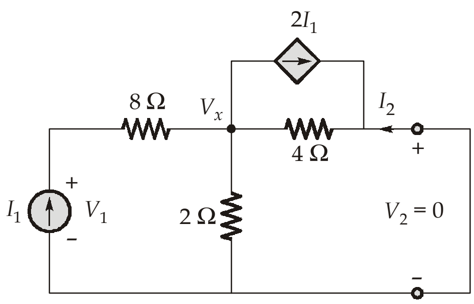
$$Z_C(5j) = -j4\,\Omega$$

$$\therefore \quad Z_{eq}(j5) = j5 + 2 \| (-j4) = 1.6 + j4.2\,\Omega$$

Now, $I(j\omega) \propto \dfrac{1}{Z(j\omega)}$

$$\therefore \quad \left|\frac{I_o(j)}{I_o(j5)}\right| = \frac{Z(j5)}{Z(j)} = \left|\frac{1.6 + j4.2}{1.98 + j0.802}\right| = 2.104$$

9. (a)



$$I_1 = \frac{V_x}{2} + \frac{V_x}{4} + 2I_1$$

$$-I_1 = 0.75V_x \qquad \text{...(i)}$$

$$I_2 = -\frac{V_x}{4} - 2I_1 = -\frac{V_x}{4} + 1.5V_x = 1.25\,V_x$$

Now, $V_1 = 8I_1 + V_1$

$V_1 = -6V_x + V_x = -5V_x$

$\therefore \quad \dfrac{I_2}{V_1} = -\dfrac{1.25}{5} = -0.25\ \text{S}$

10. (c)

Voltage across capacitor

$$v_c(t) = V_{final} + (V_{initial} - V_{final})e^{-t/RC}$$
$$= 0 + (5 - 0)e^{-t/RC}$$

But given, $v_c(t) = \dfrac{5}{e} = 5e^{-t/RC}$

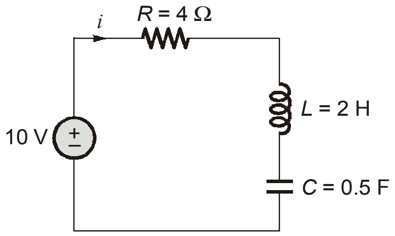
$$\frac{5}{e} = 5e^{-(0.1/40\ \text{k}\Omega \times C)}$$

$$\frac{0.1}{40\ \text{k}\Omega \times C} = 1$$

$\therefore \quad C = 2.5\ \mu\text{F} \quad \text{(or)} \quad 2.5 \times 10^{-6}\ \text{F}$

11. (c)

At ($t = 0^-$)



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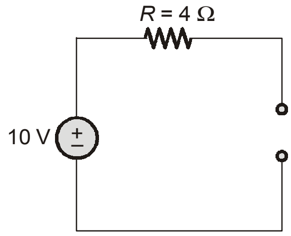
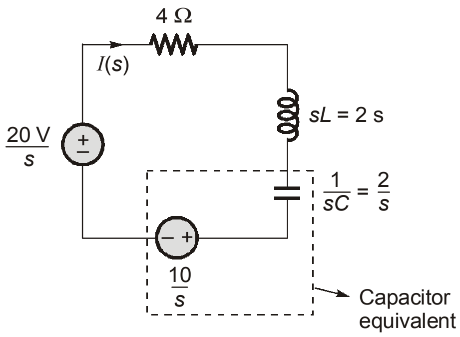


$V_C(0^-) = 10\ \text{V}$

$i_L(0^-) = 0\ \text{A}$

At ($t = 0^+$)

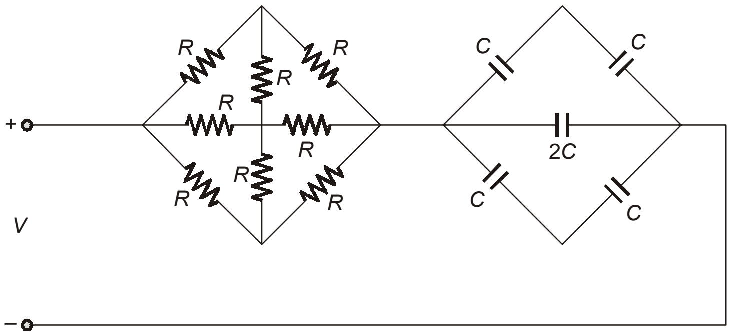


$$I(s) = \frac{10/s}{4 + 2s + \dfrac{2}{s}} = \frac{10/s}{\dfrac{4s + 2s^2 + 2}{s}} = \frac{10}{2(s^2 + 2s + 1)}$$

$$I(s) = \frac{5}{(s+1)^2}$$

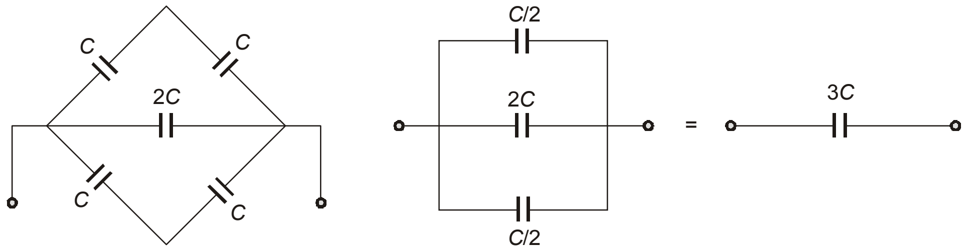
$$i(t) = 5te^{-t}\,u(t)\ \text{A}$$

12. (b)

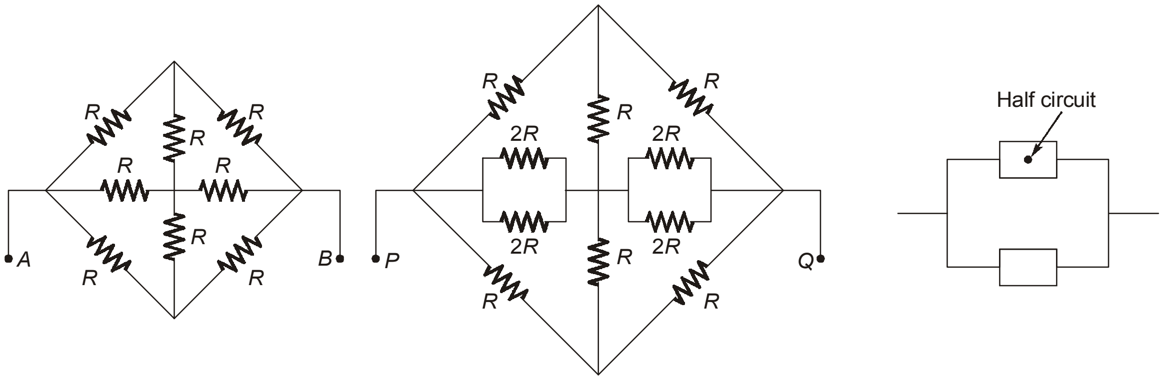


The time constant of an RC circuit is $\tau = R_{eq}C_{eq}$

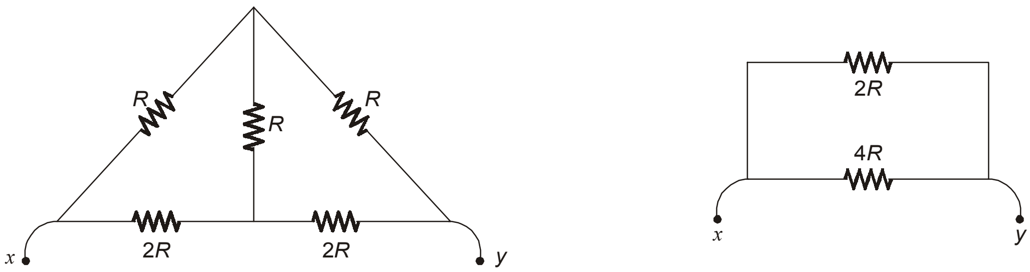
Calculation of C_{eq}



Calculation of R_{eq}



It is Wheatstone bridge.



$$(R_{eq})_{\text{Half circuit}} = \frac{4R \times 2R}{6R} = \frac{4R}{3}$$

$$R_{eq} = (R_{eq})_{\text{Half circuit}} \,||\, (R_{eq})_{\text{Half circuit}}$$

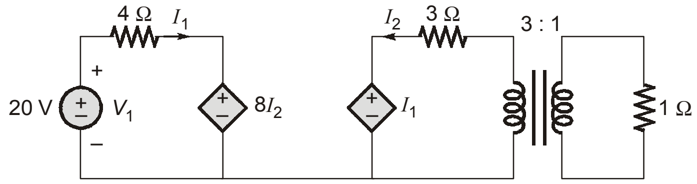
$$= \left(\frac{4R}{3}\right) \Bigg\| \left(\frac{4R}{3}\right) = \left(\frac{2R}{3}\right)$$

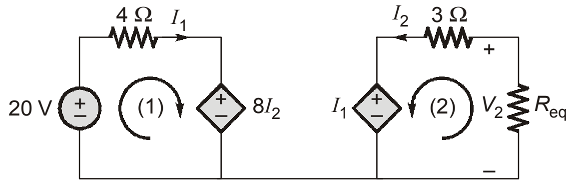
$$\therefore \quad \tau = R_{eq}C_{eq}$$

$$= \frac{2R}{3} \times 3C = 2RC$$

13. (b)

Network '*N*' can be replaced as





$$R_{eq} = (1)\times\left(\frac{3}{1}\right)^2 = 9\ \Omega$$

Applying KVL at loop (1)

$$20 = 4I_1 + 8I_2$$

also $\quad V_2 = -9I_2 = I_1 + 3I_2$

$\Rightarrow \quad I_1 = -12I_2$

$$20 = 4(-12I_2) + 8I_2$$

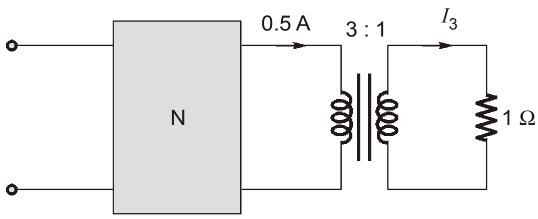
$$20 = -40I_2$$

$\Rightarrow \quad I_2 = -0.5\text{ A}$

$$I_1 = -12(-0.5) = 6\text{ A}$$

Now,

$$\frac{I_{primary}}{I_{secondary}} = \frac{1}{3}$$



$\Rightarrow \quad 3I_{primary} = I_{secondary}$

$$3(-I_2) = I_3$$

$\Rightarrow \quad I_3 = 1.5\text{ A}$

Power delivered to 1 Ω $= (I_3)^2 \times R_L = (1.5)^2 \times 1$

$= 2.25$ Watts

14. (d)

The *h*-parameter

$$V_1 = 2I_1 + 4V_2$$

$$I_2 = -5I_1 + 2V_2$$

Power dissipated in R_L

$$P_L = \frac{V_2^2}{R_L} = 25\text{ W}$$

$\therefore \quad V_2 = \sqrt{25\times 4} = 10\text{ V}$

$\because \quad V_2 = -I_2R_L$

$\therefore \quad I_2 = -2.5\text{ A}$

substituting the values

we get $\quad V_1 = 2I_1 + 40$

$$-2.5 = -5I_1 + 20$$

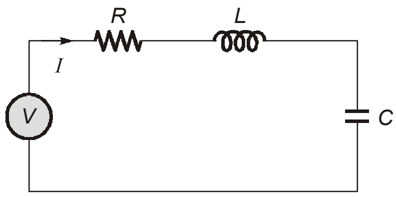
$\therefore \quad I_1 = 4.5\text{ A} \quad$ and $\quad V_1 =$

49 V

$\because \quad I_1 = \frac{V_s - V_1}{2} = 4.5$

$\Rightarrow \quad V_s = 58$ V

15. (c)



For series RLC circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The circuit current

$$I = \frac{V}{Z}$$

$\therefore$ The drop across the capacitor C

$$V_C = IX_C = \frac{V}{Z}X_C$$

i.e.
$$V_C^2 = \frac{V^2X_C^2}{Z^2} = \frac{V^2}{(\omega C)^2\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

For maximum voltage drop

$$V_{C\max} = \frac{dV_C}{d\omega} = 0$$

$$= \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} \text{ rad/sec}$$

16. (a)

Using KCL

$$100 = R\frac{dQ}{dt} + \frac{Q}{C}$$

$$100\,C = RC\frac{dQ}{dt} + Q$$

$$\int_{Q_0}^{Q}\frac{dQ}{100C - Q} = \frac{1}{RC}\int_0^t dt$$

$$(-In(100C - Q))_{Q_0}^{Q} = \frac{t}{RC}$$

$$In\frac{(100C - Q)}{(100C - Q_0)} = -\frac{t}{RC}$$

$$100\,C - Q = (100\,C - Q_0)\,e^{-t/RC}$$

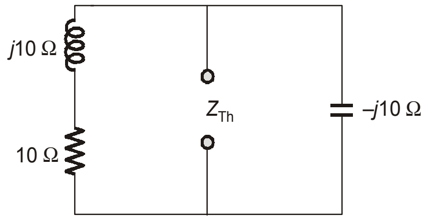
$$i = \frac{dQ}{dt} = \left(\frac{100C - Q_0}{RC}\right)e^{-t/RC} = \left(\frac{50 - 10}{1}\right)e^{-t/RC}$$

$$i = 40\,e^{-t/RC}$$

at $t = 2$ sec
$$i = 40\,e^{-2} = 5.413 \text{ A}$$

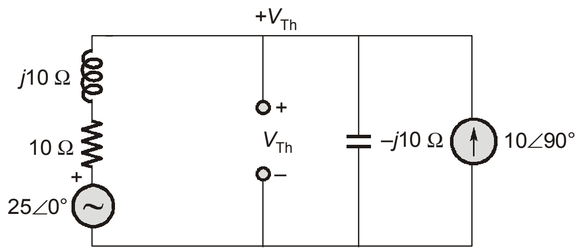
17. (a)

Z_{Th} :



$$Z_{Th} = \frac{(10 + j10)(-j10)}{10 + j10 - j10} = 10 - j10$$

V_{Th} :



$$\frac{V_{Th} - 25}{10 + j10} + \frac{V_{Th}}{-j10} = 10\angle 90°$$

$$\frac{V_{Th}}{10 + j10} - \frac{25}{10 + j10} - \frac{V_{Th}}{j10} = 10j$$

$$(V_{Th})j10 - 250j - 10\,V_{Th} - 10j\,V_{Th} = -100(10 + 10j)$$

$$j10\,V_{Th} - 250j - 10\,V_{Th} - 10j\,V_{Th} = -1000(1 + j)$$

$$-10\,V_{Th} = -1000 - 1000\,j + 250\,j$$

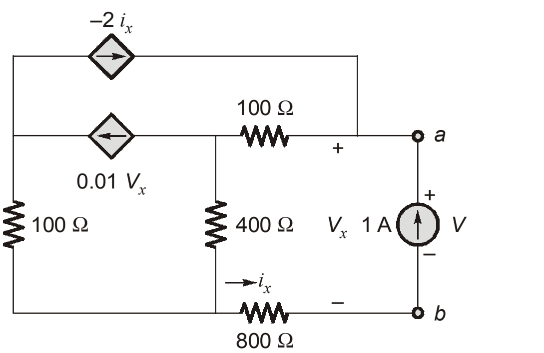
$$-10\,V_{Th} = -1000 - 750\,j$$

$$V_{Th} = 100 + j75$$

$$Z_L = Z_{Th}^{*} = 10 + j10$$

$$P_{max} = \frac{|V_{Th}|^2}{4\text{Re}\{Z_{Th}\}} = \frac{\left(\sqrt{100^2 + 75^2}\right)^2}{4 \times 10} = 390.625\text{ W}$$

18. (d)



$$V = 100\,(1 - 2i_x) + 400\,(1 - 2i_x - 0.01\,V_x) + 800\,i_x$$

$$i_x = 1\text{ A and } V_x = V$$

$$5V = 1300 - 1000 = 300$$

∴ $V = 60\text{ V}$

and $R_{TH} = 60\ \Omega$

19. (b)

Given,

$$f_0 = 5 \text{ kHz}$$
$$Z = Z_{source} + Z_{load}$$
$$= 2 + j4 + 10 - jX_C$$
$$= [12 + j(4 - X_C)]\ \Omega$$

at resonance, imaginary part of Z is zero.

$$\because \quad X_L = X_C$$
$$\therefore \quad X_C = \frac{1}{\omega_0 C} = 4$$

or $\quad C = 7.95\ \mu\text{F}$

As at resonance current is maximum and thus, maximum power transferred to R.

20. (c)

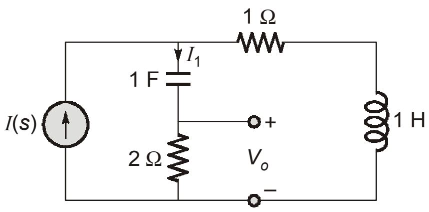
Condition for Transient free response is given by

$$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta + \frac{\pi}{2}$$

here, $\quad \omega = \frac{1}{2}, \qquad \theta = 45°$

$$\frac{t_0}{2} = \tan^{-1}\left(\frac{\frac{1}{2}\times 2}{1}\right) - \frac{\pi}{4} + \frac{\pi}{2}$$
$$\frac{t_0}{2} = \frac{\pi}{2}$$
$$t_0 = \pi = 3.1416 \text{ sec}$$

21. (a)

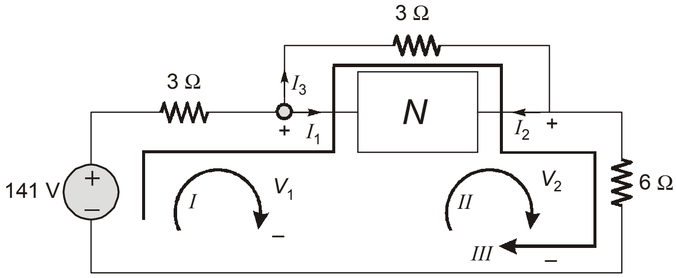


$$I_1(s) = \frac{I(s)\,(1+s)}{\frac{1}{s} + 2 + 1 + s} = \frac{I(s)\,(s+1)\,s}{s^2 + 3s + 1}$$

$$V_o(s) = 2I_1(s) = \frac{I(s)\,2s(s+1)}{s^2 + 3s + 1}$$

$$\frac{V_o(s)}{I(s)} = \frac{2s(s+1)}{s^2 + 3s + 1}$$

22. (c)



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Rightarrow \begin{array}{l} V_1 = 2I_1 + I_2 \\ V_2 = I_1 + 4I_2 \end{array}$$

Applying KVL

Loop I

$$3(I_1 + I_3) + V_1 = 141$$
$$3I_1 + 3I_3 + 2I_1 + I_2 = 141$$
$$5I_1 + I_2 + 3I_3 = 141 \quad ...(i)$$

Loop II

$$V_2 = (I_3 - I_2)\cdot 6$$
$$V_2 = I_1 + 4I_2$$
$$I_1 + 4I_2 = 6I_3 - 6I_2$$
$$I_1 + 10I_2 - 6I_3 = 0 \quad ...(ii)$$

Loop III

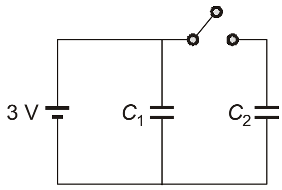
$$3(I_1 + I_3) + 3I_3 + 6(I_3 - I_2) = 141$$
$$3I_1 - 6I_2 + 12I_3 = 141 \quad ...(iii)$$

By equation (*i*), (*ii*) and (*iii*)

$I_1 = 24$ A $\qquad I_2 = 1.5$ A $\qquad I_3 = 6.5$ A

23. (b)

At $t = 0^-$ S_1 is closed. S_2 is open



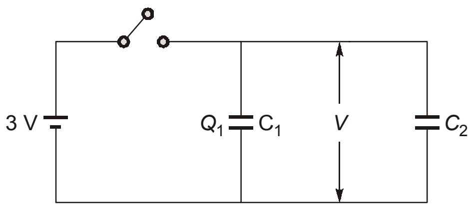
C_1 gets charged upto 3 V charge stored in C_1

$$Q_0 = C_1 V = 1 \times 3 = 3C$$

Voltage across C_2 is zero at $t = 0^-$, so no charge stored in C_2.

At $t > 0$, S_1 is open and S_2 is closed.

charge stored (Q_0) initially in C_1 gets redistributed between C_1 and C_2



Let Charge stored in $C_1 = Q_1$

Charge stored in $C_2 = Q_2$

According to conservation of charge

$$Q_1 + Q_2 = Q_0 = 3 \quad \text{...(1)}$$

Voltage across C_1 = Voltage across C_2

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \Rightarrow \frac{Q_1}{1} = \frac{Q_2}{2}$$

$$Q_2 = 2Q_1 \quad \text{...(2)}$$

from (1) and (2)

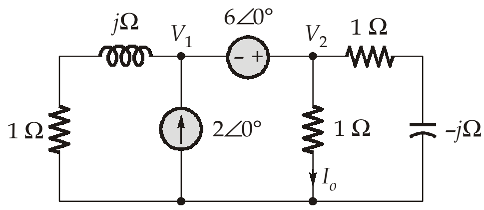
$$Q_1 = 1\,C$$

and $$Q_2 = 2\,C$$

$\therefore$ Voltage across the combination $= \dfrac{Q_1}{C_1} = \dfrac{1}{1} = 1\text{ V}$

24. (d)

Applying KCL on supernode



$$\frac{V_1}{1+j} - 2 + \frac{V_2}{1} + \frac{V_2}{1-j} = 0$$

and $$V_1 + 6 = V_2$$

$$\therefore \quad \begin{bmatrix} 0.5-0.5j & 1.5+0.5j \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$\therefore \quad V_2 = \frac{\begin{bmatrix} 0.5-0.5j & 2 \\ 1 & -6 \end{bmatrix}}{\begin{bmatrix} 0.5-0.5j & j0.5+0.5j \\ 1 & -1 \end{bmatrix}}$$

$$V_2 = \frac{5.83095\angle 149.036}{2\angle 180°}$$

$$V_2 \approx 2.915\angle{-30.96°}$$

Thus, $$I_o = \frac{V_2}{1\,\Omega} = 2.915\angle -30.96°$$

25. (b)

$$Z_{AB} = \left(\frac{23}{6}\right) + \left[(3+j4)\,\|\,(3-j4)\right]$$

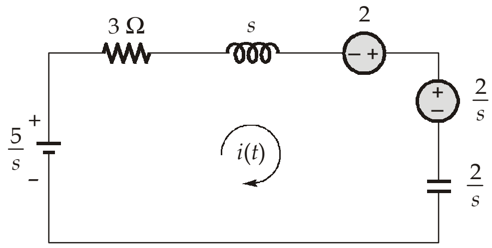
$$= \frac{23}{6} + \frac{(3+j4)\,(3-j4)}{6} = \frac{23+25}{6} = \frac{48}{6}\,\Omega = 8\,\Omega$$

$$\therefore \quad Z_{AB} = 8\,\Omega$$

26. (c)

At $t = 0$, switch is closed

For $t > 0$, the circuit in s-domain becomes,



Applying KVL, we get,

$$\frac{5}{s}-\frac{2}{s}+2 = \left(3+s+\frac{2}{s}\right)I(s)$$

$$I(s) = \frac{2s+3}{(s+1)(s+2)}$$

Using partial fractions, $I(s) = \frac{1}{(s+1)}+\frac{1}{(s+2)}$

or $i(t) = L^{-1}[I(s)] = (e^{-t}+e^{-2t})$ A ; for $t > 0$

27. (d)

Given, $f = 1.5$ MHz

$C = 150$ pF

BW $= 10$ kHz

For series *RLC* circuit, $Q = \frac{f_o}{BW} = \frac{1.5\times10^6}{10\times10^3} = 150$

$$Q = \frac{1}{\omega RC}$$

$$\frac{1}{150} = 2\pi\times1.5\times10^6\times150\times10^{-12}\times R$$

$$R = \frac{10^6}{2\pi\times1.5\times150\times150} = 4.71\,\Omega$$

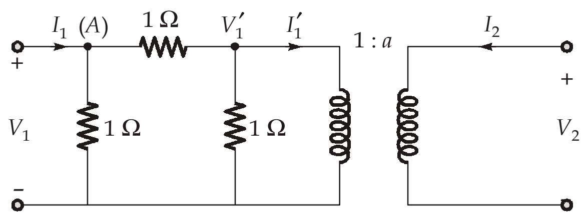
28. (c)

For a capacitor $i(t) = \frac{cdv(t)}{dt} = 10\times10^{-6}\frac{dv(t)}{dt}\times10^3$ A

$$= 10^{-2}\frac{dv(t)}{dt}\text{ A} = 10\frac{dv(t)}{dt}\text{ mA}$$

29. (d)

Now, applying KCL at node *A*, we get,



$$I_1 = V_1 + (V_1 - V_1')$$
$$= 2V_1 - V_1'$$
$$I_1 = 2V_1 - \frac{1}{a}V_2$$

For I_2, we can write

$$I_2 = -\frac{1}{a}I_1' = -\frac{1}{a}\left[-V_1' + (V_1 - V_1')\right]$$

$$= -\frac{1}{a}V_1 + \frac{2}{a^2}V_2$$

$$\therefore \qquad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 & -\frac{1}{a} \\ -\frac{1}{a} & \frac{2}{a^2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

For the Z-parameter to not exist.

$$|Y| = 0$$

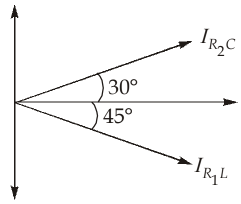
$$\therefore \qquad |Y| = \frac{4}{a^2} - \frac{1}{a^2} = \frac{3}{a^2}$$

$$\because \qquad |Y| \neq 0$$

Thus, no such value exist for which $|Y| = 0$.

30. (a)

From phasor, we can write



$$\tan 30° = \frac{X_C}{R_2}$$

$$\Rightarrow \qquad R_2 = X_C\sqrt{3} = \frac{\sqrt{3}}{\omega C}$$

$$\tan 45° = \frac{X_L}{R_1}$$

$$\Rightarrow \qquad R_1 = X_L = \omega L$$

$$R_1R_2 = \frac{\sqrt{3}}{\omega C} \times \omega L = \frac{L}{C}\sqrt{3}$$

$$R_1R_2 = \sqrt{3} = 1.732$$

we know

$$\frac{R_1 + R_2}{2} \geq \sqrt{R_1R_2}$$

as arithmetic mean ≥ geometric mean ; (for non-negative real numbers)

$$R_1 + R_2 \geq 2\sqrt{\sqrt{3}}$$

$$R_1 + R_2 \geq 2(3)^{1/4}$$

Minimum value of $R_1 + R_2 = 2.63\ \Omega$

■■■■