RSSB-JE 2020

Rajasthan Staff Selection Board

Combined Junior Engineer Direct Recruitment Examination

Civil Engineering

Theory of Structures (SOM)

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY Publications

Theory of Structures (SOM)

Contents

CHAPTER 8

Shear Stress

8.1 Introduction

Assumption:

(i) The material is homogenous, isotropic and elastic in which Hooke's law is valid.

(ii) The shear stress is constant along the width but very along the depth.

8.2 Shear Stress Distribution in Beams

Let τ be the shear stress in a layer at a distance y from $N.A$ where a particular section is subjected to $S.F = V$

$$\tau = \frac{VA\bar{y}}{Ib}$$

V = SF (Shear force) at the section where shear stress is to be found out

$A\bar{y}$ = Moment of area of section above the level at which shear stress is to be found out

I = Moment of inertia of complete section about NA

b = Width of the section at the level where shear stress is to be found out

NOTE

Shear force per unit length of beam is called shear flow. (q)

$$q = \frac{VA\bar{y}}{I} = \tau.b$$

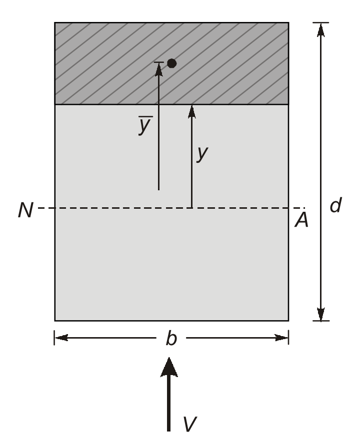
8.3 Shear Stress Distribution in Rectangular Section

$$\tau = \frac{VA\bar{y}}{Ib}$$

$$A\bar{y} = b.\left(\frac{d}{2} - y\right).\left[y + \frac{\frac{d}{2} - y}{2}\right]$$

$$= b\left(\frac{d}{2} - y\right).\left(\frac{y + d/2}{2}\right)$$

$$= \frac{b}{2}.\left(\frac{d^2}{4} - y^2\right)$$



⇒ Shear stress,

$$t = \frac{VA\bar{y}}{Ib} = \frac{V\frac{b}{2}\left(\frac{d^2}{4} - y^2\right)}{\left(\frac{bd^3}{12}\right) \times b}$$

$$\boxed{\tau = \frac{6V}{bd^3}\cdot\left(\frac{d^2}{4} - y^2\right)}$$

From above equation, it is clear that variation of shear stress is parabolic.

Also,

From above equation:

At $y = \pm\, d/2\ ;\ \tau = 0$

At $y = 0$ (*i.e.* at neutral axis)

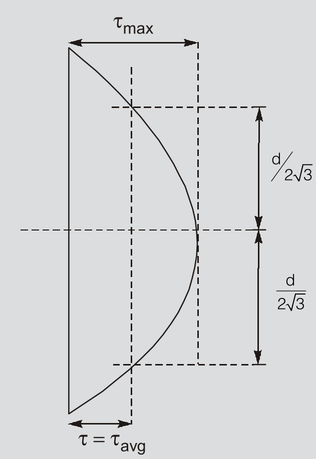
$$\tau = \tau_{max}$$

$$\tau_{max} = \frac{6V}{bd^3}\cdot\frac{d^2}{4} = \frac{3}{2}\cdot\frac{V}{bd} = \frac{3}{2}\cdot\tau_{avg}$$

$$\tau_{max} = \frac{3}{2}\cdot\tau_{avg}$$

$$\boxed{\tau_{max} = 1.5\tau_{avg}}$$

Shear Stress Distribution in Rectangular Section



- Let at a distance y' from *N.A* where

$$\tau_{av} = \tau$$

$$\frac{V}{bd} = \frac{6V}{bd^3}\cdot\left(\frac{d^2}{4} - y'^2\right)$$

$$\frac{d^2}{6} = \frac{d^2}{4} - (y')^2$$

$$(y')^2 = \frac{d^2}{4} - d^2/6 = \frac{d^2}{12}$$

$$\left(y' = \pm\frac{d}{2\sqrt{3}}\right)$$

⇒ Shear Stress will be equal average shear stress at $\frac{d}{2\sqrt{3}}$ distance from neutral axis.

Example - 8.1 A rectangular beam of width 100 mm is subjected to maximum shear force of 60 kN. The corresponding maximum shear stress in the cross-section is 4 N/mm². What is the depth of beam?

(a) 150 mm (b) 225 mm (c) 200 mm (d) 100 mm

Ans. (b)

Given

$$\tau_{max} = 4\ \text{N/mm}^2$$

$$\tau_{max} = 1.5 \times \tau_{avg}$$

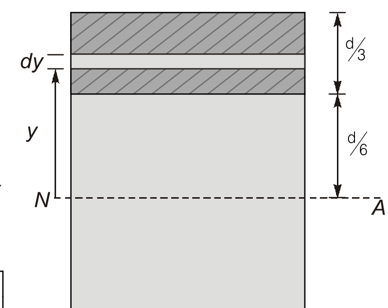
$$\therefore \quad 4\frac{N}{mm^2} = 1.5 \times \frac{60 \times 10^3 N}{100mm \times d(mm)}$$

$$\Rightarrow \quad d = 225 \text{ mm}$$

Example - 8.2 If a beam of rectangular cross-section is subjected to a vertical shear force *S*, then how much shear force will be carried by the upper one third of the section?

(a) zero (b) $\frac{7V}{27}$ (c) $\frac{8V}{27}$ (d) $\frac{V}{3}$

Ans. (b)



S.F resisted by small strips, $dF = \int \tau.(bdy)$

S.F resisted by upper one third portion :

$$F = \int_{d/6}^{d/2} \tau.(bdy) = \int_{d/6}^{d/2} \frac{6V}{bd^3}.\left(\frac{d^2}{4} - y^2\right).bdy$$

$$= \frac{6V}{d^3}.\left[\frac{d^2}{4}.\left(\frac{d}{2} - \frac{d}{6}\right) - \frac{1}{3}.\left(\frac{d^3}{8} - \frac{d^3}{216}\right)\right]$$

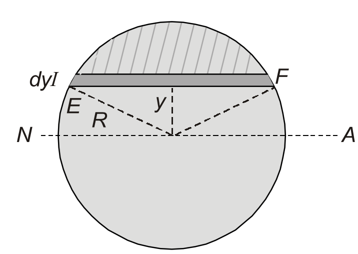
$$= \frac{6V}{d^3}\left[\frac{d^3}{12} - \frac{26d^3}{3 \times 216}\right]$$

$$F = 6V\left[\frac{1}{12} - \frac{26}{648}\right]$$

$$F = \frac{7V}{27}$$

$\therefore$ S.F resisted by top $1/3^{rd}$ portion = $\frac{7V}{27}$

8.4 Shear Stress Distribution in Circular Sections



$$b = EF = 2\sqrt{R^2 - y^2}$$

$$A\bar{y} = \int_y^R ydA$$

$$= \int_y^R (bdy).y.$$

$$= \int_y^R y\left[2\sqrt{R^2 - y^2}\right]dy$$

$$= \frac{2}{3}\left(R^2 - y^2\right)^{3/2}$$

$$\tau = \frac{V\frac{2}{3}\left(R^2 - y^2\right)^{3/2}}{\left(\frac{\pi R^4}{4}\right) \times 2\sqrt{R^2 - y^2}} = \frac{4V}{3\pi R^4} \times \left(R^2 - y^2\right)$$

$$\text{At } y = \pm R$$
$$\tau = 0$$

Also, variation of shear stress is parabolic.

At $y = 0$ (*i.e.* at *NA*)

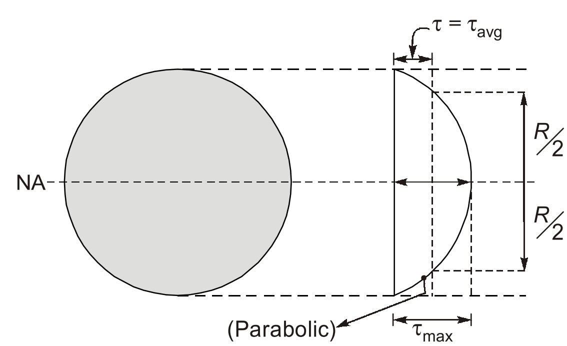
$$\tau = \tau_{max} = \frac{4V}{3\pi R^4}\left(R^2\right) = \frac{4V}{3\pi R^2} = \frac{4}{3}.\frac{V}{\pi R^2} = \frac{4}{3}\tau_{avg}$$

$$\boxed{\tau_{max} = \frac{4}{3}.\tau_{avg}}$$

Let at a distance y' from NA (Neutral axis) Where.

$$\tau_{avg} = \tau$$
$$\frac{V}{\pi R^2} = \frac{4V}{3\pi R^4}\left(R^2 - y'^2\right)$$
$$\frac{4V}{3R^4}\left(R^2 - y'^2\right) = 1$$
$$4R^2 - 4y'^2 = 3R^2$$
$$R^2 = 4y'^2$$
$$y'^2 = R^2/4$$
$$\boxed{y' = \pm R/2}$$

Shear Stress Distribution



1. $\tau_{max} = 4/3.\tau_{avg}$
2. τ_{max} occur at $y = 0$ *i.e.* at *NA*.
3. τ_{avg} occur at $y = R/2$ from *NA*.

NOTE

For Hollow circular section,

R_1 = Internal radius

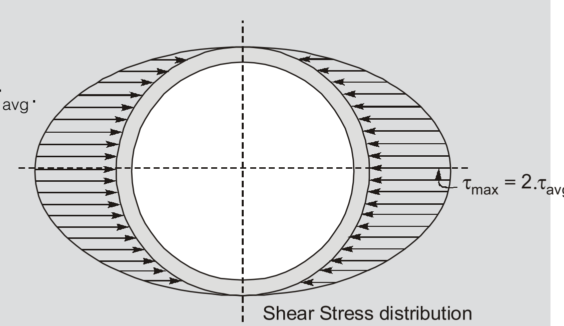
R_2 = Outer radius.

$$\tau_{max} = \frac{4}{3}\left(\frac{R_1^2 + R_1R_2 + R_2^2}{R_1^2 + R_2^2}\right).\tau_{avg}.$$

For thin circular tube ($R_1 \approx R_2$)

$$\tau_{max} = \frac{4}{3}\left(\frac{3R_1^2}{2R_1^2}\right).\tau_{avg}.$$

$$\boxed{\tau_{max} = 2.\tau_{avg}}$$



Shear Stress distribution

8.5 Shear Stress Distribution in Triangular Section

Let τ be the shear stress at a distance y from vertex.

$$EF = b'$$

$$A\bar{y} = \left(\frac{1}{2}b'y\right)\left(\frac{2}{3}h - \frac{2y}{3}\right)$$

$$\tau = \frac{VA\bar{y}}{Ib}$$

$$= \frac{V\left(\frac{1}{2}b'y\right)\left\{\frac{2}{3}(h-y)\right\}}{\left(\frac{bh^3}{36}\right)b'}$$

$$\tau = \frac{12V}{bh^3}\left(hy = y^2\right)$$

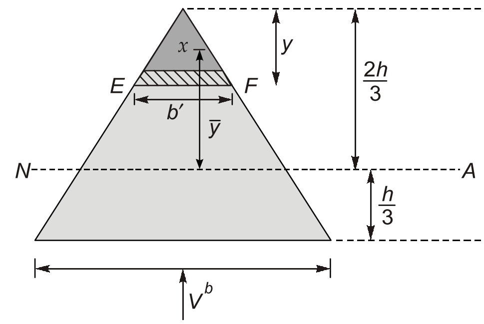


Fig.9.6

From above equation, it is clear that variation is parabolic

Also, At $y = 0$; $\tau = 0$

$y = h$; $\tau = 0$

for τ to be maximum

$$\frac{d\tau}{dy} = 0$$

$$h - 2y = 0$$

$$y = h/2$$

$$\therefore \quad \tau_{max} = \frac{12V}{bh^3}\cdot\left(\frac{h^2}{2} - \frac{h^2}{4}\right) = \frac{3V}{bh} = \frac{3}{2}\times\frac{V}{\frac{1}{2}.bh} = \frac{3}{2}\tau_{avg}$$

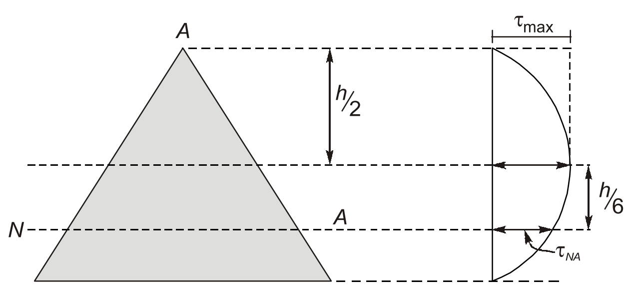
$$\boxed{\tau_{max} = 1.5\,\tau_{avg}}$$

$$\tau_{N.A} = \frac{12V}{bh^3}\left(\frac{2h^2}{3} - \frac{4h^2}{9}\right) = \frac{12V}{bh^3}\left(\frac{6h^2 - 4h^2}{9}\right)$$

$$= \frac{4}{3}\cdot\frac{V}{bh}\times 2 = \frac{4}{3}\times\frac{V}{1/2\,bh}$$

$$\boxed{\tau_{NA} = \frac{4}{3}\tau_{avg}}$$

Shear Stress distribution



$$\tau_{max} = 1.5\,\tau_{avg} \text{ (at } y = h/2 \text{ from vertex)}$$

$$\tau_{NA} = 4/3\,\tau_{avg}$$

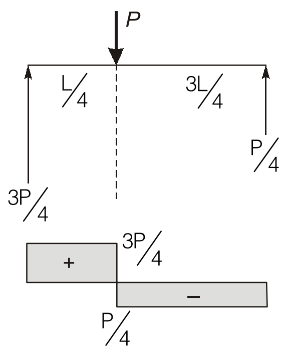
Distance between NA and τ_{max} location = $h/6$

ANSWER KEY — STUDENT'S ASSIGNMENTS

1. (c)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (d)
6. (c)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (b)
11. (c)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (a)

HINTS & SOLUTIONS — STUDENT'S ASSIGNMENTS

2. (b)



maximum shear force in the beam = $3P/4$

$$\therefore \quad \tau_{max} = \frac{3}{2}.\tau_{avg}$$

$$= \frac{3}{2}.\left(\frac{3P/4}{bh}\right) = \frac{9P}{8bh}$$

4. (c)

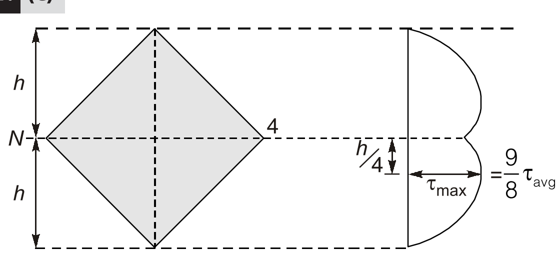
$$\tau_{NA} = \frac{4}{3}.\tau_{avg} = \frac{4}{3}.\frac{F}{\left(\frac{1}{2}bh\right)} = \frac{8F}{3bh}$$

10. (c)

$$\frac{(\tau_{web})\text{at junction}}{(\tau_{fienge})\text{at junction}} = \frac{B}{b} = \frac{100}{20} = 5$$

$$(\tau_{web})\text{ at junction} = 5 \times 12 = 60 \text{ MPa}$$

11. (c)



$$2h = B\sqrt{2} \Rightarrow h = \frac{B\sqrt{2}}{2}$$

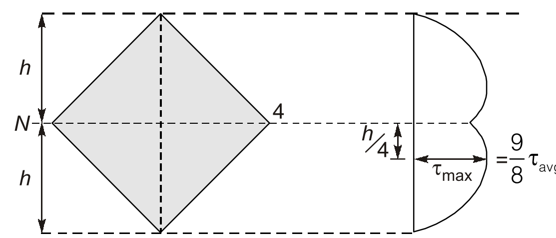
τ_{max} occur at $h/4$ from NA

$$= \frac{B\sqrt{2}}{2\times4} = \frac{B\sqrt{2}}{2\times4}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{B}{4\sqrt{2}}$$

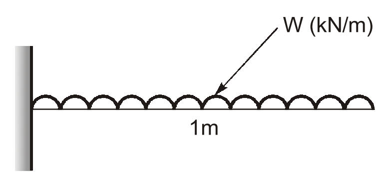
13. (c)

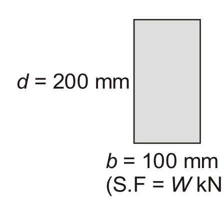
$$\tau_{max} = \frac{3}{2}.\tau_{avg} = 1.5\times\frac{200\times10^3}{200\times300} = 5 \text{ MPa}$$

14. (a)



15. (a)





$$d = 200$$

$$\tau_{max} = 1.5 \text{ N/mm2}$$

$$1.5 = \frac{3}{2}\times\frac{(W\times1)\times10^3}{100\times200}$$

$$W = \frac{400\times100\times1.5}{3\times10^3} = 20kN/m$$

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