CLASS TEST

S.No. : 01_KT_CE_19082022

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# STRENGTH OF MATERIALS

## CIVIL ENGINEERING

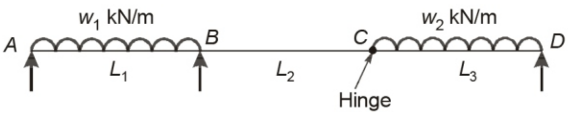
**Date of Test : 19/08/2022**

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (a) | 7. | (a) | 13. | (a) | 19. | (d) | 25. | (b) |
| 2. | (b) | 8. | (d) | 14. | (d) | 20. | (c) | 26. | (b) |
| 3. | (c) | 9. | (d) | 15. | (d) | 21 | (d) | 27. | (a) |
| 4. | (d) | 10. | (d) | 16. | (b) | 22. | (b) | 28. | (c) |
| 5. | (c) | 11. | (a) | 17. | (c) | 23. | (b) | 29. | (d) |
| 6. | (b) | 12. | (a) | 18. | (a) | 24. | (b) | 30. | (c) |

# DETAILED EXPLANATIONS

1. (a)



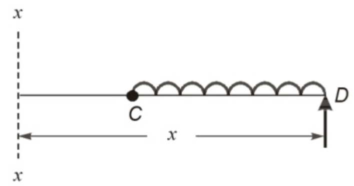

As details of loading are not given, just predict the probable shape of it.

Consider region *BC*

$$R_D \times L_3 = \frac{w_2 L_3^2}{2}$$

$$\Rightarrow \quad R_D = \frac{w_2 L_3}{2}$$

Consider section *x-x* at a distance *x* from *D*




$$(\text{BM})_{x\text{-}x} = R_D x - w_2 L_3\left(x - \frac{L_3}{2}\right)$$

$$(\text{BM})_{x\text{-}x} = \frac{-w_2 L_3 x}{2} + \frac{w_2 L_3^2}{2}$$

So, BM in region BC is linearly varying.

So correct option is (a)

2. (b)

$$\because \quad \frac{\tau_{max}}{D/2} = \frac{T}{J}$$

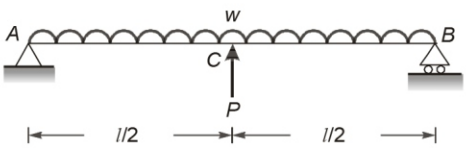
$$\Rightarrow \quad T = \frac{\tau_{max}}{D/2} \times J$$

$$\Rightarrow \quad T = \tau_{max}\frac{\pi}{16 D_0}\left[D_0^4 - D_i^4\right]$$

$$\Rightarrow \quad T = \frac{90 \times 10^3 \times \pi}{16 \times 0.25} \times \left[0.25^4 - 0.15^4\right] \text{ kNm}$$

$$\Rightarrow \quad T \simeq 240 \text{ kNm}$$

3. (c)




Deflection at *C* due to uniformly distributed load = $\frac{5wl^4}{384EI}(\downarrow)$

Deflection at $C$ due to prop support = $\frac{Pl^3}{48EI}(\uparrow)$

Applying compatibility equation at the prop,

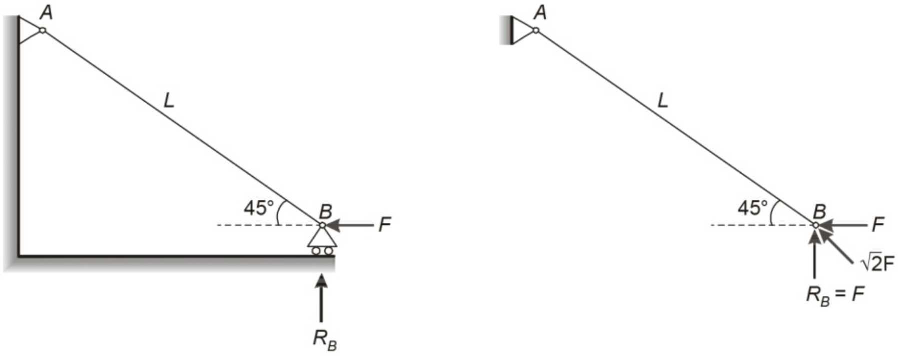
$$\frac{5wl^4}{384EI} = \frac{Pl^3}{48EI}$$

$$\Rightarrow \quad P = \frac{5wl}{8}$$

4. (d)

With the increase in carbon percentage in steel, hardness of steel does increase.

5. (c)




$$\because \quad \Sigma M_A = 0$$

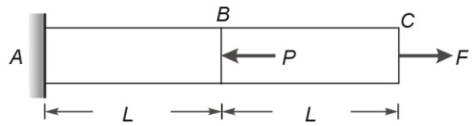
$$\Rightarrow \quad R_B \times L_1 - F \times L_1 = 0$$

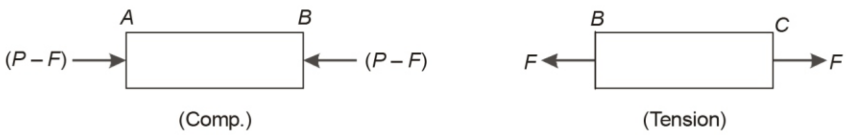
$$\Rightarrow \quad R_B = F$$

Buckling load for column, $$P_e = \frac{\pi^2 EI}{L^2} = \sqrt{2}F$$

$$\therefore \quad F = \frac{\pi^2 EI}{\sqrt{2}L^2}$$

6. (b)




$$\Delta_C = 0$$

$$\Rightarrow \quad \Delta_{BC} + \Delta_{AB} = 0$$

$$\Rightarrow \quad \frac{FL}{AE} - \frac{(P-F)L}{A \times 3E} = 0$$

$$\Rightarrow \quad F + \frac{F-P}{3} = 0$$

$$\Rightarrow \quad 4F = P$$

$$\Rightarrow \quad \frac{P}{F} = 4$$

7. (a)

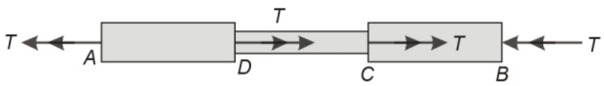
Thin plate of uniform thickness pertains to plane stress condition. So, stress out of plane will be zero.

8. (d)

$$\because \quad \frac{T}{I_P} = \frac{G\theta}{L}$$

where $\frac{\theta}{L}$ is rate of angle of twist

$$\therefore \quad \frac{\theta}{L} = \frac{T}{GI_P}$$

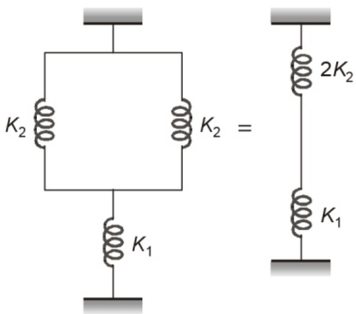



For segment $BC$, $\frac{\theta}{L}$ remains constant

For segment $CD$, $\frac{\theta}{L} = 0$, since $T = 0$

For segment $AD$, $\frac{\theta}{L}$ remains constant but with opposite sign


9. (d)




$$\equiv K_{eq} = \frac{2K_1K_2}{K_1 + 2K_2}$$

10. (d)

$$\Delta_A = \frac{M\left(\frac{L}{2}\right)}{EI} \times \frac{L}{2} + \frac{M\left(\frac{L}{2}\right)}{EI} \times \frac{L}{4}$$

$$= \frac{ML^2}{4EI} + \frac{ML^2}{8EI} = \frac{3ML^2}{8EI}$$

11. (a)

Free extension of bar *AB* under the load of 10 kN

$$\Delta L = \frac{PL}{AE} = \frac{10\times10^3\times0.5\times10^3}{100\times2\times10^5}$$
$$= 0.25 \text{ mm}$$

$\Delta L > 0.1$ mm (gap) and therefore free extension is prevented.

Compressive strain is developed in portion *BC*

Let stress in portion *AB* is $\sigma_1$ and that in portion *BC* is $\sigma_2$.

$$\therefore \quad \Delta_{AB} - \Delta_{BC} = 0.1 \text{ mm}$$

$$\Rightarrow \quad \frac{\sigma_1}{E}\times500 - \frac{\sigma_2}{E}\times500 = 0.1$$

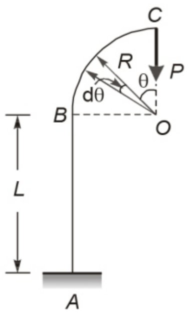
$$\Rightarrow \quad \sigma_1 - \sigma_2 = \frac{0.1\times2\times10^5}{500} = 40 \quad \text{...(i)}$$

$$\text{and} \quad R_A + R_C = 10 \text{ kN}$$
$$\text{But} \quad \sigma_1 A_1 = R_A$$
$$\text{and} \quad \sigma_2 A_2 = R_B$$
$$\therefore \quad \sigma_1 A_1 + \sigma_2 A_2 = 10\times10^3 \text{ N}$$
$$\Rightarrow \quad 100\sigma_2 + 200\sigma_2 = 10000$$
$$\Rightarrow \quad \sigma_1 + 2\sigma_2 = 100 \quad \text{...(ii)}$$

On solving (i) and (ii), we get

$$\sigma_2 = 20 \text{ N/mm}^2 \text{ (C)}$$
$$\text{and} \quad \sigma_1 = 60 \text{ N/mm}^2 \text{ (T)}$$

12. (a)




Using unit load method,

$$dx = Rd\theta$$

| Segment | Origin | $M$ | $m$ |
|---|---|---|---|
| *AB* | *B* | $PR$ | $R$ |
| *BC* | *C* | $PR\sin\theta$ | $R\sin\theta$ |

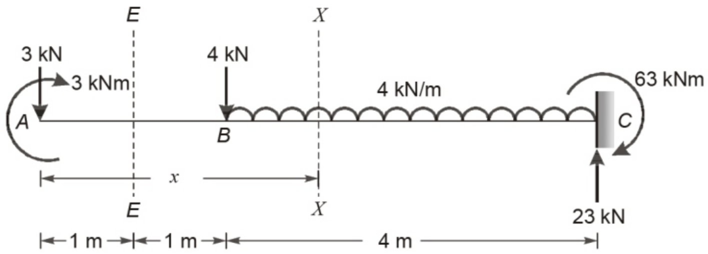
where, $m$ = moment due to unit load applied vertically at *C*

$$\therefore \quad \Delta_{Cv} = \int_0^{\pi/2} \frac{PR\sin\theta\cdot R\sin\theta(Rd\theta)}{EI} + \int_0^{L} \frac{PR\cdot R\,dx}{EI}$$

$$= \frac{PR^3}{2EI}\int_0^{\pi/2}(1-\cos2\theta)d\theta + \frac{PR^2L}{EI}$$

$$= \frac{PR^3\pi}{4EI} + \frac{PR^2L}{EI}$$

13. (a)




Portion AB:

$M_A = +3$ kNm (at $x = 0$)

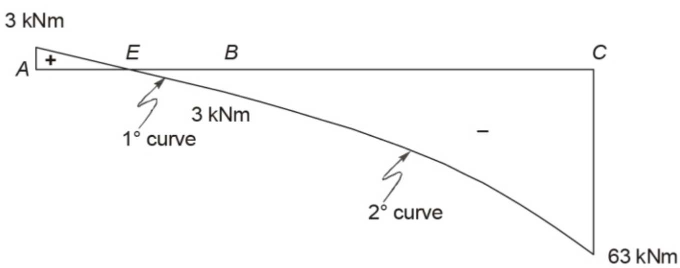
$M_B = +3 - 3 \times 2 = -3$ kNm (at $x = 2$ m)

$M_E = +3 - 3 \times 1 = 0$ (at $x = 1$ m)

Portion BC:

$$M_x = 3 - 3x - 4(x-2) - \frac{4}{2}(x-2)^2$$

$\therefore$ $M_c = -63$ kNm (at $x = 6$ m)




14. (d)


Maximum bending moment,

$M = PL = 12 \times 2 = 24$ kNm

Moment of inertia, $I = \dfrac{bd^3}{12}$

$\therefore$Maximum flexural stress, $f = \dfrac{M}{I} \cdot y = \dfrac{24 \times 10^6}{bd^3/12} \times \dfrac{d}{2}$

$= \dfrac{144 \times 10^6}{bd^2} = \dfrac{288 \times 10^6}{d^3}$ ($\because$ $b = 0.5\ d$)

But $f \leq 18$ MPa

$\Rightarrow$ $\dfrac{288 \times 10^6}{d^3} \leq 18$

$\Rightarrow$ $d \geq 251.98$ mm $\simeq 252$ mm

$\therefore$ $b = 0.5\ d = 0.5 \times 251.98 = 125.99$ mm $\simeq 126$ mm

$\therefore$ Most suitable section is 126 mm × 252 mm.

15. (d)

$$k = \frac{Gd^4}{8D^3N}$$

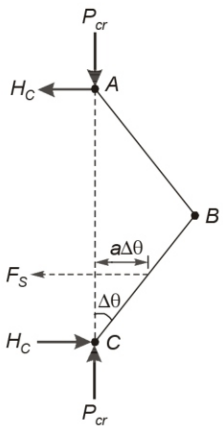
$$\therefore \quad \frac{k_2}{k_1} = \left(\frac{D_1}{D_2}\right)^3$$

$$\Rightarrow \quad \frac{k_2}{k_1} = \left(\frac{20}{10}\right)^3 = 8$$

∴ Stiffness of spring increases by 8 times.

16. (b)

Free body diagram of the entire system




$$\Sigma M_A = 0$$

$$\Rightarrow \quad H_C \times 4a - (ka\Delta\theta) \times 3a = 0$$

$$\therefore \quad H_C = \frac{3ka(\Delta\theta)}{4}$$

Now, $\Sigma M_B = 0$

$$\Rightarrow H_C \times 2a - P_{cr} \times (2a\Delta\theta) - ka\,(\Delta\theta) \times a = 0$$

$$\Rightarrow \quad P_{cr} = \frac{ka}{4} \qquad \left[\because H_C = \frac{3ka(\Delta\theta)}{4}\right]$$

17. (c)


Total strain energy stored is given by

$$U = 2\int_0^L \frac{(Px)^2\,dx}{2EI} + \frac{(PL)^2 \cdot 2L}{2EI}$$

$$= \frac{2P^2}{2EI}\int_0^L x^2 dx + \frac{P^2L^3}{EI}$$

$$= \frac{P^2}{EI} \times \frac{L^3}{3} + \frac{P^2L^3}{EI}$$

$$= \frac{P^2L^3}{3EI} + \frac{P^2L^3}{EI} = \frac{4}{3}\frac{P^2L^3}{EI}$$

18. (a)

Rise in temperature to expand the steel beam by 1.5 mm,

$$\Delta l = L\alpha\Delta T_0$$

$$1.5 = 4000 \times 0.000012 \times \Delta T_0$$

$$\Rightarrow \quad \Delta T_0 = 31.25°\text{C}$$

$\therefore$ Temperature responsible for development of stress in beam,

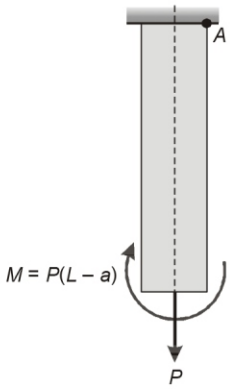
$$\Delta T_f = (315 - 31.25 - 15)°\text{C} = 268.75°\text{C}$$

$$\therefore \quad \text{Stress} = E\alpha\Delta T_f$$

$$= 2.1 \times 10^5 \times 0.000012 \times 268.75°\text{C}$$

$$= 677.25 \text{ N/mm}^2$$

19. (d)




Section modulus, $Z = \frac{2a(2a)^2}{6} = \frac{8a^3}{6}$

Bending stress, $\sigma_b = \frac{6P(L-a)}{8a^3}$

Direct stress, $\sigma_d = \frac{P}{4a^2}$

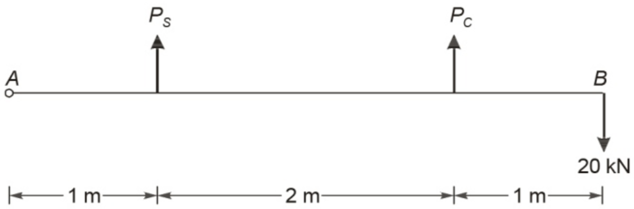
Total stress at $A = \sigma_b + \sigma_d$

$$= \frac{6P(L-a)}{8a^3} + \frac{P}{4a^2}$$

$$= \frac{6P(L-a) + P \times 2a}{8a^3}$$

$$= \frac{P(3L-2a)}{4a^3}$$

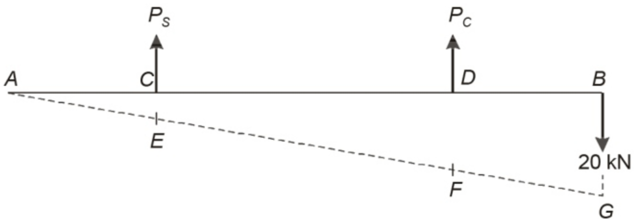
20. (c)




$\because \quad \Sigma M_A = 0$

$\Rightarrow \quad 1 \times P_s + 3 \times P_c = 20 \times 4$

$\Rightarrow \quad P_s + 3P_c = 80 \quad$ ...(i)




$\Delta ACE \sim \Delta ADF$, we get

$$\frac{AC}{CE} = \frac{AD}{DF}$$

$$\Rightarrow \quad \frac{1}{\Delta_s} = \frac{3}{\Delta_c}$$

$$\therefore \quad \Delta_c = 3\Delta_s$$

$$\Rightarrow \quad \frac{P_c L_c}{A_c E_c} = 3 \times \frac{P_s L_s}{A_s E_s}$$

$$\Rightarrow \quad \frac{P_c \times 2000}{400 \times 100 \times 10^3} = 3 \times \frac{P_s \times 1000}{200 \times 200 \times 10^3}$$

$$\therefore \quad 2P_c = 3P_s \quad \text{...(ii)}$$

Solving eq. (i) and (ii), we get

$$P_s + 3 \times 3\frac{P_s}{2} = 80$$

$$\therefore \quad P_s = 14.545 \text{ kN}$$

and $\quad P_c = 21.82 \text{ kN}$

So, Stress in steel rod, $\sigma_s = \frac{P_s}{A_s} = \frac{14.545 \times 10^3}{200} = 72.73 \text{ MPa}$

Stress in copper rod, $\sigma_c = \frac{P_c}{A_c} = \frac{21.82 \times 10^3}{400} = 54.55 \text{ MPa}$

21 (d)

Using the principle of superposition:

$$\theta_A = \frac{M_0 L}{3EI} + \frac{2M_0 L}{6EI} = \frac{2M_0 L}{3EI}$$

$$\theta_B = \frac{M_0 L}{6EI} + \frac{2M_0 L}{3EI} = \frac{5M_0 L}{6EI}$$

22. (b)
Due to increase in temperature, the bar *AB* will tend to expand, but since it is fixed at both of its ends, therefore compressive stress will be induced in the bar.
Expansion due to temperature rise,

$$\Delta l_t = l \alpha t$$

Compression due to compressive stress,

$$\Delta l = \frac{4Pl}{\pi E d_1 d_2}$$

$$\Rightarrow \quad l \alpha t = \frac{4Pl}{\pi E d_1 d_2}$$

$$\therefore \quad P = \frac{\pi E d_1 d_2 \alpha t}{4}$$

Maximum stress, $\sigma_{max} = \dfrac{P}{A_{min}}$

$$\sigma_{max} = \frac{\dfrac{\pi E d_1 d_2 \alpha t}{4}}{\dfrac{\pi}{4} d_2^2} = \frac{\alpha t E d_1}{d_2}$$

$$\therefore \quad \sigma_{max} = \frac{\left(12 \times 10^{-6}\right) \times 26 \times 200 \times 10^3 \times 75}{50}$$

$$= 93.6 \text{ MPa}$$

23. (b)

$$\because \quad \tau_{max} = \frac{16}{\pi D^3}\sqrt{M^2 + T^2}$$

$$= \frac{16}{\pi D^3}\sqrt{(30)^2 + (60)^2} \times 10^6$$

$$= \frac{341.65 \times 10^6}{D^3}$$

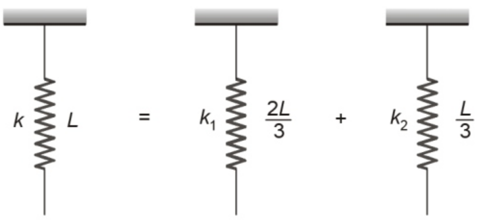
According to Tresca's theory,

$$\tau_{max} \leq \frac{\sigma_y}{\text{FOS} \times 2}$$

$$\Rightarrow \quad \frac{341.65 \times 10^6}{D^3} \leq \frac{250}{3 \times 2}$$

$$\therefore \quad D \simeq 202 \text{ mm}$$

24. (b)




Force constant, $k \propto \dfrac{1}{L}$

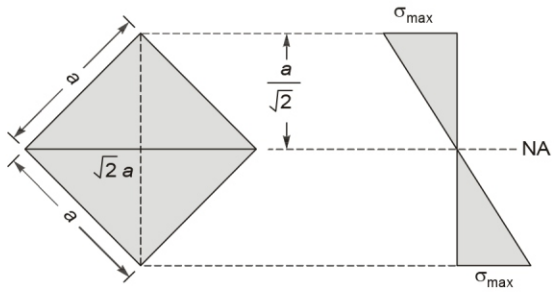
$$\Rightarrow \quad \frac{k_1}{k} = \frac{L}{\frac{2L}{3}}$$

$$\therefore \quad k_1 = \frac{3k}{2}$$

**25. (b)**

The yielding is peculiar to structural steel and other materials do not possess well defined yield point. Yield strength is defined as the lowest stress at which extension of test piece increases without further increase in load.

**26. (b)**




The bending stress in a beam is governed by its section modulus,

$$I_{NA} = 2\times\frac{bh^3}{12} = \frac{2\times\left(\sqrt{2}a\right)\times\left(\frac{4}{\sqrt{2}}\right)^3}{12} = \frac{a^4}{12}$$

$$y_{max} = \frac{a}{\sqrt{2}}$$

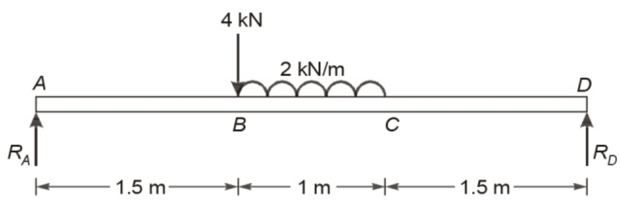
$$\text{Section modulus, } Z = \frac{I_{NA}}{y_{max}} = \frac{a^4/12}{a/\sqrt{2}} = \frac{a^3}{6\sqrt{2}}$$

$\therefore$ Maximum bending stress,

$$\sigma_{max} = \frac{M}{Z} = \frac{M}{a^3/6\sqrt{2}}$$

$$\Rightarrow \quad \sigma_{max} = \frac{6\sqrt{2}M}{a^3}$$

**27. (a)**




$\because \quad \Sigma F_y = 0$

$\Rightarrow \quad R_A + R_D = 4 + 2\times 1$

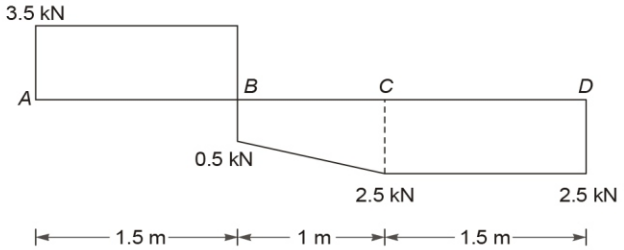
$\therefore \quad R_A + R_D = 6$ ...(i)

$\because \quad \Sigma M_D = 0$

$\Rightarrow \quad R_A \times 4 - 4\times 2.5 - 2\times 1\times 2 = 0$

$$\Rightarrow \qquad R_A = 3.5 \text{ kN}$$

$$R_D = 6 - 3.5 = 2.5 \text{ kN} \qquad \text{(Using equation (i))}$$




**SFD**

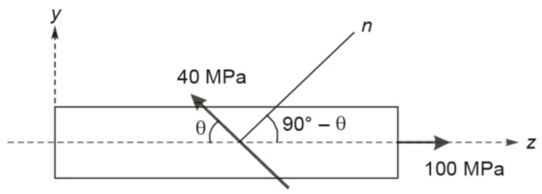
The maximum bending moment will occur at $B$ because the shear force diagram changes its sign at $B$ i.e., the distance of point $B$ from $C$ = 1 m.

**28. (c)**

$$\because \qquad \sigma_n = \frac{\sigma_z + \sigma_y}{2} + \left(\frac{\sigma_z - \sigma_y}{2}\right)\cos 2\theta + \tau_{zy}\sin 2\theta$$

and

$$\tau = \left(\frac{\sigma_z - \sigma_y}{2}\right)\sin 2\theta - \tau_{zy}\cos 2\theta$$




$$\therefore \qquad \tau = \left(\frac{100 - 0}{2}\right)\sin 2\theta$$

$$\Rightarrow \qquad 40 = 50\sin 2\theta$$

$$\therefore \qquad \theta = 26.56^\circ$$

and

$$90^\circ - \theta = 63.44^\circ$$

**29. (d)**

Beam is supported at $B$ and $D$

$$R_B = 2P + \frac{P}{3} = \frac{7P}{3}(\uparrow)$$

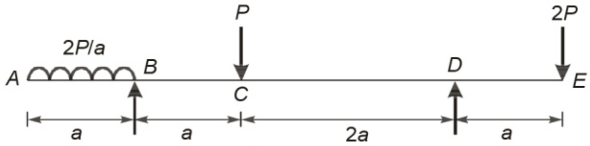
$$R_D = 2P + \frac{2P}{3} = \frac{8P}{3}(\uparrow)$$

$$\text{Total load on beam} = \frac{7P}{3} + \frac{8P}{3} = 5P(\downarrow)$$

$$\text{Rate of loading between } A \text{ and } B = \frac{2P - 0}{a} = \frac{2P}{a}(\downarrow)$$

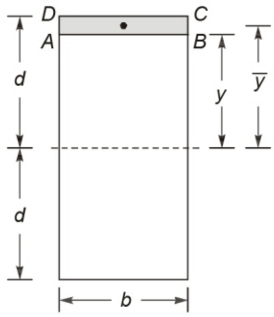
$$\text{Vertical load at } C = \frac{P}{3} + \frac{2P}{3} = P(\downarrow)$$

$$\text{Vertical load at } E = 2P(\downarrow)$$




Loading diagram

30. (c)




Area of shaded portion, $A = b(d-y)$

Distance of centroid of shaded area from neutral axis,

$$\bar{y} = y + \frac{d-y}{2} = \frac{d+y}{2}$$

$\because$ Shear stress, $\tau = F \times \frac{A\bar{y}}{Ib}$

$$\Rightarrow \quad \tau = F\frac{b(d-y)}{Ib} \times \left(\frac{d+y}{2}\right)$$

$$= \frac{F(d^2 - y^2)}{2I}$$

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