Civil Engineering

Fluid Mechanics including Hydraulic Machines

Comprehensive Theory

with Solved Examples and Practice Questions

MADE EASY
Publications

MADE EASY Publications Pvt. Ltd.

Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016
E-mail: infomep@madeeasy.in
Contact: 011-45124660, 8860378007
Visit us at: www.madeeasypublications.org

Fluid Mechanics including Hydraulic Machines

First Edition: 2015
Second Edition: 2016
Third Edition: 2017
Fourth Edition: 2018
Fifth Edition: 2019
Sixth Edition: 2020
Seventh Edition: 2021
Eighth Edition: 2022

Contents

Fluid Mechanics including Hydraulic Machines

Chapter 6

Chapter 7

Chapter 8

Chapter 9

Chapter 10

Chapter 11

Chapter 12

Turbulent Flow in Pipes 306

Chapter 13

Dimensional Analysis 327

Chapter 14

External Flow-Drag and Lift 354

Chapter 15

Open Channel Flow 376

■■■■

Chapter 01

Fluid Properties

1.1 Introduction

- A fluid is a substance which deforms continuously under the influence of shearing forces no matter how small the forces may be.
- Fluid is a substance that is capable of flowing and conforms to the shape of the containing vessel.
- This property of continuous deformation in technical terms is known as 'flow property', whereas this property is absent in solids.
- If a fluid is at rest, there can be no shearing forces acting and, therefore, all forces in the fluid must be perpendicular to the planes upon which they act.
- Fluids are classified as ideal fluids and practical or real fluids.
- Ideal fluids are those fluids which have neither viscosity nor surface tension and they are incompressible. In nature, the ideal fluids do not exist and therefore, they are only imaginary fluids.
- Practical or real fluids are those fluids which possess viscosity, surface tension and compressibility.
- Fluids are considered to be continuum i.e., a continuous distribution of matter with no voids or empty spaces.

1.2 Fluid Mechanics

- Fluid mechanics is the study of fluids at rest or in motion.
- It has been applied in areas such as the design of canal, levee, dam system, pumps etc.
- The basic laws which are applicable to any fluid for analysis of any problem in fluid mechanics, are
  (i) The law of conservation of mass
  (ii) Newton's second law of motion
  (iii) The principle of angular momentum
  (iv) The first law of thermodynamics
  (v) The second law of thermodynamics

1.3 Fluid as a Continuum

- In a fluid system, the intermolecular spacing between the fluid particles is treated as negligible and the entire fluid mass system is assumed as continuous distribution of mass, which is known as Continuum.

- This assumption is valid only if the fluid system is very large as compared to the spacing between the particles.
- As a consequence of the continuum, each fluid property is assumed to have a definite value at every point in space. Thus, the fluid properties such as density, temperature and velocity etc., are considered as continuous functions of position and time.

For Example:

Velocity field, $\vec{V} = \vec{V}(x, y, z, t)$ or $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

where, each component, *u, v, w* will be a function of *x, y, z* and *t*.

$\vec{V}(x, y, z, t)$ indicates the velocity of a fluid particle that is passing through the point *x, y, z* at time instant *t*.

Thus, the velocity is measured at the same location at different points of time.

In case of steady flow,

$$\frac{\partial \vec{V}}{\partial t} = 0$$

Therefore, $$\vec{V} = \vec{V}(x, y, z)$$

1.4 Fluid Properties

- Any characteristic of a fluid system is called a fluid property.
- Fluid properties are of two types:
  (i) **Intensive Properties:** Intensive properties are those that are independent of the size of the system or the amount of material in it. **Example:** Temperature, pressure, density etc.
  (ii) **Extensive Properties:** Extensive properties are those whose values depend on the size or extent of the system. **Example:** Total mass, total volume, total momentum etc.
- Following are some of the intensive and extensive properties of a fluid system.
  (i) Viscosity (ii) Surface tension (iii) Vapour pressure (iv) Compressibility and elasticity

1.4.1 Some other Important Properties

1. **Mass Density:** Mass density (or specific mass) of a fluid is the mass which it possesses per unit volume. It is denoted by the Greek symbol ρ. In SI system, the unit of ρ is kg/m^3.
2. **Specific Gravity:** Specific gravity (*S*) is the ratio of specific weight (or mass density) of a fluid to the specific weight (or mass density) of a standard fluid. The standard fluid chosen for comparison is pure water at 4°C.

   $$\text{Specific gravity of liquid (S)} = \frac{\text{Specific weight of liquid}}{\text{Specific weight of water}} = \frac{\text{Specific weight of liquid}}{9810 \text{ N/m}^3}.$$
3. **Relative Density (R.D.):** It is defined as ratio of density of one substance with respect to other substance. $\rho_{1/2} = \dfrac{\rho_1}{\rho_2}$.

   where, $\rho_{1/2}$ = Relative density of substance '1' with respect to substance '2'.
4. **Specific Weight:** Specific weight (also called weight density) of a fluid is the weight it possesses per unit volume. It is denoted by the Greek symbol γ (gamma). For water, it is denoted by γ_w. Its SI unit is N/m^3. The mass density and specific weight γ has following relationship $\gamma = \rho g$; $\rho = \gamma/g$. Both mass density and specific weight depend upon temperature and pressure.

5. **Specific Volume:** Specific volume of a fluid is the volume of the fluid per unit mass. Thus it is the reciprocal of density. It is generally denoted by v. In SI unit specific volume is expressed in cubic meter per kilogram, i.e., m^3/kg.

Example 1.1 Three litres of petrol weigh 23.7 N. Calculate the mass density, specific weight, specific volume and specific gravity of petrol.

Solution :

Mass density of petrol, $\rho_p = \frac{M}{V} = \frac{W/g}{V} = \frac{W}{gV} = \frac{23.7}{9.81 \times 3} = 0.805 \text{ kg/litre} = 805 \text{ kg/m}^3$

Mass density of water, $\rho_w = 1000 \text{ kg/m}^3$

$$\text{Specific gravity of petrol} = \frac{\rho_p}{\rho_w} = \frac{805}{1000} = 0.805$$

$$\text{Specific weight of petrol} = \frac{W}{V} = \frac{23.7}{3.0} = 7.9 \text{ N/litre} = 7.9 \text{ kN/m}^3$$

$$\text{Specific volume} = \frac{V}{M} = \frac{1}{\rho_p} = \frac{1}{805} = 1.242 \times 10^{-3} \text{ m}^3/\text{kg}$$

1.4.2 Viscosity

- Viscosity is a property of the fluids by virtue of which they offer resistance to shear or angular deformation.
- It is primarily due to cohesion and molecular momentum exchange between fluid layers, and as flow occurs, these effects appear as shearing stresses between the moving layers.

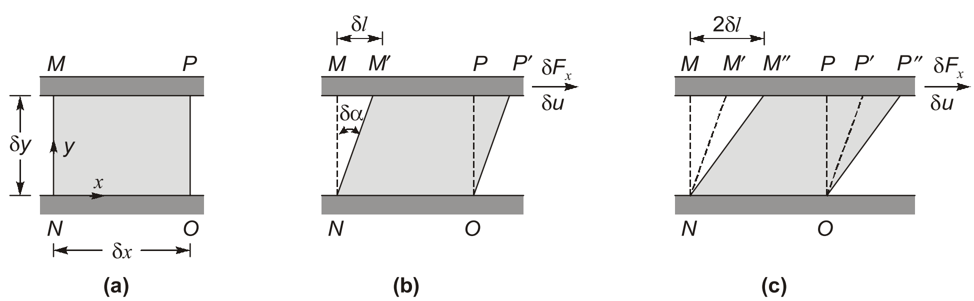


Figure 1.1 **(a)** Fluid element at time t, **(b)** deformation of fluid element at time $t + \delta t$, and **(c)** deformation of fluid element at time $t + 2\delta t$.

- Consider the behavior of a fluid element between the two infinite plates as shown in Figure 1.1 (a). The rectangular fluid element is initially at rest at time t. Let us now suppose a constant rightward force δF_x is applied to the upper plate so that it is dragged across the fluid at constant velocity δu. The relative shearing action of the plates produces a shear stress, τ_{yx}, which acts on the fluid element and is given by $\tau_{yx} = \lim_{\delta A_y \to 0} \frac{\delta F_x}{\delta A_y} = \frac{dF_x}{dA_y}$, where δA_y is the area of contact of the fluid element with the plate and δF_x is the force exerted by the plate on that element.

Various positions of the fluid element, shown in Figure (1.1) illustrate the deformation of the fluid element from position *MNOP* at time *t*, to *M′NOP′* at time $t + \delta t$, to *M″NOP″* at time $t + 2\delta t$, due to the imposed shear stress. The deformation of the fluid is given by

$$\text{Deformation rate} = \lim_{\delta t \to 0} \frac{\delta\alpha}{\delta t} = \frac{d\alpha}{dt}$$

Distance between the points *M* and *M′* is given by,

$$\delta l = \delta u \delta t \quad ...(1)$$

Alternatively, for small angles, $\delta l = \delta y \delta\alpha$ ...(2)

Equating Equations (1) and (2), $\delta u \delta t = \delta y \delta\alpha$

or $$\frac{\delta\alpha}{\delta t} = \frac{\delta u}{\delta y}$$

Taking the limits of both sides

$$\lim_{\delta t \to 0} \frac{\delta\alpha}{\delta t} = \lim_{\delta t \to 0} \frac{\delta u}{\delta y}$$

$$\frac{d\alpha}{dt} = \frac{du}{dy}$$

Thus, the rate of angular deformation is equal to velocity gradient across the flow.

- On the basis of relation between the applied shear stresses and the flow or rate of deformation, fluids can be categorized as Newtonian and non - Newtonian fluids.

1.4.2.1 Newtonian Fluids

- Fluids which obey Newton's law of viscosity are known as Newtonian fluids.
- According to Newton's law of viscosity, shear stress is directly proportional to the rate of deformation or velocity gradient across the flow.

  Thus, $\tau \propto \frac{du}{dy}$ or $\tau = \mu \frac{du}{dy}$

  where, μ = coefficient of dynamic viscosity
- Water, air, and gasoline are Newtonian under normal conditions.

Dynamic Viscosity (μ)

- Dimension of $\mu = [M L^{-1} T^{-1}]$
- Unit of μ = Ns/m^2 or Pa.s
- In c. g. s. units, μ is expressed as 'poise', 1 poise = 0.1 N-s/m^2
- $(\mu)_{water} \approx 10^{-3}$ Ns/m^2;
  $(\mu)_{air} \approx 1.81 \times 10^{-5}$ Ns/m^2 (Both at 20°C and at standard atmospheric pressure)

NOTE: Water is nearly 55 times viscous than air.

Kinematic Viscosity (ν)

- The kinematic viscosity (ν) is defined as the ratio of dynamic viscosity to mass density of the fluid therefore, $\nu = \mu/\rho$
- Dimension of $\nu = [L^2 T^{-1}]$
- Unit of ν = m^2/s or cm^2/s (stoke)
- 1 stoke = 10^{-4} m^2/s
- At 20°C and at standard atmospheric pressure $\nu_{water} = 1 \times 10^{-6}$ m^2/s, $\nu_{air} = 15 \times 10^{-6}$ m^2/s

NOTE: Kinematic viscosity of air is about 15 times greater than the corresponding value of water.

Variation of Viscosity with Temperature

- Increase in temperature causes a decrease in the viscosity of a liquid, whereas viscosity of gases increases with temperature growth (Figure 1.2).
- The reason for the above phenomena is that; in liquids; viscosity is primarily due to molecular cohesion which decreases with increase in volume due to temperature increment, while in gases, viscosity is due to molecular momentum transfer which increases with increase in number of collision between gas molecules.

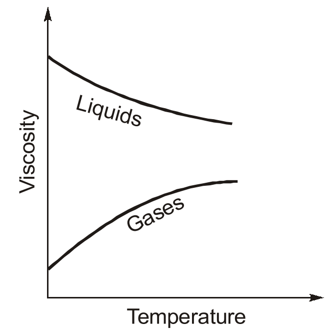


Figure 1.2 Variation of Viscosity with Temperature

1.4.2.2 Non-Newtonian Fluids

- Fluids for which shear stress is not directly proportional to deformation rate are non-Newtonian. Toothpaste and paint are the examples of non-Newtonian fluids.
- Non-Newtonian fluids are commonly classified as having time-independent or time-dependent behavior.

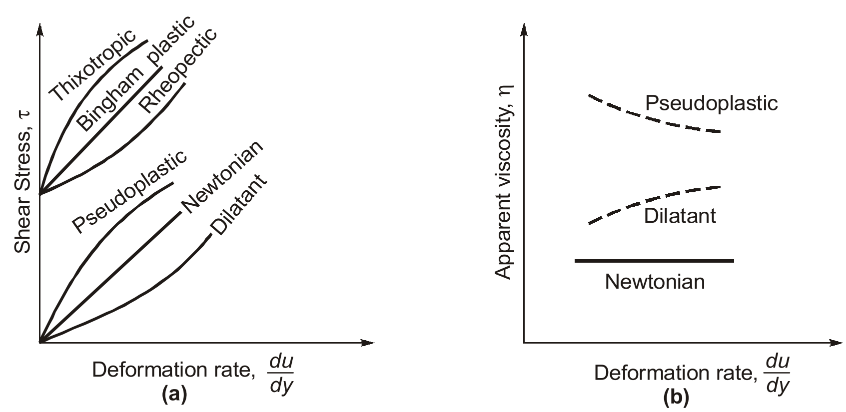


Figure 1.3 **(a)** Shear stress, τ and **(b)** Apparent viscosity, η

- Relation between shear stress and rate of deformation for non-Newtonian fluid can be represented as

$$\tau = k\left(\frac{du}{dy}\right)^n$$

where, n = flow behavior index; k = consistency index

For Newtonian fluid, $n = 1$; $k = \mu$

Above equation can also be represented as

$$\tau = k\left(\frac{du}{dy}\right)^{n-1}\left(\frac{du}{dy}\right) = \eta\frac{du}{dy}$$

where, $\eta = k\left(\frac{du}{dy}\right)^{n-1}$ is referred as the apparent viscosity

NOTE: Dynamic viscosity (μ) is constant (except for temperature effects) while apparent viscosity (η) depends on the shear rate.

- Various types of non-Newtonian fluids are :
  1. **Pseudoplastic**: Fluids in which the apparent viscosity decreases with increasing deformation rate ($n < 1$) are called pseudoplastic fluids (or shear thinning). Most non-Newtonian fluids fall into this group.
     Example: Polymer solutions, colloidal suspensions, milk, blood and paper pulp in water.
  2. **Dilatant:** If the apparent viscosity increases with increasing deformation rate ($n > 1$), the fluid is termed as dilatant (or shear thickening).
     Example: Suspensions of starch, saturated sugar solution.
  3. **Bingham Plastic:** Fluids that behave as a solid until a minimum yield stress, τ_y, and flow after crossing this limit are known as ideal plastic or Bingham plastic. The corresponding shear stress model is $\tau = \tau_y + \mu \dfrac{du}{dy}$.
     Example: Clay suspensions, drilling muds, creams and toothpaste.
  4. **Thixotropic:** Apparent viscosity (η) for thixotropic fluids decreases with time under a constant applied shear stress.
     Example: Paints, printer inks
  5. **Rheopectic:** Apparent viscosity (η) for rheopectic fluids increases with time under constant shear stress.
     Example: Gypsum pastes.

NOTE

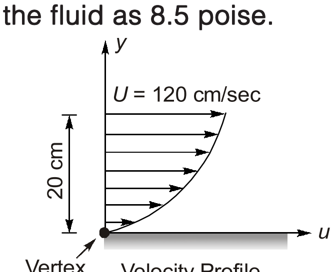
(i) There is no relative movement between fluid attached to the solid boundary and solid boundary i.e. the fluid layer just adjacent to the solid surface will have same velocity as of the solid surface.

(ii) Viscoelastic : Fluids which after some deformation partially return to their original shape when the applied stress is released such fluids are called viscoelastic.

(iii) Rheology : Branch of science which deals with the studies of different types of fluid behaviours.

Example 1.2 If the velocity profile of a fluid over a plate is parabolic with free stream velocity of 120 cm/s occurring at 20 cm from the plate, calculate the velocity gradients and shear stress at a distance of 0, 10 and 20 cm from the plate. Take the viscosity of the fluid as 8.5 poise.

Solution:



Given:

Distance of surface from plate = 20 cm

Velocity at surface, $U = 120$ cm/s

Viscosity, $\mu = 8.5 \text{ poise} = \dfrac{8.5}{10} \text{Ns/m}^2 = 0.85 \text{ Ns/m}^2$

The velocity profile is given parabolic. Hence equation of velocity profile is

$$u = ay^2 + by + c \quad \text{...(i)}$$

where a, b and c are constants. Their values are determined from boundary conditions as:

(a) at $y = 0$, $u = 0$

(b) at $y = 20$ cm, $u = 120$ cm/s

(c) at $y = 20$ cm, $\dfrac{du}{dy} = 0$

Substituting boundary condition (*a*) in equation (i), we get

$$c = 0$$

Boundary condition (*b*) on substitution in (i) gives

$$120 = a(20)^2 + b(20) = 400a + 20b \quad \text{...(ii)}$$

Boundary condition (*c*) on substitution in equation (i) gives

$$\frac{du}{dy} = 2ay + b \quad \text{...(iii)}$$

or $$0 = 2 \times a \times 20 + b = 40\,a + b$$

Solving equations (ii) and (iii) for *a* and *b*

From equation (iii), $b = -40\,a$

Substituting this value in equation (ii), we get

$$120 = 400\,a + 20 \times (-40a)$$
$$= 400a - 800a = -400a$$

$\therefore$ $$a = \frac{120}{-400} = -\frac{3}{10} = -0.3$$

$\therefore$ $$b = (-40) \times (-0.3) = 12.0$$

Substituting the values of *a*, *b* and *c* in equation (i)

$$u = -0.3\,y^2 + 12\,y$$

Velocity Gradient

$$\frac{du}{dy} = -0.3 \times 2y + 12 = -0.6y + 12$$

at $y = 0$, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=0} = -0.6 \times 0 + 12 = 12$ per sec

at $y = 10$ cm, $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = -6 + 12 = 6$ per sec

at $y = 20$ cm, $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = -12 + 12 = 0$

Shear Stresses

Shear stress is given by $$\tau = \mu \frac{du}{dy}$$

(i) Shear stress at $y = 0$, $\tau = \mu\left(\frac{du}{dy}\right)_{y=0} = 0.85 \times 12.0 = 10.2\ \text{N/m}^2$

(ii) Shear stress at $y = 10$, $\tau = \mu\left(\frac{du}{dy}\right)_{y=10} = 0.85 \times 6.0 = 5.1\ \text{N/m}^2$

(iii) Shear stress at $y = 20$, $\tau = \mu\left(\frac{du}{dy}\right)_{y=20} = 0.85 \times 0 = 0$

Example 1.3 Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerin. What force is required to drag a very thin plate of surface area 0.5 square metre between the two large plane surfaces at a speed of 0.6 m/s, if:

(i) the thin plate is in the middle of the two plane surfaces, and

(ii) the thin plate is at a distance of 0.8 cm from one of the plane surfaces?

Take the dynamic viscosity of glycerin = 8.10×10^{-1} Ns/m^2. Assume linear velocity distribution in transverse direction.

Solution :

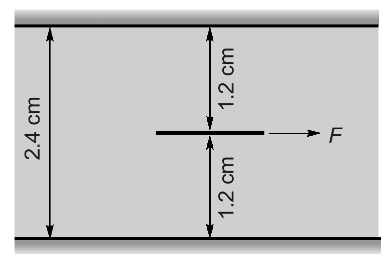
Given:

Distance between two large surfaces = 2.4 cm

Area of thin plate, $A = 0.5$ m^2

Velocity of thin plate, $u = 0.6$ m/s

Viscosity of glycerin, $\mu = 8.10 \times 10^{-1}$ Ns/m^2



Case-I: When the thin plate is in the middle of the two plane surfaces as shown in Figure.

Let, F_1 = Shear force on the upper side of the thin plate

F_2 = Shear force on the lower side of the thin plate

F = Total force required to drag the plate

Then, $F = F_1 + F_2$

The shear stress (τ_1) on the upper side of the thin plate is given by equation,

$$\tau_1 = \mu\left(\frac{du}{dy}\right)_1$$

where, du = Relative velocity between thin plate and upper large plane surface = 0.6 m/s.

dy = Distance between thin plate and upper large plane surface

= 1.2 cm = 0.012 m (plate is a thin one and hence thickness of plate is neglected)

Assuming linear velocity distribution between large plane surfaces and thin plate.

$$\therefore \quad \tau_1 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.012}\right) = 40.5 \text{ N/m}^2$$

Now shear force, F_1 = Shear stress × Area

$= \tau_1 \times A = 40.5 \times 0.5 = 20.25$ N

Similarly shear stress (τ_2) on the lower side of the thin plate is given by

$$\tau_2 = \mu\left(\frac{du}{dy}\right)_2 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.012}\right) = 40.5 \text{ N/m}^2$$

$\therefore$ Shear force, $F_2 = \tau_2 \times A = 40.5 \times 0.5 = 20.25$ N

$\therefore$ Total force, $F = F_1 + F_2 = 20.25 + 20.25 = 40.5$ N

Case II: When the thin plate is at a distance of 0.8 cm from one of the plane surfaces, as shown in the Figure below.

Let the thin plate is at a distance 0.8 cm from the lower plane surface.

Then distance of the plate from the upper plane surface

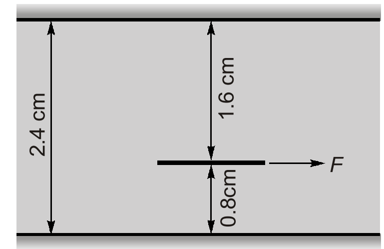
= 2.4 – 0.8 = 1.6 cm = 0.016 m (Neglecting thickness of the plate)

The shear force on the upper side of the thin plate,

$$F_1 = \text{Shear stress} \times \text{Area} = \tau_1 \times A$$

$$= \mu\left(\frac{du}{dy}\right)_1 \times A$$

$$= 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.016}\right) \times 0.5 = 15.19 \text{ N}$$



The shear force on the lower side of the thin plate,

$$F_2 = \tau_2 \times A = \mu\left(\frac{du}{dy}\right)_2 \times A$$

$$= 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.8/100}\right) \times 0.5 = 30.38 \text{ N}$$

$\therefore$ Total force required $= F_1 + F_2 = 15.19 + 30.38 = 45.57$ N **Ans.**

1.4.3 Surface Tension

- The property of the liquid surface film to exert tension is called the surface tension.
- Surface tension is due to "cohesion" between the liquid particles and the surface.
- Whenever a liquid is in contact with other liquids or gases, or solid surface, an interface develops that acts like a stretched elastic membrane, creating surface tension.
- There are two features to this membrane : the contact angle θ, and the magnitude of the surface tension, σ (N/m). Both of these, depend on the type of liquid and the type of solid surface (or other liquid or gas) with which it shares an interface. For example, the car's surface will get wetted when water is applied to the surface. If before applying water, waxing is done to the car's surface and then water is applied, the car's surface will not get wet.

  This is because of the change of the contact angle from being smaller than 90°, to larger than 90° because, in effect, the waxing changed the nature of the solid surface.

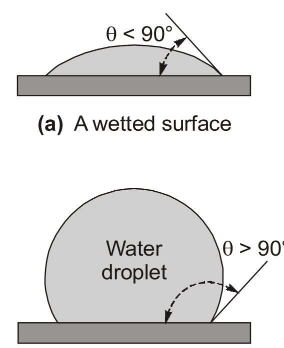


Figure 1.4 Surface tension effect on water droplets

- For liquids, surface tension decreases with increase in temperature.
- It is due to surface tension, the liquid droplets take the form of a sphere, which is the shape for minimum surface area.
- Unit of surface tension – N/m.
- Due to surface tension, pressure change occurs across a curved interface.

Remember

For air-water interface, $\sigma = 0.073$ N/m.
For water glass interface, contact angle, $\theta \approx 0$.
For air-mercury interface, $\sigma = 0.480$ N/m.
For mercury glass interface, contact angle, $\theta = 130°$.

Droplet and Jet

- When a droplet is separated initially from the surface of the main body of liquid, then due to surface tension there is a net inward force exerted over the entire surface of the droplet which causes the surface of the droplet to contract from all the sides and results in increasing the internal pressure within the droplet.
- The contraction of the droplet continues till the inward force due to surface tension is in balance with the internal pressure and the droplet forms into sphere which is the shape for minimum surface area.
- The internal pressure within a jet of liquid is also increased due to surface tension.
- The internal pressure intensity within a droplet and a jet of liquid in excess of the outside pressure intensity may be determined by the expressions derived below.

(i) **Pressure intensity inside a droplet :** Consider a spherical droplet (Figure 1.5 (a)) of radius r having internal pressure intensity p in excess of the outside pressure intensity. If the droplet is cut into two halves, then the forces acting on one half (Figure.1.5 (b)) will be those due to pressure intensity (p) on the projected area (πr^2) and the tensile force due to surface tension (σ) acting around the circumference ($2\pi r$). These two forces will be equal and opposite for equilibrium and hence we have

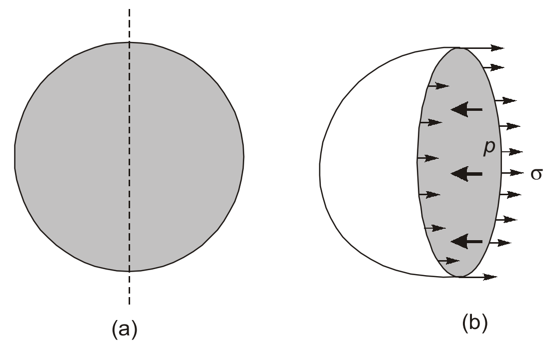


Figure 1.5 Surface Tension (σ) and Internal Pressure (p) in a droplet

$$p(\pi r^2) = \sigma(2\pi r)$$

or
$$p = \frac{2\sigma}{r}$$

NOTE: Above equation indicates that the internal pressure intensity increase with the decrease in the size of droplet.

(ii) **Pressure intensity inside a soap bubble :** A spherical soap bubble has two surfaces in contact with air, one inside and the other outside, each one of which contributes the same amount of tensile force due to surface tension (Figure 1.6). As such on a hemispherical section of a soap bubble of radius r, the tensile force due to surface tension is equal to $2\sigma(2\pi r)$. However, the pressure force acting on the hemispherical section of the soap bubble is same as in the case of a droplet and it is equal to $p(\pi r^2)$. Thus equating these two forces for equilibrium, we have

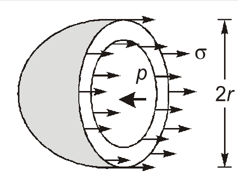


Figure 1.6 Soap Bubble

$$p(\pi r^2) = 2\sigma(2\pi r)$$

or
$$p = \frac{4\sigma}{r}$$

(iii) **Pressure intensity inside a liquid jet :** Consider a jet of liquid of radius r, length l and having internal pressure intensity p in excess of outside pressure intensity. If the jet is cut into two halves, then the forces acting on one half will be those due to pressure intensity p on the projected area

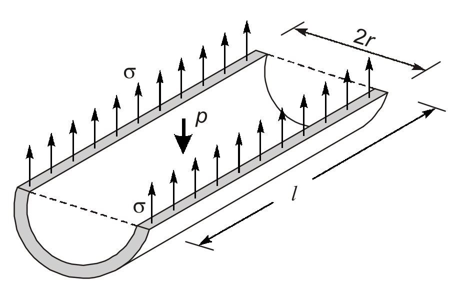


Figure 1.7 Liquid Jet

($2rl$) and the tensile force due to surface tension (σ) acting along the two sides ($2l$). These two forces will be equal and opposite for equilibrium and hence we have (Figure 1.7),

$$p(2rl) = \sigma(2l)$$

or
$$p = \frac{\sigma}{r}$$

Example 1.4 If the surface tension at the air-water interface is 0.073 N/m, estimate the pressure difference between inside and outside of an air bubble of diameter 0.01 mm.

Solution :

An air bubble has only one surface.

Hence
$$\Delta p = \frac{2\sigma}{R} = \frac{2 \times 0.073}{\left(\frac{0.01}{2}\right) \times 10^{-3}}$$

$$= 29200 \text{ N/m}^2 = 29.2 \text{ kPa}$$

Capillarity

- For a liquid in contact with a surface, if adhesion predominates cohesion, then the liquid will wet the surface with which it is in contact and tend to rise at the point of contact.
- The free surface of the fluid will be concave upward and the contact angle (θ) will be less than 90°.
  Example : Immersion of a glass tube in water.
- On the other hand, if for any liquid in contact with a surface, cohesion predominates, the liquid will not wet the surface and the liquid surface will be depressed at the point of contact.
- The liquid surface will be concave downward and the angle of contact θ will be greater than 90°.
  Example : Immersion of the glass tube in mercury.
- Such a phenomenon of rise or fall of liquid surface relative to the adjacent general level of liquid is known as capillarity.
- Let the level of liquid rises (or fall) by h above (or below) the general liquid surface when a tube of radius r is inserted in a liquid having specific gravity 'S' as shown in Figure 1.8. By equilibrium condition, the weight of liquid column of height h (or the total internal pressure in the case of capillary depression) must be balanced by the force, at surface of the liquid, due to surface tension σ.

  Thus,
  $$S\gamma_w \pi r^2 h = 2\pi r \sigma \cos\theta$$

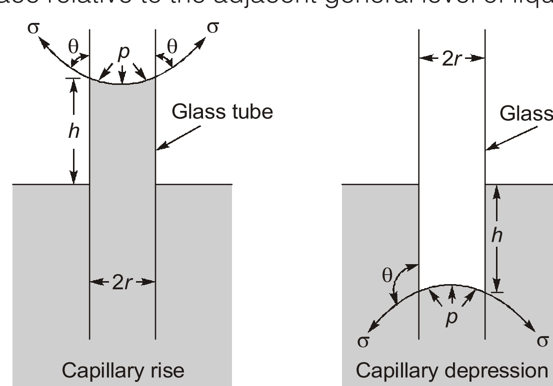


Figure 1.8 Capillarity in circular glass tubes

where, S = specific gravity of liquid, γ_w = specific weight of water, θ = contact angle

$$h = \frac{2\sigma\cos\theta}{S\gamma_w r}$$

Since, the contact angle θ for water and glass is equal to zero, $h = \dfrac{2\sigma}{\gamma_w r}$

Assumptions in deriving above equations

(a) The meniscus of the curved liquid surface is a section of sphere.

(b) The liquid and tube surface are extremely clean.

Remember: At 20°C and for water, $h = \dfrac{0.30}{d}$ m

where, d = diameter of tube in cm

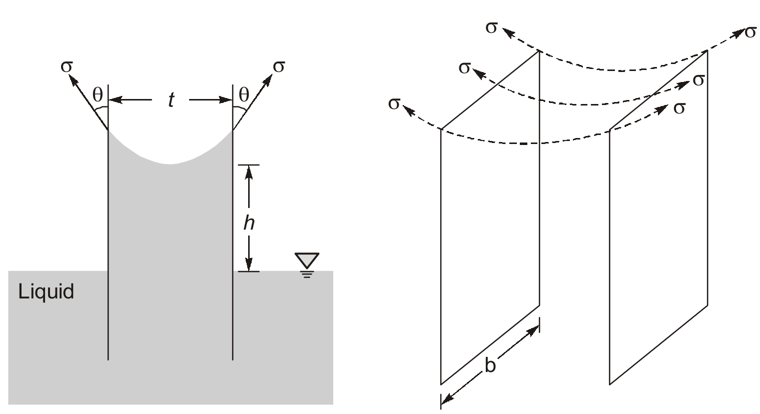
NOTE: With increase in diameter of the tube, capillary rise decreases. For tube of diameter more than 6 mm (radius > 3 mm) the capillary rise is negligible.

- If an annular tube, is immersed in a liquid, with outer radius r_o and inner radius r_i, then capillary rise is given by, $h = 2\sigma \cos\theta/(r_o - r_i)S\gamma_w$.
- If a tube of radius 'r' is inserted in mercury (Specific gravity, S_1) above which a liquid of specific gravity, S_2 lies, then the capillary depression h is given by, $h = \dfrac{2\sigma\cos\theta}{r\gamma_w(S_1 - S_2)}$.
- If two vertical plates 't' distance apart are held partially immersed in a liquid of surface tension σ and specific gravity, S, then capillary rise or depression h is given by, $h = \dfrac{2\sigma\cos\theta}{S\gamma_w t}$.

NOTE: On reducing the height of capillary tube than the required height, the curvature of the liquid surface inside the capillary tube rearranges itself. The contact angle increases for a interface and it decreases for a non-wetting interface.

Example 1.5 **Derive an expression for the capillary rise between two vertical parallel plates of width *b* partially immersed in a liquid of specific gravity *S* in terms of the distance *t* between the plates, surface tension σ and the contact angle θ between the liquid and the plates.**

Solution :



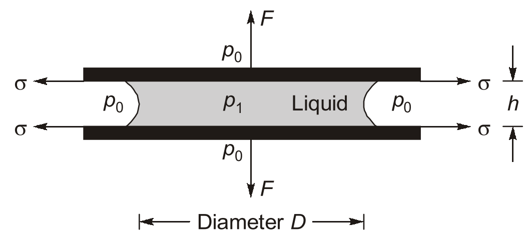
Upward surface tension force $= \sigma\cos\theta \times 2b$ ...(i)

Weight of the liquid column $= S\rho_w g\, h\, t\, b$ ...(ii)

Since, for equilibrium condition, the surface tension force and weight of the liquid column are equal, therefore

Solution :

Let p_0 and p_1 be the ambient pressure and pressure inside the liquid and Δp be the difference between them.



i.e. $\Delta p = p_1 - p_0$ {where, $p_0 > p_1$}

The circular spot is stable if

Total force on the liquid surface due to

$$\text{difference in pressure} = \text{Total surface tension force}$$

$$\pi Dh(\Delta p) = 2\sigma(\pi D)$$

or

$$\Delta p = \frac{2\sigma}{h}$$

So, force required to pull the plates apart, $F = \left(\frac{\pi D^2}{4}\right)(\Delta p) = \frac{2\pi D^2\sigma}{4h} = \frac{\pi}{2}\left(\frac{D}{h}\right)(\sigma D)$

Summary

- A fluid is a substance that deforms continuously when subjected to even an infinitesimal shear stress.
- Fluid mechanics is the study of fluids at rest or in motion.
- The concept of a continuum assumes a continuous distribution of mass within the matter or system with no empty space.
- Viscosity is the property of a fluid by virtue of which it offers resistance to flow.
- The rate of deformation of any fluid element in a fluid flow is equal to the velocity gradient across the flow.
- Fluids which obey Newton's law of viscosity are called Newtonian fluids and which do not obey are called non-Newtonian fluids.
- It is due to surface tension that a curved liquid interface, in equilibrium, results in a greater pressure at concave side than that at its convex side.
- A liquid wets a solid surface and results in a capillary rise when the forces of cohesion between the liquid molecules are lower than the forces of adhesion between the molecules of the liquid and the solid in contact.
- Cavitation occurs when the local pressure reduces below vapour pressure of the liquid.
- Compressibility of a substance is the measure of its change in volume or density under the action of external forces.

Important Expressions

- Deformation rate $\left(\frac{d\alpha}{dt}\right)$ = velocity gradient $\left(\frac{du}{dy}\right)$
- Shear stress for Newtonian fluids : $\tau = \mu\left(\frac{du}{dy}\right)$

- Shear stress for non-Newtonian fluids: $\tau = k\left(\frac{du}{dy}\right)^n = k\left(\frac{du}{dy}\right)^{n-1}\frac{du}{dy} = \eta\frac{du}{dy}$
- Shear stress for Bingham plastics : $\tau = \tau_y + \mu\frac{du}{dy}$
- Pressure intensity p in excess of the outside pressure intensity in a droplet : $p = \frac{2\sigma}{r}$
- Pressure intensity p in excess of the outside pressure intensity in a soap bubble: $p = \frac{4\sigma}{r}$
- Pressure intensity p in excess of the outside pressure intensity in a liquid jet : $p = \frac{\sigma}{r}$
- Capillary rise or fall : $h = \frac{2\sigma\cos\theta}{S\gamma_w r}$
- Compressibility : $\beta = \frac{1}{K} = \frac{-(\Delta V/V)}{dp} = \frac{(d\rho/\rho)}{dp}$

Objective Brain Teasers

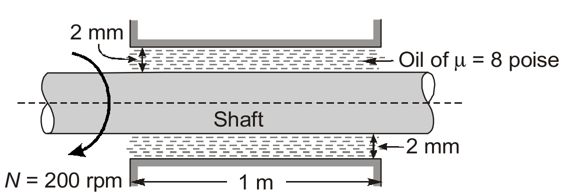
Q.1 If the dynamic viscosity of a liquid is 0.012 poise and its R.D. is 0.79, then its kinematic viscosity in stoke is

(a) 0.0152 (b) 0.152
(c) 1.52 (d) 15.20

Q.2 The velocity distribution, in m/s near the solid wall at a section in a laminar flow is given by $u = 5\sin(5\pi y)$. If $\mu = 5$ poise, the shear stress at $y = 0.05$m, in N/m^2 is

(a) 39.27 (b) 27.77
(c) 38.9 (d) 26.66

Q.3 A solid shaft diameter of 350 mm, rotates at 200 rpm inside a fixed sleeve bearing as shown in figure. The dynamic viscosity of oil is 8 poise.



The power lost (in kW) due to viscosity in bearing is

(a) 4.9 (b) 5.9
(c) 11.8 (d) 2.95

Q.4 A fluid indicated the following shear stress and deformation rates :

$\frac{du}{dy}$ (units)	0	1	2	4
τ (units)	10	15	20	30

This fluid is classified as

(a) Newtonian (b) Bingham Plastic
(c) Dilatant (d) Pseudoplastic

Q.5 Kerosene is known to have a bulk modulus of elasticity $K = 1.43 \times 10^9$ N/m^2 and a relative density of 0.806. The speed of sound in kerosene, (in m/s) is

(a) 1332 (b) 1075
(c) 1197 (d) 184

Q.6 If 5.66 m^3 of oil weighs 4765 kg, then its mass density, specific weight and specific gravity respectively are

(a) 841.87 kg/m^3, 8.26 kN/m^3 and 0.842
(b) 8.26 kg/m^3, 841 kN/m^3 and 8.42

(c) 841.87 kg/m^3, 841 kN/m^3 and 8.42
(d) None of these

Q.7 A reservoir of capacity 0.01 m^3 is completely filled with a fluid of coefficient of compressibility 0.75×10^{-9} m^2/N. The amount of fluid that spill over (in m^3), if pressure in the reservoir is reduced by 2×10^7 N/m^2 is
(a) 0.15×10^{-4}
(b) 1×10^{-4}
(c) 1.5×10^{-4}
(d) None of these

Q.8 Assuming that sap in trees has the same characteristic as water and that it rises purely due to capillary phenomenon, what will be the average diameter of capillary tubes in a tree if the sap is carried to a height of 10 m? (Take surface tension of water = 0.0735 N/m & $\theta = 0°$)
(a) 0.003 mm
(b) 0.03 mm
(c) 0.3 mm
(d) 0.006 mm

Q.9 A small circular jet of mercury 0.1 mm in diameter issue from an opening. What is the pressure difference between the inside and outside of the jet when at 20°C? (Surface tension of mercury at 20°C is 0.514 N/m)
(a) 41 kPa
(b) 21.5 kPa
(c) 10.28 kPa
(d) 5.14 kPa

Q.10 An apparatus produces water droplets of diameter 70 μm. If the coefficient of surface tension of water in air is 0.07 N/m, the excess pressure in these droplets, in kPa, is
(a) 5.6
(b) 4.0
(c) 8.0
(d) 13.2

Q.11 If the surface tension of water air interface is 0.073 N/m, the gauge pressure inside a rain drop of 1 mm diameter is
(a) 146.0 N/m^2
(b) 0.146 N/m^2
(c) 73.0 N/m^2
(d) 292.0 N/m^2

Q.12 The capillary rise in a 3 mm tube immersed in a liquid is 15 mm. If another tube of diameter 4 mm is immersed in the same liquid, the capillary rise would be
(a) 11.25 mm
(b) 20.00 mm
(c) 8.44 mm
(d) 26.67 mm

Q.13 Which of the following is the correct expression for the bulk modulus of elasticity of a fluid?
(a) $\rho \dfrac{d\rho}{dp}$
(b) $\rho \dfrac{dp}{d\rho}$
(c) $\dfrac{d\rho}{\rho\, dp}$
(d) $\dfrac{dp}{\rho\, d\rho}$

Q.14 A Newtonian fluid fills the clearance between a shaft and a sleeve. When a force of 800 N is applied to the shaft, parallel to the sleeve, the shaft attains a speed of 1.5 cm/s. If a force of 2.4 kN is applied instead, the shaft would move with a speed of
(a) 1.5 cm/s
(b) 13.5 cm/s
(c) 0.5 cm/s
(d) 4.5 cm/s

Q.15 If the shear stress τ and shear rate (du/dy) relationship of a material is plotted with τ on the Y-axis and du/dy on the X-axis, the behaviour of an ideal fluid is exhibited by
(a) a straight line passing through the origin and inclined to the X-axis
(b) the positive X-axis
(c) the positive Y-axis
(d) a curved line passing through the origin

■ ANSWERS

1. (a)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (a)
6. (a)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (b)
11. (d)	**12.** (a)	**13.** (b)	**14.** (d)	**15.** (b)

Conventional Practice Questions

Q.1 A glass tube 0.25 mm in diameter contains a mercury column with water above the mercury. The temperature is 20°C at which the surface tension of mercury in contact with water is 0.037 kg(f)/m. What will be the capillary depression of the mercury? Take angle of contact θ = 130°.

Ans. 0.02 cm

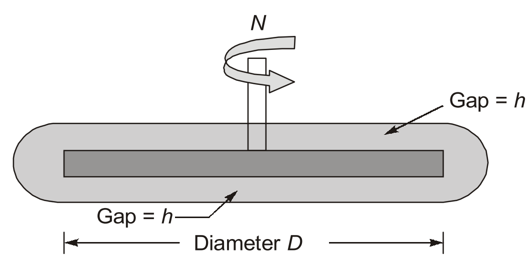
Q.2 The velocity distribution in the flow of a thin film of oil down an inclined channel is given by $u = \frac{\gamma}{2\mu}\left(d^2 - y^2\right)\sin\alpha$, where, d = depth of flow, α = angle of inclination of the channel to the horizontal, u = velocity at a depth y below the free surface, γ = unit weight of oil and μ = dynamic viscosity of oil. Calculate the shear stress : (i) on the bottom of the channel, (ii) at mid - depth, and (iii) at the free surface.

Ans. At $y = d$, $\tau_0 = -\gamma d \sin\alpha$; at $y = \frac{d}{2}$,

$\tau_{1/2} = -\gamma\left(\frac{d}{2}\right)\sin\alpha$; at $y = 0$, $\tau_t = 0$)

Q.3 A disc of diameter D rotates at a speed of N rpm inside an oil bath as shown in figure. Assuming a linear velocity profile between the disc surface and the walls of the bath and neglecting the shear on the outer edge of the disc, obtain an expression for viscous torque on the disc as $T = \frac{\pi^2 \mu N D^4}{480h}$.

In this μ = dynamic viscosity of the oil in the bath, h = thickness of the gap between the wall of the bath and the disk surface.



Q.4 Droplets of kerosene having diameter of 0.04 mm are produced in an atomizer. What is the pressure within these droplets? [Take surface tension for kerosene as 0.026 N/m]

Ans. Δp = 2600 N/m^2

■■■■