POSTAL Book Package 2023

CIVIL ENGINEERING

Strength of Materials

Conventional Practice Sets

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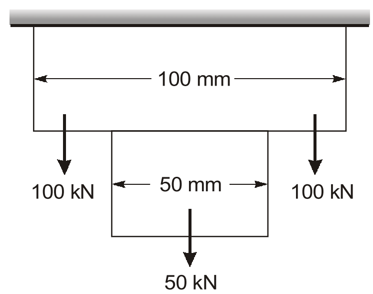
MADE EASY Publications

Simple Stress Strain and Elastic Contants

Q.1 A bar of varying square cross-section is loaded symmetrically as shown in the figure. Loads shown are placed on one of the axes of symmetry of cross-section. Ignoring self-weight, calculate the maximum tensile stress anywhere in the section

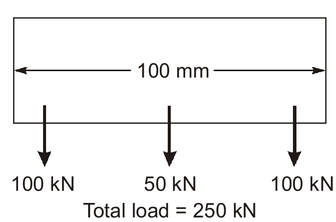


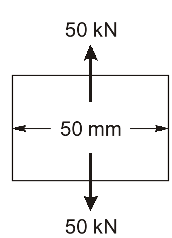
Solution:

$$\text{The stress in lower bar} = \frac{50 \times 1000}{50 \times 50} = 20 \text{ N/mm}^2$$

$$\text{The stress in upper bar} = \frac{250 \times 1000}{100 \times 100} = 25 \text{ N/mm}^2$$

Thus the maximum tensile stress anywhere in the bar is 25 N/mm^2.





Q.2 A metal bar of length 100 mm is inserted between two rigid supports and its temperature is increased by 10°C. If the coefficient of thermal expansion is 12×10^{-6} per °C and the Young's modulus is 2×10^5 MPa, then calcultae the stress in the bar

Solution:

Method-I

$$\boxed{\text{Temperature stress} = \alpha TE}$$

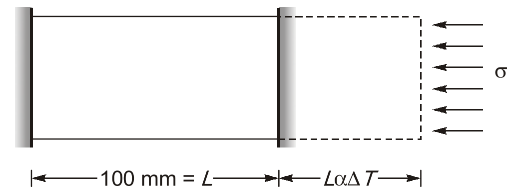
$$= 12 \times 10^{-6} \times 10 \times 2 \times 10^5 = 24 \text{ MPa}$$

Method-II

Due to temperature,

$$\Delta L = L\alpha\Delta T$$

But since support in fixed so, expansion is not allowed so stress is developed in the bar which is compressive in nature.



Now,

$$\text{Expansion due to temperature} = \text{Compression due to stress}$$

$$L\alpha\Delta T = \frac{\sigma}{E}\times L$$

$$\sigma = E\alpha\Delta T$$

$$= 1\times 10^5 \times 12\times 10^{-6}\times 10$$

$$= 24 \text{ MPa}$$

Q.3 A mild steel specimen is under uniaxial tensile stress. Young's modulus and yield stress for mild steel are 2×10^5 MPa and 250 MPa respectively. Calculate the maximum amount of strain energy per unit volume that can be stored in this specimen without permanent set

Solution:

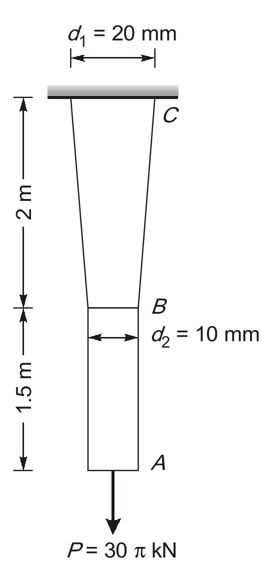
The strain energy per unit volume may be given as

$$u = \frac{1}{2}\times \text{Stresses}\times\text{Strain}$$

$$u = \frac{1}{2}\times\frac{\sigma_y^2}{E} = \frac{1}{2}\times\frac{(250)^2}{2\times 10^5}$$

$$= 0.156 \text{ N-mm/mm}^3$$

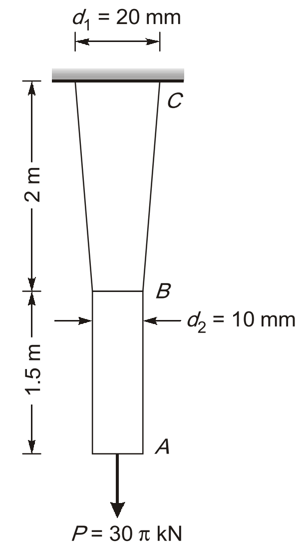
Q.4 A tapered circular rod of diameter varying from 20 mm to 10 mm is connected to another uniform circular rod of diameter 10 mm as shown in the following figure. Both bars are made of same material with the modulus of elasticity, $E = 2 \times 10^5$ MPa. If load subjected is 30π kN, then calculate deflection at point A (in mm)



Solution:

Total elongation,

AB is uniform



$$\text{So,}\Delta = \frac{PL}{AE}$$

BC is tapered

$$\boxed{\Delta = \frac{PL}{\frac{\pi}{4} d_1 d_2 E}}$$

$$\Delta = \Delta_{AB} + \Delta_{BC}$$

$$= \frac{PL}{AE} + \frac{4PL}{\pi d_1 d_2 E}$$

$$= \frac{30\pi \times 10^3 \times 1.5 \times 10^3}{\frac{\pi}{4} \times (10)^2 \times 2 \times 10^5} + + \frac{30\pi \times 10^3 \times 2 \times 10^3}{\frac{\pi}{4} \times 10 \times 20 \times 2 \times 10^5}$$

$$= (9 + 6) \text{ mm} = 15 \text{ mm}$$

Q.5 A steel specimen of 12 mm diameter extends by 6.31×10^{-2} mm over a gauge length of 150 mm when subjected to an axial load of 10 kN. The same specimen undergoes a twist of 0.5° an a length of 150 mm over a twisting moment of 10 N-m. Using the above data, determine the elastic constants E, μ, G and K.

Solution:

Tensile Test: $P = 10$ kN

Length of specimen, $L = 150$ mm

Cross-sectional area, $A = \frac{\pi}{4} \times 12^2 = 113.09 \text{ mm}^2$

Change in length of specimen, $\Delta = 6.31 \times 10^{-2}$ mm

Let E N/mm^2 is modulus of elasticity of material.

We know, axial deformation due to axial load is given by

$$\Delta = \frac{PL}{AE}$$

$$\therefore \quad E = \frac{PL}{A\Delta} = \frac{10 \times 1000 \times 150}{113.09 \times 6.31 \times 10^{-2}} = 2.10 \times 10^5 \text{ N/mm}^2$$

Torsion test:

We know, $$\frac{T}{I_P} = \frac{G\theta}{L} \quad \text{...(i)}$$

$\therefore$ Modulus of rigidity, $$G = \frac{TL}{I_P \theta}$$

$$I_P = \frac{\pi}{32} D^4 = \frac{\pi}{32} \times (12)^4 = 2035.75 \text{ mm}^4$$

Angle of twist, $$\theta = \frac{0.5 \times \pi}{180} \text{ radian} = 8.73 \times 10^{-3} \text{ radian}$$

From eq. (i), we get

$$G = \frac{10\times10^3\times150}{2035.75\times8.73\times10^{-3}} = 8.44\times10^4 \text{ N/mm}^2$$

We know, $\boxed{E = 2G(1+\mu)}$

$$\frac{E}{2G} = 1+\mu$$

$\therefore$ $$\mu = \frac{E}{2G}-1 = \frac{2.10\times10^5}{2\times8.44\times10^4}-1 = 1.24-1 = 0.24$$

Also $$E = 3k(1-2\mu)$$

$$k = \frac{E}{3(1-2\mu)} = \frac{2.10\times10^5}{3(1-2\times0.24)} = 1.35\times10^5 \text{ N/mm}^2$$

Q.6 A vertical tapered rod of length L has its diameter varying linearly from 'd' at lower end to D at the upper end which has fixed support. Young's modulus of the material is E. Show that the elongation of the rod at its lower end when subjected to a longitudinal force F is given by $\delta = \frac{4FL}{\pi dDE}$.

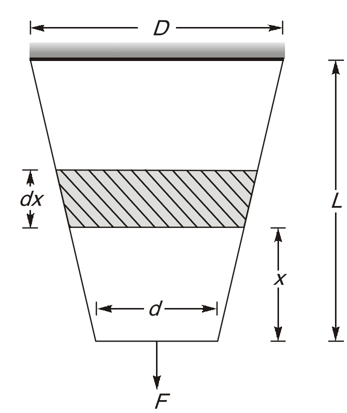
Solution:

Diameter of lower end = d, diameter of upper end = D, Length of tapering bar = L

Now, the diameter of the tapered bar at a distance 'x' from the smaller end may be given as

$$D_x = d+\left(\frac{D-d}{L}\right)x \Rightarrow D_x = d+kx \text{ where } k = \frac{D-d}{L}$$

Let the width of strip at distance x from lower end is dx. Then increase in length of the elemental strip $x = \frac{Fdx}{A_xE}$ $\left[\text{using } \Delta = \frac{PL}{AE}\right]$



where, A_x is area of cross-section of elemental strip and E is modulus of elasticity

$\therefore$ Total increase in length, $$\delta = \int_0^L \frac{Fdx}{A_xE} = \int_0^L \frac{Fdx}{\left(\frac{\pi}{4}\right)D_x^2E} \qquad \left[\because A_x = \frac{\pi}{4}D_x^2\right]$$

$$= \int_0^L \frac{4Fdx}{\pi ED_x^2} = \frac{4F}{\pi E}\int_0^L (d+kx)^{-2}dx$$

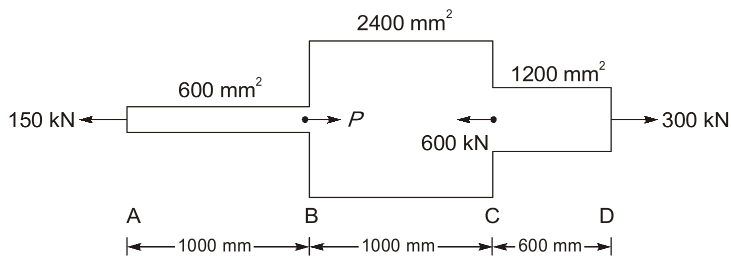
$$= \frac{4F}{\pi E}\left[\frac{(d+kx)^{-2+1}}{(-2+1)k}\right]_0^L$$

$$= \frac{-4F}{\pi E}\left[\frac{1}{k(d+kx)}\right]_0^L$$

$$= \frac{-4F}{\pi E}\left(\frac{-L}{dD}\right)$$

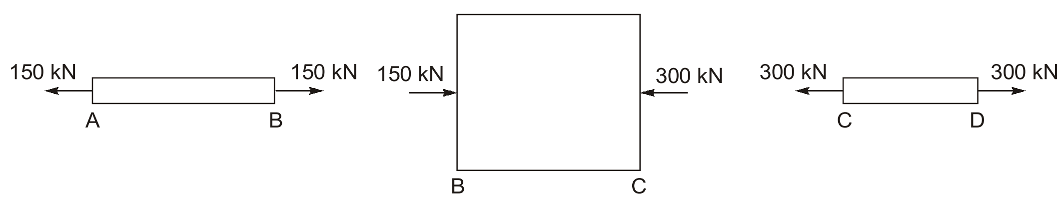
$\Rightarrow$ $$\delta = \frac{4FL}{\pi EdD} \quad \text{(Ans.)}$$

Q.7 A member *ABCD* is subjected to concentrated loads as shown. Calculate
(i) Force *P* necessary for equilibrium
(ii) Total elongation of bar
$E = 2 \times 10^5$ N/mm^2



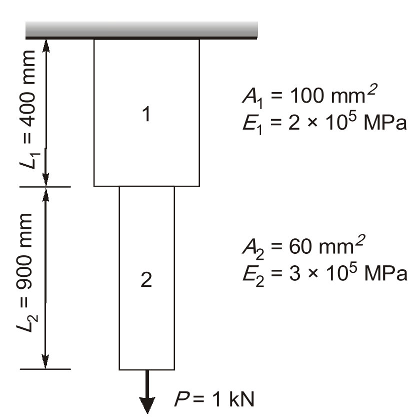
Solution:

(i)
$$\Sigma F = 0$$
$$(P + 300) - (150 + 600) = 0$$
$$P = 450 \text{ kN}$$



(ii)
$$\Delta_{Total} = \Delta_{AB} + \Delta_{BC} + \Delta_{CD}$$
$$= \frac{150 \times 1000 \times 1000}{600 \times 2 \times 10^5} - \frac{300 \times 1000 \times 1000}{2400 \times 2 \times 10^5} + \frac{300 \times 600 \times 1000}{1200 \times 2 \times 10^5}$$
$$= 1.25 - 0.625 + 0.75$$
$$= 1.375 \text{ mm (elongation)}$$

Q.8 Consider the stepped bar made with a linear elastic material and subjected to an axial load of 1 kN, as shown in the figure.



Segments 1 and 2 have cross-sectional area of 100 mm^2 and 60 mm^2. Young's modulus of 2×10^5 MPa and 3×10^5 MPa, and length of 400 mm and 900 mm, respectively. Calculate the strain energy stored in the bar (in N-mm) due to the axial load