

Civil Engineering

Strength of Materials

Comprehensive Theory

with Solved Examples and Practice Questions



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Strength of Materials

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Properties of Materials

1.1 Introduction

Strength of material is a branch of applied mechanics that deals with the behaviour of solid bodies subjected to various types of loading and internal forces developed due to these loading. A thorough understanding of mechanical behaviour is essential for the safe design of all structures, whether buildings, bridges, machines, motors, submarines or airplanes. Hence, strength of material is a basic subject in many engineering fields.

So the objective of our analysis will be to determine the stresses, strains and deflections produced by the loads in different structures. Theoretical analysis and experimental results have equally important role in the study of strength of materials. So these quantities are found for all values of load upto the failure load, and then we will have a complete picture of the mechanical behaviour of the body.

The behaviour of a member subjected to forces depends not only on the fundamental law of Newtonian mechanics that govern the equilibrium of the forces but also on the mechanical characteristics of materials of which the member is fabricated. Sometimes, to predict the behaviour of material some necessary information regarding the characteristics of material comes from laboratory tests.

1.2 Normal Stress

The fundamental concept of stress can be understood by considering a prismatic bar that is loaded by axial force P at the ends as shown in the figure 1.1.

A prismatic bar is a straight structural member having constant cross-sectional area throughout its length. In the figure 1.1(a), axial force produces a uniform stretching of the bar, hence the bar is said to be in tension and in figure 1.1(c), force produces uniform compression of the bar, hence the bar is said to be in compression.

To investigate the internal stresses produced in the bar by axial forces, we make an imaginary cut at section mn (figure 1.1(b) & (d)). This section is taken perpendicular to the longitudinal axis of bar. Hence it is known as cross-section.

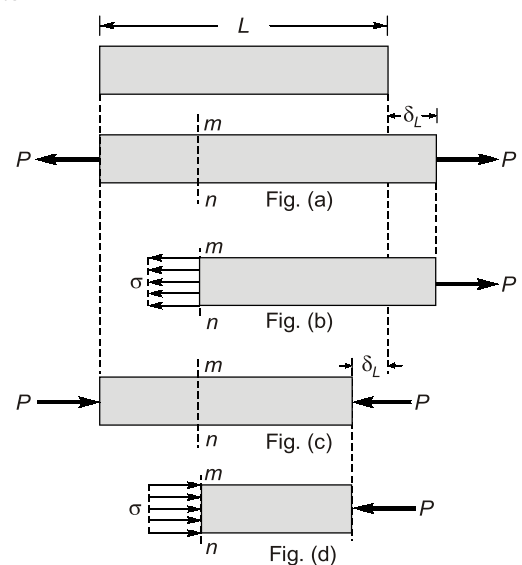


Fig. 1.1

Now isolate the part of the bar to the right of the cut and consider the right of the cut as a free body. The force P has a tendency to move free body in the direction of load, so to restrict the motion of bar an internal force is induced which is uniformly distributed over cross-sectional area. The intensity of force that is force per unit area is called the **stress**.

Thus, stress can be defined as – “**Stress is internal resistance of material offered against deformation which is force per unit area**”.

Stress induced in material depends upon the nature of force, point of application and cross-sectional area of material. Stress can be **tensile** or **compressive** in nature depending on the nature of load. Generally, stress is represented by the Greek letter σ . We can calculate stress mathematically as

$$\sigma = \frac{P}{A}$$

Sign Convention:

Tensile stresses = +ve

Compressive stresses = -ve

Unit: (i) N/m² or Pa

(ii) N/mm² or MPa

NOTE



- Stresses are induced only when motion of bar is restricted either by some force or reaction induced. If body or bar is free to move or free expansion is allowed, then no stresses will be induced.
- Pressure has same unit but pressure is different physical quantity than stress. Pressure is external normal force distributed over surface.

On the basis of cross-sectional area considered during calculation of stresses, direct stresses can be of following two types:

1. Engineering or nominal stress
2. True or actual stress

Engineering or nominal stress

Mathematically,

$$\sigma = \frac{P}{A_0} \quad \text{where, } A_0 = \text{Original cross-sectional area of specimen taken}$$

True or actual stress

Mathematically,

$$\sigma = \frac{P}{A_a} \quad \text{where, } A_a = \text{Actual cross-sectional area of specimen at any time of loading i.e. changed area of cross-section due to loading}$$

$$A_a = A_0 \pm \Delta A \quad \text{'+' for compression, '-' for tension}$$

- Remember:**
- In tension, true or actual stress is always greater than engineering or nominal stress.
 - In compression, true or actual stress is always less than engineering or nominal stress.

1.3 Strain

An axially loaded bar undergoes a change in length, becoming longer when in tension and shorter when in compression. Thus, the elongation or shortening in axially loaded member per unit length is known as strain. Strain is represented by ϵ .

Mathematically strain can be calculated as

$$\epsilon = \frac{\Delta L}{L}$$

Unit: Strain is dimensionless quantity. It is always expressed in the form of number. If the member is in tension then the strain is called tensile strain. If the member is in compression, then the strain is called compressive strain.

On the basis of length of member used in calculation of strain, strain can be of following two types:

1. Engineering or nominal strain
2. True or actual strain

Engineering or Nominal Strain

Engineering or nominal strain is strain calculated, when length of member is taken as original length

$$\epsilon_0 = \frac{\Delta l}{l_0} \quad l_0 = \text{original length of member}$$

True or Actual Strain

True or actual strain is strain calculated, when length of member is taken as actual length of member at loading

$$\epsilon_a = \frac{\Delta l}{l_a} \quad l_a = \text{Actual length of member}$$

$$l_a = l_0 \pm \Delta l \quad \text{'+' sign for tension, '-' sign for compression}$$

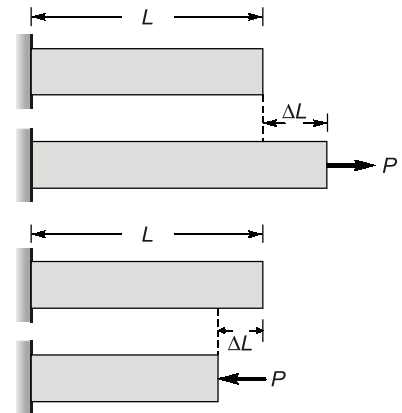
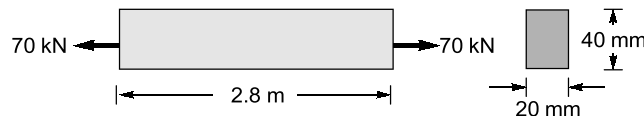


Fig. 1.2

Example 1.1

A prismatic bar with rectangular cross-section (20 mm × 40 mm), length $L = 2.8$ m is subjected to an axial tensile force of 70 kN. The measured elongation of the bar is 1.2 mm. Calculate the tensile stress and strain in the bar.



Solution:

Assuming that force acts at CG of section. We know that,

$$\text{Stress, } \sigma = \frac{P}{A} = \frac{70 \times 10^3 \text{ N}}{20 \times 40 \text{ mm}^2} = 87.5 \text{ N/mm}^2 = 87.5 \text{ MPa}$$

and

$$\text{Strain, } \epsilon = \frac{\Delta L}{L} = \frac{1.2 \text{ mm}}{2.8 \times 1000 \text{ mm}} = 4.286 \times 10^{-4}$$

1.4 Tension Test for Mild Steel

The mechanical properties of materials used in engineering are determined by experiments performed on small specimen. These experiments are conducted in laboratories equipped with testing machines that are capable of loading in tension or compression.

The American Society for Testing and Materials (ASTM) has published guidelines for conducting test. Tension test is conducted on Universal Testing Machine (UTM).

1.4.1 Specifications of Specimen

- Specimen is solid cylindrical rod
- Gauge length 2.0" (inches)
- Diameter of middle section 0.5" (inches)
- L/D ratio = 4.0

1.4.2 Stress Strain Curve for Tension

- A is limit of proportionality:** Beyond this linear variation ceases. Hooke's law is valid in OA.

- B is elastic limit:** The maximum stress upto which a specimen regains its original length on removal of applied load. For mild steel, B is very near to A. However, for other materials B may be greater than A.

- C' is upper yield point:** The magnitude of the stress corresponding to C' depends on the cross-sectional area, shape of the specimen and the type of the equipment used to perform the test. It has no practical significance.

- C is lower yield point:** This is also called actual yield point. The stress at C is the yield stress (σ_y) with a typical value of $\sigma_y = 250 \text{ N/mm}^2$ (for mild steel). The yielding begins at this stress.
- CD represents perfectly plastic region:** It is the strain which occurs after the yielding point C, without any increase in stress. The strain corresponding to point D is about 1.4% and corresponding to C is about 0.12% for mild steel. Hence, plastic strain is 10 to 15 times of elastic strain.
- DE represents strain hardening:** In this range further addition of stress gives additional strain. However, strain increases with faster rate in this region. The material in this range undergoes change in its crystalline structure, resulting in increased resistance to further deformation. This portion is not used for structural design.
- E is ultimate point:** The stress corresponding to this point is ultimate stress (σ_u) and the corresponding strain is about 20% for mild steel.
- F is fracture point:** Stress corresponding to this is called breaking stress and strain is called fracture strain. It is about 25% for mild steel.
- Region between E and F is the necking region in which area of cross-section is drastically decreased.

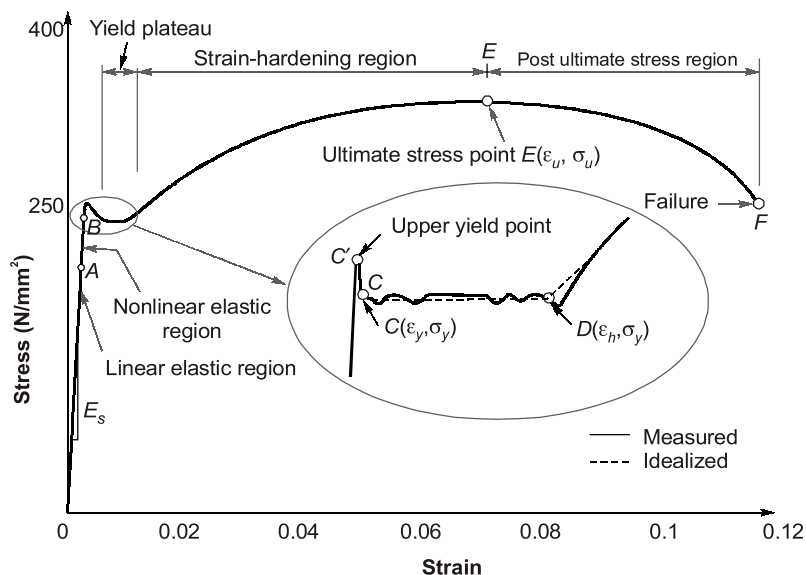


Fig. 1.3 Tensile stress-strain diagram for Mild Steel

**Do
You
Know**

- Strain that occurs before the yield point is called elastic strain and that which occurs after yield point with no increase in stress is called plastic strain. For mild steel, plastic strain is 10 to 15 times of elastic strain.
- Ideal curve for tension is shown in the figure 1.4. However actual behaviour is different and indicates apparently reduced yield stress in compression after strain hardening in tension. The divergence between tension and compression results is explained by Bauschinger and is called **Bauschinger effect**.

1.4.3 Actual Curve v/s Engineering Curve in Tension

NOTE



- The fracture strain depends upon % carbon present in steel.
- With increase in percentage carbon, fracture strain reduces.
- With increase carbon content, steel has higher yield stress and higher ultimate stress.

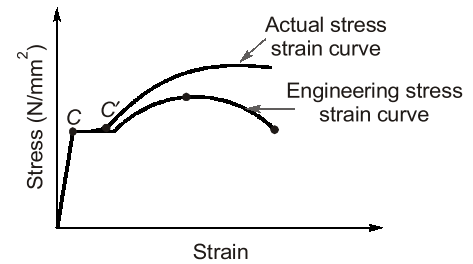


Fig. 1.4 Tension curve for mild steel

1.4.4 Compression Curve for Mild Steel

- ◆ In compression, engineering stress-strain curve lies above the actual stress-strain curve.
- ◆ In compression mild steel has yield stress $\sigma_y = 263 \text{ N/mm}^2$, slightly greater than tension.
- ◆ Mild steel has same Young's modulus of elasticity in compression and tension, $E = 2.1 \times 10^5 \text{ N/mm}^2$.

Relation between engineering and actual stress

$$\sigma_a = \sigma_0(1 \pm \epsilon_0)$$

where, σ_a = Actual stress

σ_0 = Engineering stress

ϵ_0 = Engineering strain

For tension, take positive (+ve) sign and take negative (–ve) sign for compression.

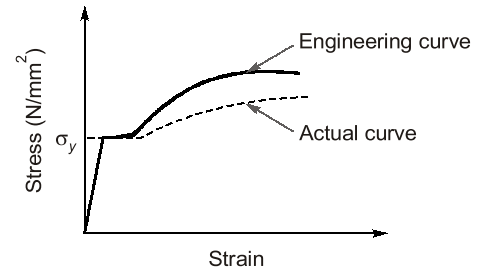


Fig. 1.5 Compression curve for Mild steel

NOTE: While deriving above equation volume changes is neglected which is true in plastic region (Non-elastic region).

1.4.5 Stress-strain Curve for other Grades of Steel in Tension

Remember



- All the grades of steel have same Young's modulus of elasticity.
- Among all steel grades high tension steel (HTS) is more brittle and mild steel is more ductile.
- High tension steel has higher ultimate strength than other grades of steel.

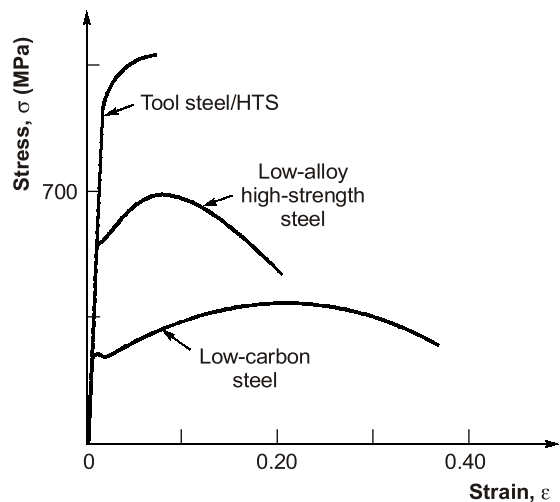


Fig. 1.6 Tensile stress-strain diagram for different grades of steel

1.4.6 Stress-strain Curve for Various Materials (fig. 1.7)

1.5 Properties of Metals

1.5.1 Ductility

Ductility is the property by which material can be stretched. Large deformations are thus possible in ductile materials before the absolute failure or rupture takes place. These materials have post-elastic strain [Plastic strain] greater than 5%. Some of the examples are mild steel, aluminium, copper, manganese, lead, nickel, brass, bronze etc.

1.5.2 Brittleness

Brittleness is the lack of ductility i.e. materials can not be stretched. In brittle materials, fracture takes place immediately after elastic limit with a relatively smaller deformation. For the brittle materials fracture and ultimate points are same and after proportional limit very small strain is seen. Brittle materials have post elastic strain less than 5%. Examples are cast iron, concrete and glass.

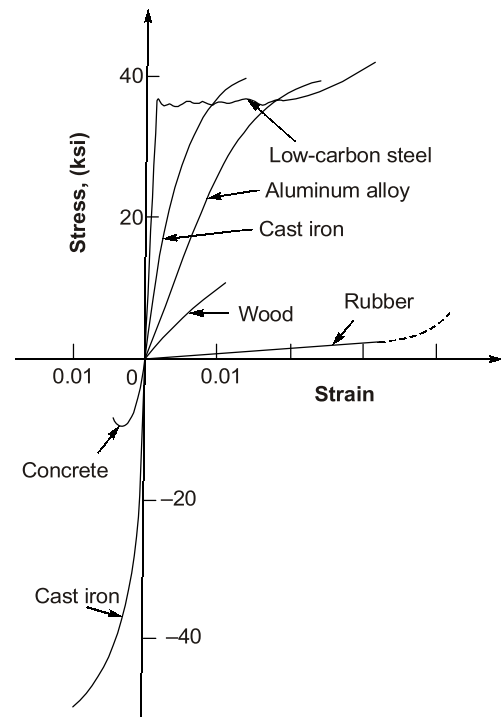


Fig.1.7 Stress-strain diagram for different material

Remember



To distinguish between these two type of materials, materials with post elastic strain less than 5% at fracture point are regarded as brittle and those having post elastic strain greater than 5% at fracture point are called ductile (this value for mild steel at fracture is about 25%).

1.5.3 Malleability

It is that property of metal due to which a piece of metal can be converted into a thin sheet by pressing it. A malleable material possess a high degree of plasticity. This property is of great use in operations like forging, hot rolling, drop (stamping) etc.

1.5.4 Hardness

- ♦ Hardness is resistance to the scratch or abrasion.
- ♦ There are two methods of hardness measurements:
 1. Scratch hardness-commonly measured by Mohr's test
 2. Indentation hardness (abrasion) measured by
 - Brinell hardness method
 - Vickers hardness
 - Rockwell hardness
 - Knoop hardness

1.6 Creep

where, Δ_e = Elastic deflection = $\frac{PL}{AE}$
 P = Static load
 Δ_c = Deformation due to creep

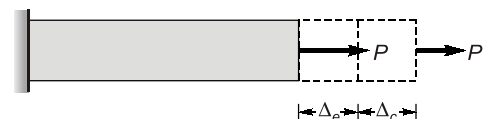


Fig. 1.8

Creep is permanent deformation which is recorded with passage of time at constant loading. Total creep deformation continues to increase with time asymptotically.

Factors affecting creep are as follows:

1. Magnitude of load
2. Type of loading (static or dynamic)
3. Time or age of loading
4. Temperature
 - ♦ At higher temperature, due to greater mobility of atoms most of the materials lose their strength and elastic constants also get reduced. Hence, greater deformation at elevated temperature results, even under constant loading. Therefore, creep is more pronounced at higher temperature, and thus it must be considered for design of engines and furnaces.
 - ♦ Temperature at which the creep becomes very appreciable is half of the melting point temperature on absolute scale and known as **homologous temperature**.

1.7 Stress Relaxation

If a wire of metal is stretched between two immovable supports, so that it has an initial tension σ_0 . The stress in the wire gradually diminishes, eventually reaching a constant value. This process, which is manifestation of creep is called **stress relaxation**. (This is the reason why electric wires sag after long time)

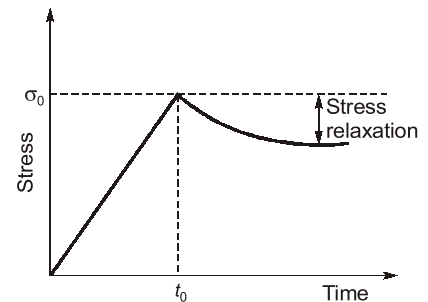


Fig. 1.9

1.8 Elasticity

Assume for instance, we apply a tensile load to a specimen so that strain and stress follow path from O to B on stress-strain curve shown in figure 1.10. Further, when the load is removed, the material follows exactly the same curve back to the origin O . So, the property by which original dimensions (i.e. length and cross-section) can be recovered after unloading is known as **elasticity**.

Within elastic limit curve may be linear or non-linear. In figure, during loading material store elastic strain energy. The total strain energy which can be stored in the given volume of the metal and can be released after unloading is called **resilience**. It is also equal to area under load deflection curve within elastic limit (B). When elastic limit coincides with yield point, the maximum elastic energy per unit volume is known as **modulus of resilience**.

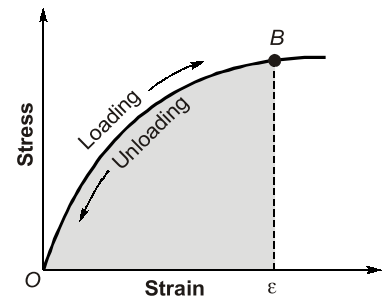


Fig. 1.10

Modulus of resilience is equal to area under the stress-strain curve within elastic limit for mild steel

$$U_r = \frac{1}{2} \times \sigma_y \times \epsilon_y = \frac{1}{2} \times \sigma_y \times \frac{\sigma_y}{E} \quad \left[\because \epsilon_y = \frac{\sigma_y}{E} \right]$$

$$U_r = \frac{\sigma_y^2}{2E}$$

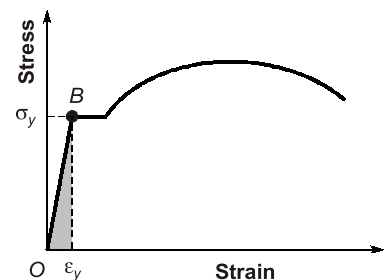


Fig. 1.11

- ♦ The modulus of resilience depends upon yield strength and hence a material with higher yield strength will have higher modulus of resilience.
- ♦ Higher resilience is desirable in suspension spring and where the load absorption is required.

NOTE: HTS (High Tension Steel) has more yield stress than mild steel so it has more modulus of resilience. Thus springs are made from high tension steel.

1.8.1 Proof Stress

Some of the ductile metals like Aluminium (Al), Copper (Cu) and Silver (Ag) do not show clear yield point in tension test, therefore yield stress (σ_y) is not clearly known. For such metals, design stress is calculated by offset method. An offset of permanent plastic strain equals to 0.2% is marked on x -axis and a straight line is drawn which is parallel to initial portion of stress-strain curve. The point of intersection of stress-strain curve with straight line is proof point and corresponding stress at that point is **Proof stress**.

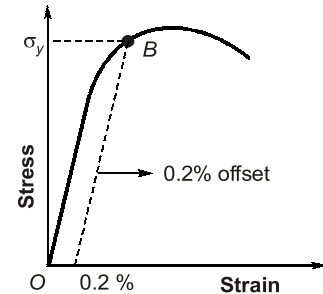


Fig. 1.12

1.8.2 Elasto-Plastic Behaviour of Metals

Now, suppose that material is loaded to a much higher level than elastic limit (B), so that point P is reached on stress-strain diagram. When unloading occurs the material follows path PC on the diagram, which is parallel to the initial portion of stress-strain curve. When point C is reached, the load has been entirely removed but a permanent strain or residual strain OC remains in material. The corresponding residual elongation of the specimen is called **permanent set**.

During unloading only CPD part of strain energy is recovered and a large part OPC is lost in permanent deformation which is called **inelastic strain energy** Fig. (1.13).

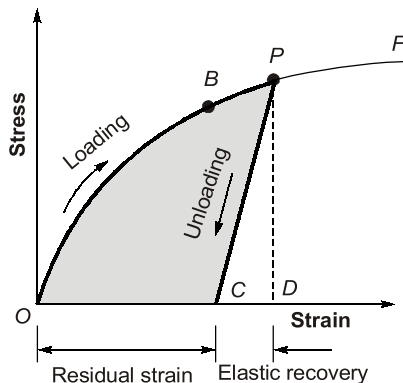


Fig. 1.13 Partially elastic behaviour

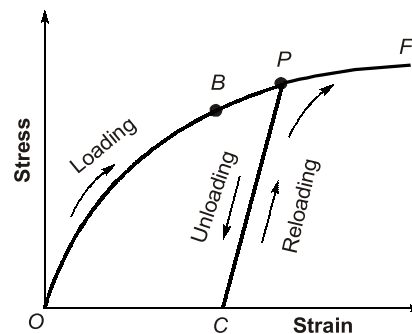
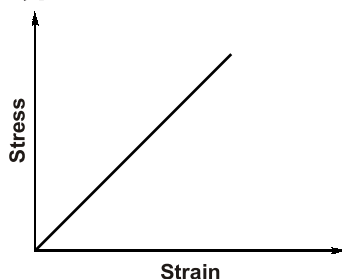


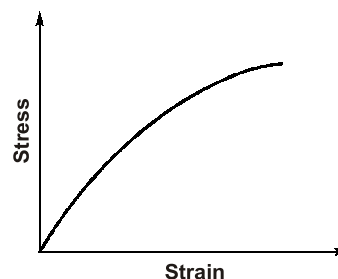
Fig. 1.14 Reloading of a material and raising of the yield stress

Beyond elastic limit, if material undergoes continuous cyclic loading and unloading, then yield limit of material continuously increases. This concept is used in cold working of mild steel bar to avoid yield plateau (Fig. 1.14).

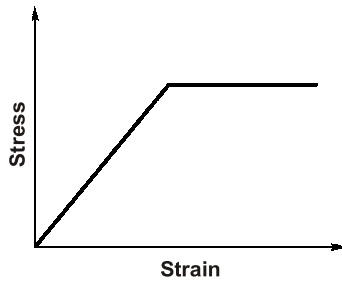
1.8.3 Types of Material Behaviour



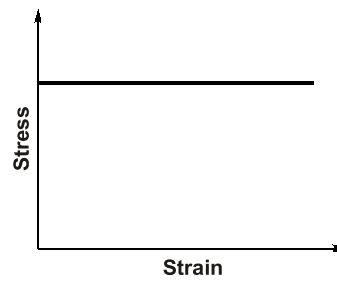
(i) Linear elastic



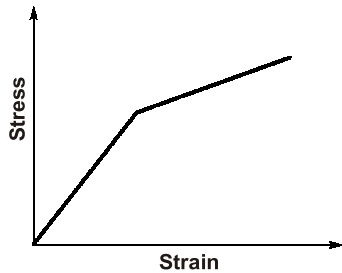
(ii) Non-linear elastic



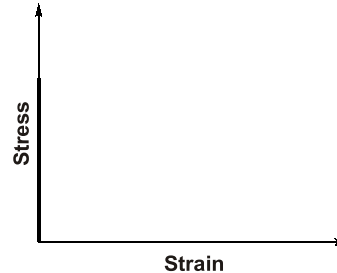
(iii) Elasto plastic or visco-plastic



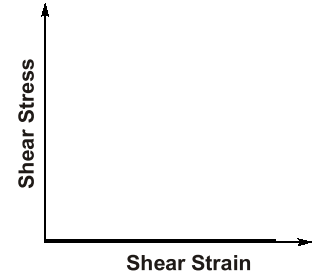
(iv) Perfectly plastic



(v) Elasto-plastic with strain hardening



(vi) Ideal rigid



(vii) Ideal fluid

1.9 Toughness

The property which enables material to absorb energy without fracture. This property is very desirable in case of cyclic loading or shock loading. If a metal is tough then it has ability to store large strain energy before fracture.

Modulus of toughness is total strain energy per unit volume upto fracture stage. It is equal to total area under stress-strain curve upto fracture.

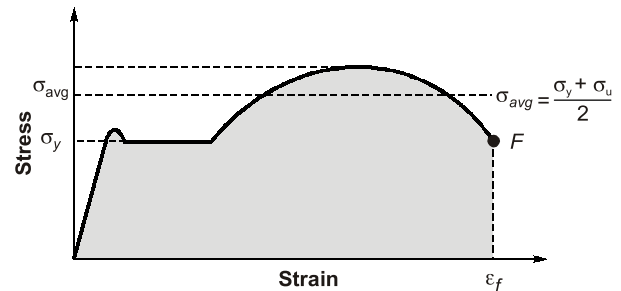


Fig. 1.15

$$\text{Modulus of toughness} = \left(\frac{\sigma_y + \sigma_u}{2} \right) \times \epsilon_f$$

where, σ_y = yield tensile strength, σ_u = ultimate tensile strength, ϵ_f = strain at fracture point

The modulus of toughness depends upon ultimate tensile strength and strain at failure (Fracture strain). Hence, the material which is very ductile will exhibit a higher modulus of toughness as is the case with mild steel.

NOTE: It is noted that ductile materials are tough and brittle materials are hard.

1.10 Fatigue

It has been found that material behave differently under the static and dynamic loading. In cyclic or reverse cyclic loading, if total accumulated strain energy exceed the toughness then fracture failure may occur.

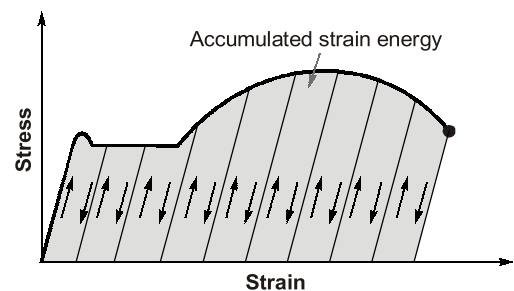


Fig. 1.16

Factors affecting fatigue are:

1. Loading condition
2. Frequency of loading
3. Corrosion
4. Temperature
5. Stress concentration

The number of load cycles required to initiate surface crack is called **fatigue initiation life** and additional number of load cycle required to propagate surface crack is called **fatigue propagation life**.

To prevent fatigue failure the developed stress should be kept below endurance limit. **Endurance limit** is that stress below which a material has high probability of no failure even at infinite number of load cycles.

For, mild steel, endurance limit = 186 N/mm^2 and Aluminium, endurance limit = 131 N/mm^2

Remember: Endurance limit is lower than proportional limit.

Examples of fatigue failure:

1. Crashing of aircraft due to crack in turbine blade
2. Failure of fly wheels
3. Breaking of wire due to cyclic bending

1.11 Failure of Materials in Tension and Compression

1.11.1 Ductile Metals in Tension Test

Ductile metals are weak in shear and failure is due to shear strain along the plane forming 45° angles with the axis of the specimen. In ductile material, cup and cone fracture take place. Failure surface is rough.

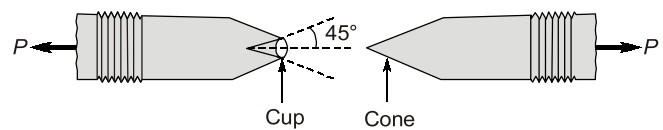


Fig. 1.17

NOTE: In ductile metals, necking is form before fracture.

1.11.2 Brittle Metals in Tension Test

Brittle metals are very weak in tension. Brittle metals fail due to separation of particles along the surface which is at 90° to the direction of load. Failure surface is rough.

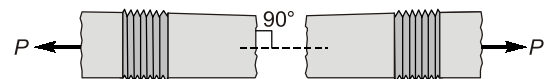


Fig. 1.18 Brittle failure of metal

1.11.3 Ductile Metals in Compression Test

Short compression members fail in compression yielding. Failure plane is parallel to the compressive load. In compression yielding, bulging of material occurs (Fig. 1.19) which leads to crack formation in a direction to compressive load.

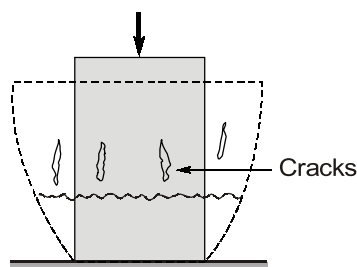


Fig. 1.19

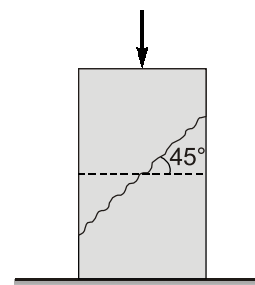


Fig. 1.20

1.11.4 Brittle Metals in Compression Test

In compression, brittle metals fail in shear, failure plane is at 45° to the direction of loading (Fig. 1.20).

Example 1.2

A bar of length 5.0 m is made of a structural steel having the stress-strain diagram shown in the Figure (a). The yield stress of the steel is 250 MPa and the slope of the initial linear part of the stress-strain curve (Modulus of elasticity) is 200 GPa. The bar is loaded axially until it elongates 7.5 mm and then the load is removed. How does the final length of the bar compare with its original length?

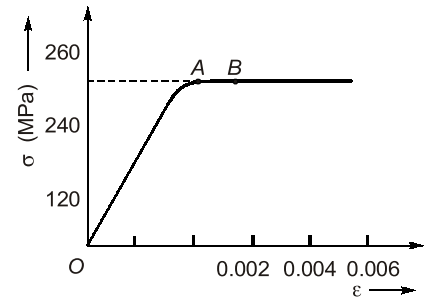


Fig. (a)

Solution:

Given:

Length of specimen, $L = 5000 \text{ mm}$

Yield stress, $\sigma_y = 250 \text{ MPa}$

Modulus of elasticity, $E = 200 \text{ GPa}$

Elongation, $\Delta L = 7.5 \text{ mm}$

Total strain at B due to axial loading,

$$\begin{aligned}\epsilon_B &= \frac{\Delta L}{L} = \frac{7.5 \text{ mm}}{5000 \text{ mm}} \\ &= 0.0015\end{aligned}$$

Elastic strain at A,

$$\begin{aligned}\epsilon_A &= \frac{\sigma_y}{E} = \frac{250 \text{ MPa}}{2 \times 10^5 \text{ MPa}} \\ &= 0.00125\end{aligned}$$

It is clear that $\epsilon_B > \epsilon_A$, it means specimen is loaded beyond elastic limit and on unloading total dimension will not recover and permanent elongations will occur.

So, elastic recovery due to unloading

$$\text{Elastic recovery} = \frac{\sigma_y}{E} = 0.00125$$

so permanent strain or residual strain

$$\begin{aligned}\epsilon_C &= \epsilon_B - \epsilon_{\text{Recovery}} \\ &= 0.0015 - 0.00125 \\ &= 0.00025\end{aligned}$$

so permanent deformation,

$$\begin{aligned}\Delta L_P &= \epsilon_C \times L \\ &= 0.00025 \times 5000 \\ &= 1.25 \text{ mm (longer)}\end{aligned}$$

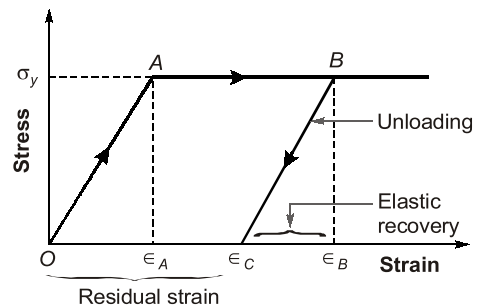


Fig.(b)

Summary



- Stress is internal resistance per unit area offered by material against deformation.
- Strain is deformation per unit length. It is unit less quantity.
- In stress-strain curve lower yield point is considered for designing purpose.
- Ductile materials are tough and brittle materials are hard.
- In general, ductile materials fail in shear and brittle metals fail in principal tension.
- For no fatigue failure, stress should be below endurance limit.


Objective Brain Teasers

Q.1 Match **List-I (Type of material)** with **List-II (Characteristics)** and select the correct answer using the codes given below the lists:

List-I

- A. Elastic material
- B. Rigid material
- C. Plastic material
- D. Resilient material

List-II

- 1. Does not store energy
- 2. Has no plastic region in stress strain curve
- 3. Behave as a spring
- 4. Offers resistance to deformation
- 5. Does not offer resistance to deformation

Codes:

	A	B	C	D
(a)	4	1	5	3
(b)	4	1	5	2
(c)	2	3	4	5
(d)	1	3	4	5

Q.2 A structure is said to be linearly elastic if

- (a) Load \propto displacement
- (b) Load $\propto \frac{1}{\text{Displacement}}$
- (c) Energy \propto displacement
- (d) Energy \propto Load

Q.3 Which one is not the characteristics of fatigue fracture?

- (a) Rough fracture surface
- (b) Rough and smooth areas on fracture surface
- (c) Plastic deformation
- (d) Conchoidal markings on fracture surface

Q.4 A fatigue crack in a sound and smooth specimen takes

- (a) longer time in initiation than propagation
- (b) longer time in propagation than initiation
- (c) equal time in initiation and propagation
- (d) no time in propagation

Q.5 Stress curve is always a straight line for

- (a) elastic material
- (b) materials obeying Hooke's law
- (c) elasto plastic materials
- (d) none of the above

Q.6 The term nominal stress in stress-strain curve for mild steel implies

- (a) average stress
- (b) actual stress
- (c) yield stress
- (d) stress at necking

Q.7 Consider the following statements:

The principle of superposition is applied to

- 1. Linear elastic bodies
- 2. Bodies subjected to small deformations

Which of these statements is/are correct?

- (a) 1 alone
- (b) 1 and 2
- (c) 2 alone
- (d) neither 1 nor 2

Q.8 The strain at a point is a

- (a) Scalar
- (b) Vector
- (c) Tensor
- (d) None of these

Q.9 If the value of Poisson's ratio is zero, then it means that

- (a) the material is rigid
- (b) the material is perfectly plastic
- (c) there is no longitudinal strain in the material
- (d) the longitudinal strain in the material is infinite

Q.10 Consider the following statements

- 1. Strength of steel increases with carbon content
- 2. Young's modulus of steel increase with carbon content
- 3. Young's modulus of steel remain unchanged with variation of carbon content

Which of these statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) 1 and 2
- (d) 1 and 3

Q.11 True stress σ is related with conventional stress σ_0 as

(a) $\frac{\sigma}{\sigma_0} = (1 + \epsilon)^2$ (b) $\frac{\sigma}{\sigma_0} = \frac{1}{(1 + \epsilon)^2}$

(c) $\frac{\sigma}{\sigma_0} = \frac{1}{(1 + \epsilon)}$ (d) $\frac{\sigma}{\sigma_0} = 1 + \epsilon$

Q.12 Steel has its yield strength of 400 N/mm² and modulus of elasticity of 2×10^5 MPa. Assuming the material to obey Hooke's law up to yielding, what is its proof resilience?

- (a) 0.8 N/mm² (b) 0.4 N/mm²
(c) 0.6 N/mm² (d) 0.7 N/mm²

Q.13 What would be the shape of the failure surface of a standard cast iron specimen subjected to torque?

- (a) Cup and cone shape at the center.
(b) Plane surface perpendicular to the axis of the specimen.
(c) Pyramid type wedge-shaped surface perpendicular to the axis of the specimen.
(d) Helicoidal surface at 45° to the axis of the specimen.

Directions: The following items consists of two statements; one labelled as '**Assertion (A)**' and the other as '**Reason (R)**'. You are to examine these two statements carefully and select the answers to these items using the codes given below:

- (a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q.14 Assertion (A): Many materials do not have well defined yield point.

Reason (R): 0.2% offset parallel to the initial tangent of the stress-strain curve intersects the curve at yield stress.

Q.15 Assertion (A): Strain is a fundamental behaviour of the material, while the stress is a derived concept.

Reason (R): Strain does not have a unit while the stress has a unit.

Q.16 Assertion (A): The amount of elastic deformation at a certain point, which an elastic body undergoes, under given stress is the same irrespective of the stresses being tensile or compressive.

Reason (R): The modulus of elasticity and Poisson's ratio are assumed to be the same in tension as well as compression.

Q.17 Assertion (A): A mild steel tension specimen has a cup and cone fracture at failure.

Reason (R): Mild steel is weak in shear and failure of the specimen in shear takes place at 45° to the direction of the applied tensile force.

Q.18 Assertion (A): In a tensile test on a specimen, true stress in the specimen is more than nominal stress.

Reason (R): Grip of universal testing machine introduces stress concentrations.

Q.19 Assertion (A): The failure surface of a mild steel specimen subject to a torque about its axis is along a surface perpendicular to its axis.

Reason (R): Mild steel is relatively weaker in shear than in tension and the plane of maximum shear is perpendicular to its axis.

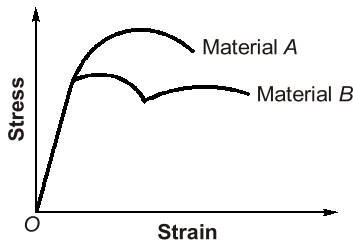
Q.20 Assertion (A): In a tension test on a cast iron specimen, the failure of the specimen is on a cross-section perpendicular to the axis of the specimen.

Reason (R): The failure of the specimen is on a plane subjected to maximum tensile-stress and cast iron is relatively weak in tension.

Q.21 Assertion (A): In a tension test on a mild steel specimen, the failure of the specimen is along a plane at 45° to cross-section.

Reason (R): The failure of the specimen is on a plane subjected to maximum shear stress and mild steel is relatively weak in shear.

Q.22 The stress strain diagram for two materials *A* and *B* is shown below:



Assertion (A): Material *A* is more brittle than material *B*.

Reason (R): The ultimate strength of material *B* is more than that of *A*.

Answers

1. (a) 2. (a) 3. (a) 4. (a) 5. (b)
6. (a) 7. (b) 8. (c) 9. (c) 10. (d)
11. (d) 12. (b) 13. (d) 14. (b) 15. (b)
16. (a) 17. (a) 18. (b) 19. (a) 20. (a)
21. (a) 22. (c)

Hints and Explanations:

5. (b)
Stress \propto strain (Hooke's law)
Which is valid within proportional limit.
Within elastic limit stress – strain curve may be linear or nonlinear. For e.g. Rubber.

6. (a)

$$\text{Nominal stress} = \frac{\text{Load}}{\text{Original area}} = \frac{P}{A_0}$$

Nominal stress is also called engineering stress or average stress.

$$\text{Actual stress} = \frac{\text{Load}}{\text{Actual area}} = \frac{P}{A_a}$$

Actual area at instant of loading does not remain constant and decreases with increase in elongation. Actual stress is also called true stress.

8. (c)
Stress, strain and MOI are tensor quantities.
10. (d)
Strength of steel increases with carbon content but Young's modulus remains constant.

12. (b)

$$\text{Proof resilience} = \frac{\sigma_y^2}{2E}$$

$$= \frac{400^2}{2 \times 2 \times 10^5} = 0.4 \text{ N/mm}^2$$

13. (d)
Brittle materials fail in a plane at 45° from the axis when subjected to torque because they are weak in tension compared to shear. If ductile materials are subjected to torque, then the failure surface will be in a plane at 90° from the axis of shaft.

15. (b)
During experiment in laboratory, strain is measured that is why it is called fundamental quantity. While stress is derived from strain.

17. (a)
Mild steel is enough strong in tension and compression but it is weak in shear. Hence, the failure of the specimen takes place due to shear.

18. (b)
For true stress, the actual area at any time used is less than original area, due to elongation in specimen, therefore, true stress is more than nominal or engineering stress.

■■■■