Chemical Engineering

Fluid Mechanics

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Fluid Mechanics

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CHAPTER 02

Fluid Pressure & its Measurement

2.1 Introduction

- Intensity of pressure at any point is defined as force exerted per unit area. However if force is not uniformly distributed, the expression will give the average value only.
- When a certain mass of fluid is confined within solid boundary. The force exerted always acts in normal to surface.
- A fluid at rest is characterised by absence of relative motion between adjacent fluid layers. Viscosity of fluid has no effect on fluid at rest and therefore the ideal and real fluid behave exactly same.
- There are no tangential force as shear stress is zero because of no relative motion.
- In a fluid at rest the normal stress is called pressure.
- Fluid at rest cannot support shear stress. Normal stress on any plane through fluid element at rest is a point property called the fluid pressure, taken positive for compression by common convention.

2.2 Pascal's Law for Pressure at a Point

- According to Pascal's law, pressure at a point in a fluid system is equally distributed in all directions (Fig.).
- It means that the pressure at a point in a fluid at rest, or in motion, is independent of direction as there are no shearing stresses present.
- Pressure in a fluid system has magnitude but not a specific direction and thus, it is a scalar quantity.

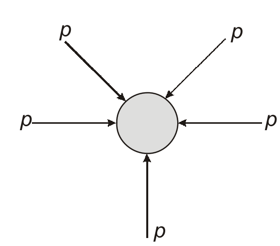


Fig. *A point in a fluid system*

- It applies to a fluid at rest.
- In case of flowing fluid, shear stresses will be set up as a result of relative motion between particles of the fluid.

  The pressure at a point is then considered to be the mean of the normal forces per unit area (stresses) on three mutually perpendicular planes. Since, these normal stresses are usually large compared to shear stresses, it is generally assumed that Pascal's law still applies.

Validation of the Law: Consider a small wedge-shaped fluid element of unit length in equilibrium as shown in Fig. The mean pressure at the three surfaces are p_1, p_2 and p_3 and the force acting on a surface is the product of mean pressure and the surface area. From Newton's second law, a force balance in the x-direction and z-direction gives

$$\Sigma F_x = ma_x = 0; \quad p_1 \Delta y \Delta z - p_3 \Delta y\, l \sin\theta = 0 \qquad \text{...(i)}$$

$$\Sigma F_z = ma_z = 0; \quad p_2 \Delta y \Delta x - p_3 \Delta y\, l \cos\theta - \frac{1}{2}\rho g \Delta x\, \Delta y\, \Delta z = 0 \qquad \text{...(ii)}$$

Geometric relations are : $\Delta x = l\cos\theta$, $\Delta z = l\sin\theta$

Apply geometric relations in Eq. (i) and (ii), we get

$$p_1 - p_3 = 0$$

$$\Rightarrow \quad p_1 = p_3$$

$$p_2 - p_3 - \frac{1}{2}\rho g\, \Delta z = 0$$

For infinitesimal element, $\Delta z \to 0$

then, $\quad p_2 = p_3$

$$\therefore \quad p_1 = p_2 = p_3 = p \qquad \text{...(iii)}$$

Thus, we conclude that the pressure at a point in a fluid has the same magnitude in all directions.

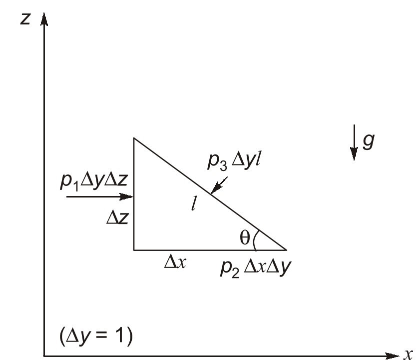


Fig. *Fluid Element*

2.3 Absolute and Gauge Pressure

- Pressures can be expressed in two different systems. The difference between two system is that their assumed datum is different.
- In absolute pressure system, the pressure is measured above absolute zero or complete vacuum.
- In gauge pressure system, the pressure is measured and expressed as difference between absolute value and local atmospheric pressure.
- If the pressure is below local atmospheric pressure, it is known as negative or vacuum or suction pressure.

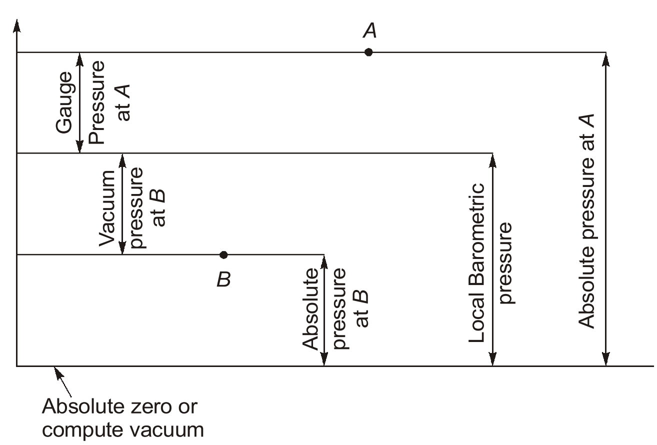


Fig. *Relation between gauge and Absolute pressure*

2.4 Variation of Pressure in a Fluid

- Consider a small fluid element of size $\delta x \times \delta y \times \delta z$ at any point in a static mass of fluid as shown in Fig. The forces acting on the element are the pressure forces on its faces and the self-weight of the element. Since, the element is in equilibrium under these forces, the algebraic sum of the forces acting on it in any direction must be zero.

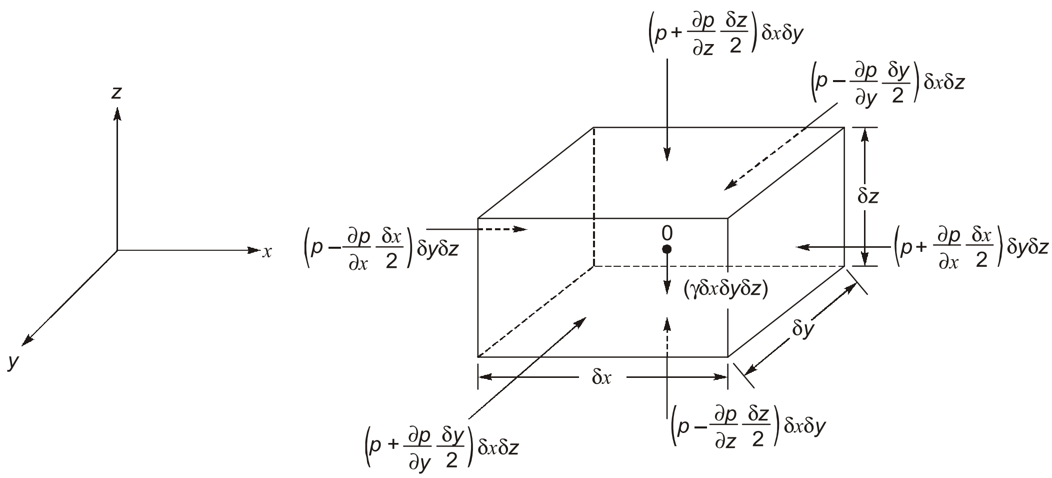


Fig. *Fluid Element*

i.e. $$\Sigma F_x = 0$$

or $$\left(p-\frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\cdot\delta y\delta z - \left(p+\frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\cdot\delta y\delta z = 0$$

or $$\frac{\partial p}{\partial x} = 0 \qquad \text{...(i)}$$

Also, $$\Sigma F_y = 0$$

or $$\left(p-\frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\cdot\delta x\delta z - \left(p+\frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\cdot\delta x\delta z = 0$$

or $$\frac{\partial p}{\partial y} = 0 \qquad \text{...(ii)}$$

Again, $$\Sigma F_z = 0$$

or $$\left(p-\frac{\partial p}{\partial z}\frac{\delta z}{2}\right)\cdot\delta x\delta y - \left(p+\frac{\partial p}{\partial z}\frac{\delta z}{2}\right)\cdot\delta x\delta y - \gamma(\delta x\delta y\delta z) = 0$$

or $$\frac{\partial p}{\partial z} = -\gamma \qquad \text{...(iii)}$$

Thus, Equations (i), (ii) and (iii) indicate that the pressure intensity 'p' at any point in a static fluid does not vary in x and y-directions and it varies only in z - direction. Partial derivative of Eq. (iii) can be reduced to total (or exact) derivative as follows

$$\frac{dp}{dz} = -\gamma = -\rho g \qquad \text{...(iv)}$$

- The minus sign(–) indicates that the pressure decreases in the direction in which z increases, i.e. in the upward direction.
- The above Eq. (iv) holds for both compressible and incompressible fluids and indicates that within a body of fluid at rest the pressure increases in the downward direction at the rate equivalent to the specific weight 'γ' of the liquid.

- If $dz = 0$, then, dp is also equal to zero; which means that the pressure remains constant over any horizontal plane in a fluid.
- It shows that in a incompressible fluid mass pressure changes according to the change in vertical column of liquid above the considered point.

$$\therefore \quad \frac{\partial p}{\partial h} = \rho \times g = \gamma \quad (\because \rho \times g = \gamma) \quad ...(v)$$

where γ = weight density of fluid.

Equation (v) states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is Hydrostatic law.

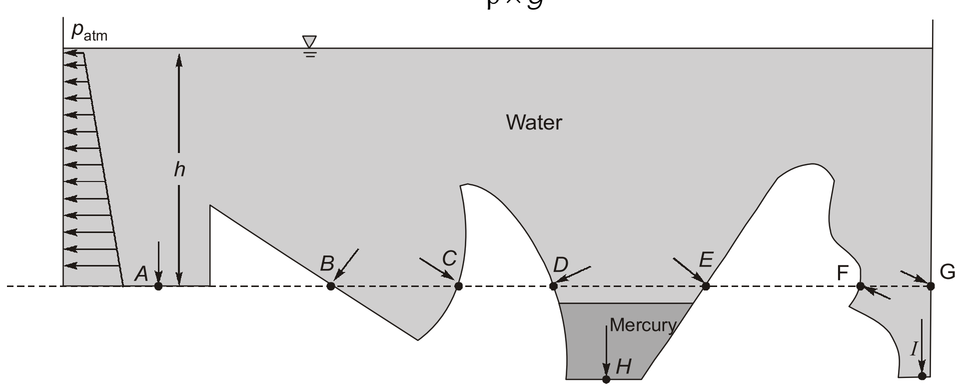
By integrating the above equation (8) for liquid, we get

$$\int dp = \int \rho g dh$$

$$\text{or} \quad p = \rho g h \quad ...(vi)$$

where p is the pressure above atmospheric pressure and h is the depth of the point from free surfaces.

$$\text{From Eq. (vi), we have} \quad h = \frac{p}{\rho \times g} \quad ...(vii)$$



$p_A = p_B = p_C = p_D = p_E = p_F = p_G = p_{atm} + \rho g h$

$p_H \neq p_I$

Fig. *Pressure at different points lying at same depth*

- In Fig., pressures at points *A*, *B*, C, *D* , *E* ,*F and G* lying on the same horizontal level and at the same vertical height *h* below the free surface of the liquid, will be same. But, the pressure at *H* and *I* are not same even if they are at same level.

2.5 Hydrostatic Paradox

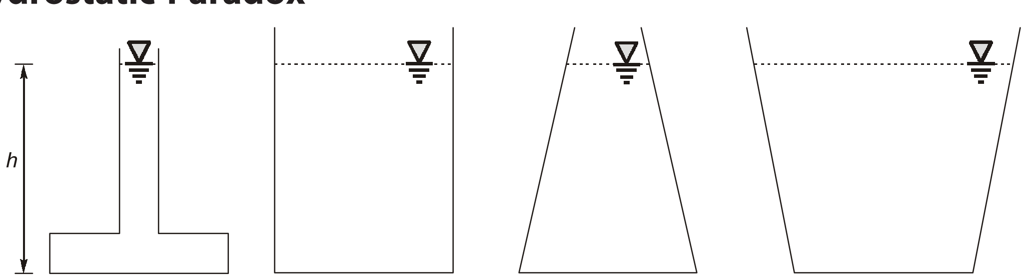


Fig. *Different shape of containers*

- Fig. shows four containers of different shape having same base area A which are filled by same liquid of specific weight of γ upto same height h.

  Gauge pressure at bottom $= \gamma h$ for all container

  So force exerted by liquid on the base of each container,

  $$F = \gamma h \times A$$
- Since shape of container is different so it has different weight of liquid but force exerted by liquid on the bottom of each container is same.
- This apparent contradiction in hydrostatic forces on the container base and weight of liquid inside the container is known as hydrostatic paradox.

2.6 Pressure Head

- The vertical height of the free surface above any point in a liquid at rest is known as pressure head for that point.

$$h = \frac{p}{\rho g} = \frac{p}{\gamma} \qquad \text{...(i)}$$

- Relationship between the heights of columns of different liquids which would develop the same pressure at any point, $p = \gamma_1 h_1 = \gamma_2 h_2$. If S_1 and S_2 are specific gravities of the two liquids then,

$$p = S_1\gamma_w h_1 = S_2\gamma_w h_2$$

$$\Rightarrow \qquad S_1 h_1 = S_2 h_2$$

Remember

Pressure in a Compressible Fluid

For a compressible fluid, the density varies with the pressure, therefore

$$\int_{p_1}^{p_2} \frac{dp}{\gamma} = -\int_{z_1}^{z_2} dz$$

where, p_1 = pressure at elevation z_1

p_2 = pressure at elevation z_2

2.7 Pressure Measurement Devices

TYPES OF DEVICES

- Barometer
- Manometers
  - Simple Manometers
    1. Piezometer
    2. U-tube manometer
    3. Single column Manometer
  - Differential Manometers
    1. Two Piezometer Manometer
    2. Inverted U-Tube Manometer
    3. U-Tube Differential Manometer
    4. Micromanometer
- Mechanical Gauges
  1. Bourdon Tube Pressure Gauge
  2. Diaphragm Pressure Gauge
  3. Bellows Pressure Gauge
  4. Dead-weight Pressure Gauge

2.7.1 Barometer

- Atmospheric pressure is measured by a device called barometer, thus, the atmospheric pressure is often referred to as the barometric pressure.
- The barometer consists of a inverted mercury-filled tube into a mercury container that is open to the atmosphere as shown in fig.
- The pressure at point *B* is equal to the atmospheric pressure, and the pressure at *C* can be taken to be zero since there is only mercury vapour above point *C* and the pressure is very low relative to p_{atm} and can be neglected.
- Writing a force balance in the vertical direction gives

$$p_{atm} = \rho g h$$

- In barometer, Hg is used because of its two important properties :
  - (i) Hg is a high density fluid.
  - (ii) Hg has very low vapour pressure.

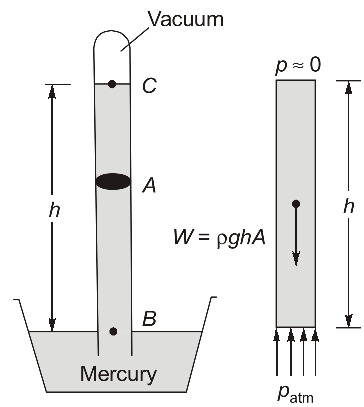


Fig. *Barometer*

> **Do You Know?**
> - The atmospheric pressure at a location is the weight of the air above that point. So, as one goes up in atmosphere, feels reduction in pressure as the air above that person continuously reduces.
> - Barometer was invented by Torricelli. To honour him, pressure is represented in unit of 'torr, where 1 torr = 1 mm Hg.

2.7.2 Manometers

- Manometers are those pressure measuring devices which are based on the principle of balancing the column of liquid (whose pressure is to be found) by the same or another column of liquid.
- Manometers are classified as :
  1. Simple manometers
  2. Differential manometers

2.7.2.1 Simple Manometers

- A simple manometer consists of a glass tube having one of its ends connected to the gauge point where the pressure is to be measured and the other remains open to atmosphere. Following are the types of simple manometers:

  (i) Piezometers:
  - A piezometer is the simplest form of manometer which can be used for measuring moderate pressures of liquids.
  - It consists of a glass tube inserted in the wall of a pipe or a vessel, containing a liquid whose pressure is to be measured. The tube extends vertically upward to such a height that liquid can freely rise in it without overflowing.

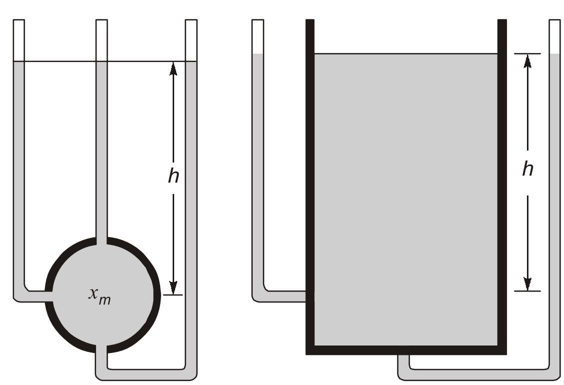


Fig. *Piezometers*

- The pressure at any point in the liquid is indicated by the height of the liquid in the tube above that point.
- The pressure measured correspond to gauge pressure. To find absolute pressure at the point, atmospheric pressure is added to the gauge pressure.
- Location of the point of insertion of a piezometer makes no difference in reading.
- To avoid the effect of capillarity, pipe-diameter of piezometric tube should be sufficiently large.

Limitations:

- Cannot be used when large pressure in lighter liquids is to be measured.
- Gas pressure cannot be measured, because gas forms no free atmospheric surface.

(ii) U-tube Manometer:

- A U-tube manometer consists of a glass tube in U-shape, one end of which is connected to the gauge point and the other end open to the atmosphere.
- The tube contains a liquid of specific gravity greater than that of the fluid of which the pressure is to be measured.
- Limitations imposed by piezometer are removed by use of U-tube manometers.
- The choice of the manometric liquid depends on the range of pressure to be measured. For low pressure range, liquids of lower specific gravity are used and for high range, generally mercury is employed.
- Consider a U-tube simple manometer is measuring pressure of a fluid of specific gravity S_1 (see fig.). To write an equation for the pressure of the fluid following points should be kept in mind.
  - Start from one end of gauge to another.
  - Write the pressure at one end. Add the change in pressure while moving from one level to another.
  - Use positive sign if the next level of contact is lower than the first and negative if it is higher.
- For the Figure

$$p_A - \rho_1 g h_1 - \rho_2 g h_2 = 0$$

$$\rho_1 = S_1 \rho_w$$

$$\rho_2 = S_2 \rho_w$$

where, ρ_w is density of water

Divide by $\rho_w g$

$$\frac{p_A}{\rho_w g} - S_1 h_1 - S_2 h_2 = 0$$

$$\frac{p_A}{\rho_w g} = S_1 h_1 + S_2 h_2$$

$$\frac{p_A}{\gamma_w} = S_1 h_1 + S_2 h_2$$

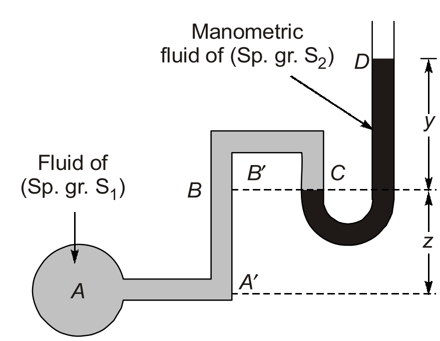


Fig. *U-tube Simple Manometer*

If, A contains gas, $S_1 << S_2$

$$\frac{p_A}{\gamma_w} = yS_2 \qquad ...(12)$$

- A U-tube manometer can also be used to measure negative or vacuum pressure. For measurement of small negative pressure, a U-tube manometer without any manometric fluid may be used.

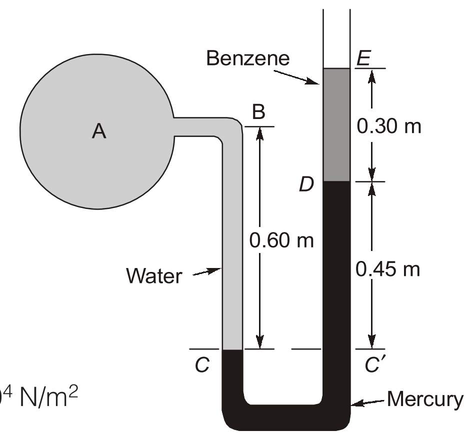
Limitations

- This method requires reading of fluid level at two or more points, since a change in pressure causes a rise of liquid in one limb of the manometer and a drop in the other.

Example 2.1 The left leg of U-tube mercury manometer is connected to a pipe-line conveying water. The level of mercury in the leg is 0.6 m below the center of pipe-line and the right leg is open to atmosphere. The level of mercury in the right leg is 0.45 m above that in the left leg and the space above mercury in the right leg contains Benzene (specific gravity 0.88) to a height of 0.3 m. Find the pressure in the pipe.

Solution :

In the accompanying figure, the pressures at C and C' are equal. Thus computing the pressure heads at C and C' from either side and equating the same, we get



$$\underbrace{\frac{p_A}{\gamma_w}+0.6}_{\text{(Left Leg)}} = \underbrace{0.45\times13.6+0.3\times0.88}_{\text{(Right Leg)}}$$

or $$\frac{p_A}{\gamma_w} = 5.784 \text{ m of water}$$

$\therefore$ $$p_A = (5.784\times9810) = 5.674\times10^4 \text{ N/m}^2$$

(iii) Single Column Manometer

- The limitation of U-tube manometer is removed in single column manometer.
- It is a modified form of U-tube manometer in which a shallow reservoir having a large cross-sectional area (about 100 times) as compared to the area of the tube is introduced into one limb of the manometer.
- For any change in pressure, the change in the liquid level in the reservoir will be so small that it may be neglected, and the pressure is indicated approximately by the height of the liquid in the other limb.
- Only one reading in the narrow limb of the manometer need to be taken for pressure measurement.

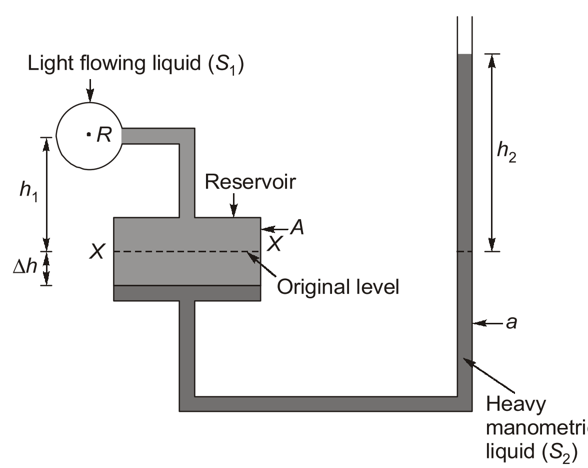


Fig. *Single column manometer*

- Narrow limb may be straight or inclined.
- The inclined type is useful for the measurement of small pressures as they are more sensitive than the vertical type.

$$A\Delta h = ah_2$$

or $$\Delta h = \frac{ah_2}{A}$$

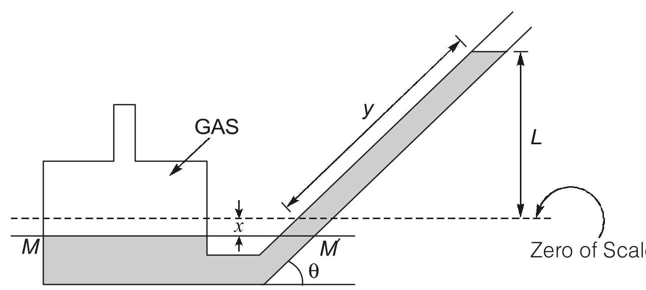
Now, $$p_R + \rho_1 g(h_1 + \Delta h) = \rho_2 g(h_2 + \Delta h)$$

$$p_R = \rho_2 gh_2 - \rho_1 gh_1 + \Delta hg(\rho_2 - \rho_1)$$

If Δh is very small, $$p_R = \rho_2 gh_2 - \rho_1 gh_1$$

Example 2.2 A manometer consists of an inclined glass tube which is connected to a metal cylinder standing upright. A manometric liquid fills the apparatus to a fixed zero mark on the tube when both cylinder and the tube are open to atmosphere. The upper end of the cylinder is then connected to a gas supply at a pressure p and the liquid rises in the tube.

Find an expression for the pressure p in cm of water when the liquid reads y cm in the tube, in terms of the inclination θ of the tube, the specific gravity of the liquid S, and the ratio a of the diameter of the cylinder to the diameter of the tube. Hence, determine the value of a so that the error due to disregarding the change in level in the cylinder will not exceed 0.1%, when θ = 30°.



Solution :

Diameter ratio = a

So, Area ratio = a^2

Now, $$x\left(\frac{\pi D^2}{4}\right) = y\left(\frac{\pi d^2}{4}\right)$$

$$xa^2 = y$$

$$x = \frac{y}{a^2}$$

$$p_{(\text{incorrect})} = \rho_M g(y\sin 30°)$$

$$p_{(\text{correct})} = \rho_M g(y\sin 30° + x)$$

$$= \rho_M g\left(y\sin 30° + \frac{y}{a^2}\right)$$

Now, $$\% \text{ Error} = \frac{p_{(\text{correct})} - p_{(\text{incorrect})}}{p_{(\text{correct})}} \times 100$$

$$0.1 = \frac{\frac{y}{a^2}}{y\sin 30° + \frac{y}{a^2}} \times 100$$

$\Rightarrow$ $$a^2 = 2000$$

$\Rightarrow$ $$a = 44.72$$

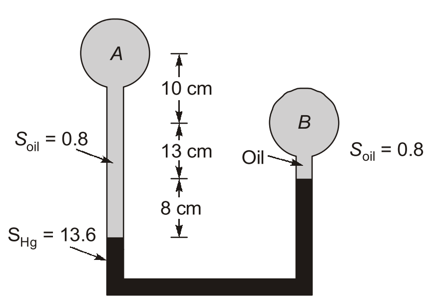
Objective Brain Teasers

Ex.1 If a Mohr circle is drawn for a fluid element inside a fluid body at rest, it would be :
(a) a circle not touching the origin
(b) a circle touching the origin
(c) a point on the normal stress axis
(d) a point on the shear stress axis

Ex.2 The pressure in meters of oil of specific gravity 0.8 equivalent to 80 m of water is :
(a) 64 m (b) 88 m
(c) 80 m (d) 100 m

Ex.3 The mass density of a liquid with variable density is given by $\rho = 1000 + 0.008\ y^{3/2}$, where ρ is in kg/m^3 ; y is measured in meters. The depth at which the pressure intensity will be 900 kPa, is
(a) 91.5 m (b) 101.5 m
(c) 112.5 m (d) 114.5 m

Ex.4 The pressure difference between point *A* and *B* for the set up shown in figure in kPa is

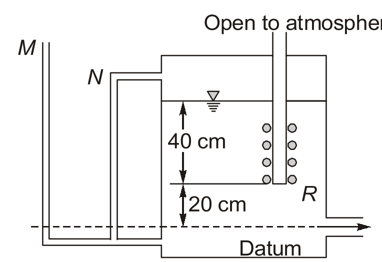


(a) 9.26 (b) 10.54
(c) 10.65 (d) 11.66

Ex.5 In a mercury column-type barometer, the correct local atmospheric pressure is obtained by considering correction due to vapour pressure of mercury as follows ; H_a =
(a) $H - h_v$ (b) $H_0 + h_v$
(c) H_0 / h_v (d) $h_v - H_0$
[where, H_a = correct local pressure in mm of mercury, H_0 = observed barometer reading in mm of mercury and h_v = vapour pressure of mercury in mm.]

Ex.6 The standard atmospheric pressure is 101.32 kPa. The local atmospheric pressure at a location was 91.52 kPa. If a pressure is recorded as 22.48 kPa (gauge), it is equivalent to
(a) 123.80 kPa (abs) (b) 88.84 kPa(*abs*)
(c) 114.00 kPa(*abs*) (d) 69.04 kPa(*abs*)

Ex.7 The tank shown in figure discharge water at constant rate for all water levels above the air inlet *R*. The height above datum to which water would rise in manometer tubes *M* and *N* respectively, are

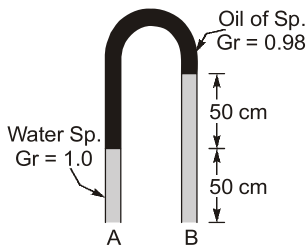


(a) (60 cm, 20 cm) (b) (40 cm, 40 cm)
(c) (20 cm, 20 cm) (d) (20 cm, 60 cm)

Ex.8 Normal stresses are of the same magnitude in all directions at a point in a fluid
(a) only when the fluid is frictionless
(b) only when the fluid is at rest
(c) only when there is no shear stress
(d) in all cases of fluid motion

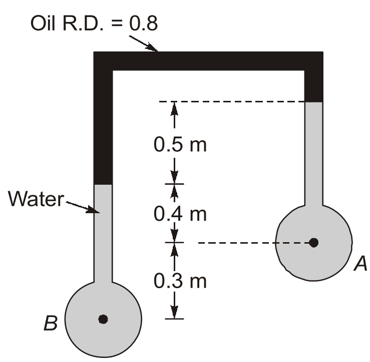
Ex.9 Identify the CORRECT statement:
(a) Local atmospheric pressure is always less than the standard atmospheric pressure
(b) Local atmospheric pressure depends only on the elevation of the place
(c) A barometer reads the difference between the local and standard atmospheric pressure
(d) Standard atmospheric pressure is 760 mm of mercury

Ex.10 In the setup shown in given figure assuming the specific weight of water as 10 kN/m^3, the pressure difference between the two points *A* and *B* will be



(a) 100 N/m^2 (b) – 100 N/m^2
(c) 200 N/m^2 (d) – 200 N/m^2

Ex.11 An inverted differential manometer is shown in given figure. The differential pressure ($p_B - p_A$) in terms of column height of oil of relative density 0.8 is



(a) 0.25 m (b) 0.5 m
(c) 0.85 m (d) None of these

ANSWERS

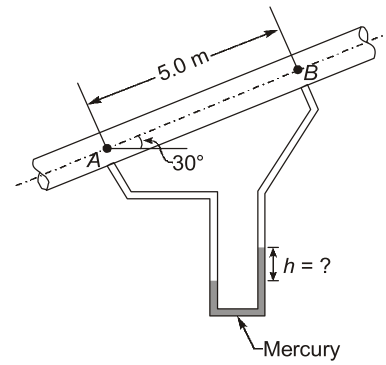
1. (c)	2. (d)	3. (a)	4. (a)	5. (b)
6. (c)	7. (d)	8. (b)	9. (d)	10. (a)
11. (a)				

Student's Assignments

Ex.1 A certain fluid of specific gravity 0.8 flows upwards through a vertical pipe. *A* and *B* are two points on the pipe, *B* being 0.3 m higher than *A*. A U-tube mercury manometer is connected at points *A* and *B*. If the difference in pressure between *A* and *B* is 5 kPa, find the difference in the heights of the mercury columns in the manometer.

Ans. h = 21.4 mm

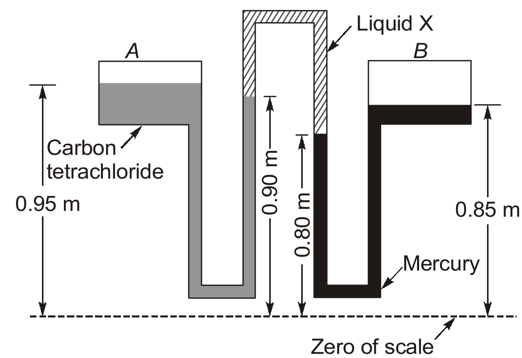
Ex.2 If the pipe in the given figure contains water and there is no flow, calculate the value of the manometer reading h.



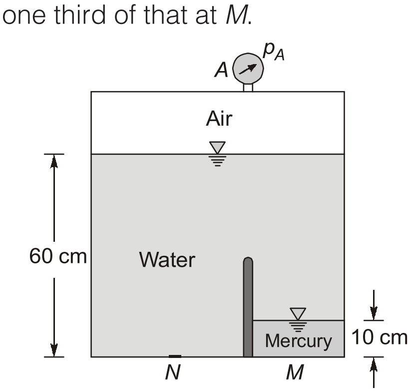
Ans. h = 0

Ex.3 In the manometer shown in the given figure, the liquid on the left side is carbon - tetrachloride of specific gravity 1.60 and liquid on the right side is mercury. If ($p_A - p_B$) is 525 kg(f)/m^2 (5150.25 N/m^2), find the specific gravity of the liquid X.

Ans. 0.75



Ex.4 For the system shown in given figure calculate the air pressure p_A to make the pressure at *N* one third of that at *M*.



Ans. p_A = 0.294 kPa

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