Chemical Engineering

Instrumentation and Process Control

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Instrumentation and Process Control

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Chapter 1

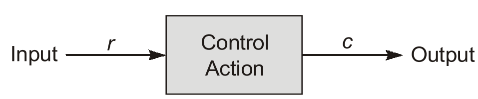
Introduction

Control System:
Control system is a means by which any quantity of interest in a machine, mechanism or other equipment is maintained or altered in accordance with a desired manner.
Control system can also be defined as the combination of elements arranged in a planned manner wherein each element causes an effect to produce a desired output.

Control systems are classified into two general categories as Open-loop and close-loop systems.

1.1 Open Loop Control Systems

An open loop control system is one in which the control action is independent of the output.



Open-loop control system

This is the simplest and most economical type of control system and does not have any feedback arrangement.

Some common examples of open-loop control systems are
(a) Traffic light controller
(b) Electric washing machine
(c) Automatic coffee server
(d) Bread toaster

Advantages of Open Loop Control Systems

(a) Simple and economic
(b) No stability problem

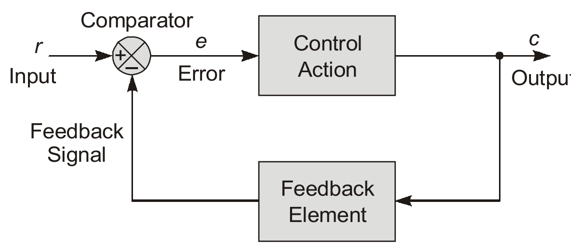
Disadvantages of Open Loop Control Systems

(a) Inaccurate
(b) Unrealisable
(c) The effect of parameter variation and external noise is more

Note: Open loop control systems does not require performance analysis.

1.2 Closed Loop Control Systems

A *closed loop control system* is one in which the control action is some how dependent on the output.



Closed loop control system

The closed loop system has same basic features as of open loop system with an additional feedback feature. The actual output is measured and a signal corresponding to this measurement is fedback to the input section, where it is with the input to obtain the desired output.

Some common examples of closed loop control systems are:

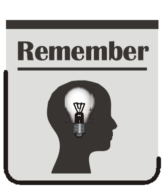
(a) Electric iron
(b) DC motor speed control
(c) A missile launching system (direction of missile changes with the location of moving target)
(d) Radar tracking system
(e) Human respiratory system
(f) Autopilot system
(g) Economic inflation

Advantages of Closed Loop Control Systems

(a) Accurate and reliable
(b) Reduced effect of parameter variation
(c) Bandwidth of the system can be increased with negative feedback
(d) Reduced effect of non-linearities

Disadvantages of Closed Loop Control Systems

(a) The system is complex and costly
(b) System may become unstable
(c) Gain of the system reduces with negative feedback



- Feedback is not used for improving stability
- An open loop stable system may also become unstable when negative feedback is applied
- Except oscillators, in positive feedback, we have always unstable systems.

1.3 Comparison Between Open Loop and Closed Loop Control Systems

Open Loop System	Closed Loop System
1. So long as the calibration is good, open-loop system will be accurate	1. Due to feedback, the close-loop system is more accurate
2. Organization is simple and easy to construct	2. Complicated and difficult
3. Generally stable in operation	3. Stability depends on system components
4. If non-linearity is present, system operation degenerates	4. Comparatively, the performance is better than open-loop system if non-linearity is present

Example-1.1 Match List-I (Physical action or activity) with List-II (Category of system) and select the correct code:

List-I
A. Human respiration system
B. Pointing of an object with a finger
C. A man driving a car
D. A thermostatically controlled room heater

List II
1. Man-made control system
2. Natural including biological control system
3. Control system whose components are both man-made and natural

Codes:

	A	B	C	D
(a)	2	2	3	1
(b)	3	1	2	1
(c)	3	2	2	3
(d)	2	1	3	3

***Solution:* (*a*)**

1.4 Laplace Transformation

In order to transform a given function of time $f(t)$ into its corresponding Laplace transform first multiply $f(t)$ by e^{-st}, s being a complex number ($s = \sigma + j\omega$). Integrate this product with respect to time with limits from zero to ∞. This integration results in Laplace transform of f(t), which is denoted by $F(s)$ or $\mathcal{L} f[(t)]$.

The mathematical expression for Laplace transform is,

$$\mathcal{L} f[(t)] = F(s),\ t \geq 0$$

where,

$$F(s) = \int_0^{\infty} f(t).e^{-st} dt$$

The original time function f(t) is obtained back from the Laplace transform by a process called inverse Laplace transformation and denoted as $\mathcal{L}^{-1}$

Thus,

$$\mathcal{L}^{-1}[Lf(t)] = \mathcal{L}^{-1}[F(s)] = f(t)$$

The time function $f(t)$ and its Laplace transform $F(s)$ form a transform pair.

S.No.	$f(t)$	$F(s) = L[f(t)]$
1.	$\delta(t)$ unit impulse at $t = 0$	1
2.	$u(t)$ unit step at $t = 0$	$\frac{1}{s}$
3.	$u(t-T)$ unit step at $t = T$	$\frac{1}{s}e^{-sT}$
4.	t	$\frac{1}{s^2}$
5.	$\frac{t^2}{2}$	$\frac{1}{s^3}$
6.	t^n	$\frac{n!}{s^{n+1}}$
7.	e^{at}	$\frac{1}{s-a}$
8.	e^{-at}	$\frac{1}{s+a}$
9.	$t\,e^{at}$	$\frac{1}{(s-a)^2}$
10.	$t\,e^{-at}$	$\frac{1}{(s+a)^2}$
11.	$t^n\,e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
12.	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
13.	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$

Table of Laplace Transform Pairs

Basic Laplace Transform Theorems

Basic theorems of Laplace transform are given below:

(a) Laplace Transform of Linear Combination:

$$\mathcal{L}[af_1(t) + bf_2(t)] = aF_1(s) + bF_2(s)$$

where $f_1(t)$, $f_2(t)$ are functions of time and a, b are constants.

(b) If the Laplace Transform of $f(t)$ is $F(s)$, then:

(i) $$\mathcal{L}\left[\frac{df(t)}{dt}\right] = [sF(s) - f(0^+)]$$

(ii) $$\mathcal{L}\left[\frac{d^2f(t)}{dt^2}\right] = [s^2F(s) - sf(0^+) - f'(0^+)]$$

(iii) $$\mathcal{L}\left[\frac{d^3f(t)}{dt^3}\right] = [s^3F(s) - s^2 f(0^+) - sf'(0^+) - f''(0^+)]$$

where $f(0^+)$, $f'(0^+)$, $f''(0^+)$ ... are the values of $f(t)$, $\frac{df(t)}{dt}$, $\frac{d^2f(t)}{dt^2}$ ... at $t = (0^+)$.

(*c*) If the Laplace Transform of $f(t)$ is $F(s)$, then:

(*i*) $$\mathcal{L}\left[\int f(t)\right] = \left[\frac{F(s)}{s} + \frac{f^{-1}(0^+)}{s}\right]$$

(*ii*) $$\mathcal{L}\left[\iint f(t)\right] = \left[\frac{F(s)}{s^2} + \frac{f^{-1}(0^+)}{s^2} + \frac{f^{-2}(0^+)}{s}\right]$$

(*iii*) $$\mathcal{L}\left[\iiint f(t)\right] = \left[\frac{F(s)}{s^3} + \frac{f^{-1}(0^+)}{s^3} + \frac{f^{-2}(0^+)}{s^2} + \frac{f^{-3}(0^+)}{s}\right]$$

where $f^{-1}(0^+)$, $f^{-2}(0^+)$, $f^{-3}(0^+)$ ... are the values of $\int f(t)$, $\iint f(t)$, $\iiint f(t)$ ... at $t = (0^+)$.

(*d*) If the Laplace Transform of f(t) is $F(s)$, then:

$$\mathcal{L}[e^{\pm at} f(t)] = F(s \mp a)$$

(*e*) If the Laplace Transform of $f(t)$ is $F(s)$, then:

$$\mathcal{L}[t\, f(t)] = -\frac{d}{ds} F(s)$$

(*f*) Initial Value Theorem:

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} s\mathcal{L}[f(t)]$$

or

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$$

(*g*) Final Value Theorem:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s\mathcal{L}[f(t)]$$

or

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

The final value theorem gives the final value ($t \to \infty$) of a time function using its Laplace transform and as such very useful in the analysis of control systems. However, if the denominator of $sF(s)$ has any root having real part as zero or positive, then the final value theorem is not valid.

Example-1.2 Laplace transform of $\sin(\omega t + \alpha)$ is

(a) $\dfrac{s\cos\alpha + \omega \sin\alpha}{s^2 + \omega^2}$

(b) $\dfrac{\omega}{s^2 + \omega^2}\cos\alpha$

(c) $\dfrac{s}{s^2 + \omega^2}\sin\alpha$

(d) $\dfrac{s\sin\alpha + \omega\cos\alpha}{s^2 + \omega^2}$

Solution: **(d)**

$$\sin(\omega t + \alpha) = \sin\omega t\cos\alpha + \cos\omega t\sin\alpha$$

$$\mathcal{L}\{\sin(\omega t + \alpha)\} = \frac{\omega\cos\alpha}{s^2 + \omega^2} + \frac{s\sin\alpha}{s^2 + \omega^2}$$

$$= \frac{s\sin\alpha + \omega\cos\alpha}{s^2 + \omega^2}$$

Example-1.3 Given $\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)e^{-st}dt$, which of the following expressions are correct?

1. $\mathcal{L}\{f(t-a)\ u(t-a)\} = F(s)e^{-sa}$
2. $\mathcal{L}\{t\ f(t)\} = \dfrac{-dF(s)}{ds}$
3. $\mathcal{L}\{(t-a)f(t)\} = as\ F(s)\}$
4. $\mathcal{L}\left\{\dfrac{df(t)}{dt}\right\} = sF(s) - f(0^+)$

Select the correct answer using the codes given below

(a) 1, 2 and 3
(b) 1, 2 and 4
(c) 2, 3 and 4
(d) 1, 3 and 4

***Solution:* (b)**

These are the properties of Laplace transform.

Example-1.4 Match List-I [Function in time domain f(t)] with List -II [Property] and select the correct answer using the code given below the lists:

List-I	List-II
A. $\sin \omega_0 t u(t-t_0)$	1. $\dfrac{\omega_0}{s^2+\omega_0^2}$
B. $\sin \omega_0 (t-t_0)\ u(t-t_0)$	2. $\left\{\dfrac{\omega_0}{s^2+\omega_0^2}\right\}e^{-t_0 s}$
C. $\sin \omega_0 (t-t_0)\ u(t)$	3. $\dfrac{e^{-t_0 s}}{\sqrt{s^2+\omega_0^2}}\sin\left(\omega_0 t_0 + \tan^{-1}\dfrac{\omega_0}{s}\right)$
D. $\sin \omega_0 t\ u(t)$	4. $-\dfrac{1}{\sqrt{s^2+\omega_0^2}}\sin\left(\omega_0 t_0 - \tan^{-1}\dfrac{\omega_0}{s}\right)$

Codes:

	A	B	C	D
(a)	3	1	4	2
(b)	4	2	3	1
(c)	3	2	4	1
(d)	4	1	3	2

***Solution:* (c)**

Example-1.5 Find the inverse Laplace transform of the following functions:

(i) $F(s) = \dfrac{s+2}{s^2+4s+6}$ (ii) $F(s) = \dfrac{5}{s(s^2+4s+5)}$ (iii) $F(s) = \dfrac{s^2+2s+3}{s^3+6s^2+12s+8}$

Solution:

(i)

$$F(s) = \frac{s+2}{s^2+4s+6}$$

$\because$ The term $(s^2 + 4s + 6)$ can be expressed as $[(s+2)^2 + (\sqrt{2})^2]$

$$\therefore \qquad F(s) = \frac{s+2}{(s+2)^2 + (\sqrt{2})^2}$$

Taking inverse Laplace transform on both sides

$$\mathcal{L}^{-1} F(s) = \mathcal{L}^{-1} \frac{s+2}{(s+2)^2 + (\sqrt{2})^2}$$

$$\therefore \qquad f(t) = e^{-2t} \cos\sqrt{2}t$$

(ii)
$$F(s) = \frac{5}{s(s^2 + 4s + 5)}$$

Using partial fraction expansion,

$$\frac{5}{s(s^2 + 4s + 5)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 4s + 5}$$

The coefficients are determined as $A = 1$, $B = -1$ and $C = -4$

$$\therefore \qquad F(s) = \frac{1}{s} - \frac{s+4}{s^2 + 4s + 5}$$

$\because$ The term $(s^2 + 4s + 5)$ can be expressed as $[(s + 2)^2 + (1)^2]$

$$\therefore \qquad F(s) = \frac{1}{s} - \frac{s+2}{[(s+2)^2 + (1)^2]} - 2\frac{1}{[(s+2)^2 + (1)^2]}$$

Taking inverse Laplace transform on both sides

$$\mathcal{L}^{-1} F(s) = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{s+2}{(s+2)^2 + (1)^2} - 2\frac{1}{(s+2)^2 + (1)^2}\right]$$

or
$$\mathcal{L}^{-1} F(s) = \mathcal{L}^{-1}\frac{1}{s} - \mathcal{L}^{-1}\frac{s+2}{[(s+2)^2 + (1)^2]} - \mathcal{L}^{-1}\, 2\frac{1}{[(s+2)^2 + (1)^2]}$$

$$\therefore \qquad f(t) = (1 - e^{-2t}\cos t - 2e^{-2t}\sin t)$$

(iii)
$$F(s) = \frac{s^2 + 2s + 3}{s^3 + 6s^2 + 12s + 8}$$

$\because$ The denominator $(s^3 + 6s^2 + 12s + 8)$ can be expressed as $(s + 2)^3$

$$\therefore \qquad F(s) = \frac{s^2 + 2s + 3}{(s+2)^3}$$

Using partial fraction expansion

$$F(s) = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(s+2)^3}$$

The coefficients are determined as $A = 1$, $B = -2$ and $C = 3$

$$\therefore \qquad F(s) = \frac{1}{s+2} - \frac{2}{(s+2)^2} + \frac{3}{(s+2)^3}$$

Taking inverse Laplace transform on both sides

$$\mathcal{L}^{-1} F(s) = \mathcal{L}^{-1}\left[\frac{1}{s+2} - \frac{2}{(s+2)^2} + \frac{3}{(s+2)^3}\right]$$

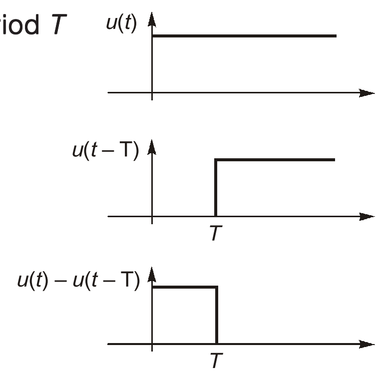
or $$\mathcal{L}^{-1} F(s) = \mathcal{L}^{-1}\frac{1}{s+2} - \mathcal{L}^{-1}\frac{2}{(s+2)^2} + \mathcal{L}^{-1}\frac{3}{(s+2)^3}$$

$\therefore$ $$f(t) = e^{-2t} - 2t\,e^{-2t} + \frac{3}{2}t^2\,e^{-2t}$$

or $$f(t) = e^{-2t}\left[1 - t\left(2 - \frac{3}{2}t\right)\right]$$

Example-1.6 $F(s) = \dfrac{(1-e^{-sT})}{s}$ is the Laplace transform of

(a) a pulse of width T
(b) a square wave of period T
(c) a unit step delayed by T
(d) a ramp delayed by T



Solution: (a)

$$F(s) = \frac{1-e^{-sT}}{s}$$

$$= \frac{1}{s} - e^{-sT}\left(\frac{1}{s}\right)$$

$\therefore$ $$f(t) = u(t) - u(t-T)$$

Example-1.7 Consider the rational function

$$F(s) = \frac{1}{(s+1)^2\,(s+2)}$$

find $f(t)$.

Solution:

The partial fraction expansion of $F(s)$ is

$$F(s) = A + \frac{B}{s+1} + \frac{C}{(s+1)^2} + \frac{D}{s+2}$$

The coefficients A, B, C, D are given by

$$A = 0$$

$$B = \frac{d}{ds}(s+1)^2\,F(s)\Big|_{s=-1} = \frac{d}{ds}\left(\frac{1}{s+2}\right)\Big|_{s=-1} = -1$$

$$C = (s+1)^2\,F(s)\Big|_{s=-1} = \frac{1}{s+2}\Big|_{s=-1} = 1$$

$$D = (s+2)\,F(s)\Big|_{s=-2} = 1$$

Thus, $$F(s) = -\frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{s+2}$$

$\Rightarrow$ $$f(t) = -e^{-t}\,u(t) + e^{-t}\,r(t) + e^{-2t}\,u(t)$$

1.5 Transfer Function and Impulse Response Function

In control theory, transfer functions are commonly used to characterise the input-output relationships of components or systems that can be described by linear, time-invariant differential equations.

Transfer Function

The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

Linear Systems

A system is called linear if the principle of superposition and principle of homogeneity apply. The principle of superposition states that the response produced by the simultaneous application of two different forcing functions is the sum of the two individual responses. Hence, for the linear system, the response to several inputs can be calculated by transferring one input at a time and adding the results. It is the principle that allows one to build up complicated solutions to the linear differential equations from simple solutions.

In an experimental investigation of a dynamic system, if cause and effect are proportional, thus implying that the principle of superposition holds, then the system can be considered as linear.

Linear Time-Invariant Systems and Linear-Time Varying Systems

A differential equation is linear if the coefficients are constants or functions only of the independent variable. Dynamic systems that are composed of linear time-invariant lumped-parameter components may be described by linear time-invariant differential equations i.e. constant-coefficient differential equations. Such systems are called linear time-invariant (or linear constant-coefficient) systems. Systems that are represented by differential equations whose coefficients are function of time are called linear time varying systems. An example of a time-varying control system is a space craft control system (the mass of a space craft changes due to fuel consumption).

The definition of transfer function is easily extended to a system with multiple inputs and outputs (i.e. a multivariable system). In a multivariable system, a linear differential equation may be used to describe the relationship between a pair of input and output variables, when all other inputs are set to zero. Since the principle of superposition is valid for linear systems, the total effect (on any output) due to all the inputs acting simultaneously is obtained by adding up the outputs due to each input acting alone.

Example-1.8 When deriving the transfer function of a linear element

(a) both initial conditions and loading are taken into account
(b) initial conditions are taken into account but the element is assumed to be not loaded.
(c) initial conditions are assumed to be zero but loading is taken into account
(d) initial conditions are assumed to be zero and the element is assumed to be not loaded.

Solution: (c)

While deriving the transfer function of a linear element only initial conditions are assumed to be zero, loading (or input) can't assume to be zero.

Example-1.9 If the initial conditions for a system are inherently zero, what does it physically mean?

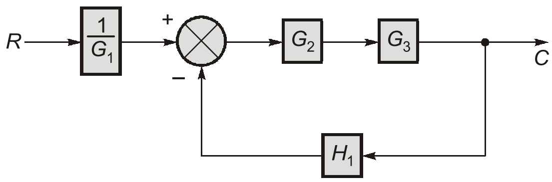
(a) The system is at rest but stores energy
(b) The system is working but does not store energy
(c) The system is at rest or no energy is stored in any of its part
(d) The system is working with zero reference input

Solution: (c)

A system with zero initial conditions is said to be at rest since there is no stored energy.

Student's Assignments

Q.1 A feedback control system is shown below . Find the transfer function for this system.

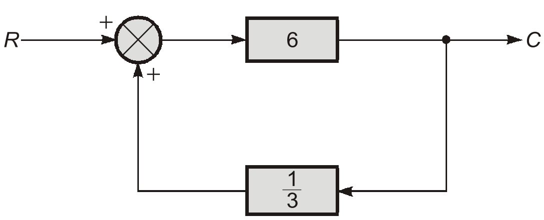


Q.2 The step response of a system is given as

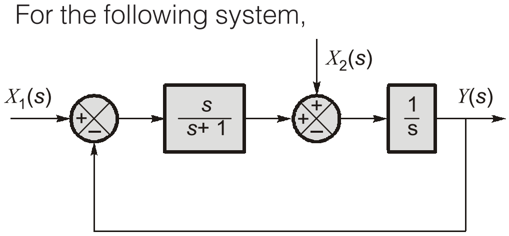
$y = 1 - \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}$. If the transfer function of this system is $\frac{(s+a)}{(s+b)(s+c)(s+d)}$ then $a + b + c + d$ is __________.

Q.3 A system has the transfer function $\frac{(1-s)}{(1+s)}$. Its gain at ω = 1 rad/sec is ______.

Q.4 The close loop gain of the system shown below is



Q.5 For the following system,

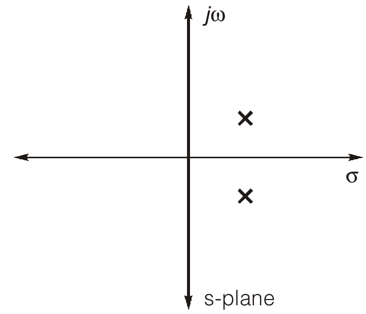


when $X_1(s)$ = 0, the transfer function $\frac{Y(s)}{X_2(s)}$ is

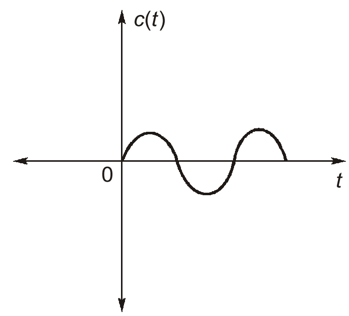
(a) $\frac{s+1}{s^2}$ (b) $\frac{1}{s+1}$

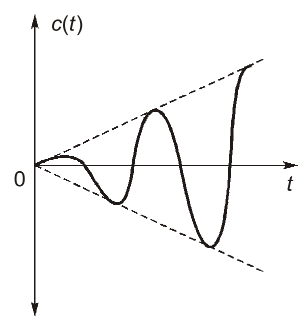
(c) $\frac{s+2}{s(s+1)}$ (d) $\frac{s+1}{s(s+2)}$

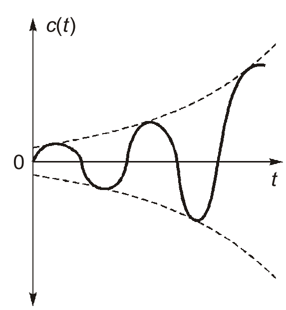
Q.6 If closed-loop transfer function poles shown below

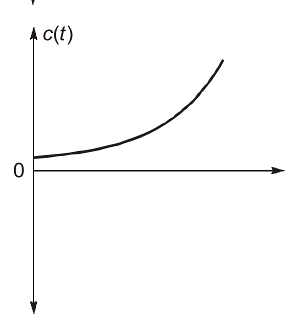


Impulse response is

(a)


(b)


(c)


(d)


Q.7 The impulse response of several continuous systems are given below. Which is/are stable?
1. $h(t) = te^{-t}$
2. $h(t) = 1$
3. $h(t) = e^{-t} \sin 3t$
4. $h(t) = \sin \omega t$

(a) 1 only (b) 1 and 3
(c) 3 and 4 (d) 2 and 4

Q.8 Ramp response of the transfer function $F(s) = \dfrac{s+1}{s+2}$ is

(a) $\frac{1}{4} - \frac{1}{4}e^{-2t} + \frac{1}{2}t$ (b) $\frac{1}{4}e^{-2t} + \frac{1}{4} + \frac{1}{2}t$
(c) $\frac{1}{2} - \frac{1}{2}e^{-2t} + t$ (d) $\frac{1}{2}e^{-2t} + \frac{1}{2} - t$

Q.9 Which of the following statements are correct?
1. Transfer function can be obtained from the signal flow graph of the system.
2. Transfer function typically characterizes to linear time invariant systems.
3. Transfer function gives the ratio of output to input in frequency domain of the system.

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

Q.10 Which of the following is not a desirable feature of a modern control system?
(a) Quick response
(b) Accuracy
(c) Correct power level
(d) Oscillations

Q.11 In regenerating feedback, the transfer function is given by

(a) $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$
(b) $\dfrac{C(s)}{R(s)} = \dfrac{G(s)\,H(s)}{1-G(s)\,H(s)}$
(c) $\dfrac{C(s)}{R(s)} = \dfrac{G(s)H(s)}{1+G(s)H(s)}$
(d) $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1-G(s)H(s)}$

Q.12 Consider the following statements regarding the advantages of closed loop negative feedback control systems over open-loop systems:
1. The overall reliability of the closed loop systems is more than that of open-loop system.
2. The transient response in the closed loop system decays more quickly than in open-loop system.
3. In an open-loop system, closing of the loop increases the overall gain of the system.
4. In the closed-loop system, the effect of variation of component parameters on its performance is reduced.

Of these statements
(a) 1 and 3 are correct
(b) 1, 2 and 4 are correct
(c) 2 and 4 are correct
(d) 3 and 4 are correct

Q.13 Match **List-I** (Time function) with **List-II** (Laplace transforms) and select the correct answer using the codes given below lists:

List-I	List-II
A. $[af_1(t) + bf_2(t)]$	1. $aF_1(s) + bF_2(s)$
B. $[e^{-at}f(t)]$	2. $sF(s) + f(0)$
C. $\left[\dfrac{df(t)}{dt}\right]$	3. $\dfrac{1}{s}F(s)$
D. $\left[\int_0^t f(x)dx\right]$	4. $sF(s) - f(0^-)$
	5. $F(s+a)$

Codes:

	A	B	C	D
(a)	5	2	3	4
(b)	1	5	4	3
(c)	2	1	3	4
(d)	1	5	3	4

Q.14 If a system is represented by the differential equation, is of the form $\dfrac{d^2y}{dt^2} + 6\dfrac{dy}{dt} + 9y = r(t)$

(a) $k_1\, e^{-t} + k_2 e^{-9t}$ (b) $(k_1 + k_2)\, e^{-3t}$
(c) $ke^{-3t} \sin(t + \phi)$ (d) $te^{-3t}\, u(t)$

Q.15 A linear system initially at rest, is subject to an input signal $r(t) = 1 - e^{-t}\,(t \geq 0)$. The response of the system for $t > 0$ is given by $c(t) = 1 - e^{-2t}$. The transfer function of the system is

(a) $\dfrac{(s+2)}{(s+1)}$ (b) $\dfrac{(s+1)}{(s+2)}$

(c) $\dfrac{2(s+1)}{(s+2)}$ (d) $\dfrac{(s+1)}{2(s+2)}$

ANSWERS

1. (sol.)	**2.** (15)	**3.** (1)	**4.** (–6)	**5.** (d)
6. (c)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (...)
11. (d)	**12.** (b)	**13.** (...)	**14.** (d)	**15.** (c)

Explanation

1. $\left(\dfrac{G_2G_3}{G_1(1+H_1G_2G_3)}\right)$

Multiply G_2 and G_3 and apply feedback formula and then again multiply with $\dfrac{1}{G_1}$

$$T(s) = \frac{G_2G_3}{G_1(1+G_2G_3H_1)}$$

2. (15)

$$y = 1-\frac{7}{3}e^{-t}+\frac{3}{2}e^{-2t}-\frac{1}{6}e^{-4t}$$

$$p(t) = \frac{dy}{dt}$$

$$= \frac{7}{3}e^{-t}+\frac{3}{2}\times(-2)\times e^{-2t}-\left(\frac{1}{6}\right)(-4)e^{-4t}$$

Laplace transform of p(t)

$$p(s) = \frac{7/3}{s+1}+\frac{-3}{s+2}+\frac{2/3}{s+4}$$

$$= \frac{s+8}{(s+1)(s+2)(s+4)}$$

$\Rightarrow \quad a + b + c + d = 15$

3. (1)

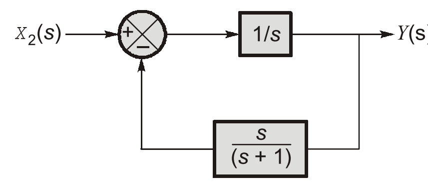
For all pass system, gain = '1' at all frequencies.

4. (–6)

$$\text{C.L.T.F.} = \frac{6}{1-6\times\frac{1}{3}} = \frac{6}{-1} = -6$$

5. (d)

Redrawing the block diagram with $X_1(s) = 0$



The transfer function

$$T(s) = \frac{Y(s)}{X_2(s)} = \frac{G(s)}{1+G(s)H(s)} \qquad \text{...(i)}$$

Here, $G(s) = \dfrac{1}{s}$ and $H(s) = \dfrac{s}{s+1}$

$$\frac{Y(s)}{X_2(s)} = \frac{1/s}{1+\frac{1}{s}\times\frac{s}{s+1}} = \frac{(s+1)}{s(s+2)}$$

6. (c)

$$\text{T.F.} = \frac{1}{[s-(\sigma+j\omega)][s-(\sigma-j\omega)]}$$

$$= \frac{1}{[(s-\sigma)-j\omega)][(s-\sigma)+j\omega)]}$$

$$= \frac{1}{\left[(s-\sigma)^2-(j\omega)^2\right]} = \frac{1}{\left[(s-\sigma)^2+\omega^2\right]}$$

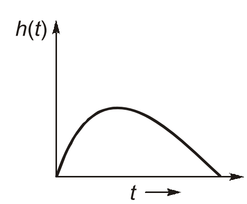
For impulse response, taking its inverse Laplace transformation we get,

$c(t) = e^{\sigma t}\sin\omega t$. So, option (c) is correct.

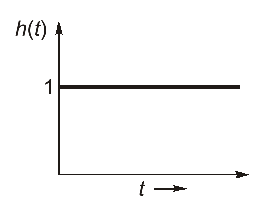
7. (b)

If the impulse response decays to zero as time approaches infinity, the system is stable.

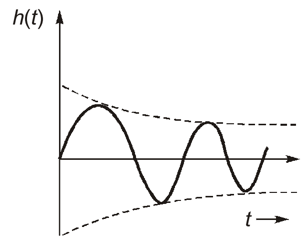
(1)



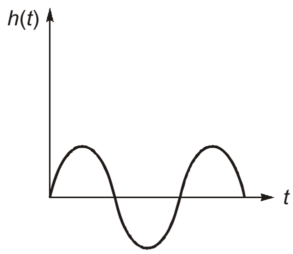
(2)



(3)



(4)



8. (a)

$$\frac{C(s)}{R(s)} = \frac{s+1}{s+2}$$

$$\therefore \quad C(s) = R(s)\cdot\frac{s+1}{s+2}$$

$$= \frac{1}{s^2}\cdot\frac{s+1}{s+2} = \frac{1}{s^2}\left(1-\frac{1}{s+2}\right)$$

$$= \frac{1}{s^2} - \frac{1}{s^2(s+2)}$$

$$\frac{1}{s^2}\cdot\left(\frac{s+1}{s+2}\right) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2}$$

$$s + 1 = As(s + 2) + B(s + 2) + Cs^2$$

$$= As^2 + 2As + Bs + 2B + Cs^2$$

$\therefore$ $A + C = 0$, $2A + B = 1$ and $2B = 1$

$$\therefore \quad A = \frac{1}{2}, \quad B = \frac{1}{4}, \quad C = -\frac{1}{4}$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{1}{4s} + \frac{1}{2s^2} + \left(-\frac{1}{4}\right)\frac{1}{s+2}$$

$$= \frac{1}{4}u(t) + \frac{1}{2}tu(t) - \frac{1}{4}e^{-2t}u(t)$$

9. (d)

(i) Transfer function can be obtained from signal flow graph of the system.

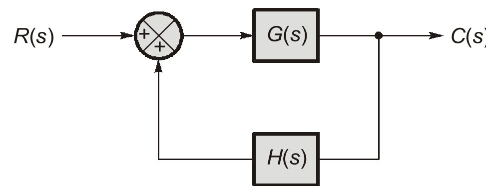
(ii) Transfer function typically characterizes to LTI systems.

(iii) Transfer function gives the ratio of output to input in s-domain of system.

$$\text{TF} = \left.\frac{L[\text{Output}]}{L[\text{Input}]}\right|_{\text{Initial conditions} = 0}$$

11. (d)

Block diagram of regenerating feedback system



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)}$$

12. (b)

Closed loop control system with negative feedback will have

- Less gain
- More accuracy
- More bandwidth hence more speed

14. (d)

Let $R(s)$ is the input Laplace transform of given differential equation is

$s^2Y(s) + 6sY(s) + 9Y(s) = R(s)$

$(s^2 + 6s + 9)\ Y(s) = R(s)$

$$\text{TF} = \frac{Y(s)}{R(s)} = \frac{1}{(s+3)^2}$$

$$IR = L^{-1}\left[\frac{1}{(s+3)^2}\right] = te^{-3t}u(t)$$

15. (c)

Given that,

$$r(t) = 1 - e^{-t},$$

$$c(t) = 1 - e^{-2t}$$

$$R(s) = \frac{1}{s} - \frac{1}{s+1} = \frac{1}{s(s+1)}$$

$$C(s) = \frac{1}{s} - \frac{1}{(s+2)} = \frac{2}{s(s+2)}$$

$$\text{TF} = \frac{C(s)}{R(s)} = \frac{2s(s+1)}{s(s+2)} = \frac{2(s+1)}{s+2}$$

■■■■