POSTAL
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ESE
Electronics Engineering
Conventional Practice Sets

Control Systems

Contents

MADE EASY
Publications

1
CHAPTER

Introduction

Q.1 (a) A control system is defined by following mathematical relationship

$$\frac{d^2x}{dt^2} + \frac{6dx}{dt} + 5x = 12(1 - e^{-2t})$$

Find the response of the system at $t \to \infty$

(b) A function $y(t)$ satisfies the following differential equation

$$\frac{dy(t)}{dt} + y(t) = \delta(t)$$

Where $\delta(t)$ is delta function. Assuming zero initial condition and denoting unit step function by $u(t)$. Find $y(t)$.

Solution:

(a) Taking *LT* on both sides

$$(s^2 + 6s + 5)\, X(s) = 12\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

$$(s + 1)(s + 5)\, X(s) = \frac{24}{s(s+2)}$$

$$X(s) = \frac{24}{s(s+1)(s+2)(s+5)}$$

Response at $t \to \infty$

Using final value theorem,

$$\boxed{\lim_{t \to \infty} x(t) = \lim_{s \to 0}[sX(s)]} = \lim_{s \to 0} \frac{s \times 24}{s(s+1)(s+2)(s+5)} = 2.4$$

(b) Taking Laplace transform on both sides

$$Y(s)[s + 1] = 1$$

$$Y(s) = \frac{1}{s+1}$$

By taking inverse Laplace transform

$$y(t) = e^{-t}u(t)$$

Q.2 The response $h(t)$ of a linear time invariant system to an impulse $\delta(t)$, under initially relaxed condition is $h(t) = e^{-t} + e^{-2t}$. Find the response of this system for a unit step input $u(t)$?

Solution:

Transfer function is given by

$$H(s) = L\{e^{-t} + e^{-2t}\} = \frac{1}{s+1} + \frac{1}{s+2}$$

$$H(s) = \frac{C(s)}{R(s)} = \frac{1}{s+1} + \frac{1}{s+2}$$

$$R(s) = \frac{1}{s} \text{ (step input)} \qquad\qquad [\because \ r(t) = u(t)]$$

$$C(s) = R(s) \cdot H(s) = \frac{1}{s}\left[\frac{1}{s+1} + \frac{1}{s+2}\right] = \frac{1}{s(s+1)} + \frac{1}{(s+2)(s)}$$

$$= \frac{1}{s} - \frac{1}{s+1} + \frac{1}{2}\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

$$= \frac{1.5}{s} - \frac{1}{s+1} - \frac{0.5}{s+2}$$

Response will be
$$c(t) = L^{-1}\{C(s)\}$$
$$c(t) = (1.5 - e^{-t} - 0.5e^{-2t})\, u(t)$$

Q.3 A system is represented by a relation given below:

$$X(s) = R(s) \cdot \frac{100}{s^2 + 2s + 50}$$

if $r(t) = 1.0$ unit, find the value of $x(t)$ when $t \to \infty$.

Solution:

Since, $\qquad\qquad r(t) = 1$

Taking Laplace transform,

$\therefore \qquad\qquad R(s) = \dfrac{1}{s}$

Applying final value theorem,

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} s\, X(s)$$

$$= \lim_{s\to 0} s \cdot \frac{1}{s} \cdot \frac{100}{s^2 + 2s + 50} = 2.0 \text{ units}$$

Q.4 (a) The Laplace equation for the charging current, $i(t)$ of a capacitor arranged in series with a resistance is given by

$$I(s) = \frac{sC}{1 + sRC} \cdot E(s)$$

The circuit is connected to a supply voltage of E. If $E = 100$ V, $R = 2$ MΩ, $C = 1$ μF. Calculate the initial value of the charging current.

(b) A series circuit consisting of resistance R and an inductance of L is connected to a d.c. supply voltage of E. Derive an expression for the steady-state value of the current flowing in the circuit using final value theorem.

Solution:

(a) Since, $\qquad\qquad E = 100\, v(t)$

Taking Laplace Transform, $\qquad E = 100\,(t)$ volts,

$\therefore \qquad\qquad E(s) = \dfrac{100}{s}$

Substituting the given values,

$$I(s) = \frac{1\times 10^{-6}s}{(2\times 10^6 \times 1\times 10^{-6}\, s + 1)} \cdot \frac{100}{s} = \frac{10^{-6}\, s}{2s+1} \cdot \frac{100}{s}$$

Applying the initial value theorem,

$$\boxed{i(0^+) = \lim_{t \to 0} i(t) = \lim_{s \to \infty} s\,I(s)}$$

$$i(0^+) = \lim_{s \to \infty} s \cdot \frac{10^{-4}}{1+2s} = \lim_{s \to \infty} \cdot \frac{10^{-4}}{\frac{1}{s}+2} = 50\ \mu A$$

(b) The differential equation relating the current $i(t)$ flowing in the circuit and the input voltage E is given by

$$E = R\,i(t) + L\frac{di(t)}{dt}$$

Taking Laplace transform of the equation yields,

$$E(s) = R\,I(s) + L[(sI(s) - i(0^+))]$$

Assume, $\qquad\qquad i(0^+) = 0$

$$\therefore \qquad\qquad E(s) = R\,I(s) + Ls\,I(s)$$
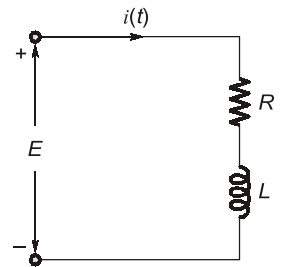
$\because\ E$ is constant (d.c. voltage)

$$E(s) = \frac{E}{s} = RI(s) + Ls\,I(s)$$

$$I(s) = \frac{E}{s(R+sL)}$$

Applying the final value theorem,

$$i_{ss} = \lim_{t \to \infty} i(t) = \lim_{s \to 0} s\,I(s) = \lim_{s \to 0} \frac{sE}{s(R+sL)}$$

$$i_{ss} = \frac{E}{R}$$

Q.5 Determine the mechanical time constant of rotor of an electrical machine in terms of its moment of inertia J kg-m^2 and windage cum friction coefficient f N-m/rad/s. Also explain the method to determine mechanical time constant experimentally in laboratory.

Solution:

Consider a field controlled separately excited DC motor.

Constant armature in field into the motor,

$$\phi_f \propto I_f$$
$$\phi_f = k_f I_f$$
$$T_m \propto \phi_f I_a$$
$$T_m = K\phi_f I_a$$
$$T_m = K'k_f I_f I_a$$
$$T_m = k_m k_f I_f$$

where, $\quad k_m = K'I_a =$ constant
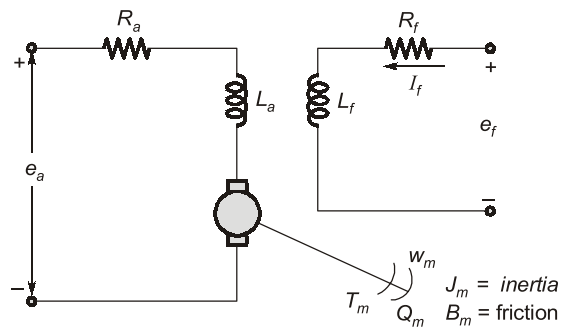
$$e_f = L_f \frac{di_f}{dt} + R_f I_f$$

$$T_m = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt}$$
$$T_m(s) = J_m s^2 \theta_m(s) + B_m s\,\theta_m(s)$$
$$T_m(s) = (J_m s^2 + B_m s)\theta_m(s)$$
$$E_f(s) = (sL_f + R_f)\,I_f(s)$$
$$= (sL_f + R_f)\frac{T_m(s)}{k_f k_m}$$

$$E_f(s) = \frac{(sL_f + R_f)(J_m s^2 + B_m s)\,\theta_m(s)}{k_f k_m}$$

$$\frac{\theta_m(s)}{E_f(s)} = \frac{k_m k_f}{s(sL_f + R_f)(J_m s + B_m)} = \frac{k_m k_f}{B_m R_f s\left(1 + \frac{J_m}{B_m}s\right)\left(1 + \frac{sL_f}{R_f}\right)}$$

$$\frac{\theta_m(s)}{E_f(s)} = \frac{k_m k_f}{s B_m R_f(1 + \tau_m s)(1 + \tau_f s)}$$

$$\tau_m = \text{motor time constant} = J_m/B_m$$
$$\tau_f = \text{field time constant} = L_f/R_f$$

Q.6 The impulse response of a system S_1 is given by $y_1(t) = 4e^{-2t}$. The step response of a system S_2 is given by $y_2(t) = 2(1 - e^{-3t})$. The two systems are cascaded together without any interaction. Find response of the cascaded system for unit ramp input.

Solution:

(a) Taking the Laplace transform of the response of S_1, we get

$$Y_1(s) = \frac{4}{s+2},$$

$$X_1(s) = 1 \ ... \ (x(t) = \delta(t))$$

Therefore, $\quad G_1(s) = \frac{Y_1(s)}{X_1(s)} = \frac{4}{s+2}$ $\qquad [\because Y_1(s) = 1]$

Taking the Laplace transform of the response of S_2, we get

$$Y_2(s) = 2\left(\frac{1}{s} - \frac{1}{s+3}\right) = \frac{6}{s(s+3)}$$

$$Y_2(s) = \frac{1}{s} \ ... \ (x_2(t) = u(t))$$

Thus, $\quad G_2(s) = \frac{Y_2(s)}{X_2(s)} = \frac{6}{s(s+3)} \cdot s = \frac{6}{s+3}$

(b) The transfer function of the cascaded system is

$$G(s) = G_1(s)G_2(s) = \frac{24}{(s+2)(s+3)}$$

The Laplace transform of unit ramp is $R(s) = \frac{1}{s^2}$. Therefore,

$$G(s) = \frac{C(s)}{R(s)}$$

$$C(s) = \frac{24}{(s+2)(s+3)} \cdot \frac{1}{s^2}$$

$$\equiv \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+2} + \frac{D}{s+3}$$

$$A = \frac{24}{(s+2)(s+3)}\bigg|_{s=0} = 4$$