POSTAL Book Package 2023

ESE

Electronics Engineering

Conventional Practice Sets

Digital Circuits

Contents

MADE EASY Publications

1 CHAPTER
Number Systems and Codes

Q.1 (i) Convert octal 756 to decimal.
(ii) Convert hexadecimal 3B2 to decimal.
(iii) Convert the long binary number 1001001101010001 to octal and to hexadecimal.

Solution:

(i) $(756)_8$

$$= 7 \times 8^2 + 5 \times 8^1 + 6 \times 8^0$$
$$= 448 + 40 + 6$$
$$= (494)_{10}$$

(ii) $(3B2)_{16}$

$$= 3 \times 16^2 + 11 \times 16^1 + 2 \times 16^0 \quad \text{(put B = 11)}$$
$$= 768 + 176 + 2$$
$$= (946)_{10}$$

(iii) $\underbrace{0\,0\,1}_{1}\ \underbrace{0\,0\,1}_{1}\ \underbrace{0\,0\,1}_{1}\ \underbrace{1\,0\,1}_{5}\ \underbrace{0\,1\,0}_{2}\ \underbrace{0\,0\,1}_{1}$

$$= (111521)_8$$

and $\underbrace{1\,0\,0\,1}_{9}\ \underbrace{0\,0\,1\,1}_{3}\ \underbrace{0\,1\,0\,1}_{5}\ \underbrace{0\,0\,0\,1}_{1}$

$$= (9351)_{16}$$

Q.2 Show the value of all bits of a 12-bit register that holds the number equivalent to decimal 215 in
(a) binary (b) binary coded octal (c) binary coded hexadecimal and (d) binary coded decimal.

Solution:

(a) Binary

$$(215)_{10} = (11010111)_2$$

2	215	
2	107	1
2	53	1
2	26	1
2	13	0
2	6	1
2	3	0
	1	1

In a 12-bit register, it will be stored as: "0 0 0 0 1 1 0 1 0 1 1 1"

(b) Binary Coded Octal

$$(215)_{10} = (0327)_8 = 000\ \ 011\ \ 010\ \ 111$$

(c) Binary Coded Hexadecimal

$$(215)_{10} = (0D7)_{16} = 0000\ \ 1101\ \ 0111$$

(d) Binary Coded Decimal

In binary coded decimal, each decimal (0 to 9) digit is represented by 4-bit binary code.

$$(215)_{10} = 0010\ \ 0001\ \ 0101$$

Q.3 Consider the addition of numbers with different bases
$(x)_7 + (y)_8 + (w)_{10} + (z)_5 = (k)_9$
If $x = 36$, $y = 67$, $w = 98$ and $k = 241$, then z is

Solution:

$$(36)_7 = (27)_{10}$$
$$(67)_8 = (55)_{10}$$
$$(98)_{10} = (98)_{10}$$
$$(z)_5 = (z)_5$$
$$(241)_9 = (199)_{10}$$
$$(z)_5 = (199)_{10} - (27)_{10} - (55)_{10} - (98)_{10}$$

5 | 19 | 4
  | 3

$$(z)_5 = (19)_{10}$$
$$(z)_5 = (34)_5$$
$$\therefore \quad z = 34$$

Q.4 (a) Convert the below binary code into Gray code.

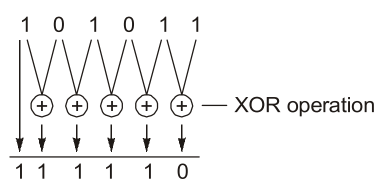
(i) 101011 (ii) 110110

(b) Convert below gray code into binary

110011

Solution:

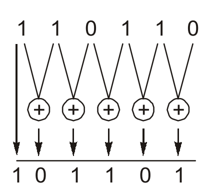
(a) (i) Binary code:



Write the most significant as it is then obtain the remaining bits by doing XOR operation between adjacent bits.

Hence Gray code will be 111110.

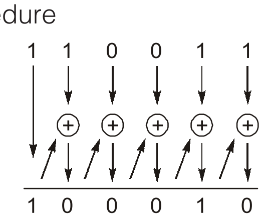
(ii) 110110



Hence Gray code will be 101101

(b) Given Gray code is 110011.

Now to convert it into binary, write most significant bit as it is and then XOR this bit to the next most significant bit and repeat the procedure



Hence result binary code will be $(100010)_2$.

Q.5 (a) Represent the 8620 into following codes:

(i) BCD (ii) Excess-3 (iii) 2421

(b) Find 7's complement of the given number $(2365)_7$

Solution:

(a) (i) Write binary equivalent of each decimal

8620 ⇒ 1000 0110 0010 0000

(ii) **Excess-3:** For excess 3, add 3 (binary 0011) to each BCD part.

Hence,

1000	0110	0010	0000
+0011	+0011	+0011	+0011
1011	1001	0101	0011

(iii) 2421: It is a weighted binary code

Decimal digit	2	4	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	0	1
8	1	1	1	0
9	1	1	1	1

There codes are minor image from the given dotted line.

As $(4)_{10}$ and $(5)_{10}$ make complementary pair.

Similarly $(3)_{10}$ and $(6)_{10}$ .... make the complementary pair.

Hence, 1110 1100 0010 0000.

(b) For a value/number having a base of r, then r's complement = (r – 1)'s complement + 1

Hence, 7's complement of $(2365)_7$ = 6's complement + 1

$$\begin{array}{rl} 6\,6\,6\,6 & \\ -\,2\,3\,6\,5 & \\ \hline 4\,3\,0\,1 & \text{6's complement} \\ +\,1 & \\ \hline 4\,3\,0\,2 & \text{7's complement} \end{array}$$

Q.6 **Perform the following conversions:**

(i) $(3287.5100098)_{10}$ into octal (ii) $(675.625)_{10}$ into hexadecimal (iii) $(A72E)_{16}$ into octal

Solution:

(i) **To convert $(3287.5100098)_{10}$ into octal:**

- Integer part conversion,

$$\begin{array}{r|l} 8 & 3287 \\ \hline 8 & 410 - 7 \\ \hline 8 & 51 - 2 \\ \hline 8 & 6 - 3 \\ \hline & 0 - 6 \end{array}$$

$$(3287)_{10} = (6327)_8$$

- Fractional part conversion,

$$0.5100098 \times 8 = 4.0800784 \rightarrow 4$$
$$0.0800784 \times 8 = 0.6406272 \rightarrow 0$$
$$0.6406272 \times 8 = 5.1250176 \rightarrow 5$$
$$0.1250176 \times 8 = 1.0001408 \rightarrow 1$$
$$(0.5100098)_{10} = (0.4051...)_8$$

So, $(3287.5100098)_{10} = (6327.4051...)_8$

(ii) To convert $(675.625)_{10}$ into hexadecimal:

- Integer part conversion,

16	675	
16	42 – 3	↑
16	2 – A	
	0 – 2	

$$(675)_{10} = (2A3)_{16}$$

- Fractional part conversion,

$$0.625 \times 16 = 10.000 \rightarrow A$$

$$(0.625)_{10} = (0.A)_{16}$$

So, $$(675.625)_{10} = (2A3.A)_{16}$$

(iii) To convert $(A72E)_{16}$ into octal:

- Hexadecimal to binary conversion,

$$(A72E)_{16} = (1010\ 0111\ 0010\ 1110)_2$$

- Binary to octal conversion,

$$(1010\ 0111\ 0010\ 1110)_2 = (001\ 010\ 011\ 100\ 101\ 110)_2 = (123456)_8$$

So, $$(A72E)_{16} = (123456)_8$$

Q.7 If $X = 111.101$ and $Y = 101.110$ calculate $X + Y$ and $\left.\begin{matrix} X - Y \\ Y - X \end{matrix}\right\}$ by 2's complement method.

Solution:

Given $$X = 111.101$$ $$Y = 101.110$$

Now
$$X + Y = \begin{array}{r} 111.101 \\ 101.110 \\ \hline 1101.001 \\ \hline \end{array}$$

For
$$X - Y = X + \text{2's complement of } Y$$
$$= 111.101 + 010.010$$

Discard the carry $= \overset{\curvearrowleft}{}\ (1)\,001.111$ as number will be positive

$$= 001.111$$
$$= 1.111$$

For
$$Y - X = Y + \text{2's complement of } X$$
$$= 101.110 + 000.011$$
$$= 110.001$$

∵ There is no carry generated its a negative number.

∴ Difference = – (2's complement of 110.001) = – 1.111

Q.8 Perform the following addition and subtraction of excess-3 numbers:

(i) 0100 1000 + 0101 1000 (ii) 1100 1011 – 0100 1001

Check the results obtained, by performing the above operations in decimal format.

Solution:

(i) 0100 1000 + 0101 1000 in excess-3 format:

	0100	1000	
(+)	0101	1000	
	1001 ①	0000	There is a carry from lower nibble, which is to be propagated
	1 ↵		
	1010	0000	Add "0011" to lower nibble
(–)	0011	(+) 0011	Subtract "0011" from higher nibble
	0111	0011	← Final result in excess-3 format

Checking the above result in decimal format:

$$0100\ 1000 \xrightarrow{\text{To 8421 BCD}} 00010101 \xrightarrow{\text{To decimal}} (15)_{10}$$

$$01011000 \xrightarrow{\text{To 8421 BCD}} 0010\ 0101 \xrightarrow{\text{To decimal}} (25)_{10}$$

$$(15)_{10} + (25)_{10} = (40)_{10}$$

$$(40)_{10} \xrightarrow{\text{To 8421 BCD}} 0100\ 0000 \xrightarrow{\text{To excess-3}} 01110011$$

(ii) 1100 1011 – 0100 1001 in excess-3 format:

	1100	1011	
(–)	0100	1001	
	1000	0010	Add "0011" to both the nibbles
(+)	0011	(+) 0011	
	1011	0101	← Final result in excess-3 format

Checking the above result in decimal format:

$$1100\ 1011 \xrightarrow{\text{To 8421 BCD}} 10011000 \xrightarrow{\text{To decimal}} (98)_{10}$$

$$0100\ 1001 \xrightarrow{\text{To 8421 BCD}} 00010110 \xrightarrow{\text{To decimal}} (16)_{10}$$

$$(98)_{10} - (16)_{10} = (82)_{10}$$

$$(82)_{10} \xrightarrow{\text{To 8421 BCD}} 1000\ 0010 \xrightarrow{\text{To excess-3}} 10110101$$

Q.9 (i) Each of the following arithmetic operations is correct in atleast one number system. Calculate the minimum non-zero base for which the following operations are true.

1. $\frac{54}{4} = 13$ 2. $\sqrt{41} = 5$ 3. $\frac{302}{20} = 12.1$ 4. $3 \times 11 = 33$

(ii) Calculate the minimum non-zero base of x which satisfies the quadratic equation $x^2 - 11x + 22 = 0$, whose roots are $x = 3$ and $x = 6$.

Solution:

(i) 1. Let the base of the expression be 'x'.

thus, $$\frac{(54)_x}{(4)_x} = (13)_x$$

$$\Rightarrow \quad \frac{5x + 4}{4} = x + 3$$

$$\Rightarrow \quad 5x + 4 = 4x + 12$$

$$x = 8$$

Hence, the minimum non-zero base is equal to '8'.

2. Let the base of the expression be equal to 'x'.

$$\sqrt{(41)_x} = (5)_x$$

$$\sqrt{4x + 1} = 5$$

$$4x + 1 = 25$$
$$x = 6$$

Hence, the minimum non-zero base for the expression is equal to 6.

3. Let the base of the expression be equal to x.

$$\frac{(302)_x}{(20)_x} = (12.1)_x$$
$$\frac{3x^2+2}{2x} = x+2+\frac{1}{x}$$
$$\frac{3x^2+2}{2} = x^2 + 2x + 1$$
$$x^2 - 4x = 0$$
$$x = 0, 4$$

Thus, the non-zero base is equal to 4.

4. Let the base of the expression be equal to 'x'.

$$(3)_x \times (11)_x = (33)_x$$
$$3(x + 1) = 3x + 3$$
$$3x + 3 = 3x + 3$$

Thus, the equation is valid for any value of x. So the minimum non-zero base will be equal to $x = 4$.

(ii) The given quadratic equation,

$$x^2 - 11x + 22 = 0 \quad \text{...(i)}$$

The factor of solution are 3 and 6.

Thus, the equation can also be written as

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$
$$x^2 - [(3)_b + (6)_b]x + (3)_b \times (6)_b = 0 \quad \text{...(ii)}$$

Equating equation (i) and (ii), we get,

$$(3)_b + (6)_b = (11)_b$$

and $$(3)_b \times (6)_b = (22)_b$$
$$3 + 6 = b + 1$$

$\therefore$ $$b = 8$$

$\therefore$ The minimum non-zero base = 8.

Q.10 What are error-correcting codes? For Hamming code write an expression for describing the location of possible error. Also, find out the value of 'K' for converting BCD code into Hamming code and the bit positions of the resulting Hamming code?

Solution:

- When the digital data is transmitted through the channel, the noise signal changes the original data and false output is obtained at the receiver. To correct this data, it is required to detect the error which is done by error detecting codes and the errors are corrected using error correction codes such as hamming code.
- Hamming codes are a family of linear error-correcting codes. Hamming codes can detect upto two-bit errors or correct one-bit error with detection of uncorrected errors.
- Hamming code is constructed by adding number of parity bits to each group of n-bit information or message in such a way so as to able to locate the bit position in which error occurs. Let us assume that k-parity bits p_1, p_2, p_3 ...., p_k are added to the n-bit message to form an $(n + k)$-bit code. The value of 'k' must be chosen in such a way so as to be able to describe the location of any of the $(n + k)$ possible error bit locations and also "no error" condition consequently, k must satisfy the inequality.

$$\boxed{2^k \geq n + k + 1} \quad \text{...(i)}$$