

POSTAL Book Package

2023

GATE • ESE

Electronics Engineering

Objective Practice Sets

Electromagnetics

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Vector Analysis

MCQ and NAT Questions

Q.1 For a given vector field $\vec{A} = 5x^2 \left(\sin \frac{\pi x}{2} \right) \hat{a}_x$. The divergence $\vec{\nabla} \cdot \vec{A}$ at $x = 2$ is _____.

Q.2 A field $\vec{A} = 3x^2yz\hat{a}_x + x^3z\hat{a}_y + (x^3y - 2z)\hat{a}_z$ can be termed as
 (a) Irrotational (b) Divergence less
 (c) Solenoidal (d) Rotational

Q.3 The angle θ_{AB} between the vectors $A = 3a_x + 4a_y + a_z$ and $B = 2a_y - 5a_z$ is nearly
 (a) 83.7° (b) 73.7°
 (c) 63.7° (d) 53.7°

Q.4 The total length of the curve $\mu = \cos^2\theta$ (Cylindrical co-ordinates) from $\theta = 0$ to $\theta = \pi$ is
 (a) 0.5π (b) 1.0
 (c) 2.0 (d) π

Q.5 Laplacian of a scalar function V is
 (a) Gradient of V
 (b) Divergence of V
 (c) Gradient of the gradient of V
 (d) Divergence of the gradient of V

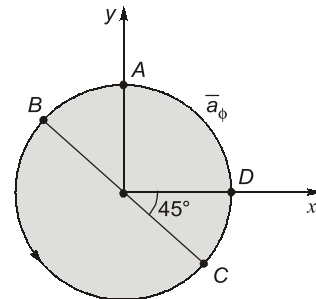
Q.6 For a solenoidal vector field $\vec{F} = (x + 3y)\hat{a}_x + (5y + 2z)\hat{a}_y + (x - Qz)\hat{a}_z$, the value of Q must be _____.

Q.7 Divergence of a vector D in the cylindrical coordinate system is

- (a) $\frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$
 (b) $\frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho} \frac{\partial(\phi D_\phi)}{\partial \phi} + \frac{1}{z} \frac{\partial(ZD_z)}{\partial z}$
 (c) $\frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$
 (d) $\frac{\partial D_\rho}{\partial \rho} + \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$

Q.8 Given a vector \vec{A} in spherical coordinates as $\vec{A} = 5 \sin \theta a_\theta + 5 \sin \phi a_\phi$. The divergence of \vec{A} i.e. $\nabla \cdot \vec{A}$ at $\left(r = 1, \theta = \frac{\pi}{2}, \phi = \frac{\pi}{3} \right)$ is _____.

Q.9 Consider points A, B, C and D on a circle of radius 2 units as in the below figure. The items in List-II are the values of \hat{a}_ϕ at different points on the circle. Match List-I with List-II and select the correct answer using the codes given below the lists:



- | List-I | List-II |
|--------|--|
| A. | 1. \hat{a}_x |
| B. | 2. \hat{a}_y |
| C. | 3. $-\hat{a}_x$ |
| D. | 4. $(\hat{a}_x + \hat{a}_y)/\sqrt{2}$ |
| | 5. $-(\hat{a}_x + \hat{a}_y)/\sqrt{2}$ |
| | 6. $(\hat{a}_x - \hat{a}_y)/\sqrt{2}$ |

Codes:

- | | A | B | C | D |
|-----|---|---|---|---|
| (a) | 3 | 4 | 5 | 2 |
| (b) | 1 | 6 | 5 | 2 |
| (c) | 1 | 6 | 2 | 4 |
| (d) | 3 | 5 | 4 | 2 |

Q.10 The unit vector extending from origin toward the point $G(2, -2, -1)$ is

- (a) $\frac{2}{3}\hat{a}_x + \frac{2}{3}\hat{a}_y + \frac{1}{3}\hat{a}_z$ (b) $-\frac{2}{3}\hat{a}_x + \frac{2}{3}\hat{a}_y + \frac{1}{3}\hat{a}_z$
(c) $\frac{2}{3}\hat{a}_x - \frac{2}{3}\hat{a}_y - \frac{1}{3}\hat{a}_z$ (d) $-\frac{2}{3}\hat{a}_x - \frac{2}{3}\hat{a}_y - \frac{1}{3}\hat{a}_z$

Q.11 For vector field $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$,

1. $\nabla \cdot (\nabla \times \vec{r}) = 1$
2. $\nabla \times \vec{r} = 0$
3. $\nabla \cdot \vec{r} \neq 0$
4. $\nabla(\vec{r} \cdot \vec{r}) = \vec{r}$

Which of the above relations are true?

- (a) 1 and 3 (b) 1 and 4
(c) 2, 3 and 4 (d) 2 and 3

Q.12 If $A = -\nabla f = (x+z)\hat{a}_x - 3z\hat{a}_y + (x-3y-z)\hat{a}_z$.

Then the scalar field, f is

- (a) $\frac{x^2}{2} + xz + \frac{z^2}{2}$
(b) $-\frac{x^2}{2} - 2xz + 6yz + \frac{z^2}{2}$
(c) $-xz + 3yz + \frac{z^2}{2}$
(d) $-\frac{x^2}{2} - xz + 3yz + \frac{z^2}{2}$

Q.13 If $V = \sinh x \cdot \cos ky \cdot e^{pz}$ is a solution of Laplace's equation, what will be the value of k ?

- (a) $\frac{1}{\sqrt{1+p^2}}$ (b) $\sqrt{1+p^2}$
(c) $\frac{1}{\sqrt{1-p^2}}$ (d) $\sqrt{1-p^2}$

Q.14 Laplace equation in cylindrical coordinates is given by

- (a) $\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \left(\frac{\partial^2 V}{\partial \phi^2} \right) + \frac{\partial^2 V}{\partial z^2} = 0$
(b) $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$
(c) $\nabla^2 V = \frac{-\rho}{\epsilon}$
(d) $\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \left(-\frac{1}{r^2 \sin \theta} \right) \frac{\partial}{\partial \theta}$

$$\left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} = 0$$

Q.15 The vector R_{AB} extends from $A(1, 2, 3)$ to B . If the length of R_{AB} is 10 units and its direction is given by

$$a = 0.6\hat{a}_x + 0.64\hat{a}_y + 0.48\hat{a}_z$$

the coordinates of B will be

- (a) $7\hat{a}_x + 4.8\hat{a}_y + 4.8\hat{a}_z$
(b) $6\hat{a}_x + 6.4\hat{a}_y + 4.8\hat{a}_z$
(c) $7\hat{a}_x + 8.4\hat{a}_y + 7.8\hat{a}_z$
(d) $6\hat{a}_x + 8.4\hat{a}_y + 7.8\hat{a}_z$

Q.16 Given $\vec{V} = x \cos^2 y \hat{i} + x^2 e^{z^2} \hat{j} + z \sin^2 y \hat{k}$ and S

the surface of unit cube with one corner at the origin and edges parallel to the coordinate axis,

the value of the integral $\iint_C \vec{V} \cdot \hat{n} dS$ is _____.

Q.17 Consider the following statements:

Stokes' theorem is valid irrespective of

1. shape of closed curve C
2. type of vector A
3. type of coordinate system
4. whether the surface is closed or open

Which of the above statements are correct?

- (a) 1, 2 and 4 (b) 1, 3 and 4
(c) 2, 3 and 4 (d) 1, 2 and 3

Q.18 Consider the vector field $\vec{A} = y\hat{a}_x + x\hat{a}_y$. The

scalar line integral of this vector along the parabola $x = 2y^2$ from point $(2, 1, -1)$ to $(4, 2, -1)$

is

- (a) 7 (b) 14
(c) 5 (d) 28

Multiple Select Questions (MSQs)

Q.19 $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ represents a position vector and

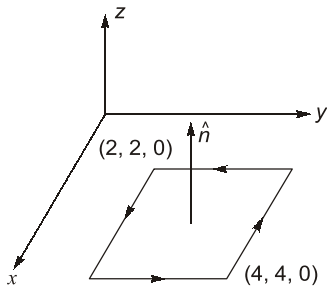
$\|\vec{r}\|$ represents the normal of vector \vec{r} , then which of the below statements is/are true?

- (a) Divergence of \vec{r} is 3.
(b) Gradient of $\|\vec{r}\|^2$ is $3\vec{r}$
(c) Curl of \vec{r} is 0
(d) Laplacian of $\|\vec{r}\|^2$ is 6.

Q.20 If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = \sqrt{x^2 + y^2 + z^2}$, then which of the below relations are correct?

- (a) $\nabla(\log r) = \frac{\vec{r}}{r}$ (b) $\nabla\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$
 (c) $\nabla \cdot \left(\frac{\vec{r}}{r^3}\right) = 1$ (d) $\nabla \cdot (3\vec{r}) = 9$

Q.21 Let $\vec{F} = xy^2\hat{a}_x + y^3\hat{a}_y + x^2y\hat{a}_z$ and the surface S consists of a square of length 2 lying in the xy plane as shown below:



Which of the following options is/are correct?

- (a) $\iint_S \vec{F} \cdot \hat{n} ds = 80$
 (b) $\iint_S (\vec{F} \times \hat{n}) ds = 120\hat{a}_x - 112\hat{a}_y$
 (c) $\nabla \times \vec{F} = x^2\hat{a}_x - 2xy\hat{a}_y - 2xy\hat{a}_z$
 (d) $\iint_S (\nabla \times \vec{F}) \cdot \hat{n} ds = -120$

Q.22 If $[\vec{a}, \vec{b}, \vec{c}]$ represents the scalar triple product of vectors \vec{a}, \vec{b} and \vec{c} , then which of the below statements is/are true?

- (a) $[\vec{a}, \vec{b}, \vec{c}] = [\vec{c}, \vec{b}, \vec{a}]$
 (b) $[\vec{a}, \vec{b} + \vec{a}, \vec{c}] = 0$
 (c) $[3\vec{b}, \vec{c}, \vec{a}] = 3[\vec{a}, \vec{b}, \vec{c}]$
 (d) If $[\vec{a}, \vec{b}, \vec{c}] = 0$, the vectors \vec{a}, \vec{b} and \vec{c} are coplanar.

Q.23 The values of α for which the vectors $\vec{A} = \alpha\hat{a}_x + 2\hat{a}_y + 10\hat{a}_z$ and $\vec{B} = 4\alpha\hat{a}_x + 8\hat{a}_y - 2\alpha\hat{a}_z$ are perpendicular is/are

- (a) 1 (b) 2
 (c) 3 (d) 4

Q.24 Which of the below vector identities are true?

- (a) $A \times (B \times C) = (A \times B) \times C$
 (b) $A \times (B \times C) + C \times (A \times B) + B \times (C \times A) = 0$
 (c) $(B \times C) \times (C \times A) = C(A \cdot B \times C)$
 (d) $(A \times B) \cdot (C \times D) = (A \cdot C)(B \cdot D) - (A \cdot D)(B \cdot C)$

Q.25 For the scalar function, $\phi = x^2yz^3$, which of the below statements is/are correct?

- (a) From the point $(2, 1, -1)$ the directional derivative of ϕ is maximum in the direction represented by vector $-12\hat{i} - 4\hat{j} + 4\hat{k}$.
 (b) The magnitude of greatest rate of change of ϕ from the point $(2, 1, -1)$ is $4\sqrt{11}$.
 (c) $(x-2) + (y-1) - 3(z+1) = 0$ represents the tangent plane to the surface $\phi = 0$ at point $(2, 1, -1)$.
 (d) ϕ satisfies the Laplacian equation.

■■■■

Answers Vector Analysis

1. (-31.42) 2. (a) 3. (a) 4. (a) 5. (d) 6. (6) 7. (c)
 8. (2.5) 9. (d) 10. (c) 11. (d) 12. (d) 13. (b) 14. (a)
 15. (c) 16. (1) 17. (d) 18. (b) 19. (a, c, d) 20. (b, d) 21. (b, c)
 22. (c, d) 23. (a, d) 24. (b, c, d) 25. (b, c)

Explanations Vector Analysis

1. (-31.42)

$$\begin{aligned} \text{Div } \vec{A} &= \frac{\partial}{\partial x} \left[5x^2 \left(\sin \frac{\pi x}{2} \right) \right] \\ &= 5x^2 \left(\cos \frac{\pi x}{2} \right) \cdot \frac{\pi}{2} + 10x \sin \left(\frac{\pi x}{2} \right) \\ \text{Div } \vec{A} \Big|_{x=2} &= \frac{\pi}{2} \times 5(2)^2 \cos \pi + 10(2) \sin(\pi) \\ &= -5 \times 4 \times \frac{\pi}{2} = -10\pi = -31.42 \end{aligned}$$

2. (a)

$$\begin{aligned} \vec{A} &= 3x^2yz\hat{a}_x + x^3z\hat{a}_y + (x^3y - 2z)\hat{a}_z \\ \nabla \cdot \vec{A} &= \frac{\partial}{\partial x}(3x^2yz) + \frac{\partial}{\partial y}(x^3z) + \frac{\partial}{\partial z}(x^3y - 2z) \\ &= 6xyz - 2 \\ \nabla \cdot \vec{A} \neq 0 &\Rightarrow \vec{A} \text{ is not a solenoidal.} \\ \nabla \times \vec{A} &= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2yz & x^3z & x^3y - 2z \end{vmatrix} \\ &= (x^3 - x^3)\hat{a}_x - (3x^2y - 3x^2y)\hat{a}_y + (3x^2z - 3x^2z)\hat{a}_z \\ &= 0 \\ \Rightarrow \vec{A} &\text{ is irrotational.} \end{aligned}$$

3. (a)

$$\begin{aligned} \vec{A} \cdot \vec{B} &= |\vec{A}| |\vec{B}| \cos \theta_{AB} \\ |\vec{A}| &= \sqrt{3^2 + 4^2 + 1^2} = \sqrt{26} \\ |\vec{B}| &= \sqrt{2^2 + 5^2} = \sqrt{29} \\ \vec{A} \cdot \vec{B} &= (4 \times 2) - (1 \times 5) = 3 \\ \cos \theta_{AB} &= \frac{3}{\sqrt{26 \times 29}} \\ \theta_{AB} &= \cos^{-1} \left[\frac{3}{\sqrt{26 \times 29}} \right] = 83.7^\circ \end{aligned}$$

4. (a)

$$\int_{\theta=0}^{\pi} \cos^2 \theta d\theta = \int_{\theta=0}^{\pi} \left(\frac{1 + \cos 2\theta}{2} \right) d\theta = \frac{\pi}{2}$$

5. (d)

$$\begin{aligned} \nabla^2 V &= \nabla \cdot (\nabla V) \\ &= \text{divergence of gradient of } V \end{aligned}$$

6. (6)

The vector will be solenoidal if its divergence is zero

$$\begin{aligned} \nabla \cdot \vec{F} &= 0 \\ \Rightarrow \frac{\partial}{\partial x}(x + 3y) + \frac{\partial}{\partial y}(5y + 2z) + \frac{\partial}{\partial z}(x - Qz) &= 0 \\ \Rightarrow 1 + 5 - Q &= 0 \\ \Rightarrow Q &= 6 \end{aligned}$$

7. (c)

Divergence of vector \vec{D} in different coordinates system is given by:

Cartesian coordinates:

$$\nabla \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

Cylindrical coordinates:

$$\nabla \cdot \vec{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho D_\rho) + \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

Spherical coordinates:

$$\nabla \cdot \vec{D} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (D_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial D_\phi}{\partial \phi}$$

8. (2.5)

In spherical coordinates,

$$\begin{aligned} \nabla \cdot \vec{A} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) \\ &\quad + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi} \\ \nabla \cdot \vec{A} &= \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (5 \sin^2 \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (5 \sin \phi) \\ \nabla \cdot \vec{A} &= \frac{10 \cos \theta}{r} + \frac{5 \cos \phi}{r \sin \theta} \\ &= \frac{5}{1} \times \frac{1}{2} \times \frac{1}{1} = 2.5 \end{aligned}$$

9. (d)

