

# Electrical Engineering

## Electrical Machines

Comprehensive Theory

*with* Solved Examples and Practice Questions



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### **Electrical Machines**

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# Transformer

## Introduction

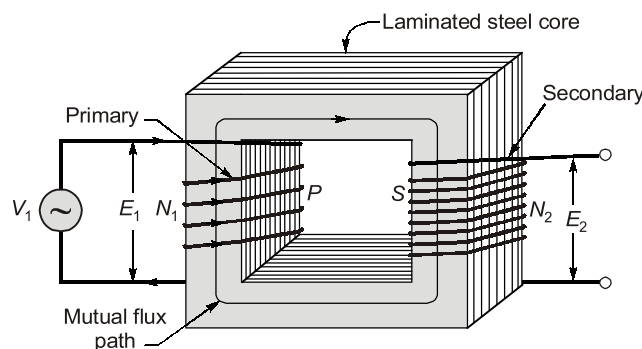
The transformer is a static device for transferring electrical energy from one alternating current circuit to another without a change in frequency. A transformer may receive energy at one voltage and deliver it at a higher voltage, in which case it is called a **step-up** transformer. When the energy is received at a higher voltage and delivered at a lower voltage, it is called a **step-down** transformer.

A transformer has no rotating parts; therefore, it requires little attention and its maintenance cost is low; its efficiency is much higher when compared to other electrical machines.

## 2.1 Operating Principle

The transformer is based on the principle that energy may be efficiently transferred by induction from one set of coils to another by means of a varying magnetic flux, provided that both sets of coils are on a combined magnetic circuit. Electromotive forces are induced by change in flux linkages.

In the transformer, coils and magnetic circuit are all stationary with respect to one another. The emfs are induced by the change in the magnitude of the flux with time. This is called 'transformer emf'. This is illustrated in Figure (2.1).



**Figure-2.1 :** A simple transformer, with secondary open circuited

Referring to Figure (2.1), an a.c. voltage is applied to the primary winding 'P'. As this winding is linked with an iron core, its m.m.f. produces an alternating flux  $\phi$  in the core. This alternating flux links the turns of the winding 'S'. As this flux is alternating, it induces in the winding 'S' an e.m.f. of the same frequency as the flux. Because of this induced e.m.f. the secondary winding 'S' is capable of delivering current and energy. The energy therefore, is transferred from 'P', the primary, to 'S', the secondary, by means of magnetic flux. In a transformer, either winding may be the primary, the other being the secondary, depending upon which winding receives and which delivers energy.

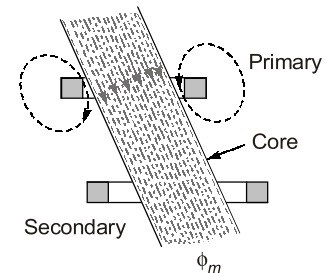
**NOTE**

In the generator, flux is substantially constant in magnitude. The flux linking the armature coils is changed by the relative mechanical motion of flux and coils. This is called 'motional emf'.

## 2.2 Primary and Secondary

The more closely the primary and secondary circuits are mutually linked, the more direct becomes the exchange of energy between them. If the two circuits link a common iron core Figure 2.2, the effects are:

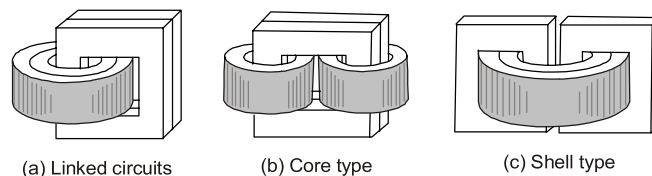
- A great increase in the total flux by virtue of the improved permeance of the magnetic circuit.
- A smaller magnetizing current (i.e. primary current with secondary open-circuited), since the increased flux per ampere induces more primary e.m.f.
- A much greater proportion of mutual to non-mutual or leakage flux : the latter has air paths whereas the former occupies the permeable iron core.
- The introduction of losses in the core, so that the field can no longer be established without loss.



**Figure-2.2 :** Increase of mutual inductance by iron core

## 2.3 Linked Electric and Magnetic Circuits in Power Transformers

The power transformer is required to pass electrical energy from one circuit to another, via the medium of the pulsating mutual magnetic field, as efficiently and economically as possible. Our knowledge of magnetic materials indicates the use of iron or steel for the conveyance of the flux with much greater ease than any other known material. The coils are therefore made to embrace an iron core, which serves as a good conducting path for the mutual magnetic flux, ensuring that the flux links each coil fairly completely.

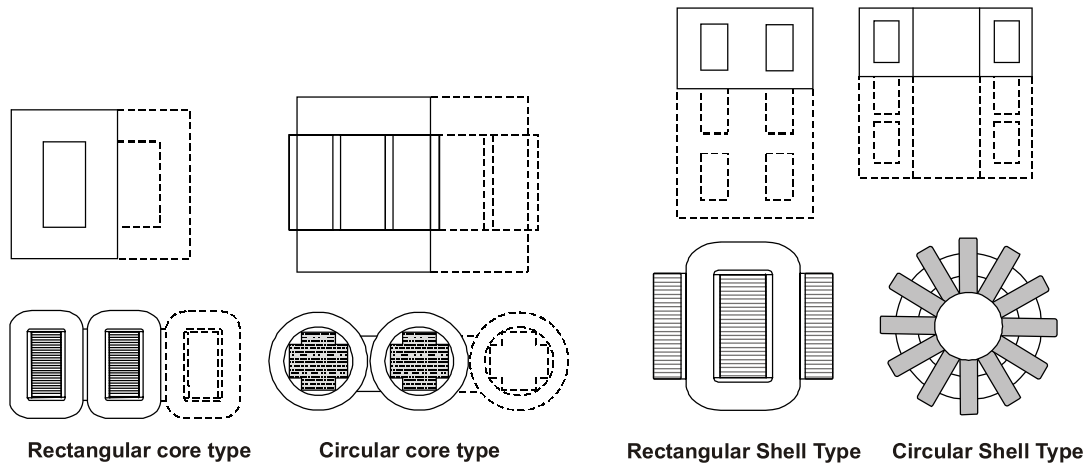


**Figure-2.3 :** Linked Electric and Magnetic Circuits

The elementary linked circuits are shown diagrammatically in Figure (2.3). The use of an iron core permits of much greater freedom in the shape and arrangement of the primary and secondary coils, since the majority of the flux will be conveyed by the core almost regardless of the relative position of the two sets of coils-primary and secondary – that link it. In practice, two general forms are usual : these are obtained from the simple linked circuits (a) of Figure (2.3) by splitting either the coils (b) or the core (c) to give the core and shell types, of which the elementary forms are shown for single-phase transformation.

In core types, to avoid undue leakage flux, it is usual to have half the primary and half the secondary winding side-by-side or concentrically on each limb; not primary on one limb and secondary on the other. The forms of the transformer construction are determined by the constructional methods employed and by the control of the leakage flux.

Three-phase transformers are developed from 1-phase types as in Figure 2.4(a) and 2.4(b). The three-phase shell arrangement is merely three single-phase transformers assembled together. The three-phase core type, on the other hand, embodies the principle that the sum of the fluxes in each phase in a given direction along the cores is zero, i.e. the flux going up one limb can be returned down the other two. Thus only one-half of a complete magnetic circuit is necessary for each phase. Each set of phase windings occupies one limb only.



**Figure-2.4(a): Core-type transformers**

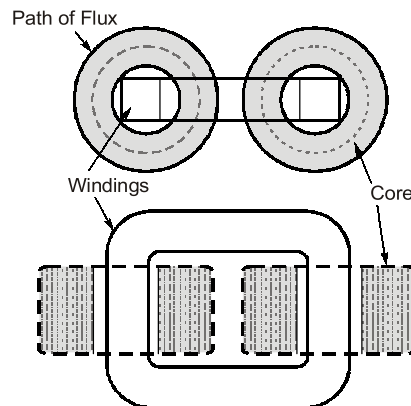
**Figure-2.4(b): Shell-type transformers**

The core type is more easily repaired on site, by removing the yoke, which permits the inspection of the coils and cores. The shell type is more robust mechanically since the coils are more readily braced. The radial shell type employs simple round coils, and the cooling is good, particularly for the iron.

#### NOTE



A method of construction developed in America for small distribution transformers upto about 5 kVA employs cores comprising long continuous strips of sheet steel, wound round the coils as shown in Figure-2.4(c). The core winding requires special machinery, but the advantages include reduction of joints and the use of the grain-direction of the steel for the flux-path.



**Figure-2.4(c): Wound-Core Transformers**



## 2.4 E.m.f. Equation of Transformer

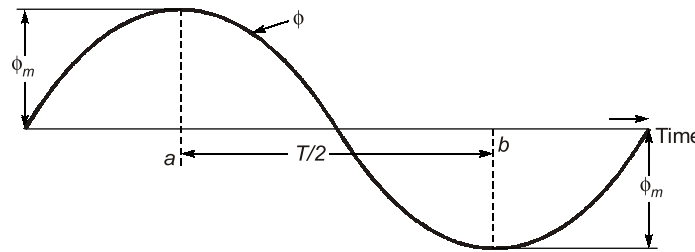
Referring to Figure (2.5), the mutual flux  $\phi$ , is passing through the magnetic circuit formed by the iron core, links both the primary winding 'P' and the secondary winding 'S'. Hence, it must induce the same e.m.f. per turn in each winding. The total induced e.m.f. in each winding then must be proportional to the number of turns in that winding; that is,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

where  $E_1$  and  $E_2$  are the primary and secondary induced e.m.fs and  $N_1$  and  $N_2$  are the number of turns in primary and secondary, respectively.

Figure (2.5) shows the mutual flux,  $\phi$ , varying sinusoidally with time. Between points  $a$  and  $b$ , the total change of flux is  $2\phi_m$  webers.

This change of flux occurs in a half cycle, or in a time  $T/2$  sec, where  $T$  is the period, or the time required for the wave to complete one cycle. The time  $T/2$  is equal to  $1/2f$  seconds.



**Figure-2.5:** Sinusoidal variation of flux with time

**Method-1 :** The average induced e.m.f. in the primary winding is equal to the total change of flux divided by the time. i.e.,

$$e_1 = -N_1 \frac{2\phi_m}{T/2} \text{ Volts} = -N_1 \frac{2\phi_m}{1/2f} \text{ Volts} = -4f N_1 \phi_m \text{ Volts}$$

Since, the form factor for a sine wave is 1.11, the r.m.s induced e.m.f. is

$$E_1 = -4.44f N_1 \phi_m$$

**Method-2 :**

$$\phi = \phi_m \sin \omega t$$

$$e_1 = -N_1 \cdot \frac{d\phi}{dt} = -N_1 \phi_m \omega \cdot \cos \omega t \text{ Volts}$$

The maximum e.m.f. is,  $E_m = -N_1 \phi_m \omega \text{ volts} = -2\pi f N_1 \phi_m \text{ volts}$

$$E_{\text{rms}} = -\frac{2\pi}{\sqrt{2}} f N_1 \phi_m \text{ volts} \quad \text{or} \quad -\sqrt{2} \pi f N_1 \phi_m \text{ volts} = -4.44f N_1 \phi_m \text{ volts}$$

In terms of the maximum flux density  $B_m$  (Wb/m<sup>2</sup>) in the core and the area of cross-section of the core,  $A$ (m<sup>2</sup>),

$$E_{\text{rms}} = -4.44f N B_m A \text{ volts}$$

In the same manner, the voltage induced in the secondary winding  $N_2$  by flux linkage  $\phi_m$  is

$$E_2 = -4.44f N_2 \phi_m \text{ volts}$$

The applied voltage must balance the induced primary voltage  $E_1$  and hence,

$$V_1 = 4.44 N_1 f \phi_m \text{ volts}$$

Thus we see that the applied voltage  $V_1$  and the secondary induced voltage  $-E_2$  are  $\pi$  radians out of phase ( $+V_1$  and  $-E_2$ ).

The ratio of the induced voltages in the two windings is called the transformation ratio  $K$ .

$$K = \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

**NOTE**

Students are advised to use transformation ratio,

$$K \text{ or } a = \frac{E_{H.V.}}{E_{L.V.}} = \frac{V_{H.V.}}{V_{L.V.}}$$

**Example 2.1**

The low voltage winding of a 400/230 V single phase 50 Hz transformer is to be connected to a 25 Hz supply. In order to keep the magnetization current at the same level in both the cases the voltage at 25 Hz should be:

(a) 230 V

(b) 460 V

(c) 115 V

(d) 65 V

**Solution : (c)**

To keep the magnetizing current same,  $V/f$  should be same.

## 2.5 Ampere-turns Relation

At all ordinary loads, the e.m.f.  $E_1$  induced in the primary by the mutual flux,  $\phi$ , is nearly equal in magnitude to the primary terminal voltage  $V_1$  differing only by the small impedance drop in the primary. Hence, since  $V_1$  is constant, the induced e.m.f.  $E_1$  must also be nearly constant.

It follows then, that since  $E_1$  is nearly constant the mutual flux  $\phi$  also must be nearly constant at all normal loads and therefore the m.m.f. producing it as well as the iron losses must be nearly constant. Thus the exciting current  $I_0$  must be constant at all normal loads on the transformer. Also  $I_0$  is small in magnitude, ordinarily being 2 to 6% of the rated current.

The exciting current is of small magnitude and generally differs considerably in phase from the total primary current. It is normally neglected, therefore, in comparison with the total primary current. Now, let the secondary be loaded and the load current be  $I_2$  Ampere. So, there is flux  $\phi_2$  produced by the secondary ampere-turns ( $N_2 I_2$ ) and as per **Lenz's law**, this flux opposes the mutual flux  $\phi_m$ . This results in decrease of the resultant flux and hence decrease in the induced e.m.f.  $E_1$ . But e.m.f.  $E_1$  is almost constant and to keep this constant, the flux  $\phi_m$  should be restored to the original value. Hence, the primary draws a current  $I_1$  from the a.c. mains, such that the primary ampere-turns ( $N_1 I_1$ ) is equal to the secondary ampere-turns ( $N_2 I_2$ ).

Therefore, under loading conditions,

$$N_1 I_1 = N_2 I_2$$

$$\therefore \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

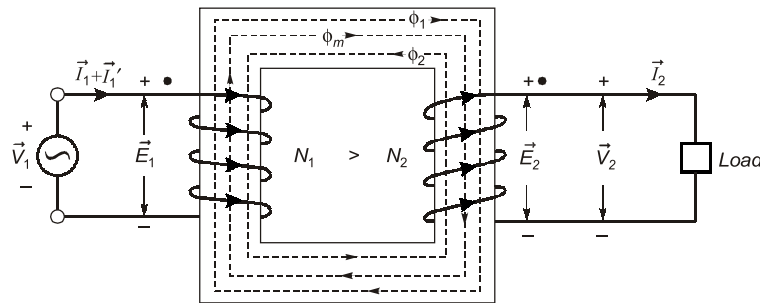
If the transformer losses be neglected and unity power-factor be assumed, then,

$$\frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

i.e.,

$$V_1 I_1 = V_2 I_2$$

**Remark**



**Figure-2.6**

$V_1$  is supply voltage source connected to the primary side of transformer.  $I_1$  current flow through primary winding and induced e.m.f. in primary winding is  $E_1$ . Let secondary terminal be loaded and the load current be  $I_2$ . So there is flux  $\phi_2$  produced by secondary ampere-turn ( $N_2 I_2$ ) and as per Lenz's law, this flux ( $\phi_2$ ) oppose the main mutual  $\phi_m$  (which is produced by primary ampere turn  $N_1 I_1$ ). This results in decrease of the resultant flux and hence decrease in the induced e.m.f.  $E_1$ . Due to the difference between  $V_1$  and  $E_1$  draw additional current through primary windings and produce additional flux  $\phi_1$  by primary ampere turns ( $N_1 I_1'$ ) in same direction of main flux.

Flux  $\phi_1$  compensate the flux  $\phi_2$  and keep the  $\phi_m$  constant. Additional current in primary winding is

$$I_1' = \frac{V - E_1}{Z_p}$$

As shown in Figure 2.6 three flux is circulated in which  $\phi_1$  and  $\phi_m$  circulates in same direction and  $\phi_2$  circulate in opposite direction. Flux  $\phi_1$  and  $\phi_2$  canceled each other and flux  $\phi_m$  remains constant.

So that, we can say transformer is a constant frequency device that is which cannot change frequency only change voltage and current level of the system or circuit.

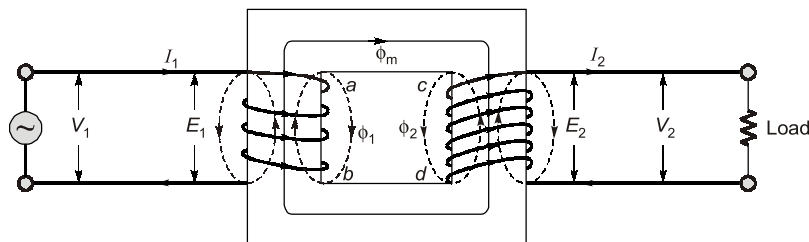
**Turn Ratio of Transformer**

The ratio of the number of turns in higher voltage winding to the lower voltage winding.

$$k = \frac{\text{Secondary voltage}}{\text{Primary voltage}} = \frac{V_1}{V_2}$$

**2.6 Leakage Reactance**

In the preceding section, it has been assumed that all the flux which links the primary also links the secondary. In practice, it is impossible to realize this condition. All the flux produced by the primary does not link the secondary, but a part completes its magnetic circuit by passing through the air rather than around through the core, as shown in Figure (2.6). That is, between planes 'a and b', there is a m.m.f. due to the primary ampere-turns.



**Figure-2.7 : Mutual flux, primary leakage flux and secondary leakage flux in a transformer**

This m.m.f. is proportional to the primary current and tends to send flux from 'a and b' through the air and around through the core. That part of the flux which passes from 'a to b' through the air and around through the core follows a magnetic circuit that is acted upon by the primary ampere-turns only. This flux  $\phi_1$  is called the primary leakage flux. It is proportional to the total ampere-turns of the primary alone, as the secondary turns do not link the magnetic circuit of  $\phi_1$  which, therefore, induces an e.m.f. in the primary but not in the secondary. Due to this leakage flux  $\phi_1$ , there is an e.m.f. induced in the primary which is denoted by  $-I_1 X_1$  where ' $X_1$ ' is the primary leakage reactance. The component of line voltage that balances this e.m.f. is  $+I_1 X_1$ . A reactance drop exists in a transformer primary therefore, in precisely the same manner that a reactance drop exists in an alternator armature. Similarly, there exists a reactance drop in the secondary, which is equal to  $I_2 X_2$ . This secondary reactance opposes the change in current following out of the secondary.

Referring to Figure 2.6 and applying Kirchhoff's law, we get,

$$\begin{aligned}\vec{V}_1 &= R_1 \vec{I}_1 + j\omega L_1 \vec{I}_1 + \vec{E}_1 \\ &= R_1 \vec{I}_1 + j \vec{I}_1 X_1 + \vec{E}_1 \\ &= \vec{I}_1 (R_1 + jX_1) + \vec{E}_1\end{aligned}$$

where  $V_1$  is the applied voltage,  $R_1$  and  $X_1$  are the primary winding resistance and leakage reactance respectively and  $E_1$  is the induced e.m.f.

Likewise, considering the secondary circuit, we get,

$$\begin{aligned}\vec{E}_2 &= \vec{V}_2 + \vec{I}_2 R_2 + j \vec{I}_2 X_2 \\ &= \vec{V}_2 + \vec{I}_2 (R_2 + jX_2)\end{aligned}$$

## 2.7 Ideal Transformer

Characteristics of Ideal transformer are:

1. No winding loss
2. No core loss
3. No leakage flux i.e. any flux in core links both winding completely
4. Constant relative permeability

### 2.7.1 Ideal Transformer on No Load

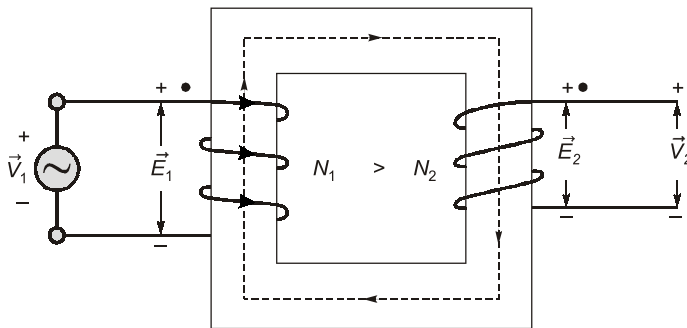


Figure-2.8 (a)

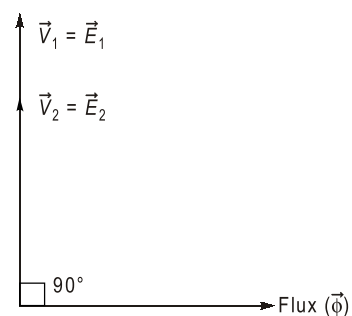


Figure-2.8 (b)

$$\frac{\vec{V}_1}{\vec{V}_2} = \frac{\vec{E}_1}{\vec{E}_2} = \frac{N_1}{N_2} = a \text{ (scalar quantity) = Voltage ratio or Turns ratio}$$

"Always induced e.m.f.  $\vec{E}_1$  and  $\vec{E}_2$  leads flux  $\vec{\Phi}$  by  $90^\circ$ "

### 2.7.2 Ideal Transformer on Load

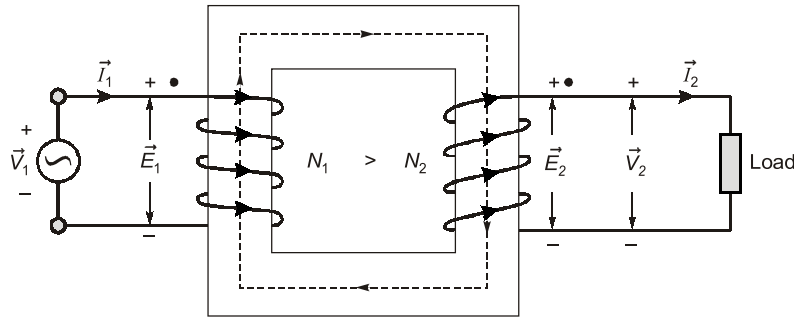


Figure-2.9

As applied to the transformers, dots indicate the same instantaneous polarity.

#### M.M.F balance equation of an Ideal Transformer

$$N_1 \vec{I}_1 - N_2 \vec{I}_2 = 0$$

$$\Rightarrow N_1 \vec{I}_1 = N_2 \vec{I}_2 \quad \dots(i)$$

$$\text{and} \quad \frac{\vec{V}_1}{\vec{V}_2} = \frac{\vec{E}_1}{\vec{E}_2} = \frac{N_1}{N_2} = a \quad \dots(ii)$$

From equation (i) and (ii),

$$\Rightarrow \vec{I}_1 = \frac{N_2}{N_1} \vec{I}_2 = \frac{\vec{I}_2}{a} = \vec{I}_2' = \text{secondary current referred to primary side}$$

$\therefore$  Turns ratio will be equal to

$$\frac{\vec{V}_1}{\vec{V}_2} = \frac{\vec{E}_1}{\vec{E}_2} = \frac{N_1}{N_2} = \frac{\vec{I}_2}{\vec{I}_1} = a$$

$$\text{or,} \quad \vec{V}_1 \vec{I}_1 = \vec{V}_2 \vec{I}_2 \text{ or } \vec{V}_1 \vec{I}_1^* = \vec{V}_2 \vec{I}_2^* \text{ or, } \vec{E}_1 \vec{I}_1 = \vec{E}_2 \vec{I}_2$$

$$\text{So,} \quad \vec{S}_1 = \vec{S}_2$$

By using these relations, we can say that electric power at one voltage/current level transforms to another level. Hence Transformer statement satisfies clearly.

### 2.7.3 Phasor Diagram of Ideal Step Down Transformer

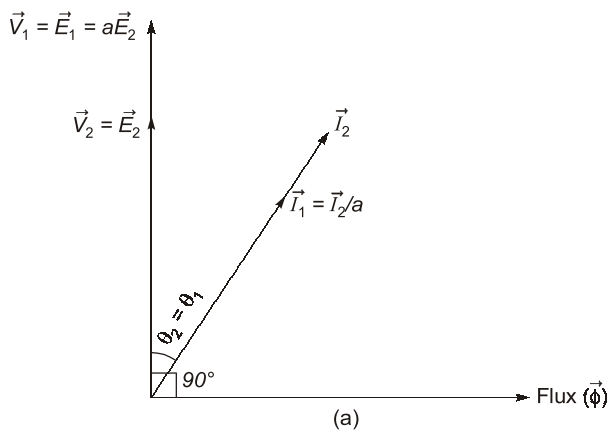


Figure-2.10 (a)

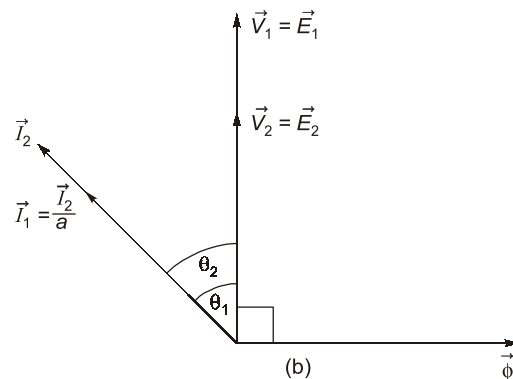


Figure-2.10 (b)

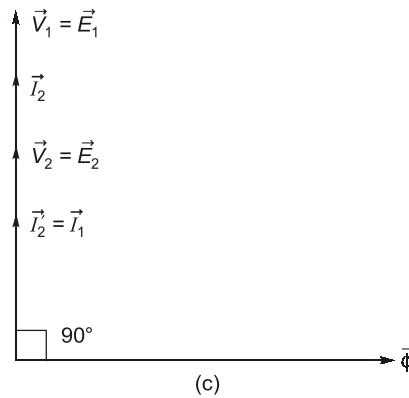


Figure-2.10 (c)

**Figure-2.10:** Phasor diagram of Ideal transformer at  
(a) lagging power factor load (b) leading power factor load (c) unity power factor load

### 2.7.4 Phasor Diagram of Different Cases of Practical Step Down Transformer

#### Case-1:

1. No winding loss
  2. No core loss
  3. No leakage flux
  4. Permeability ( $\mu \neq \infty$ ) i.e.  $I_0$  exists
- $\vec{I}_0$  lags  $\vec{V}_1$  by  $90^\circ$  because input power = 0 (lossless transformer) i.e.  $I_0$  is purely magnetising current

No Load Phasor Diagram:

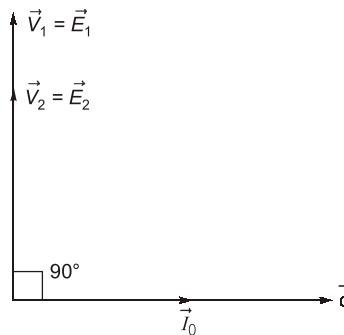


Figure-2.11

On Load Phasor Diagram:

The M.M.F. balance equation for Practical Transformer

$$N_1 \vec{I}_1 - N_2 \vec{I}_2 = N_1 \vec{I}_0$$

$$\Rightarrow \vec{I}_1 = \frac{\vec{I}_2}{a} + \vec{I}_0$$

$$= \vec{I}_2' + \vec{I}_0$$

#### Case-2:

1. No winding loss
2. Core loss present
3. No leakage flux
4.  $\mu \neq \infty$

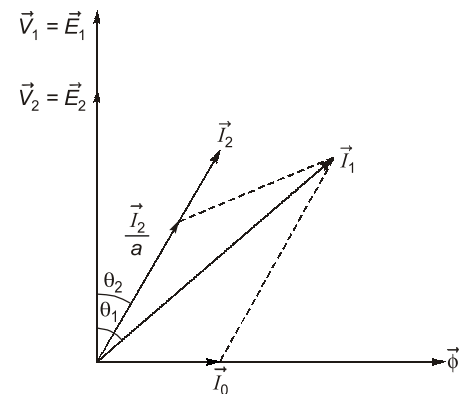
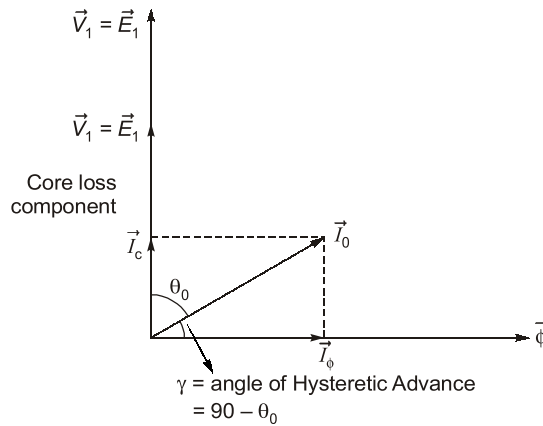


Figure-2.12

No Load Phasor Diagram:



**Figure-2.13**

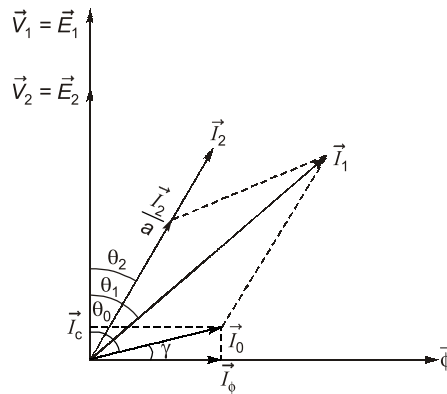
Here,  $\vec{I}_0$  cannot lag  $\vec{V}_1$  by  $90^\circ$  because of core loss

$$I_c = I_0 \cos \phi_0 = \text{core loss component of } I_0$$

$$I_\phi = I_0 \sin \phi_0 = \text{magnetizing component of } I_0$$

In practical transformer value of  $\theta_0$  is between  $80^\circ$  to  $85^\circ$ .

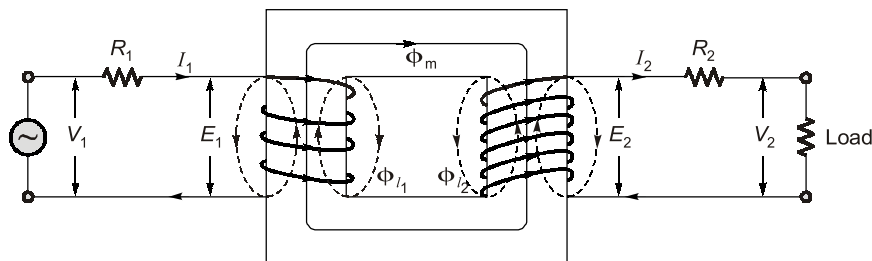
On Load Phasor Diagram:



**Figure-2.14**

**Case-3:**

1. Winding losses are present
2. Core losses are present
3. Leakage flux still available
4.  $\mu \neq \infty$  means  $I_0 \neq 0$



**Figure-2.15**

Here,

$\phi_{l_1}$  = primary leakage flux

$\phi_{l_2}$  = secondary leakage flux

$\vec{E}_{l_1} = j\vec{I}_1 X_1$  = a phasor which leads  $\vec{I}_1$  by  $90^\circ$

$\vec{E}_{l_2} = j\vec{I}_2 X_2$  = a phasor which leads  $\vec{I}_2$  by  $90^\circ$

$X_1$  = primary leakage reactance

$X_2$  = secondary leakage reactance

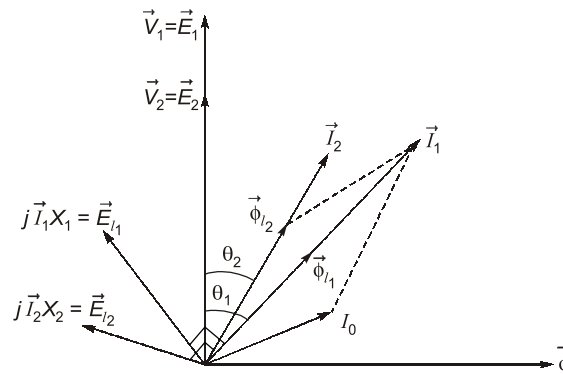


Figure-2.16

#### NOTE



Performance equation of a transformer are:

1.  $\vec{V}_1 = \vec{E}_1 + \vec{I}_1 R_1 + j\vec{I}_1 X_1$
2.  $\vec{E}_2 = I_2 R_2 + j\vec{I}_2 X_2 + \vec{V}_2$
3.  $\vec{I}_1 = \frac{\vec{I}_2}{a} = \vec{I}_2'$
4.  $\vec{E}_1 = a\vec{E}_2 = \vec{E}_2'$
5.  $\vec{I}_0 = \vec{I}_c + \vec{I}_\phi$

## 2.8 Exact Equivalent Circuit of a Transformer

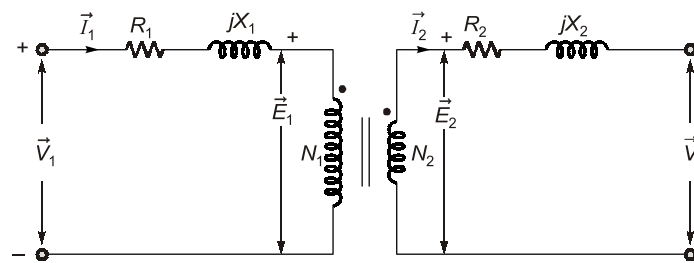


Figure-2.17



Student's  
Assignments

1

- Q.1** A 200/400 V, 20 kVA, 50 Hz transformer is connected as an auto-transformer to work off 600/200 V supplies. With a load of 20 kVA, 0.8 p.f. lagging connected to the 200 V terminals. Find the current in common winding and kVA rating of auto transformer respectively.
- Q.2** A 4 kVA, 400/200 V, 1-phase transformer has leakage impedance of  $0.02 + j0.04$  per unit. This leakage impedance in ohms, when referred to hv side is  $x + jy$  the value of  $x$  is \_\_\_\_.
- Q.3** A 100 MVA, 230/115 kV  $\Delta$ - $\Delta$ , 3- $\phi$  power transformer has a resistance of 0.02 per unit and a reactance of 0.055 per unit. The transformer supplies a load of 80 MVA at 0.85 p.f. lagging. What is the percentage voltage regulation of the transformer?
- Q.4** A 1- $\phi$ , 5 kVA, 440/220, 50 Hz transformer is connected for short circuit test, it has following constants:
- $$\begin{aligned} r_p &= 0.5 \, \Omega, & r_s &= 0.20 \, \Omega \\ x_p &= 0.6 \, \Omega, & x_s &= 0.15 \, \Omega \\ r_0 &= 600 \, \Omega, & x_0 &= 200 \, \Omega \end{aligned}$$
- Calculate the short circuit voltage  $V_{SC}$ .
- Q.5** Determine the suitable tapping on an auto-transformer starter for an induction motor required to start the motor with 40 per cent of full-load torque. The short-circuit current of the motor is 5 times the full load current and full-load slip is 0.035. Also determine the current drawn from the mains as a fraction of full-load current.
- Q.6** A 25 kVA, 230/115 V, 50 Hz transformer has the following data:
- $$r_1 = 0.12 \, \Omega, \, r_2 = 0.04 \, \Omega, \, x_1 = 0.2 \, \Omega, \, x_2 = 0.05 \, \Omega$$
- If the primary induced emf equal in magnitude to the primary terminal voltage, when the transformer is carrying the full load current then load is of \_\_\_\_\_ p.f. lead.

## Common Data for Questions (7 and 8):

A 500 kVA transformer has 95 % efficiency at full load and also at 60 % of full load both at unity p.f.

- Q.7** Sum of iron loss and copper loss at full load is \_\_\_\_\_ kW.
- Q.8** Transformer efficiency at 75 % full load and unity power factor is \_\_\_\_\_ %.
- Q.9** Two transformers connected in open delta supplies a 400 kVA balanced load operating at 0.866 p.f. (lag). The load voltage is 440 V. What is the kW supplied by each transformer?
- Q.10** An auto-transformer having 1250 turns is connected across a 250 V supply. What secondary voltage will be obtained if a tap is taken at 800<sup>th</sup> turn?
- Q.11** A 3-phase transformer has its primary connected in delta and secondary in star. It has an equivalent resistance of 1% and equivalent reactance of 6%. The primary applied voltage is 6600 V. \_\_\_\_\_ be the ratio of transformation in order that it will deliver 4800 V at full load current and 0.8 power factor lag.
- Q.12** At 400 V and 50 Hz the total core loss of a transformer was found to be 2400 W. When the transformer is supplied at 200 V and 25 Hz, the core loss is 800 W. Calculate the hysteresis and eddy current loss at 400 V and 50 Hz.
- Q.13** Two single-phase transformers in parallel connection, supplies a load of 500 A, at 0.8 p.f. lagging and at 400 V. Their equivalent impedances referred to secondary winding are  $(2 + j3)$  ohms and  $(2.5 + j5)$  ohms respectively. Calculate the kVA supplied by transformer-I.
- Q.14** A 500 kVA, 11 kV/0.43 kV, 3-phase delta/star connected transformer has HV copper loss of 2.5 kW and LV copper loss of 2 kW on rated load. The ohmic value of the equivalent resistance on the delta side is \_\_\_\_\_  $\Omega$ /ph.

**Q.15** A 5 kVA, 400/80V transformer has  $R_{eq}(HV) = 0.25 \Omega$  and  $X_{eq}(HV) = 5 \Omega$  and a lagging load is being supplied by it resulting in the following meter readings (meters are placed on the HV side).

$I_1 = 16 \text{ A}$ ,  $V_1 = 400 \text{ V}$ ,  $P_1 = 5 \text{ kW}$ . For this condition calculate what a voltmeter would read if connected across the load terminals. Assume the exciting current to be zero.



**Student's  
Assignments**

**1**

**Explanations**

**1. (30)**

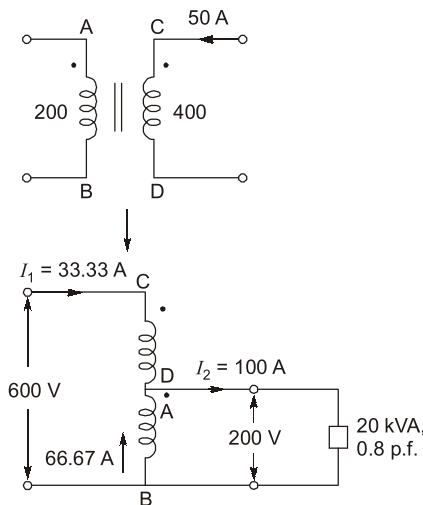
Two winding must be connected in series with the proper polarity so that 600 V can be applied across the total windings.

With 20 kVA load, load current

$$I_2 = \frac{20 \times 1000}{200} = 100 \text{ A}$$

$$I_1 = \frac{20 \times 1000}{600} = 33.33 \text{ A}$$

current in common winding =  $(100 - 33.33) \text{ A}$   
= 66.67 A.



Through C, a maximum current = 50 A  
kVA rating =  $600 \times 50 = 30 \text{ kVA}$

**2. (0.8)**

$$\text{Base impedance} = \frac{400}{\left(\frac{4000}{400}\right)} = \frac{400 \times 400}{4000} = 0.40$$

leakage impedance referred to hv side =  
 $(0.02 + j0.04) \times 40 = 0.8 + j16$

**3. (3.7%)**

$$I_{S(\text{base})} = \frac{100}{\sqrt{3} \times 115} = 502 \text{ A}$$

Since the transformer supplies a load of 80 MVA at 0.85 pf lagging, so secondary line current of the transformer is

$$I_S = \frac{80}{\sqrt{3} \times 115} = 402 \text{ A}$$

$$(I_S)_{pu} = \frac{402}{502} \angle -\cos^{-1}(0.85)$$

$$= 0.8 \angle -31.8^\circ$$

per unit no load voltage of this transformer is

$$V_{NL} = V_S + \vec{I} \vec{Z}$$

$$= 1 \angle 0^\circ + (0.8 \angle -31.8^\circ)(0.02 + j0.055)$$

$$= 1.037 \angle 1.6^\circ$$

$$\text{V.R.} = \frac{1.037 - 1}{1} \times 100 \% = 3.7 \%$$

**4. (14.77∠42.70°)**

$$I_{SC} = \frac{5000}{440} = 11.36 \text{ A}$$

$$\begin{aligned} V_{SC} &= I_{SC} \times Z_{eq} \\ &= 11.36 [(0.5 + (2)^2 \times 0.2) \\ &\quad + j \{0.6 + (2)^2 \times 0.15\}] \\ &= 11.36 [1.3 + j1.2] \\ &= 11.36 \times 1.77 \angle 42.70^\circ \\ &= 20.1 \angle 42.70^\circ \text{ V} \end{aligned}$$

**5. (2.28)**

$$\tau_{st} = x^2 \left( \frac{I_{sc}}{I_{fl}} \right)^2 S_{fl} \tau_{fl}$$

$$0.4 \tau_{fl} = x^2 (5)^2 \times 0.035 \tau_{fl}$$

$$x^2 = \frac{0.4}{(5)^2 \times 0.035} = 0.457$$

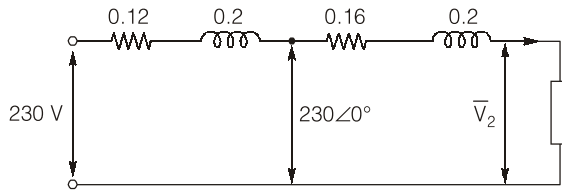
$$x = 0.676$$

Current drawn from the supply

$$= x^2 I_{sc} = 0.457 \times 5 \times I_{fl}$$

$$= 2.28 I_{fl}$$

6. (0.79)



$$\text{Turn ratio} = \frac{230}{115} = 2$$

$$r_2' = 0.16 \, \Omega, x_2' = 0.2 \, \Omega$$

$$I_1(\text{F.L.}) = \frac{25 \times 1000}{230} = 108.7 \angle \phi_1 \, \text{A}$$

$$230 - 108.7 (0.12 \cos \phi_1 + 0.2 \sin \phi_1) = 230$$

$$\text{or, } \tan \phi_1 = -\frac{0.12}{0.2} = -0.6$$

$$\text{p.f.} = \cos \phi_1 = 0.858 \text{ leading}$$

$$\therefore \phi_1 = 30.9^\circ$$

$$\bar{V}_2 = 230 - 108.7 \angle 30.9^\circ \times (0.16 + j0.2)$$

$$= 227.9 \angle -6.95^\circ$$

$$\phi_2 = 30.9^\circ + 6.9^\circ = 37.8^\circ$$

$$\cos \phi_2 = 0.79 \text{ leading}$$

7. (26.32)

$$\frac{500}{500 + P_i + P_{cu}} = 0.95$$

$$\Rightarrow P_i + P_{cu} = \left( \frac{500}{0.95} - 500 \right) = 26.315 \quad \dots(i)$$

$$\frac{500 \times 0.6}{500 \times 0.6 + P_i + 0.36 P_{cu}} = 0.95$$

$$\left( \frac{500}{0.95} - 500 \right) \times 0.6 = P_i + 0.36 P_{cu}$$

$$P_i + 0.36 P_{cu} = 15.79 \quad \dots(ii)$$

Solving equations (i) and (ii),

$$P_{cu} = 16.45 \, \text{kW}, P_i = 9.87 \, \text{kW}$$

8. (95.15%)

$$\eta = \frac{\text{kVA} \times \cos \phi}{\text{kVA} \times \cos \phi + P_i + P_{cu}}$$

$$\eta = \frac{500 \times 0.75}{500 \times 0.75 + 9.87 + 16.45(0.75)^2} \times 100 = 95.15 \%$$

9. (231, 115.5)

Secondary line current,

$$I = \frac{\text{Total load in kVA} \times 1000}{\sqrt{3} \times \text{line voltage}} = \frac{400 \times 1000}{\sqrt{3} \times 440} = 525 \, \text{A}$$

kVA supplied by each transformer

$$= \frac{V \times I}{1000} = \frac{440 \times 525}{1000} = 231 \, \text{kVA}$$

for a pf of 0.866 lag,

$$\phi = 30^\circ$$

So power delivered by one transformer,

$$P_1 = \text{kVA} \cos (30^\circ - \phi) = 231 \cos 0^\circ = 231 \, \text{kW}$$

and power delivered by the other transformer,

$$P_2 = \text{kVA} \cos (30^\circ + \phi) = 231 \cos 60^\circ = 115.5 \, \text{kW}$$

10. (160)

Method-1:

Supply voltage,  $V_1 = 250 \, \text{V}$ Primary turns,  $N_1 = 1250$ 

Secondary turns,

$$N_2 = 800$$

Secondary voltage,

$$V_2 = V_1 \times \frac{N_2}{N_1} = 250 \times \frac{800}{1250} = 160 \, \text{V}$$

Method-2:

$$\text{EMF per turn} = \frac{V_1}{N_1} = \frac{250}{1250} = 0.2 \, \text{V}$$

and secondary voltage  $V_2 = \text{EMF per turn} \times \text{secondary turns} = 0.2 \times 800 = 160 \, \text{V}$ 

11. (0.43 - 0.44)

Percentage regulation =  $v_r \cos \phi + v_x \sin \phi$ where  $v_r$  is the percentage resistive drop or percentage equivalent resistance and  $v_x$  is the percentage reactive drop or percentage equivalent reactance.Since,  $v_r = 1\%$ 

$$v_x = 6\%$$

$$\cos \phi = 0.8 \text{ and } \sin \phi = 0.6$$

$$\text{Percentage regulation} = 1 \times 0.8 + 6 \times 0.6 = 4.4\%$$

Secondary induced emf (line to line)  
= 4800 + 4.4% of 4800 = 5011.2 V  
Secondary induced emf per phase

$$= \frac{5011.2}{\sqrt{3}} = 2893.22 \text{ V}$$

Transformation ratio,

$$K = \frac{\text{Secondary phase emf}}{\text{Primary phase emf}} = \frac{2893.22}{6600} = 0.438$$

### 12. (800 W, 1600 W)

$$\frac{V_1}{f_1} = \frac{400}{50} = 8$$

$$\frac{V_2}{f_2} = \frac{200}{25} = 8$$

Since,  $\frac{V_1}{f_1} = \frac{V_2}{f_2} = 8$

the flux density  $B_m$  remains constant

Hence,  $\frac{P_i}{f} = u + vf$

$$\therefore \frac{2400}{50} = u + 50v$$

and  $\frac{800}{25} = u + 25v$

Solving these equations, we get

$$u = 16$$

$$v = 0.64$$

Therefore, at 50 Hz

$$P_h = uf = 16 \times 50 = 800 \text{ W}$$

$$P_e = vf^2 = 0.64 \times (50)^2 = 1600 \text{ W}$$

### 13. (78.57 kVA)

$$I_1 = \frac{500 \times (2.5 + j5)}{4.5 + j8} = 304.516 \angle -2.79^\circ \text{ A}$$

$$I_2 = \frac{500 \times (2 + j3)}{4.5 + j8} = 196.426 \angle -4.34^\circ \text{ A}$$

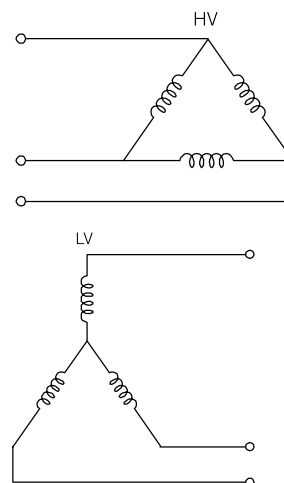
(kVA) supplied by transformer-I

$$= \frac{304.5 \times 400}{1000} = 121.8 \text{ kVA}$$

(kVA) supplied by transformer-II

$$= \frac{196.426 \times 400}{1000} = 78.57 \text{ kVA}$$

### 14. (7.2 - 7.3)



$$\text{Turn ratio} = \frac{V_{HV}}{V_{LV}} = \frac{11}{\left(\frac{0.43}{\sqrt{3}}\right)} = 44.3$$

$$(I_{\text{phase}})_Y = \frac{500}{\sqrt{3} \times 0.43} = 671.34 \text{ A}$$

$$(I_{\text{phase}})_\Delta = \frac{500}{3 \times 11} = 15.15 \text{ A}$$

$$(R_{LV})_{\text{ph}} = \frac{2500}{3 \times (671.34)^2} = 1.85 \times 10^{-3} \Omega$$

$$(R_{HV})_{\text{ph}} = \frac{2500}{3 \times (15.15)^2} = 3.63 \Omega$$

$$(R_{\text{eq}})_{HV} = 3.63 + 1.85 \times 10^{-3} \times (44.3)^2 = 7.26 \Omega/\text{ph}$$

### 15 (70.4 V)

$$\cos \phi_1 = \frac{5 \times 1000}{400 \times 16} = 0.78$$

$$\therefore \phi_1 = 38.6^\circ \text{ lagging}$$

$$\bar{I}_1 = 16 \angle -38.6^\circ \text{ A}$$

$$(\bar{V}_L)_1 = 400 \angle 0^\circ - 16 \angle -38.6^\circ (0.25 + j5) = 347.96 - j60$$

$$|(V_L)_1| = 352$$

$$\therefore |(V_L)_2| = \frac{352 \times 80}{400} = 70.4 \text{ V}$$

■■■■

Student's  
Assignments

2

**Q.1** Find the percentage loading at which maximum efficiency occurs in a given transformer with full load iron and copper losses as 3 kW and 8 kW respectively

- (a) 50% (b) 61.2%  
(c) 70.7% (d) 37.5%

**Q.2** Open circuit test was done on a transformer with rated voltage and rated frequency in order to find out core losses. If this open circuit test now conducted on the double of rated voltage and double of the rated frequency, then

- (a) both current magnitude and power factor decreases  
(b) both current magnitude and power factor increases  
(c) current magnitude increases and power factor decreases  
(d) current magnitude decreases and power factor increases

**Q.3** Match List-I (Type of Coil) with List-II (Use of Coil) and select the correct answer using the codes given below the lists:

## List-I

## List-II

- |                            |   |
|----------------------------|---|
| <b>A.</b> Sandwich coils   | <b>1.</b> Low voltage coils for currents above 100 A              |
| <b>B.</b> Disc coils       | <b>2.</b> High voltage windings of small transformers             |
| <b>C.</b> Cross-over coils | <b>3.</b> Cooling oil is in contact with each turn of the winding |
| <b>D.</b> Spiral type      | <b>4.</b> Shell-type transformer core                             |

## Codes:

- |     | A | B | C | D |
|-----|---|---|---|---|
| (a) | 2 | 3 | 4 | 1 |
| (b) | 4 | 1 | 2 | 3 |
| (c) | 2 | 1 | 4 | 3 |
| (d) | 4 | 3 | 2 | 1 |

**Q.4** Consider the following statements:

1. An auto transformer has higher efficiency than a two winding transformer of same ratings.

2. In an auto transformer, power is transferred through conduction and induction processes.  
3. In an auto transformer, power is transferred through induction process only.

Which of the above statements are correct?

- (a) 1 and 2 (b) 2 and 3  
(c) 1 and 3 (d) 3 only

**Q.5** Which of the following statements are incorrect?

1. Maximum voltage regulation of a transformer occurs at leading power factor.  
2. Voltage regulation of a transformer is maximum when load power factor (lagging) angle has the same value as the angle of equivalent impedance.  
3. Voltage regulation of a transformer at zero power factor is always zero.  
4. Voltage regulation of a transformer can be negative at leading power factor.

- (a) 1 and 3 (b) 2 and 3  
(c) 2 and 4 (d) 1 and 4

**Q.6** The main purpose of the conservator in a transformer is to

- (a) store extra oil to compensate for the loss of oil due to leakage.  
(b) achieve better cooling of the transformer.  
(c) take up the expansion of oil due to heating.  
(d) have the Buchholz relay fitted.

**Q.7** Two transformers 'A' and 'B' with identical ratings but with different leakage reactances are operated in parallel. 'A' has more leakage reactance than that of 'B'. Then, the kVA shared by 'A' is

- (a) more than that of 'B'  
(b) less than that of 'B'  
(c) equal to 'B'  
(d) cannot be determined

**Q.8** To calculate the voltage regulation of a 1- $\phi$  transformer at half the full load and 0.8 p.f. lagging, the test required is

- (a) short circuit test  
(b) open circuit test  
(c) both SC and OC tests  
(d) none of these