POSTAL Book Package 2023

Electrical Engineering

Objective Practice Sets

Electromagnetic Theory

Contents

MADE EASY Publications

1
CHAPTER

Vector Analysis

MCQ and NAT Questions

Q.1 If $\vec{G} = 15r\hat{a}_\phi$ then $\oint \vec{G}\cdot\vec{dl}$ over the circular path r = 2 m, θ = 30°, 0 < ϕ < 2π is

(a) 120π (b) 120
(c) 60π (d) 60

Q.2 Which of the following is true?

(a) $\text{Curl}(\vec{A}\cdot\vec{B}) = \text{Curl}\,\vec{A} + \text{Curl}\,\vec{B}$

(b) $\text{Div}\,(\vec{A}\cdot\vec{B}) = \text{Div}\,\vec{A}\cdot\text{Div}\,\vec{B}$

(c) $\text{Div}\,(\text{Curl}\,\vec{A}) = 0$

(d) $\text{Div}(\text{Curl}\,\vec{A}) = \Delta\cdot\vec{A}$

Q.3 Which of the following equations is correct?

1. $\hat{a}_x \times \hat{a}_x = |a_x|^2$
2. $(\hat{a}_x \times \hat{a}_y) + (\hat{a}_y \times \hat{a}_x) = 0$
3. $\hat{a}_x \times (\hat{a}_y \times \hat{a}_z) = \hat{a}_x \times (\hat{a}_z \times \hat{a}_y)$
4. $\hat{a}_r\cdot\hat{a}_\theta + \hat{a}_\theta\cdot\hat{a}_r = 0$

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 2 and 4 only

Q.4 Match **List-I** (Term) with **List-II** (Type) and select the correct answer:

List-I	List-II
A. Curl ($\bar{F}$) = 0	**1.** Laplace equation
B. Div ($\bar{F}$) = 0	**2.** Irrotational
C. Div Grad (ϕ) = 0	**3.** Solenoidal
D. Div Div (ϕ) = 0	**4.** Not defined

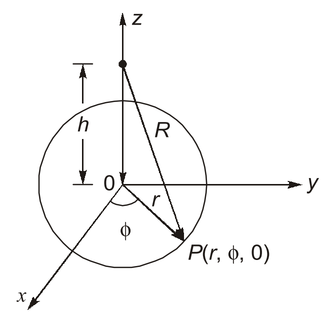
Codes:

	A	B	C	D
(a)	2	3	1	4
(b)	4	1	3	2
(c)	2	1	3	4
(d)	4	3	1	2

Q.5 Laplacian of a scalar function V is

(a) Gradient of V
(b) Divergence of V
(c) Gradient of the gradient of V
(d) Divergence of the gradient of V

Q.6 The unit vector $\vec{a}_R$ which points from $z = h$ on the z-axis towards (r, ϕ, 0) in cylindrical co-ordinates as shown below is given by



(a) $\dfrac{h\vec{a}_r - r\vec{a}_z}{\sqrt{r^2+h^2}}$ (b) $\dfrac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2+h^2}}$

(c) $\dfrac{h\vec{a}_\phi - r\vec{a}_z}{\sqrt{r^2+h^2}}$ (d) $\dfrac{r\vec{a}_z - h\vec{a}_\phi}{\sqrt{r^2+h^2}}$

Q.7 Match **List-I (Vector Identities)** with **List-II (Equivalent expression)** and select the correct answer using the codes given below the lists:

List-I

A. $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D})$

B. $\vec{A}\times(\vec{B}\times\vec{C})$

C. $(\vec{A}\times\vec{B})\times(\vec{C}\times\vec{D})$

List-II

1. $(\vec{A}\cdot\vec{C}\cdot\vec{D})\vec{B} - (\vec{B}\cdot\vec{C}\cdot\vec{D})\vec{A}$

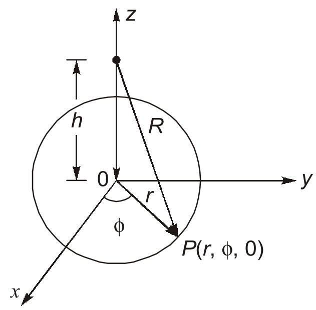
2. $[(\vec{A}\cdot\vec{C})(\vec{B}\cdot\vec{D}) - (\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C})]$

3. $(\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$

Codes:

	A	B	C
(a)	1	3	2
(b)	3	1	2
(c)	2	1	3
(d)	2	3	1

Q.50 The unit vector $\vec{a}_R$ which points from $z = h$ on the z-axis towards $(r, \phi, 0)$ in cylindrical co-ordinates as shown below is given by

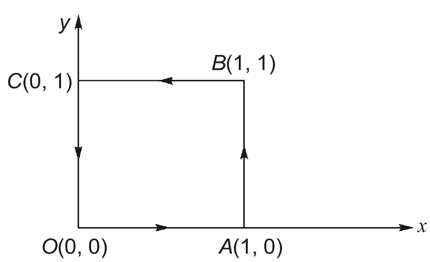


(a) $\dfrac{h\vec{a}_r - r\vec{a}_z}{\sqrt{r^2 + h^2}}$ (b) $\dfrac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$

(c) $\dfrac{h\vec{a}_\phi - r\vec{a}_z}{\sqrt{r^2 + h^2}}$ (d) $\dfrac{r\vec{a}_z - h\vec{a}_\phi}{\sqrt{r^2 + h^2}}$

Q.51 If $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ and $|\vec{r}| = r$ then find div $(r^2 \nabla(ln r))$

Q.52 Given vector $\vec{A} = x^2 y\,\hat{a}_x + 2xy^2\hat{a}_y$, find circulation of $\vec{A}$ along a closed path OABC as shown in figure below.



Q.53 For a vector field

$\vec{A} = xyz^3\hat{a}_x + xy^3z\hat{a}_y + x^3yz\hat{a}_z$. Evaluate the surface integral for a surface of unit cube by $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$.

Q.54 Verify the above question using divergence theorem.

Multiple Select Questions (MSQ)

Q.55 $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ represents a position vector and $\|\vec{r}\|$ represents the normal of vector $\vec{r}$, then which of the below statements is/are true?

(a) Divergence of $\vec{r}$ is 3.

(b) Gradient of $\|\vec{r}\|^2$ is $3\vec{r}$

(c) Curl of $\vec{r}$ is 0

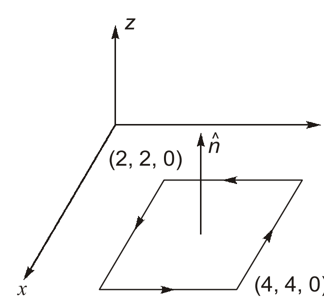
(d) Laplacian of $\|\vec{r}\|^2$ is 6.

Q.56 If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = \sqrt{x^2 + y^2 + z^2}$, then which of the below relations are correct?

(a) $\nabla(\log r) = \dfrac{\vec{r}}{r}$ (b) $\nabla\left(\dfrac{1}{r}\right) = \dfrac{-\vec{r}}{r^3}$

(c) $\nabla \cdot \left(\dfrac{\vec{r}}{r^3}\right) = 1$ (d) $\nabla \cdot (3\vec{r}) = 9$

Q.57 Let $\vec{F} = xy^2\hat{a}_x + y^3\hat{a}_y + x^2y\hat{a}_z$ and the surface S consists of a square of length 2 lying in the xy plane as shown below:



Which of the following options is/are correct?

(a) $\iint_S \vec{F} \cdot \hat{n}\, ds = 80$

(b) $\iint_S (\vec{F} \times \hat{n})ds = 120\hat{a}_x - 112\hat{a}_y$

(c) $\nabla \times \vec{F} = x^2\hat{a}_x - 2xy\hat{a}_y - 2xy\,\hat{a}_z$

(d) $\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, ds = -120$

Q.58 If $[\vec{a}, \vec{b}, \vec{c}]$ represents the scalar triple product of vectors $\vec{a}, \vec{b}$ and $\vec{c}$, then which of the below statements is/are true?

(a) $\left[\vec{a}, \vec{b}, \vec{c}\right] = \left[\vec{c}, \vec{b}, \vec{a}\right]$

(b) $\left[\vec{a}, \vec{b} + \vec{a}, \vec{c}\right] = 0$

(c) $\left[3\vec{b}, \vec{c}, \vec{a}\right] = 3\left[\vec{a}, \vec{b}, \vec{c}\right]$

(d) If $\left[\vec{a}, \vec{b}, \vec{c}\right] = 0$, the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.

Q.59 The values of α for which the vectors

$\vec{A} = \alpha\hat{a}_x + 2\hat{a}_y + 10\hat{a}_z$ and $\vec{B} = 4\alpha\hat{a}_x + 8\hat{a}_y - 2\alpha\hat{a}_z$

are perpendicular is/are

(a) 1 (b) 2
(c) 3 (d) 4

Q.60 Which of the below vector identities are true?

(a) $A \times (B \times C) = (A \times B) \times C$
(b) $A \times (B \times C) + C \times (A \times B) + B \times (C \times A) = 0$
(c) $(B \times C) \times (C \times A) = C(A \cdot B \times C)$
(d) $(A \times B) \cdot (C \times D) = (A \cdot C)(B \cdot D) - (A \cdot D)(B \cdot C)$

Q.61 For the scalar function, $\phi = x^2yz^3$, which of the below statements is/are correct?

(a) From the point (2, 1, –1) the directional derivative of ϕ is maximum in the direction represented by vector $-12\hat{i} - 4\hat{j} + 4\hat{k}$.
(b) The magnitude of greatest rate of change of ϕ from the point (2, 1, –1) is $4\sqrt{11}$.
(c) $(x - 2) + (y - 1) - 3(z + 1) = 0$ represents the tangent plane to the surface $\phi = 0$ at point (2, 1, –1).
(d) ϕ satisfies the Laplacian equation.

■■■■

Answers Vector Analysis

1. (c)	2. (c)	3. (d)	4. (a)	5. (d)	6. (b)	7. (d)	8. (c)	9. (a)
10. (b)	11. (a)	12. (a)	13. (b)	14. (c)	15. (a)	16. (b)	17. (d)	18. (c)
19. (c)	20. (c)	21. (c)	22. (d)	23. (c)	24. (c)	25. (a)	26. (62.83)	27. (5.14)
28. (–1.15)		29. (d)	30. (a)	31. (3.75)	32. (a)	33. (d)	34. (129.43)	
35. (c)	36. (d)	37. (b)	38. (2)	39. (1244)	40. (b)	41. (a)	42. (5)	43. (c)
44. (a)	45. (a)	46. (d)	47. (a)	48. (d)	49. (a)	50. (b)	51. (3)	52. (0.33)
53. (0.5)	54. (0.5)	55. (a,c,d)	56. (b,d)	57. (b,c)	58. (c,d)	59. (a,d)	60. (b,c,d)	61. (b,c)

Explanations Vector Analysis

1. (c)

For spherical coordinate systems,

$$\vec{dl} = r\sin\theta d\phi \hat{a}_\phi$$

$$\oint \vec{G}\cdot\vec{dl} = \int_0^{2\pi} 15r\hat{a}_\phi \cdot r\sin\theta d\phi\ \hat{a}_\phi$$

$$= 15 \cdot r^2 \cdot \sin\theta(2\pi)$$

$$= 15 \cdot (2)^2 \times \sin 30°(2\pi)$$

$$\oint \vec{G}\cdot\vec{dl} = 60\pi$$

2. (c)

Divergence (Curl $\vec{A}$) = 0

3. (d)

$$(\hat{a}_x \times \hat{a}_x) = 0$$

Since cross product with same vector is zero because $\theta = 0$ so $\sin\theta = 0$

$$\hat{a}_x \times \hat{a}_y = \hat{a}_z$$

$$\hat{a}_y \times \hat{a}_x = -\hat{a}_z$$

$$(\hat{a}_x \times \hat{a}_y) + (\hat{a}_y \times \hat{a}_x) = \hat{a}_z + (-\hat{a}_z) = 0$$

4. (a)

curl $\vec{F}$ = 0 that means vector $\vec{F}$ is irrotational

div $\vec{F}$ = 0 that means vector $\vec{F}$ is solenoidal since flux coming out from a solenoid is zero.

div(grad ϕ) = $\nabla \cdot (\nabla\phi) = 0 \rightarrow$ Laplace equation

div(divϕ) = not defined because div$\phi \rightarrow$ scalar quantity and div of a scalar quantity is not defined.

5. (d)

$$\nabla^2 V = \bar{\nabla}\cdot(\bar{\nabla}V)$$

= divergence of gradient of V

6. (b)

Let the unit vector be given by $\vec{a}_R$.

Now, $\vec{R}$ = Difference of two vectors