POSTAL Book Package 2023

Electrical Engineering

Conventional Practice Sets

Electrical & Electronic Measurements

Contents

MADE EASY Publications

1 CHAPTER

Errors in Measurements

Q.1 Four ammeters M1, M2, M3 and M4 with the following specifications are available:

Instrument	Type	Full scale value (A)	Accuracy % of FS
M1	$3\frac{1}{2}$ digit dual slope	20	±0.10
M2	PMMC	10	±0.20
M3	Electro-dynamic	5	±0.50
M4	Moving-iron	1	±1.00

A current of 1 A is to be measured. Calculate the error in the reading of each instruments and which meter has least error?

Solution:

Error in reading of first meter = FSD × accuracy = $20\times\frac{\pm 0.1}{100} = \pm 0.02$

Error in reading of second meter = $10\times\frac{\pm 0.2}{100} = \pm 0.02$

Error in reading of third meter = $5\times\frac{\pm 0.5}{100} = \pm 0.025$

Error in reading of fourth meter = $1\times\frac{\pm 1.00}{100} = \pm 0.01$

Fourth meter has least error.

Q.2 The dead zone of a certain pyrometer is 0.125 percent of the span. The calibration is 800°C to 1800°C. What temperature change must occur before it is detected?

Solution:

Given that,

$$\text{Span} = 1800° - 800° = 1000°\text{C}$$

$$\text{Dead zone} = \frac{0.125}{100}\times 1000° = 1.25°\text{C}$$

A change of 1.25°C must occur before it is detected.

Q.3 The limiting errors for a four dial resistance box are:
Units : ±0.2%
Tens : ±0.1%
Hundreds : ±0.05%
Thousands : ±0.02%
If the resistance value is set at 4325 Ω calculate the limiting error for this value.

Solution:

Thousand is set at 4000 Ω and error

$$= \pm 4000 \times \frac{0.02}{100} = \pm 0.8\ \Omega$$

$$\text{For hundred error} = \pm 300 \times \frac{0.05}{100} = \pm 0.15\ \Omega$$

Similarly,
$$\text{For ten error} = \pm 20 \times \frac{0.1}{100} = \pm 0.02\ \Omega$$

and
$$\text{For unit error} = \pm 5 \times \frac{0.2}{100} = \pm 0.01\ \Omega$$

Hence,
$$\text{Total error} = \pm (0.8 + 0.15 + 0.02 + 0.01)\ \Omega = \pm 0.98\ \Omega$$

$$\%\ \text{Relative error} = \frac{0.98}{4325} \times 100 = 0.0226\%$$

Q.4 Current was measured during a test as 30.4 A flowing in a resistor of 0.105 Ω. It was discovered later that the ammeter was low by 1.2% and the marked resistance was high by 0.3 %. Find the true power as percentage of the power that was originally calculated.

Solution:

Measured current, $I = 30.4$ A

Resistance, $R = 0.105\ \Omega$

Measured power, $P = I^2R = (30.4)^2 (0.105) = 97.037$ W

$$\text{True value of current} = I(1 + \in_r) = 1.2\%$$

$$\text{as, } \in_r = 0.012$$

$$= 30.4\ (1 + 0.012) = 30.765\ \text{A}$$

$$\text{True value of resistance} = R(1 - \in_r)$$

$$= 0.105\ (1 - 0.003) = 0.1047\ \Omega$$

$$\text{True power} = I^2R = (30.765)^2 (0.1047) = 99.097\ \text{W}$$

Now,
$$\frac{\text{True power}}{\text{Measured power}} \times 100 = \frac{99.097}{97.037} \times 100 = 102\%$$

Q.5 The following measurement are obtained on a single-phase load:

$$V = 200\ \text{V} \pm 1\%,\ I = 5\ \text{A} \pm 1\% \text{ and } P = 555\ \text{W} \pm 2\%$$

If the power factor is calculated using these measurements. What is the calculated power factor in the worst case error?

Solution:

Given that, Voltage, $V = 220 \pm 1\%$,

Current, $I = 5 \pm 1\%$

Power, $P = 555 \pm 2\%$

$$P = VI \cdot \cos(\phi)$$

⇒ Power factor,
$$\text{p.f} = \cos(\phi) = \frac{P}{V.I}$$

$$\text{p.f.} = \cos(\phi) = \frac{555 \pm 2\%}{(220 \pm 1\%)(5 \pm 1\%)} = \frac{555}{220 \times 5} \pm 4\%$$

$$\text{p.f.} = \cos(\phi) = 0.5 \pm 4\%$$

Q.6 An 820 Ω resistance with an accuracy of ±10% carries a current of 10 mA. The current was measured by an analog meter of 25 mA range with an accuracy of ±2% of full scale. Compute the power dissipated in the resistor and determine the accuracy of the result.

Solution:

Resistance, $R = (820 \pm 10\%)\ \Omega$

Current, $I = 10$ mA

Full scale current $= 25$ mA

Accuracy in current $= \pm 2\%$ of FSD

$= \pm 2\% \times 25 \text{ mA} = 0.5 \text{ mA}$

$\therefore \quad I = 10 \text{ mA} \pm 0.5 \text{ mA}$

or $\quad I = (10 \text{ mA} \pm 5\%)$ mA

Power, $P = I^2 R$

$P = (10 \text{ mA})^2 \cdot (820) = 0.082$ W

Taking log on both sides, $\log P = \log (I^2 R)$

Differentiating both sides, $$\frac{\partial P}{P} = 2\frac{\partial I}{I} + \frac{\partial R}{R}$$

$$\therefore \quad \frac{\partial P}{P} = 2 \times 5\% + 10\%$$

$$\frac{\partial P}{P} = 10\% + 10\%$$

$$\frac{\partial P}{P} = 20\%$$

$\therefore \quad P = 0.082 \text{ W} \pm 20\%$

Q.7 A variable w is related to three other variables x, y, z as $w = xy/z$. The variables are measured with meters of accuracy ± 0.5% reading, ± 1% of full scale value and ±1.5% reading. The actual readings of the three meters are 80, 20 and 50 with 100 being the full scale value for all three. Find the maximum limiting error in the measurement of variable w.

Solution:

Full scale reading of all three $= 100$

Readings of $x = 80$

Readings of $y = 20$

Reading of $z = 50$

$$\delta x = \pm 0.5\% \text{ of reading} = \pm\frac{0.5 \times 80}{100} = \pm 0.4$$

$$\delta y = \pm 1\% \text{ of full reading} = \pm\frac{1 \times 100}{100} = \pm 1$$

$$\delta z = \pm 1.5\% \text{ of reading} = \pm\frac{1.5 \times 50}{100} = \pm 0.75$$

Given, $$w = \frac{xy}{z}$$

Taking log, we get, $\log w = \log x + \log y - \log z$

Differentiating w.r.t. w, we get $$\frac{\delta w}{w} = \frac{\delta x}{x} + \frac{\delta y}{y} - \frac{\delta z}{z}$$

For maximum limiting error, $$\frac{\delta w}{w} = \pm\left(\frac{0.4}{80} + \frac{1}{20} + \frac{0.75}{100}\right) \times 100 = \pm 6.25\%$$

Q.8 The following readings were observed when measuring a voltage:

S.No.	1	2	3	4	5	6	7	8
Volts	532	548	543	535	546	531	543	536

Calculate:
(i) Average deviation
(ii) Standard deviation
(iii) Probable error of one reading.

Solution:

Given that:

S.No.	1	2	3	4	5	6	7	8
Volts	532	548	543	535	546	531	543	536

Deviations,

$$d_1 = 532 - 539.25 = -7.25$$
$$d_2 = 548 - 539.25 = 8.75$$
$$d_3 = 543 - 539.25 = 3.75$$
$$d_4 = 535 - 539.25 = -4.25$$
$$d_5 = 546 - 539.25 = 6.75$$
$$d_6 = 531 - 539.25 = -8.25$$
$$d_7 = 543 - 539.25 = 3.75$$
$$d_8 = 536 - 539.25 = -3.25$$

(i) Average deviation, $$\bar{d}_{avg.} = \frac{|d_1|+|d_2|+|d_3|+|d_4|+|d_5|+|d_6|+|d_7|+|d_8|}{8} = 5.75$$

(ii) Standard deviation, $$\sigma = \sqrt{\frac{(d_1)^2+(d_2)^2+(d_3)^2+(d_4)^2+(d_5)^2+(d_6)^2+(d_7)^2+(d_8)^2}{n-1}}$$

$$= \sqrt{\frac{299.5}{7}} = \sqrt{42.7857} = 6.541$$

(iii) Probable error $= 0.6745\,\sigma = 0.6745 \times 6.541 = 4.4119$

Q.9 Two resistors R_1 and R_2 are connected in series and then in parallel. The values of resistance are:

$$R_1 = 100.0 \pm 0.1\ \Omega\ ; \quad R_2 = 50 \pm 0.03\ \Omega$$

Calculate the uncertainty in the combined resistance for both series and parallel arrangements.

Solution:

When the two resistances are connected in series the resultant resistance is

$$R = R_1 + R_2$$

$$\frac{\partial R}{\partial R_1} = 1 \quad \text{and} \quad \frac{\partial R}{\partial R_2} = 1$$

Hence uncertainty in the total resistance is

$$\boxed{W_R = \pm\sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 W_{R_1}^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 W_{R_2}^2}}$$

$$W_R = \pm\sqrt{(1)^2 \times (0.1)^2 + (1)^2 \times (0.03)^2} = \pm 0.1044\ \Omega$$

The total resistance is $R = 100 + 50 = 150\ \Omega$

and can be expressed as $R = 150 \pm 0.1044\ \Omega$

When the two resistances are connected in parallel the resultant resistance is

$$R = \frac{R_1 R_2}{(R_1 + R_2)} = \frac{100 \times 50}{100 + 50} = 33.3333\ \Omega$$

$$R = \frac{R_1 R_2}{(R_1 + R_2)}$$

$$\frac{\partial R}{\partial R_1} = \frac{R_2(R_1 + R_2) - R_1 R_2(1)}{(R_1 + R_2)^2} = \frac{R_2}{R_1 + R_2} - \frac{R_1 R_2}{(R_1 + R_2)^2} = \frac{50}{150} - \frac{100 \times 50}{(150)^2} = 0.1111$$

$$\frac{\partial R}{\partial R_2} = \frac{R_1(R_1 + R_2) - R_1 - R_2(1)}{(R_1 + R_2)^2} = \frac{R_1}{R_1 + R_2} - \frac{R_1 R_2}{(R_1 + R_2)^2}$$

$$= \frac{100}{150} - \frac{100 \times 50}{(150)^2} = 0.4444$$

Hence uncertainty in total resistance is

$$W_R = \pm\sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 W_{R_1}^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 W_{R_2}^2} = \pm\sqrt{(0.1111)^2 \times (0.1)^2 + (0.4444)^2 \times (0.03)^2} = \pm 0.01735\ \Omega$$

The total resistance can be written as,

$$R = 33.3333 \pm 0.01735\ \Omega$$

Q.10 **A power transformer was tested to determine losses and efficiency. The input power was measured as 3650 W and the delivered output power was 3385 W, with each reading in doubt by ±10 W. Calculate the percentage uncertainty in the losses of the transformer and the percentage uncertainty in the efficiency of the transformer, as determined by the difference in input and output power readings.**

Solution:

Given that, Input power, $P_i = 3650$ W, Output power, $P_0 = 3385$ W

Uncertainties, $W_{Pi} = \pm 10$ W

$W_{P0} = \pm 10$ W

Losses in transformer, $P_L = P_i - P_0$

$P_L = 3650 - 3385 = 265$ W

$\therefore$ $$\frac{\partial P_L}{\partial P_i} = 1, \quad \frac{\partial P_L}{\partial P_0} = -1$$

$$\text{Uncertainty in loss} = \pm\sqrt{\left(\frac{\partial P_L}{\partial P_i}\right)^2 W_{Pi}^2 + \left(\frac{\partial P_L}{\partial P_0}\right)^2 W_{P0}^2}$$

$$\text{Uncertainty in loss} = \pm\sqrt{(1)^2 10^2 + (-1)^2 10^2} = \pm 10\sqrt{2}\ \text{W}$$

$$\%\ \text{Uncertainty in loss} = \frac{\pm 10\sqrt{2}}{265} \times 100 = \pm 5.34\%$$

Efficiency, $$\eta = \frac{P_0}{P_i}$$

$$\frac{\partial \eta}{\partial P_i} = -\frac{P_0}{P_i^2} = -\frac{3385}{(3650)^2} = -2.54 \times 10^{-4}$$