POSTAL Book Package 2023

GATE • PSUs

Instrumentation Engineering

Objective Practice Sets

Communication

Contents

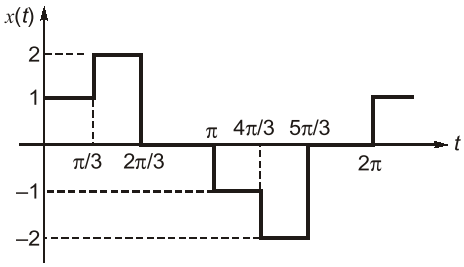
MADE EASY Publications

1 CHAPTER
Fourier Analysis of Signal Energy and Power Signals

MCQ and NAT Questions

Q.1 If $G(f)$ represents the Fourier transform of a signal $g(t)$ which is real and odd symmetric in time then
(a) $G(f)$ is complex
(b) $G(f)$ is imaginary
(c) $G(f)$ is real
(d) $G(f)$ is real and non-negative

Q.2 Compute the amplitude of the fundamental component of the waveform given in figure.



(a) 0 (b) 1.00
(c) 1.603 (d) 1.712

Q.3 Let $x(t)$ be a signal with its Fourier transform $X(j\omega)$ suppose we are given the following facts.
1. $x(t)$ is real.
2. $x(t) = 0$ for $t \le 0$.
3. $\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{Re}\{X(j\omega)\}e^{j\omega t}d\omega = 2|t|e^{-|t|}$.

then a closed form expression for $x(t)$ is
(a) $2e^{-t}u(t)$ (b) $e^{-|t|}$
(c) $te^{-2t}u(t)$ (d) $2te^{-t}u(t)$

Q.4 **Assertion (A):** If two signals are orthogonal they will also be orthonormal.
Reason (R): If two signals are orthonormal they also will be orthogonal.
(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not a correct explanation of A.
(c) A is true, but R is false.
(d) A is false, but R is true.

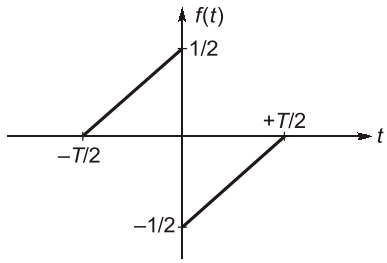
Q.5 Consider the following statements:
The normalized power, $S \equiv v^2(t)$ can be defined as the
1. instantaneous power divided by the maximum power in the circuit.
2. time average power that appears in a one ohm resistor.
3. Total power consumed by the circuit divided by the average power consumed in that circuit.
4. the mean square value of $v(t)$.

Which of the above statements is/are correct?
(a) 2 only (b) 1 and 2
(c) 2 and 3 (d) 2 and 4

Q.6 The auto correlation function of a rectangular pulse of duration T is
(a) A rectangular pulse of duration T
(b) A rectangular pulse of duration $2T$
(c) A triangular pulse of duration T
(d) A triangular pulse of duration $2T$

Q.7 The amplitude spectrum of Gaussian pulse is
(a) uniform (b) a sine function
(c) gaussian (d) an impulse function

Q.8 A function $f(t)$ is shown in figure.



The Fourier transform $F(\omega)$ of $f(t)$ is
(a) real and even function of ω
(b) real and odd function of ω
(c) imaginary and odd function of ω
(d) imaginary and even function of ω

Q.26 If $x(t)=\frac{1}{t}$, then Hilbert transform of $x(t)$ will be $-K\delta(t)$. Then the value of K will be _____.

Q.27 What will be the value of following integral _____?

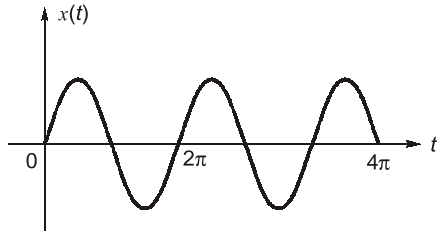
$$\int_{-\infty}^{\infty} S_a^2(2t)dt$$

where $S_a(t)$ = Sampling function $S_a(t)=\frac{\sin t}{t}$

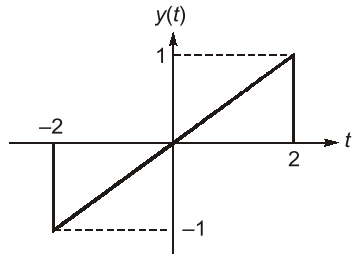
Multiple Select Questions (MSQs)

Q.28 Consider the real signals shown below:

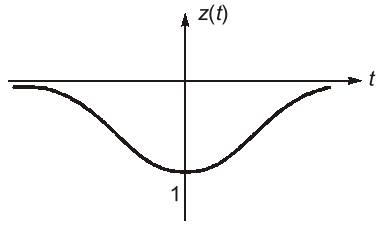
(i)



(ii)



(iii)



Which of the below statements are correct?

(a) The Fourier transform of $y(t)$ and $z(t)$ is real-valued.

(b) The Fourier transform of $x(t)$ is conjugate symmetric.

(c) $\int_{-\infty}^{\infty} X(j\omega)\cdot d\omega = 0$

(d) $\int_{-\infty}^{\infty} Z(j\omega)\cdot d\omega = 0$

Q.29 Consider a continuous-time ideal low pass filter having the frequency response

$$H(j\omega)=\begin{cases}1, & |\omega|\le 80\\ 0, & |\omega|>80\end{cases}$$

The input to this filter is a signal $x(t)$ with fundamental frequency ω_0 = 10 rad/sec and Fourier series coefficients $X[k]$. If $y(t)$ represents the output of the filter and it is given that $Y[k]=X[k]$, then the values of k for which $X[k]$ is non-zero are:

(a) 3 (b) 7

(c) 10 (d) 12

Q.30 For a periodic signal $x(t)$, the Fourier series coefficients are given as below:

$$X[k]=\begin{cases}5, & k=0\\ j\left(\frac{1}{2}\right)^{|k|}, & \text{otherwise}\end{cases}$$

Which of the below statements are correct?

(a) $x(t)$ is real signal.

(b) $x(t)$ is an even signal.

(c) $\frac{dx(t)}{dt}$ is an odd signal.

(d) $x(t)$ is an energy signal.

■■■■

Answers Fourier Analysis of Signal Energy and Power Signals

1.	(b)	2.	(a)	3.	(d)	4.	(d)	5.	(d)	6.	(d)	7.	(c)
8.	(c)	9.	(b)	10.	(a)	11.	(c)	12.	(a)	13.	(a)	14.	(d)
15.	(b)	16.	(c)	17.	(b)	18.	(b)	19.	(b)	20.	(d)	21.	(a)
22.	(c)	23.	(b)	24.	(a)	25.	(4.5)	26.	(3.14)	27.	(1.57)	28.	(b, c)
29.	(a, b)	30.	(b, c)										

Explanations Fourier Analysis of Signal Energy and Power Signals

1. (b)

Function, $g(t)$	Fourier Transform, $G(f)$
Real and odd	Imaginary and odd
Real and even	Real and even
Imaginary and odd	Real and odd
Imaginary and even	Imaginary and even

2. (a)

$$a_0 = \frac{1}{T}\cdot\int_0^T f(t)\,dt$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(t)\,dt = \frac{1}{2\pi}\times\left[\int_0^{2\pi/3} f(t)\,dt + \int_{\pi}^{5\pi/3} f(t)\,dt\right]$$

$$= \frac{2}{2\pi}\left[1\cdot\frac{\pi}{3}+2\cdot\frac{\pi}{3}-1\cdot\frac{\pi}{3}-2\cdot\frac{\pi}{3}\right]=0$$

3. (d)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

$\Rightarrow$ Real $(x(t)) = 2|t|e^{|t|}$

Since, $x(t) = 0, \quad t \le 0$

$\Rightarrow$ $x(t) = 2te^{-t} \quad t > 0$

$\Rightarrow$ $x(t) = 2te^{-t}u(t)$

4. (d)

Orthogonal: Two vector are perpendicular i.e. their dot product is zero.

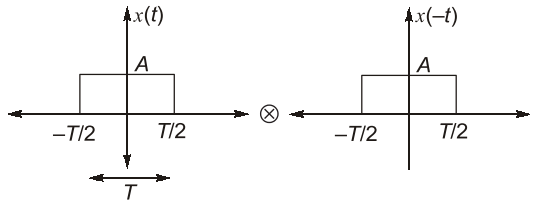
Orthonormal: Two vectors are perpendicular and are of unit length.

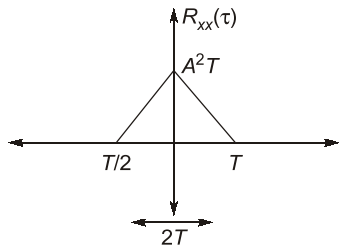
5. (d)

- Normalized power $\rightarrow$ Power in 1 Ω resistor
- $P_N = V^2(t)$, normalized power is average and mean of voltage required.
- $V_{rms} = \sqrt{P_N}$

6. (d)

ACF or Auto correlation function is nothing but convolution of $x(t)$ with time reversed form of $x(t)$, i.e. $x(t)$





ACF is a triangular pulse of duration 2T.

7. (c)

Amplitude spectrum of Gaussian pulse is Gaussian.

8. (c)

Signal is odd,

$$x(t) = -x(-t)$$

Signal is half symmetric

$$x(t) = x\left(t+\frac{T_0}{2}\right)$$

$\therefore$ contains odd harmonic.

Signal $f(t)$ is real and odd,

$\therefore$ $F(\omega)$ is imaginary and odd.

9. (b)

$$x(t) = A; \qquad -\frac{T}{2} \le t \le \frac{T}{2}$$

$$= 0; \qquad \text{for all other } t$$