

Instrumentation Engineering

Communication

Comprehensive Theory

with Solved Examples and Practice Questions



MADE EASY
Publications



MADE EASY Publications Pvt. Ltd.

Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016

E-mail: infomep@madeeasy.in

Contact: 011-45124660, 8860378007

Visit us at: www.madeeasypublications.org

Communication

© Copyright, by MADE EASY Publications Pvt. Ltd.

All rights are reserved. No part of this publication may be reproduced, stored in or introduced into a retrieval system, or transmitted in any form or by any means (electronic, mechanical, photo-copying, recording or otherwise), without the prior written permission of the above mentioned publisher of this book.

First Edition: 2015

Second Edition: 2016

Third Edition: 2017

Fourth Edition: 2018

Fifth Edition: 2019

Sixth Edition: 2020

Seventh Edition: 2021

Eighth Edition : 2022

Contents

Communication

Chapter 1

Introduction to Communication Systems 1-5

1.1	Historical Sketch	1
1.2	Why Study Communication.....	1
1.3	What is Communication	1
1.4	Communication Model	2
1.5	Modes of Communication	3
1.6	Types of Modulation.....	4
1.7	An Exam Oriented Approach	5

Chapter 2

Basics of Signal and System..... 6-45

2.1	Signal and System	6
2.2	Time Domain and Frequency Domain Representation of a Signal	12
2.3	Signals Versus Vectors.....	13
2.4	Orthogonal Signal Set	14
2.5	The Fourier Series.....	14
2.6	Fourier Transforms of Signals.....	19
2.7	Correlation of Signals.....	28
2.8	Transmission of Signals through Linear Time-Invariant Systems.....	29
2.9	Ideal Filters.....	31
2.10	Hilbert Transform.....	33
2.11	Pre-envelopes.....	38
2.12	Complex Envelopes of Band-Pass Signals	40
2.13	Canonical Representation of Band-Pass Signals	41
2.14	Complex Low-Pass Representations of Band-Pass Systems.....	42
	<i>Student Assignments-1</i>	44
	<i>Student Assignments-2</i>	45

Chapter 3

Theory of Random Variable & Noise...46-107

3.1	Randomness in the Real World.....	46
3.2	Random Experiments	46
3.3	Relation of the Model to the Real World	49
3.4	Conditional Probability	49
3.5	Statistical Independence	49
3.6	A Communication Problem	50
3.7	Random Variable.....	51
3.8	Distribution Function.....	52
3.9	Probability Density Function.....	53
3.10	Some Special Random Variables.....	55
3.11	Mean, Variance and Moment	58
3.12	Transformation of Variables.....	60
3.13	Functions of One Random Variable	62
3.14	Two Dimensional Random Variables with Extension to N-dimension.....	63
3.15	Functions of Random Variables.....	67
3.16	Statistical Independence	72
3.17	Distribution and Density of A Sum of Random Variables.....	74
3.18	Central Limit Theorem	79
3.19	Stochastic Processes.....	80
3.20	Ergodic Processes.....	84
3.21	Transmission of a Weakly Stationary Process through a Linear Time-Invariant Filter	85

3.22	Power Spectral Density of a Weakly Stationary Process ...	86
3.23	Noise	90
3.24	Noise Calculations	93
3.25	Noise Figure.....	96
3.26	Ideal Low-pass Filtered White Noise.....	99
3.27	Narrowband Noise	102
	<i>Student Assignments-1</i>	106
	<i>Student Assignments-2</i>	107

Chapter 4

Amplitude Modulation..... 108-146

4.1	Amplitude Modulation.....	108
4.2	Single Tone Amplitude Modulation.....	112
4.3	Power Relations in AM.....	113
4.4	Modulation by a Multiple Single Tone Signals (Multi-Tone Modulation)	116
4.5	Generation of AM Waves	120
4.6	Double-Sideband Suppressed-Carrier Modulation.....	126
4.7	Single Side-Band.....	136
4.8	Vestigial Side-band Modulation (VSB).....	141
4.9	Independent Single Sideband (ISB).....	142
	<i>Student Assignments-1</i>	145
	<i>Student Assignments-2</i>	146

Chapter 5

Angle Modulation..... 147-183

5.1	Time Domain Description of Angle Modulation.....	147
5.2	Single-Tone Frequency Modulation	150
5.3	Spectrum Analysis of Sinusoidal FM wave	154
5.4	Types of Frequency Modulation (FM)	158
5.5	Generation of Frequency Modulation Waves.....	160
5.6	Demodulation of Frequency Modulation Waves.....	164
5.7	PLL Characteristics	172

5.8	Limiting of FM Waves	179
5.9	Comparison between Amplitude Modulation and Frequency Modulation	181
	<i>Student Assignments-1</i>	182
	<i>Student Assignments-2</i>	182

Chapter 6

AM Transmitters and Receivers 184-221

6.1	Transmitter.....	184
6.2	Receivers.....	188
6.3	Noise Figure of the Receiver	198
6.4	Noise Performance of Continuous Wave Modulation.....	200
6.5	Preemphasis, Deemphasis & SNR Improvement.....	216
	<i>Student Assignments-1</i>	220
	<i>Student Assignments-2</i>	221

Chapter 7

Pulse Modulation..... 222-273

7.1	Analog Communication Versus Digital Communication	222
7.2	Sampling Theory.....	223
7.3	Pulse Amplitude Modulation	232
7.4	Pulse Width Modulation.....	233
7.5	Pulse Position Modulation	235
7.6	PCM (Pulse Code Modulation)	237
7.7	Companding	241
7.8	Mathematical Analysis of PCM	243
7.9	Noise in PCM	251
7.10	Encoding in PCM	251
7.11	Intersymbol Interference	254
7.12	Merits and Demerits of PCM.....	259
7.13	Delta Modulation	259
7.14	Adaptive Delta Modulation (ADM).....	263
7.15	Differential Pulse Code Modulation.....	264
7.16	Multiplexing	266
	<i>Student Assignments-1</i>	272
	<i>Student Assignments-2</i>	273

Chapter 8

Data Transmission Schemes 274-306

8.1	Geometric Representation of Signal	275
8.2	Schwarz Inequality.....	278
8.3	Digital Modulation Schemes	280
8.4	Amplitude Shift Keying (ASK)	280
8.5	Frequency Shift Keying (FSK)	283
8.6	Phase Shift Keying.....	286
8.7	Quadrature Phase Shift Keying	290
8.8	Quadrature Amplitude Modulation.....	295
8.9	Differential Phase Shift Keying (DPSK).....	298
8.10	Minimum Shift Keying(MSK)	299
	<i>Student Assignments-1</i>	304
	<i>Student Assignments-2</i>	305

Chapter 9

Optimum Receivers for AWGN Channels 307-331

9.1	Detection of Signal in Noise	308
9.2	Optimum Detection in a Binary Communication System Using MAP Criteria.....	309
9.3	Probability of Error	311
9.4	Matched Filter.....	312
9.5	Generalized Probability of Error Expression of Digital Signalling Schemes	317
9.6	Probability of Error for Binary Signalling Schemes	319
9.7	Probability of Error in PSK.....	320
9.8	Probability of Error in FSK.....	320
9.9	Probability of Error of QPSK.....	323
9.10	Calculation of Probability of Error Using Constellation Diagram	325
9.11	Probability of Error for 16-QAM.....	328
	<i>Student Assignments-1</i>	330
	<i>Student Assignments-2</i>	331

Chapter 10

Information Theory and Coding.... 332-371

10.1	Need of Information Theory	332
10.2	Information.....	333
10.3	Entropy	334
10.4	Discrete Memoryless Channels	337
10.5	Special Channels.....	338
10.6	Mutual Information	341
10.7	Channel Capacity.....	342
10.8	Entropy of Continuous Channel.....	344
10.9	Additive White Gaussian Noise Channel (AWGN)	344
10.10	Source Coding	345
10.11	Entropy Coding	347
10.12	Channel Coding Theorem	351
10.13	Error Control Coding	353
10.14	Linear Code.....	356
	<i>Student Assignments-1</i>	370
	<i>Student Assignments-2</i>	370

Chapter 11

Multiple Access Technique and Communication Standards 372-389

11.1	Multiple Access Technique.....	372
11.2	Frequency Division Multiple Access (FDMA)	373
11.3	Time Division Multiple Access (TDMA)	374
11.4	Code Division Multiple Access (CDMA)	377
11.5	Standards of Mobile Communication System	382
11.6	Global System for Mobile Communications (GSM).....	384
11.7	Bluetooth.....	387



Introduction to Communication Systems

1.1 Historical Sketch

The development of communication technology has proceeded in step with the development of electronic technology as a whole. For example, the demonstration of telegraphy by Joseph Henry in 1832 and by Samuel F.B. Morse in 1838 followed hard on the discovery of electromagnetism by Oersted and Ampere early in 1820's. Similarly, Hertz's verification late in the 1880's of Maxwell's postulation (1873) predicting the wireless propagation of electromagnetic energy led within 10 years of the radio-telegraph experiments of Marconi and Popov. The invention of diode by Fleming in 1904 and of triode by deForest in 1906 made possible the rapid development of long distance telephony, both by radio and wireless.

1.2 Why Study Communication

The rapidly changing face of technology necessitates learning of new technology. Today the question is no longer in the field of invention but of innovation. The question today in the twenty first century is not how to transmit data from point A to point B but how efficiently can we do it. To be able to answer this question, first we should be able to diagnose the problem. This can be done only by studying communication from the beginning to its modern form.

1.3 What is Communication

In the most fundamental sense, communication involves implicitly the transmission of information from one point to another through a succession of processes, as described here:

1. The generation of a message signal: voice, music, picture, or computer data.
2. The description of that message signal with a certain measure of precisions, by a set of symbols: electrical, audio, or visual.
3. The encoding of these symbols in a form that is suitable for transmission over a physical medium of interest.
4. The transmission of the encoded symbols to the desired destination.
5. The decoding of the reproduction of the original symbols.
6. The re-creation of the original message signal, with a definable degradation in quality; the degradation is caused by imperfections in the system.

There are, of course, many other forms of communication that do not directly involve the human mind in real time. For example, in computer communications involving communication between two or more computers, human decisions may enter only in setting up the programs or commands for the computer, or in monitoring the results.

1.4 Communication Model

The study of communication becomes easier, if we break the whole subject of communication in parts and then study it part by part. The whole idea of presenting the model of communication is to analyse the key concepts used in communication in isolated parts and then combining them to form the complete picture.

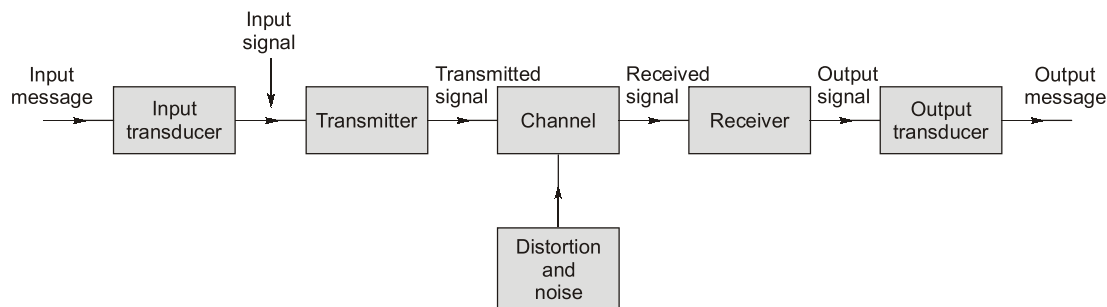


Figure-1.1: Model of communication system

Source

The source originates a message, such as a human voice, a television picture, an e-mail message, or data. If the data is non-electric (e.g., human voice, e-mail text, television video), it must be converted by an **input transducer** into an electric waveform referred to as the **baseband signal** or **message signal** through physical devices such as a microphone, a computer keyboard or a CCD camera.

Transmitter

The transmitter modifies the baseband signal for efficient transmission. The transmitter may consist of one or more subsystems: an A/D converter, an encoder and a modulator. Similarly, the receiver may consist of a demodulator, a decoder and a D/A converter.

Channel

The channel is a medium of choice that can carry the electric signals at the transmitter output over a distance. A typical channel can be a pair of twisted copper wires (telephone and DSL), coaxial cable (television and internet), an optical fibre or a radio link. Channel may be of two types.

1. **Physical channel:** When there is a physical connection between the transmitter and receiver through wires. eg. coaxial cable.
2. **Wireless channel:** When no physical channel is present and transmission is through air. eg. mobile communication.

It is inevitable that the signal will deteriorate during the process of transmission and reception as a result of some distortion in the system, or because of the introduction of noise, which is unwanted energy, usually of random character, present in a transmission system, due to a variety of causes. Since noise will be received together with the signal, it places a limitation on the transmission system as a whole. When noise is severe, it may mask a given signal so much that the signal becomes undetectable and therefore useless. Noise may interfere with signal at any point in a communications system, but it will have its greatest effect when the signal is weakest. This means that noise in the channel or at the input to the receiver is the most noticeable.

Receiver

The receiver reprocesses the signal received from the channel by reversing the signal modifications made at the transmitter and removing the distortions made by the channel. The receiver output is fed to the output transducer, which converts the electric signal to its original form i.e. the message signal.

Destination

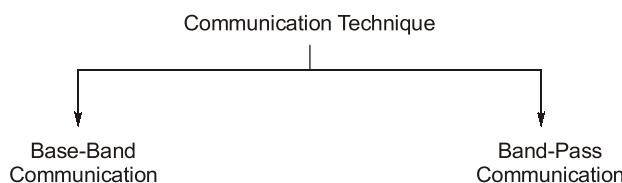
The destination is the unit to which the message is communicated.

1.5 Modes of Communication

There are two basic modes of communication:

1. **Broadcasting**, which involves the use of a single powerful transmitter and numerous receivers that are relatively inexpensive to build. Here information-bearing signals flow only in one direction.
2. **Point-to-point communication**, in which the communication process takes place over a link between a single transmitter and a receiver. In this case, there is usually a bidirectional flow of information-bearing signals, which requires the use of a transmitter and receiver at each end of the link.

1.5.1 Communication Technique



1. Base Band Communication:

It is generally used for short distance communication. In this type of communication message is directly sent to the receiver without altering its frequency.

2. Band Pass Communication:

It is used for long distance communication. In this type of communication, the message signal is mixed with another signal called as the carrier signal for the process of transmission. This process of adding a carrier to a signal is called as modulation.

1.5.2 Need of Modulation

1. To avoid the mixing of signals

All messages lie within the range of 20 Hz - 20 kHz for speech and music, few MHz for video, so that all signals from the different sources would be inseparable and mixed up. In order to avoid mixing of various signals, it is necessary to translate them all to different portions of the electromagnetic spectrum.

2. To decrease the length of transmitting and receiving antenna

For a message at 10 kHz, the antenna length 'l' for practical purposes is equal to $\lambda/4$ (from antenna theory) i.e.,

$$\lambda = \frac{3 \times 10^8}{10 \times 10^3} = 3 \times 10^4 \text{ m}$$

and
$$l = \frac{\lambda}{4} = \frac{3 \times 10^4}{4} = 7500 \text{ m}$$

An antenna of this size is impractical and for a message signal at 1 MHz

$$\lambda = \frac{3 \times 10^8}{10^6} = 300 \text{ m}$$

and
$$l = \frac{\lambda}{4} = 75 \text{ m (practicable)}$$

3. To allow the multiplexing of signals

By translating all signals from different sources to different carrier frequency, we can multiplex the signals and able to send all signals through a single channel.

4. To remove the interference
5. To improve the quality of reception i.e. increasing the value of S/N ratio
6. To increase the range of communication

1.6 Types of Modulation

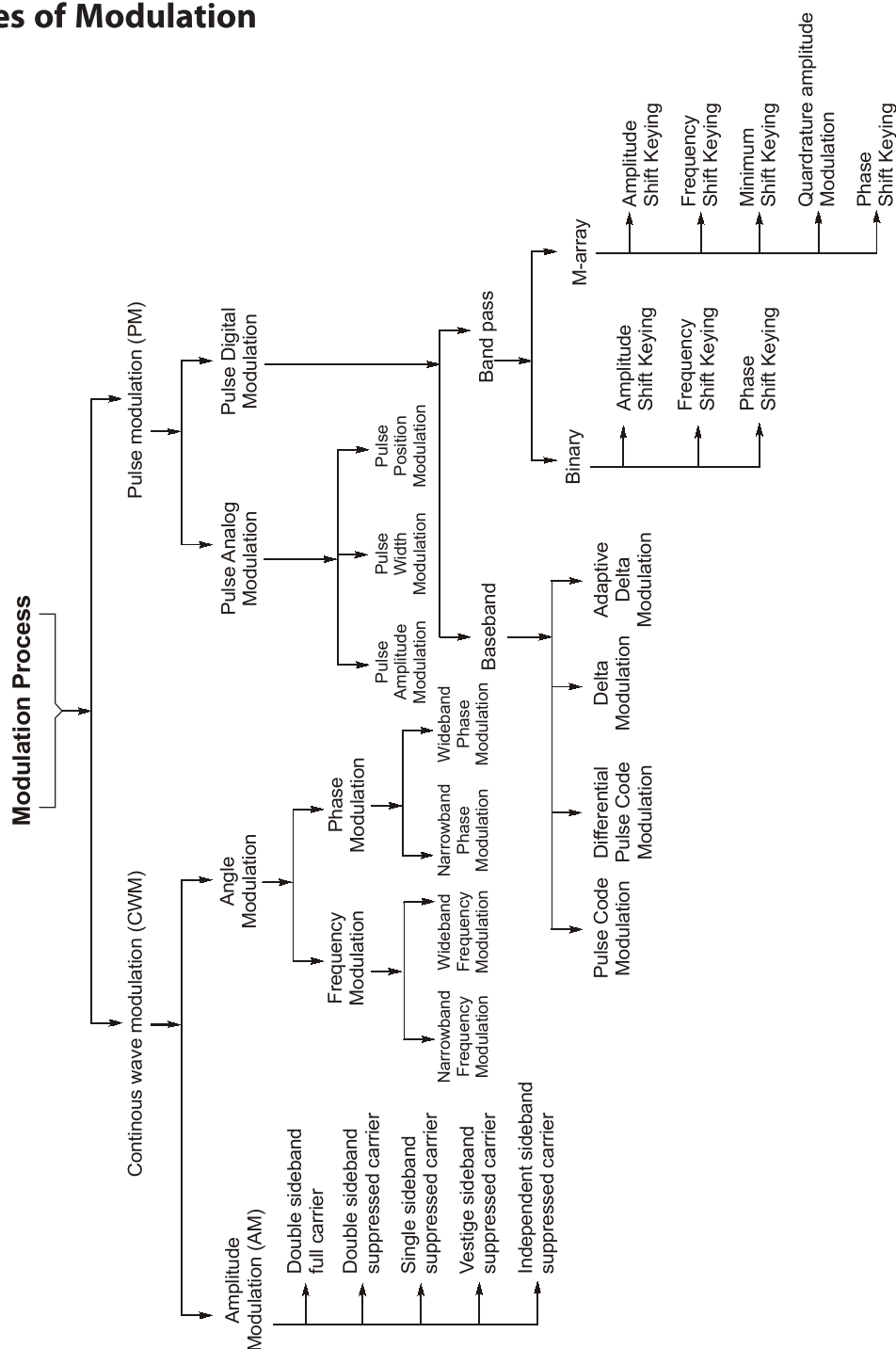


Figure-1.2

1.7 An Exam Oriented Approach

Communication is a modern technology is undergoing many changes. The main focus of a student should be to single out on optimum path in which he develops a theoretically strong background of the subject while keeping in mind that he should be able to solve questions asked in various exams using the theory they have studied. Focusing on one aspect leads to failure in written exam or in the interview. Thus this book and communication both have the same approach and that is “optimization” and being a communication engineer one should have this approach too.

Frequency (f) range	Wavelength (λ) range	EM Spectrum Nomenclature	Typical Application
30 – 300 Hz	$10^7 - 10^6$ m	Extremely low frequency (ELF)	Power line communication
0.3 – 3 kHz	$10^6 - 10^5$ m	Voice frequency (VF)	Face to face speech, communication intercom
3 – 30 kHz	$10^5 - 10^4$ m	Very low frequency (VLF)	Submarine communication
30 – 300 kHz	$10^4 - 10^3$ m	Low frequency (LF)	Marine communication
0.3 – 3 MHz	$10^3 - 10^2$ m	Medium frequency (MF)	AM broadcasting
3 – 30 MHz	$10^2 - 10^1$ m	High frequency (HF)	Landline telephony
30 – 300 MHz	$10^1 - 10^0$ m	Very high frequency (VHF)	FM broadcasting, TV
0.3 – 3 GHz	$10^0 - 10^{-1}$ m	Ultra high frequency (UHF)	TV, Cellular telephony
3 – 30 GHz	$10^{-1} - 10^{-2}$ m	Super high frequency (SHF)	Microwave oven, radar
30 – 300 GHz	$10^{-2} - 10^{-3}$ m	Extremely high frequency (EHF)	Satellite communication, radar
0.3 – 3 THz	0.1 – 1 mm	Experimental	For all new explorations
3 – 430 THz	100 – 0.7 μm	Infrared	LED, Laser, TV remote
430 – 750 THz	0.7 – 0.4 μm	Visible light	Optical communication
750 – 3000 THz	0.4 – 0.1 μm	Ultraviolet	Medical application
> 3000 THz	< 0.1 μm	X-rays, gamma rays, cosmic rays	Medical application

Table-1.1: EM Spectrum



Basics of Signal and System

Introduction

Just as a carpenter requires proper set of tools before he can sit down to make a piece of furniture, in a similar manner a communication engineer needs to know about signals before he can start the process of learning communication.

2.1 Signal and System

The communication technology can be conveniently broken down into three interacting parts.

- Signal processing operations performed.
- The device that performs these operations.
- The underlining physics.

Thus to study the basic form of modulation and signal processing used in the communication it will be fruitful to have a quick review of the concepts of signal and system.

2.1.1 Some Basic Signals

It will be very helpful to study some signals before hand, so that the analysis of the communication system becomes easier. Some important and frequently used signals and their properties are mentioned in this section.

The Impulse Signal

Impulse function is not a function in its strict sense. It is a distributed or generalized function. A generalized function is defined in terms of its effect on other function. The unit impulse function is generalised as any function that follow the following condition:

1. Impulse signal (Dirac delta function):

$$\delta(t) = \begin{cases} \infty; & t = 0 \\ 0; & t \neq 0 \end{cases}$$

$$\text{and} \quad \int_{-\infty}^{\infty} \delta(t) dt = 1$$

2. Unit impulse signal:

$$\delta[n] = \begin{cases} 1; & n = 0 \\ 0; & n \neq 0 \end{cases}$$

Properties of Impulse Function

1. Product property

$$x(t) \delta(t) = x(0) \delta(t)$$

Similarly, $x(t) \delta(t - \alpha) = x(\alpha) \delta(t - \alpha)$

2. Shifting property

$$\int_{-\infty}^{\infty} x(t) \delta(t) dt = x(0)$$

Similarly,

$$\int_{-\infty}^{\infty} x(t) \delta(t - \alpha) dt = x(\alpha)$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

3. Scaling property

$$\delta(\alpha t) = \frac{1}{|\alpha|} \delta(t)$$

Example 2.1

Find the impulse function form if $x(t) = 4t^2 \delta(2t - 4)$, where $x(t)$ is an arbitrary signal.

Solution:

$$\begin{aligned} x(t) &= 4t^2 \delta(2t - 4) \\ &= 4t^2 \delta\{2(t - 2)\} \\ &= 4t^2 \cdot \frac{1}{2} \delta(t - 2) \quad \dots \text{from scaling property} \\ &= 2t^2 \delta(t - 2) \end{aligned}$$

Now, from product property we have,

$$x(t) \delta(t - \alpha) = x(\alpha) \delta(t - \alpha)$$

So, $x(t) = 2t^2 \Big|_{t=2} \cdot \delta(t - 2) = 8 \delta(t - 2)$

Example 2.2

Let $\delta(t)$ denote the delta function. The value of the integral

$$\int_{-\infty}^{\infty} \delta(t) \cos\left(\frac{3t}{2}\right) dt \text{ is}$$

(a) 1

(b) -1

(c) 0

(d) $\pi/2$

Solution: (a)

We know,

$$\int_{-\infty}^{\infty} \delta(t) x(t) dt = x(0)$$

So here,

$$\int_{-\infty}^{\infty} \delta(t) \cos\left(\frac{3t}{2}\right) dt = \cos 0 = 1$$

Do you know? Impulse signals do not occur naturally but they are important functions providing a mathematical frame work for the representation of various processes and signals. These come under a special class of functions known as generalized functions.

Gate Function/Rectangular Pulse

Let us consider a rectangular pulse as shown in figure below:

$$x(t) = A \operatorname{rect}(t) = \begin{cases} A, & \text{for } -\frac{1}{2} < t < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

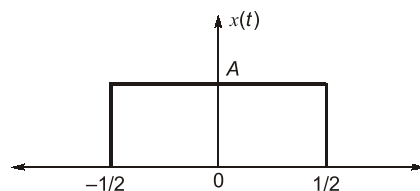


Figure-2.1

$$x(t) = A \operatorname{rect}\left(\frac{t}{\tau}\right) = \begin{cases} A, & \text{for } -\frac{\tau}{2} < t < \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

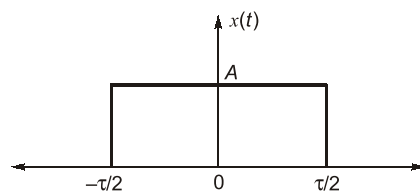


Figure-2.2

Step Signal

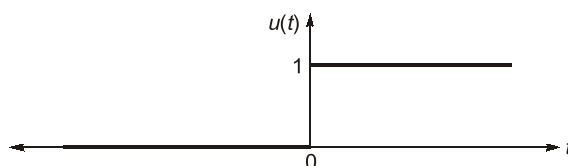


Figure-2.3: Continuous-time version of the unit-step function of unit amplitude

The continuous-time version of the unit-step function is defined by

$$u(t) = \begin{cases} 1; & t > 0 \\ 0; & t < 0 \end{cases}$$

NOTE



- Figure depicts the unit-step function $u(t)$. It is said to exhibit discontinuity at $t = 0$, since the value of $u(t)$ changes instantaneously from 0 to 1 when $t = 0$. It is for this reason that we have left out the equal sign in equation; that is $u(0)$ is undefined.
- Unit step function denote sudden change in real time and a frequency or phase selectivity in frequency domain.

There is one more definition of unit step function.

$$u(t) = \begin{cases} 0 & ; t < 0 \\ 1/2 & ; t = 0 \\ 1 & ; t > 0 \end{cases}$$

Properties of Unit-Step Function

- $u(t - t_0) = [u(t - t_0)]^2 = u[u(t - t_0)]^k$, with k being any positive integer.
- $u(at - t_0) = u\left(t - \frac{t_0}{a}\right)$; $a > 0$

$$3. \quad \delta(t) = \frac{d}{dt} u(t)$$

$$4. \quad u(t) = \int_{-\infty}^t \delta(\tau) d\tau = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

Do you know? The unit-step function $u(t)$ may also be used to construct other discontinuous waveforms. The value at $t = 0$ gives rise to Gibb's phenomenon when unit step function is constructed by sinusoidal signals.

Sampling/Interpolating/Sinc Function

The function $\frac{\sin \pi x}{\pi x}$ is the "sine over argument" function and it is denoted by "sinc (x)". It is also known as "filtering function".

Mathematically,

$$\begin{aligned} \text{sinc}(x) &= \frac{\sin \pi x}{\pi x} \\ &= \text{Sa}(\pi x) \end{aligned}$$

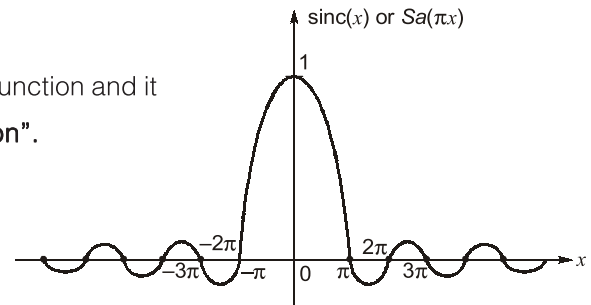


Figure-2.4 : Sinc Function

Do you know? Just like impulse function $\delta(x)$ is also a conceptual function since it can not be realized.

The Unit-Ramp Function

The ramp function $r(t)$ is a linearly growing function for positive values of independent variable t . The ramp function shown in figure is defined by

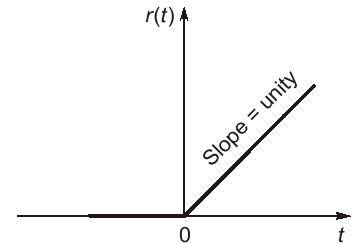
$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

or

$$r(t) = tu(t)$$

The ramp function is obtained by integrating the unit step function

$$\int_{-\infty}^t u(\tau) d\tau = r(t)$$



The relationship between the impulse, step and ramp signals are represented below:

Remember: Relationship between impulse, step and ramp signals

$$\begin{aligned} \delta(t) &\xrightarrow{\text{Integrate}} u(t) \xrightarrow{\text{Integrate}} r(t) \\ r(t) &\xrightarrow{\text{Differentiate}} u(t) \xrightarrow{\text{Differentiate}} \delta(t) \end{aligned}$$

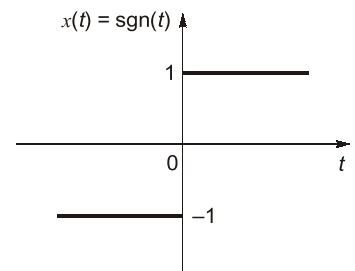
Unit Signum Function

The unit signum function shown in figure is defined as follows

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

This function can be expressed in terms of unit step function as

$$\text{sgn}(t) = -1 + 2u(t)$$



Unit Signum Function

2.1.2 Signal-Classification

Continuous-Time and Discrete-Time signals

The signals that are defined at each instant of time are known as continuous time signals. However, if the signals are defined only at certain time instants, it is called as discrete-time signals.

Based upon above discussion, four combinations are possible:

- Continuous time continuous amplitude signal (Analog signal)
- Continuous time discrete amplitude signal (Quantized signal)
- Discrete time continuous amplitude signal (Sampled signal)
- Discrete time discrete amplitude signal (Digital signal)

Analog and Digital Signal

If the amplitude of the signal can take all possible values in its dynamic range, it is called as analog signal. On the other hand, a digital signal is one whose amplitude take some specific values in its dynamic range.

Periodic and Aperiodic Signals

A signal is said to be periodic if it repeats itself after a certain time interval. For a signal to be periodic, it must satisfy the following condition.

1. It should exist for all values of 't'.
 2. $x(t) = x(t + T)$, where T is the least value after which the signal repeats itself.
 3. The value of T should be a fixed positive constant.
- ' T ' is referred as fundamental period.

Any signal which do not follow these conditions are termed as aperiodic signal.

NOTE



Periodicity of Signal $x_1(t) + x_2(t)$:

A signal $x(t)$ that is a linear combination of two periodic signals, $x_1(t)$ with fundamental period T_1 and $x_2(t)$ with fundamental period T_2 as follows:

$$x(t) = x_1(t) + x_2(t)$$

is periodic if, $\frac{T_1}{T_2} = \frac{m}{n}$ = a rational number

Period of $x(t)$, $T = nT_1 = mT_2$
or, $T = \text{LCM}(T_1, T_2)$

Deterministic and Random Signal

A signal is said to be deterministic, if they can be completely represented by a mathematical expression at any instance of time. Signals, which cannot be represented by any mathematical expression is called random signal.

Note: For analysis purpose random signal can also be approximated by their statistical property.

Energy Signals and Power Signals

$x(t)$ is an energy signal if

$$0 < E < \infty \text{ and } P = 0$$

where 'E' is the energy and 'P' is the power of the signal $x(t)$.

For a continuous-time signal (CTS),

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

For an energy signal, energy is finite while power is zero.

NOTE



If $x(t) \longrightarrow E$, [where, E is energy of $x(t)$]

then $x\left(\frac{t}{\alpha}\right) \longrightarrow \alpha E$

$$x(\alpha t) \longrightarrow \frac{E}{\alpha}$$

$$ax(t) \longrightarrow a^2 E$$

$x(t)$ is a Power Signal if

if, $0 < P < \infty$ and

$$E = \infty$$

where

E = Energy of signal $x(t)$

P = Power of signal $x(t)$

Almost all the practical periodic signals are “power signals”, since their average power is finite and non-zero. For a CTS, the average power of a signal $x(t)$ is,

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

NOTE



• If $x(t) = A \cos \omega t$ or $A \sin \omega t$, then $P_x = A^2/2$

• If $x(t) = Ae^{\pm j\omega t} \Rightarrow P_x = A^2$

• If $x(t) = A \Rightarrow P_x = A^2$

• If $x(t) \longrightarrow P$, then $x\left(\frac{t}{\alpha}\right) \longrightarrow P$

$x(\alpha t) \longrightarrow P$ and $ax(\alpha t) \longrightarrow a^2 P$

• For an **unit step signal**, $x(t) = u(t)$ and $P_x = \frac{1}{2}$

Energy Signal	Power Signal
1. The total energy is obtained using $E = \lim_{T \rightarrow \infty} \int_{-T}^T x(t) ^2 dt$	The average power is obtained $P = \lim_{T \rightarrow \infty} \int_{-T}^T \frac{1}{2T} x(t) ^2 dt$
2. For the energy signal, $0 < E < \infty$, and the average power $P = 0$	For the power signal, $0 < P < \infty$, and the energy $E = \infty$
3. Non-periodic and finite duration signals are in general energy signals.	Periodic signals are power signals. However, all power signals need not be periodic.
4. Energy signals are time limited.	Power signals exist over infinite time.

Table-2.1

NOTE

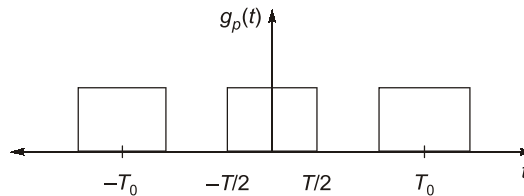


From the analysis equation in trigonometric Fourier series we conclude that:

- The trigonometric Fourier series of an even function of time contains only D.C. term and cosine terms.
- The trigonometric Fourier series of an odd function of time contains only sine terms.

Example 2.3

Given a periodic signal $g_p(t)$ as shown the figure below.



Find the complex Fourier coefficient C_n .

Solution:

$$g_p(t) = \begin{cases} A & -T/2 \leq t \leq T/2 \\ 0 & \text{for the remainder of the period} \end{cases}$$

Now,

$$C_n = \frac{1}{T_0} \int_{-T/2}^{T/2} A \exp\left(\frac{-j2\pi nt}{T_0}\right) dt$$

\therefore

$$C_n = \frac{A}{n\pi} \sin\left(\frac{n\pi T}{T_0}\right) ; n = 0, \pm 1, \pm 2 \dots$$

Example 2.4

The Fourier series representation of an impulse train denoted by

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0) \text{ is given by}$$

$$(a) \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(\frac{j2\pi nt}{T_0}\right)$$

$$(b) \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(\frac{j\pi nt}{T_0}\right)$$

$$(c) \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(\frac{j\pi nt}{T_0}\right)$$

$$(d) \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(\frac{j2\pi nt}{T_0}\right)$$

Solution: (d)

In this discrete case,

$$C_n = \frac{1}{T_0}$$

and we know that the Fourier series is represented as,

$$F.S = \sum_{n=-\infty}^{\infty} C_n e^{j\omega_0 kt} = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(\frac{j2\pi nt}{T_0}\right)$$

**Student's
Assignments****1**

- Q.1** A modulated signal is given by,
 $s(t) = m_1(t) \cos(2\pi f_c t) + m_2(t) \sin(2\pi f_c t)$
 where the baseband signal $m_1(t)$ and $m_2(t)$ have
 bandwidths of 10 kHz and 15 kHz, respectively.
 The bandwidth of the modulated signal, in kHz, is
 (a) 10 (b) 15
 (c) 25 (d) 30
- Q.2** Let $\delta(t)$ denote the delta function. The value of
 the integral $\int_{-\infty}^{\infty} \delta(t) \cos\left(\frac{3t}{2}\right) dt$ is
 (a) 1 (b) -1
 (c) 0 (d) $\frac{\pi}{2}$
- Q.3** If a signal $f(t)$ has energy E , the energy of the
 signal $f(5t)$ is equal to
 (a) E (b) $\frac{E}{5}$
 (c) $5E$ (d) $10E$
- Q.4** The trigonometric Fourier series of an even
 function of time does not have
 (a) the dc term (b) cosine terms
 (c) sine terms (d) odd harmonic terms
- Q.5** The trigonometric Fourier series of a periodic time
 function can have only
 (a) cosine terms
 (b) sine terms
 (c) cosine and sine terms
 (d) dc and cosine terms
- Q.6** The expression of trigonometrical Fourier series
 coefficient b_n in terms of exponential Fourier
 series coefficient C_n is
 (a) $j(C_n + C_{-n})$ (b) $j\left(\frac{C_n + C_{-n}}{2}\right)$
 (c) $j(C_n - C_{-n})$ (d) $j\left(\frac{C_n - C_{-n}}{2}\right)$

- Q.7** Consider a real time domain signal $x(t)$ whose
 Fourier transform is $X(j\omega)$. Which of the following
 properties are true:

(i) $\text{Even}\{x(t)\} \longleftrightarrow \text{Re}\{X(j\omega)\}$

(ii) $\text{Odd}\{x(t)\} \longleftrightarrow j\text{Im}\{X(j\omega)\}$

(iii) $x^*(t) \longleftrightarrow X^*(j\omega)$

(iv) $\int_{-\infty}^t x(\tau) d\tau \longleftrightarrow \frac{X(j\omega)}{j\omega}$

- (a) (i) and (ii) (b) (i), (ii) and (iii)
 (c) (i) and (iii) (d) All the above are true

- Q.8** Consider two periodic signal $x_1(t)$ and $x_2(t)$, these
 signal can be represented in terms of linear
 combination of complex exponential as:

$$\text{If } x_1(t) = \sum_{k=-100}^{100} \cos(k\pi) e^{jk\left(\frac{2\pi}{50}\right)t}$$

$$\text{and } x_2(t) = \sum_{k=-100}^{100} j \sin(k\pi) e^{jk\left(\frac{2\pi}{50}\right)t}$$

then which of the following option is true

- (a) $x_1(t)$ is real and even
 (b) $x_2(t)$ is real and even
 (c) $x_1(t)$ and $x_2(t)$ are real and even
 (d) $x_2(t)$ is imaginary and odd

- Q.9** If $f(t)$ is an even function, then what is its fourier
 transform $F(j\omega)$?

(a) $\int_0^{\infty} f(t) \cos(2\omega t) dt$ (b) $2 \int_0^{\infty} f(t) \cos(\omega t) dt$

(c) $2 \int_0^{\infty} f(t) \sin(\omega t) dt$ (d) $\int_0^{\infty} f(t) \sin(2\omega t) dt$

- Q.10** If the fourier transform of $f(t)$ is $f(j\omega)$, then what
 is the fourier transform of $f(-t)$?

- (a) $f(j\omega)$
 (b) $f(-j\omega)$
 (c) $-F(j\omega)$
 (d) complete conjugate of $f(j\omega)$

- Q.11** The trigonometric fourier series expansion of an odd function shall have
- only sine terms
 - only cosine terms
 - odd harmonics of both sine and cosine terms
 - none of the these

■ **ANSWERS**

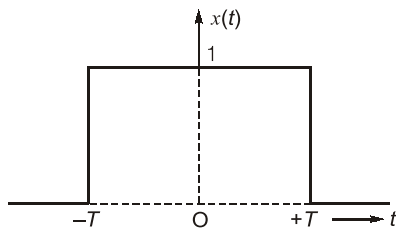
- (d)
- (a)
- (b)
- (c)
- (c)
- (c)
- (a)
- (a)
- (b)
- (b)
- (a)



**Student's
Assignments**

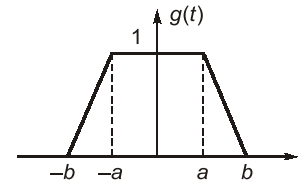
2

- Q.1** For the rectangular pulse shown in the figure below, determine the Fourier Transform of $x(t)$ and sketch the magnitude spectrum with respect to frequency.



- Q.2** State and prove convolution theorem in Fourier transform.

- Q.3** Using time shifting and time differentiation properties, find the Fourier transform of the trapezoidal signal shown.



- Q.4** State and explain Parseval's theorem.

- Q.5** A white Gaussian noise is passed through an ideal bandpass filter with power spectral density of noise being a $\eta = \frac{N_0}{2}$. Derive the expression for the autocorelation function of the input and output noise.

■ **ANSWERS**

1. $X(j\omega) = \frac{2 \sin \omega T}{\omega}$

3. $G(j\omega) = \frac{4}{\omega^2(b-a)} \sin \frac{\omega(a+b)}{2} \cdot \sin \frac{\omega(b-a)}{2}$

