Instrumentation Engineering

Electrical Machines

Comprehensive Theory

with **Solved Examples** and **Objective Practice Set**

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Electrical Machines

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Chapter 1

Magnetic Circuits

Introduction

The electromagnetic system is an essential element of all rotating electric machinery and electromechanical device and static devices like the transformer. Electromechanical energy conversion takes place via the medium of a magnetic field or electrical field, but most practical converters use magnetic field as the coupling medium between electrical and mechanical systems. In transformers, the electrical energy convert from one electrical circuit to another electrical circuit via the medium of a magnetic field as the coupling medium between one electrical circuit to another electrical circuits. This is due to fact that the energy storing capacity of magnetic field is much greater than that of the electric field.

1.1 Magnetic Circuits

- The complete closed path followed by the lines of flux is called a magnetic circuit. In low power electrical machines, magnetic field is produced by permanent magnets. But in high-power electrical machinery and transformers, coupling magnetic field is produced by electric current.

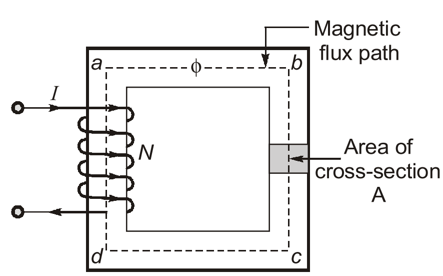


***Figure*-1.1 :** *Magnetic circuit*

- In a magnetic circuit, the magnetic flux is due to the presence of a magnetomotive force same as in an electric circuit, the current is due to the presence of a electromotive force.
- The mmf is created by a current flowing through one or more turns.

$$MMF = \text{Current} \times \text{Number of turns in the coil}$$

$$f = MMF = NI \text{ (ampere-turns) or (ATs)}$$

- The magnetic flux ϕ may be defined as the magnetomotive force per unit reluctance.

$$\phi = \frac{\text{MMF}}{\text{Reluctance}}$$

where reluctance in magnetic circuit is same as resistance in electric circuit.

- It means the opposition offered by the magnetic flux is called reluctance,

$$Rl = \frac{l}{\mu A} \text{ AT/wb}$$

where,

l = length of the magnetic path

A = area of cross-section normal to flux path, m^2.

$\mu = \mu_0 \cdot \mu_r$ = permeability of the magnetic material

μ_r = relative permeability of the magnetic material

μ_0 = permeability of free space = $4\pi \times 10^{-7}$ H/m.

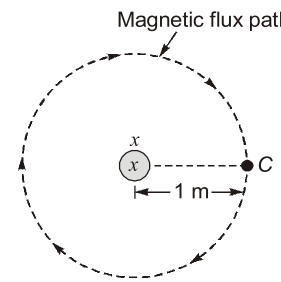


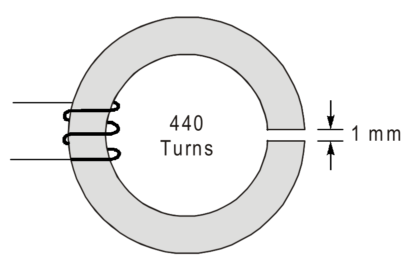
***Figure*-1.2**

Here the concept of permeability can be understand in easy way with following examples.

Suppose a current I carrying conductor in a free space. (Figure 1.2).

According to the right hand grip rule, around the current carrying conductor a magnetic flux path is generated. Actually right hand grip rule stated that grip the conductor with thumb pointing in the direction of conductor current then four fingers give the direction of magnetic flux created by the current.

Example 1.1 An iron ring with a mean length of magnetic path of 20 cm and of small cross-section has an air gap of 1 mm. It is wound uniformly with a coil of 440 turns. A current of 1 A in the coil produces a flux density of $16\pi \times 10^{-3}$ Wb/m². Neglecting leakage and fringing, calculate the relative permeability of iron.



Solution:

The above figure shows an iron ring of mean length = 20 cm = ℓ_1

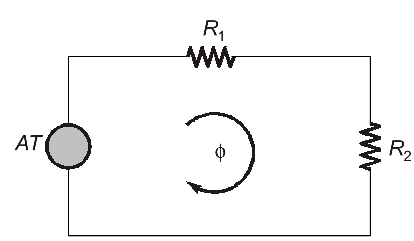
Length of air gap = 1 mm = 1×10^{-3} m = ℓ_1

Number of turns would = 440 turns = N

Current in the coil = 1 A = I

Flux density = $16\pi \times 10^{-3}$ Wb/m² = B

The electrical equivalent is as shown given figure,



Here, R_1 = Reluctance of iron.

R_2 = Reluctance of air gap.

$\therefore \quad AT = \phi(R_1 + R_2)$

$\phi = BA$ (A = Area)

$$AT = BA\left(\frac{\ell_1}{\mu_0\mu_1 A} + \frac{\ell_2}{\mu_0 A}\right) = \frac{B}{\mu_0}\left[\frac{\ell_1}{\mu_1} + \ell_2\right]$$

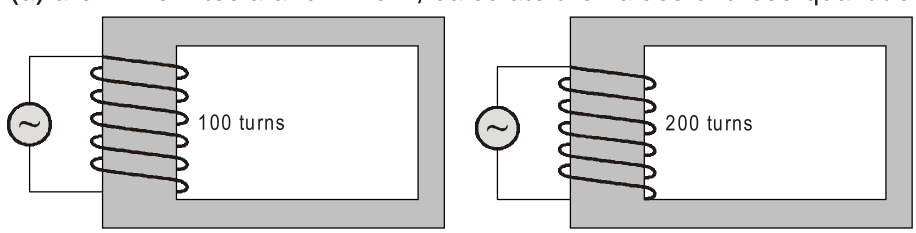
$$\therefore \quad \frac{\ell_1}{\mu_1} + \ell_2 = \frac{\mu_0 AT}{B} = \frac{4\pi \times 10^{-7} \times 440 \times 1}{16\pi \times 10^{-3}}$$

$$\frac{20 \times 10^{-2}}{\mu_r} + 1 \times 10^{-3} = 110 \times 10^{-4}$$

$$\therefore \quad \frac{20 \times 10^{-2}}{\mu_r} = 11 \times 10^{-3} - 1 \times 10^{-3} = 10 \times 10^{-3}$$

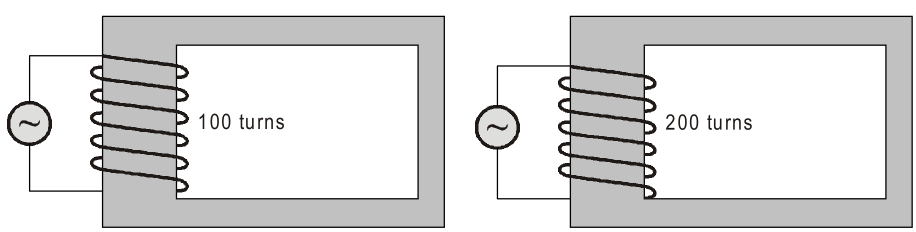
$$\therefore \quad \mu_r = \frac{20 \times 10^{-2}}{10 \times 10^{-3}} = 20$$

Example 1.2 A magnetic core is excited with two different arrangements of exciting coils as shown in figure. The resistance of the exciting coils is negligible. The same sinusoidal voltage at a specified frequency is applied to the exciting coil in each case. If the flux density and the exciting current in case (a) are B = 0.1 tesla and I = 8 A, calculate the values of these quantities in case (b).



(a) (b)

Solution:



(a) (b)

Method-1:

The resistance of exciting coil is negligible,

$$B_1 = 0.1 \text{ tesla}, \; I_1 = 8 \text{ A}$$

We know,

$$AT = \text{flux} \times \text{reluctance}$$

$$\therefore \quad AT = \phi \times \frac{\ell}{\mu_0 \mu_r A} = B \times \frac{\ell}{\mu_0 \mu_r}$$

Since,

$$B = \frac{\phi}{A}$$

$$AT = \text{Ampere-turns} = I \times T$$

$$I = \text{Number of turns.}$$

$$\therefore \quad I_1 T_1 = B_1 \times \frac{\ell}{\mu_0 \mu_r}$$

Similarly,

$$I_2 T_2 = B_2 \times \frac{\ell}{\mu_0 \mu_r}$$

$$\therefore \quad \frac{I_1 T_1}{I_2 T_2} = \frac{B_1}{B_2}$$

$$\therefore \quad \frac{8 \times 100}{I_2 \times 200} = \frac{0.1}{B_2}$$

$$\therefore \quad I_2 = 40\, B_2$$

$$\text{Induced emf} = -N\frac{d\phi}{dt}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{-N_1 \frac{d\phi_1}{dt}}{-N_2 \frac{d\phi_2}{dt}} = \frac{N_1}{N_2} \times \frac{\phi_1}{\phi_2} = \frac{N_1 B_1}{N_2 B_2}$$

Since, $E_1 = E_2$ (same)

$$N_1 B_1 = N_2 B_2$$

$$\therefore \quad 100 \times 0.1 = 200 \times B_2$$

$$\therefore \quad B_2 = \frac{100 \times 0.1}{200} = 0.05 \text{ tesla}$$

Method-2:

$$L \;\alpha\; N^2$$

$$L_2 = 4\, L_1$$

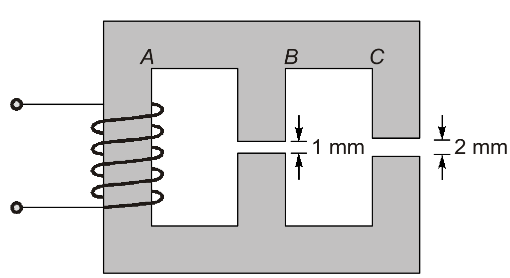
$$\Rightarrow \quad X_2 = 4\, L_1$$

$$\Rightarrow \quad I_2 = \frac{1}{4} I_1$$

$$MMF_2 = \frac{1}{2} MMF_1$$

For same magnetic circuit, $B_2 = \frac{1}{2} B_1 = \frac{0.1}{2} = 0.05$ tesla

Example 1.3 In the magnetic circuit shown in figure, the areas of cross-section of limbs *B* and *C* are respectively 0.01 m^2 and 0.02 m^2. Air gaps of lengths 1.0 mm and 2.0 mm respectively are cut in the limbs *B* and *C*. If the magnetic medium can be assumed to have infinite permeability and the flux in limb *B* is 1.0 Wb, the flux in limb *A* is



(a) 3 Wb
(b) 1.5 Wb
(c) 2 Wb
(d) 4 Wb

Solution :

Area of cross-section of limb $B = 0.01\ m^2$

Area of cross-section of limb $C = 0.02\ mm^2$

Air gap length = 1.0 mm for limb B

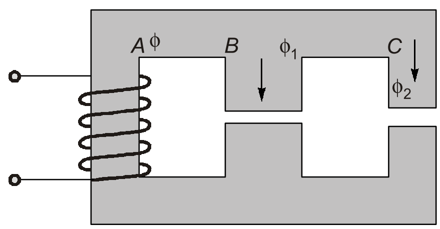
Air gap length = 2.0 mm for limb C

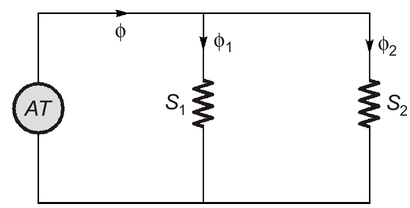
Magnetic medium is assumed as infinite permeability

$\therefore$ Reluctance of iron path is zero since

$$R = \frac{1}{\mu}$$

Flux in limb, $B = 1.0$ Wb





where, R_1 – reluctance of air gap of limb B

R_2 – reluctance of air gap of limb C

ϕ_1 – flux across air gap of limb B

ϕ_2 – flux across air gap of limb C

$$\therefore \quad R_1 \times \phi_1 = R_2 \times \phi_2$$

$$\therefore \quad \frac{\ell_1}{\mu_0 \times A_1} \times \phi_1 = \frac{\ell_2}{\mu_0 \times A_1} \times \phi_2$$

$$\therefore \quad \phi_2 = \frac{A_2}{A_1} \times \frac{\ell_1}{\ell_2} \times \phi_1 = \frac{0.02}{0.01} \times \frac{1}{2} \times 1 = 1\,\text{Wb}$$

$\therefore$ Flux in limb $A = \phi_1 + \phi_2 = 1$ Wb + 1 Wb = 2 Wb

1.2 Leakage Flux

In Ideal magnetic circuits, all the flux produced by an exciting coil is confined to the desired magnetic path of negligible reluctance. But in practical magnetic circuits, a small amount of flux does follow a path through the surrounding air. Therefore, leakage flux may be defined as that flux which does not follow the intended path in a magnetic circuit. Leakage flux does exist in all practical ferromagnetic device. Its effect on the analysis of electrical machinery is carried out by replacing it by an equivalent leakage reactance.

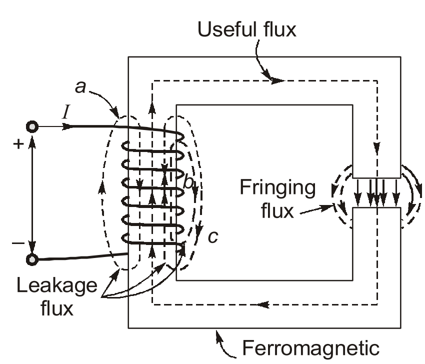


***Figure*-1.3 :** Leakage flux

Student's Assignments 1

Q.1 A cast steel electromagnet has an airgap of length 2 mm and an iron path of length 30 cm. Find the number of ampere turns necessary to produce a flux density of 0.8 Wb/m^2 in the gap. Neglect leakage and fringing. (For 0.8 Wb/m^2 cast steel requires 750 AT/m).

Q.2 A cast steel ring has a circular cross-section 3 cm in diameter and a mean circumference of 80 cm. The ring is uniformly wound with 600 turns.

(a) Estimate the current required to produce a flux of 0.5 m Wb in the ring.

(b) If a saw cut 2 mm wide is made in the ring, find approximately the flux produced by the current found in (a),

(c) Find the current value which will give the same flux as in (a).

Assume the gap density to be the same as in the iron and neglect fringing.
(For 0.705 Wb/m^2 cast steel requires 670 AT/m and 365 AT/m will produce a flux density of 0.15 Wb/m^2)

Q.3 An iron ring of mean length 50 cm has an air gap of 1 mm and a winding of 200 turns. If the permeability of the iron is 300 when a current of 1 A flows through the coil, find the flux density.

Student's Assignments 1 Explanations

1. Solution:

The required for air gap

$$\text{AT} = \frac{B}{\mu_0} l = \frac{0.8}{4\pi \times 10^{-7}} \times 2 \times 10^{-3}$$

$$= 1273$$

For 0.8 Wb/m^2 cast steel requires 750 AT/m

∴ AT required for iron path

$$= 750\, l = 750 \times 0.3 = 225 \text{ AT}$$

∴ The total AT required

$$= 1273 + 225 \approx 1498$$

2. Solution:

(a) The cross-sectional area

$$= \frac{\pi d^2}{4} \times 10^{-4} = \frac{\pi \times 9}{4} \times 10^{-4}$$

$$= 7.068 \approx 7.1 \times 10^{-4} \text{ m}^2$$

The flux density,

$$B = \frac{\phi}{A} = \frac{0.5 \times 10^{-3}}{7.1 \times 10^{-5}} = \frac{5}{7.1}$$

$$= 0.705 \text{ Wb/m}^2$$

∴ AT required = 670 × 0.8 = 536

∴ The current required

$$= \frac{\text{AT}}{N} = \frac{536}{600} = 0.89 \text{ A}$$

(b) If all the available 536 AT is used by the air gap

$$H = \frac{536}{2 \times 10^{-3}} = 268 \times 10^3$$

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 268 \times 10^3$$

$$= 0.33 \text{ Wb/m}^2$$

Since same AT are required by the iron path as well the actual flux density produced will be lower than this assume a flux density of 0.15 Wb/m^2 .

The AT required for air gap

$$= 536 \times \frac{0.15}{0.33} = 244$$

The AT available for iron

$$= 536 - 244 = 292$$

or, $\frac{292}{0.8} = 365$ AT/m

∴ The approximate flux density

$$= 0.15 \text{ Wb/m}^2$$

(c) For 0.705 Wb/m^2 AT required by the 2 mm air gap

$$= \frac{B}{\mu_0} l = \frac{0.705}{4\pi \times 10^{-7}} \times 2 \times 10^{-3}$$

$$= 1122$$

∴ The total AT required

$$= 1122 + 536 = 1658$$

∴ The current $= \frac{1658}{600} \approx 2.763$ A

3. **Solution:**

The reluctance,

$$R = \frac{l}{A\mu}$$

∴ The total reluctance

$$= R_1 + R_2$$

$$= \frac{1\times10^{-3}}{A\mu_0} + \frac{0.5}{300\,A\mu_0}$$

$$= \frac{8\times10^{-3}}{3A\mu_0}\ \text{AT/Wb}$$

∴ The total flux $= \frac{\text{Total AT}}{\text{Relutance}} = \frac{200\times1\times3A\mu_0}{8\times10^{-3}}$

∴ The flux density $= \frac{600\times\mu_0}{8\times10^{-3}}$ Wb/m^2

$$= \frac{600\times4\pi\times10^{-7}\times10^{3}}{8\times10^{-3}}$$

$$= 94.2\ \text{mWb/m}^2$$

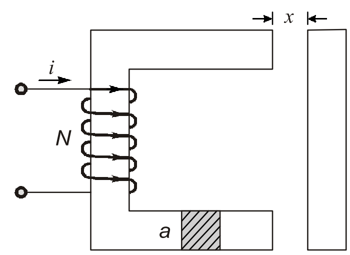
Student's Assignments 2

Q.1 Why the transformer stampings are varnished before being used to build the core?
(a) To increase air-gap between stampings
(b) To reduce hysteresis loss
(c) To reduce eddy current loss
(d) To provide strength to the core

Q.2 Maximum flux established in an AC excited iron core is determined by
(a) impressed frequency only
(b) impressed voltage only
(c) both impressed voltage and frequency
(d) reluctance of the core

Q.3 A circular iron core has an air-gap cut in it and is excited by passing direct current through a coil wound on it. The magnetic energy stored in the air-gap and the iron core is
(a) in inverse ratio of their reluctance
(b) in direct ratio of their reluctances
(c) equally divided among then
(d) energy resides wholly in the iron core

Q.4 In the electromagnetic relay of given figure below the reluctance of the iron path is negligible. The coil self-inductance is given by the expression



(a) $\mu_0 N^2 a/x$ (b) $\mu_0 N/2ax$
(c) $\mu_0 N^2 a/2x$ (d) $\mu_0 N^2/2ax$

Q.5 An iron-cored choke with 1 mm air-gap length, draws 1 A when fed from a constant voltage AC source of 220 V. If the length of air-gap is increased to 2 mm, the current drawn by the choke would
(a) become nearly one half
(b) remain nearly the same
(c) become nearly double
(d) become nearly zero

Q.6 Match **List-I (Electric and Magnetic Quantities)** with **List-II (SI Units)** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Flux	1. AT/Wb
B. Magnetomotive force	2. Wb
C. Reluctance	3. Wb/AT
D. Permeance	4. AT

Codes:

	A	B	C	D
(a)	1	3	4	2
(b)	3	1	4	2
(c)	4	3	2	1
(d)	2	4	1	3

Q.7 Match **List-I (Electric and Magnetic Quantities)** with **List-II (SI Units)** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Flux linkage	1. AT/m
B. Flux density	2. Wb T or V.s
C. Magnetic field strength	3. H/m
D. Permeability	4. Wb/m^2 or Tesla