POSTAL Book Package 2023

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Electricity and Magnetism

Contents

MADE EASY Publications

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CHAPTER

Vector Analysis

MCQ and NAT Questions

Q.1 If $\vec{G} = 15r\hat{a}_\phi$ then $\oint \vec{G} \cdot \vec{dl}$ over the circular path $r = 2$ m, $\theta = 30°$, $0 < \phi < 2\pi$ is

(a) 120π (b) 120
(c) 60π (d) 60

Q.2 Which of the following is true?

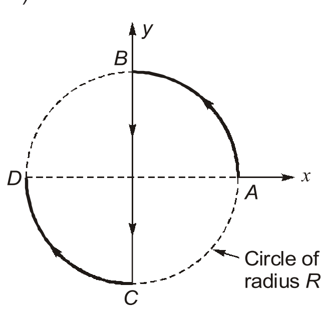
(a) $\text{Curl}(\vec{A} \cdot \vec{B}) = \text{Curl } \vec{A} + \text{Curl } \vec{B}$

(b) $\text{Div } (\vec{A} \cdot \vec{B}) = \text{Div } \vec{A} \cdot \text{Div } \vec{B}$

(c) $\text{Div } (\text{Curl } \vec{A}) = 0$

(d) $\text{Div}(\text{Curl } \vec{A}) = \Delta \cdot \vec{A}$

Q.3 What is the value of the integral $\int_c \vec{dl}$ along the curve c (c is the curve $ABCD$ in the direction of the arrow) ?



(a) $2R(\hat{a}_x + \hat{a}_y)/\sqrt{2}$ (b) $-2R(\hat{a}_x + \hat{a}_y)/\sqrt{2}$

(c) $2R\hat{a}_x$ (d) $-2R\hat{a}_y$

Q.4 If $uF = \nabla v$, where u and v are scalar fields and F is a vector field, then F. curl F is equal to

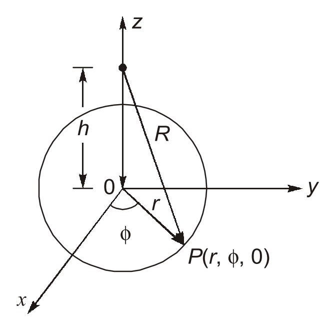
(a) zero (b) $\dfrac{\nabla^2 v}{u^2}$

(c) $\dfrac{(\nabla v \cdot \nabla)\, v}{u^2}$ (d) not defined

Q.5 Laplacian of a scalar function V is

(a) Gradient of V
(b) Divergence of V
(c) Gradient of the gradient of V
(d) Divergence of the gradient of V

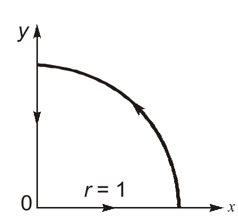
Q.6 The unit vector $\vec{a}_R$ which points from $z = h$ on the z-axis towards $(r, \phi, 0)$ in cylindrical co-ordinates as shown below is given by



(a) $\dfrac{h\vec{a}_r - r\vec{a}_z}{\sqrt{r^2 + h^2}}$ (b) $\dfrac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$

(c) $\dfrac{h\vec{a}_\phi - r\vec{a}_z}{\sqrt{r^2 + h^2}}$ (d) $\dfrac{r\vec{a}_z - h\vec{a}_\phi}{\sqrt{r^2 + h^2}}$

Q.7 Given a vector field $\bar{A} = 2r\cos\phi\,\hat{a}_r$ in cylindrical coordinates. For the contour as shown below, $\oint \bar{A} \cdot d\bar{l}$ is



(a) 1 (b) $1 - (\pi/2)$
(c) $1 + (\pi/2)$ (d) -1

(a) $a = 4$, $b = 2$ and $c = -1$
(b) $a = 2$, $b = -1$ and $c = 4$
(c) $a = 4$, $b = -1$ and $c = 2$
(d) $a = 2$, $b = 4$ and $c = -1$

Q.33 What is the value of $\iint_s \vec{F} \cdot d\vec{s}$, where $\vec{F} = 4xz\vec{i}_1 - y^2\vec{i}_2 + yz\vec{i}_3$?
Here, s is the surface bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$ and $\vec{i}_1, \vec{i}_2, \vec{i}_3$ are unit vectors along x, y and z axes respectively.
(a) 1/2 (b) 5/2
(c) 2 (d) 3/2

Q.34 Given a vector $\vec{A} = 30e^{-r}\hat{a}_r - 2z\hat{a}_z$ in cylindrical co-ordinates. If a volume is enclosed by $r = 2$, $\phi = 2\pi$ and $z = 5$ then $\int(\nabla \cdot \vec{A})dv$ = _______ .

Q.35 If $\vec{r} = x\vec{I}_x + y\vec{I}_y + z\vec{I}_z$, then which of the following relation will hold true?
(a) $\nabla\vec{r} = 3$ (b) $\nabla \times \vec{r} = 0$
(c) Both (a) and (b) (d) Neither (a) nor (b)

Q.36 If $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ is the position vector of point (x, y, z), then $\nabla(ln|r|)$ is
(a) $|r|\,\vec{r}$ (b) $|r|^2\,\vec{r}$
(c) $\frac{\vec{r}}{|r|}$ (d) $\frac{\vec{r}}{|r|^2}$

Q.37 If $B = x^2y\,\hat{a}_x + (2x^2 + y)\hat{a}_y - (y - z)\hat{a}_z$ then $\nabla(\nabla \cdot B)$ is
(a) $2\hat{a}_x + 2xy\,\hat{a}_y$ (b) $2y\,\hat{a}_x + 2x\,\hat{a}_y$
(c) $x\,\hat{a}_x + y\,\hat{a}_y$ (d) $xy\,\hat{a}_x + xy\,\hat{a}_y$

Q.38 If $H = R\sin\theta\,\hat{a}_\phi$ (in spherical coordinates) then the magnitude of curl of the vector field H at the origin is ______.

Q.39 The divergence of the vector $\vec{A}$ which is given as follows,
$\vec{A} = 2r\cos\theta \cdot \cos\phi\hat{a}_r + r^{1/2}\hat{a}_\phi$ at point $\left(1, \frac{\pi}{4}, \frac{\pi}{3}\right)$ is
(a) 1.52 (b) 2.12
(c) 3.45 (d) 2.75

Q.40 If a general vector is given by $\vec{A} = (\sin 2\phi)\hat{a}_\phi$ in cylindrical co-ordinate system, then the curl of vector $\vec{A}$ at $(4, \pi/6, 0)$ will be
(a) $\frac{\sqrt{3}}{8}\hat{a}_z$ (b) $-0.5\hat{a}_z$
(c) $\frac{\sqrt{3}}{4}\hat{a}_z$ (d) $0.5\hat{a}_z$

Q.41 The values of constants a, b and c so that
$\vec{V} = (x + 2y + az)\hat{a}_x + (bx - 3y - z)\hat{a}_y + (4x + cy + 2z)\hat{a}_z$
is irrotational, then sum of value of constant a, b and c will be _______ .

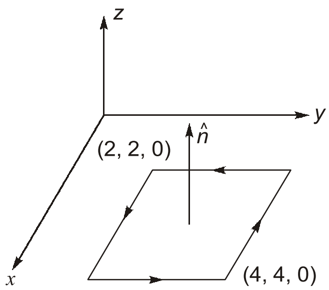
Q.42 If a vector field $\vec{A}$ is said to be solenoidal, then which one of the following relations is true?
(a) $\oint_L \vec{A} \cdot \vec{dL} = 0$ (b) $\nabla \times \vec{A} \neq 0$
(c) $\oint_s \vec{A} \cdot \vec{ds} = 0$ (d) $\nabla \times \vec{A} = 0$

Multiple Select Questions (MSQs)

Q.43 $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ represents a position vector and $\|\vec{r}\|$ represents the normal of vector $\vec{r}$, then which of the below statements is/are true?
(a) Divergence of $\vec{r}$ is 3.
(b) Gradient of $\|\vec{r}\|^2$ is $3\vec{r}$
(c) Curl of $\vec{r}$ is 0
(d) Laplacian of $\|\vec{r}\|^2$ is 6.

Q.44 If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = \sqrt{x^2 + y^2 + z^2}$, then which of the below relations are correct?
(a) $\nabla(\log r) = \frac{\vec{r}}{r}$ (b) $\nabla\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$
(c) $\nabla \cdot \left(\frac{\vec{r}}{r^3}\right) = 1$ (d) $\nabla \cdot (3\vec{r}) = 9$

Q.45 Let $\vec{F} = xy^2\hat{a}_x + y^3\hat{a}_y + x^2y\hat{a}_z$ and the surface S consists of a square of length 2 lying in the xy plane as shown below:



Which of the following options is/are correct?

(a) $\iint_S \vec{F}\cdot\hat{n}\,ds = 80$

(b) $\iint_S (\vec{F}\times\hat{n})ds = 120\hat{a}_x - 112\hat{a}_y$

(c) $\nabla\times\vec{F} = x^2\hat{a}_x - 2xy\hat{a}_y - 2xy\,\hat{a}_z$

(d) $\iint_S (\nabla\times\vec{F})\cdot\hat{n}\,ds = -120$

Q.46 If $[\vec{a}, \vec{b}, \vec{c}]$ represents the scalar triple product of vectors $\vec{a}, \vec{b}$ and $\vec{c}$, then which of the below statements is/are true?

(a) $\left[\vec{a}, \vec{b}, \vec{c}\right] = \left[\vec{c}, \vec{b}, \vec{a}\right]$

(b) $\left[\vec{a}, \vec{b}+\vec{a}, \vec{c}\right] = 0$

(c) $\left[3\vec{b}, \vec{c}, \vec{a}\right] = 3\left[\vec{a}, \vec{b}, \vec{c}\right]$

(d) If $\left[\vec{a}, \vec{b}, \vec{c}\right] = 0$, the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.

Q.47 The values of α for which the vectors $\vec{A} = \alpha\hat{a}_x + 2\hat{a}_y + 10\hat{a}_z$ and $\vec{B} = 4\alpha\hat{a}_x + 8\hat{a}_y - 2\alpha\hat{a}_z$ are perpendicular is/are

(a) 1 (b) 2

(c) 3 (d) 4

Q.48 Which of the below vector identities are true?

(a) $A\times(B\times C) = (A\times B)\times C$

(b) $A\times(B\times C) + C\times(A\times B) + B\times(C\times A) = 0$

(c) $(B\times C)\times(C\times A) = C(A\cdot B\times C)$

(d) $(A\times B)\cdot(C\times D) = (A\cdot C)(B\cdot D) - (A\cdot D)(B\cdot C)$

Q.49 For the scalar function, $\phi = x^2yz^3$, which of the below statements is/are correct?

(a) From the point (2, 1, –1) the directional derivative of ϕ is maximum in the direction represented by vector $-12\hat{i} - 4\hat{j} + 4\hat{k}$.

(b) The magnitude of greatest rate of change of ϕ from the point (2, 1, –1) is $4\sqrt{11}$.

(c) $(x-2) + (y-1) - 3(z+1) = 0$ represents the tangent plane to the surface $\phi = 0$ at point (2, 1, –1).

(d) ϕ satisfies the Laplacian equation.

■■■■

Answers **Vector Analysis**

1.	(c)	2.	(c)	3.	(d)	4.	(a)	5.	(d)	6.	(b)	7.	(a)
8.	(3)	9.	(a)	10.	(b)	11.	(0.33)	12.	(a)	13.	(b)	14.	(c)
15.	(a)	16.	(b)	17.	(d)	18.	(0.5)	19.	(c)	20.	(c)	21.	(a)
22.	(d)	23.	(c)	24.	(c)	25.	(a)	26.	(62.83)	27.	(2.09)	28.	(–1.15)
29.	(d)	30.	(a)	31.	(3.75)	32.	(a)	33.	(d)	34.	(129.43)	35.	(c)
36.	(d)	37.	(b)	38.	(2)	39.	(b)	40.	(a)	41.	(5)	42.	(c)
43.	(a, c, d)	44.	(b, d)	45.	(b, c)	46.	(c, d)	47.	(a, d)	48.	(b, c, d)	49.	(b, c)

Explanations — Vector Analysis

1. (c)

For spherical coordinate systems,

$$\vec{dl} = r\sin\theta d\phi \hat{a}_\phi$$

$$\oint \vec{G}\cdot\vec{dl} = \int_0^{2\pi} 15r\hat{a}_\phi \cdot r\sin\theta d\phi\, \hat{a}_\phi$$

$$= 15\cdot r^2\cdot \sin\theta(2\pi)$$

$$= 15\cdot(2)^2 \times \sin30°(2\pi)$$

$$\oint \vec{G}\cdot\vec{dl} = 60\pi$$

2. (c)

Divergence (Curl $\vec{A}$) = 0

3. (d)

$$\int_{AB}\vec{dl} = \int_0^{\pi/2} R\cdot d\theta\,\hat{a}_\theta = \frac{\pi}{2}R\,\hat{a}_\theta$$

$$\int_{BC}\vec{dl} = \int_{+R}^{-R} dl(-\hat{a}_y) = -2R\hat{a}_y$$

$$\int_{CD}\vec{dl} = \int_{-\pi/2}^{-\pi} R\cdot d\theta\,\hat{a}_\theta = -\frac{\pi}{2}R\,\hat{a}_\theta$$

$$\therefore \int_C \vec{dl} = \int_{AB}\vec{dl} + \int_{BC}\vec{dl} + \int_{CD}\vec{dl} = -2R\,\hat{a}_y$$

4. (a)

Given, $F = \frac{1}{u}\nabla v$

$\therefore$ Curl $F = \nabla\times\left(\frac{1}{u}\nabla v\right)$

or, Curl $F = \nabla\frac{1}{u}\times\nabla v + \frac{1}{u}\nabla\times(\nabla v)$

$$= \nabla\frac{1}{u}\times\nabla v$$

Hence, $F.$ Curl $F = \frac{1}{u}\nabla v\cdot\left(\nabla\frac{1}{u}\times\nabla v\right) = 0$

5. (d)

$$\nabla^2 V = \vec{\nabla}\cdot(\vec{\nabla}V)$$

= divergence of gradient of V

6. (b)

Let the unit vector be given by $\vec{a}_R$.

Now, $\vec{R}$ = Difference of two vectors

$$= r\vec{a}_r - h\vec{a}_z$$

$\therefore$ Unit vector, $\vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2+h^2}}$

7. (a)

$$\oint\vec{A}\cdot\vec{dl} = \int_0^1 2r\cos\phi\hat{a}_r\cdot dr\hat{a}_r + \int_0^{\pi/2} 2r\cos\phi\cdot r\,d\phi(\hat{a}_r\cdot\hat{a}_\phi) + \int_{01}^{0} 2r\cos\phi\cdot\hat{a}_r\cdot dr\hat{a}_r$$

As $\hat{a}_r\cdot\hat{a}_\phi$ and $\cos\frac{\pi}{2} = 0$

$$\therefore \oint\vec{A}\cdot\vec{dl} = 1+0+0 = 1$$

8. (3)

$$\text{div}(r^2\nabla(lnr)) = \text{div}\left(r^2\left(\frac{\partial lnr}{\partial r}\hat{r}\right)\right)$$

$$= \text{div}\left(r^2\cdot\frac{1}{|r|}\hat{r}\right) = \text{div}(|r|\hat{r})$$

Since, $\text{div}(\vec{A}) = \frac{1}{r^2}+\frac{\partial r^2 A_r}{\partial r}+\frac{1}{r^2\sin\theta} + \frac{\partial\sin\theta A_\theta}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$

$$\therefore \text{div}(r^2\nabla(lnr)) = \text{div}(\vec{r})$$

$$= \frac{1}{r^2}\frac{\partial r^2 r}{\partial r} = \frac{1}{r^2}\frac{\partial r^3}{\partial r} = \frac{3r^2}{r^2}$$

$$\text{div} = (r^2\nabla ln(r) = 3$$

9. (a)

A phasor is always a vector quantity.

10. (b)

Gradient of a scalar;

∇A = maximum rate of change of scalar A with respect to given coordinates system.

11. (0.33)

In a closed path the circulation of vector $\vec{A}$ is given as,

$$\oint\vec{A}\cdot\vec{dl} = \int_{OA}+\int_{AB}+\int_{AB}+\int_{BC}+\int_{CO}(\vec{A}\cdot\vec{dl})$$

$$\because \quad \vec{A} = x^2y\hat{a}_x + 2xy^2\hat{a}_y$$