

# Instrumentation Engineering

Section A  
**Sensors and Industrial Instrumentation**

Section B  
**Optical Instrumentation**

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Comprehensive Theory

*with* Solved Examples and Practice Questions



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**Sensors and Industrial Instrumentation + Optical Instrumentation**

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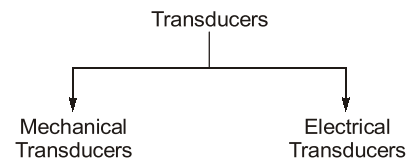
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# Introduction to Transducers

## 1.1 Transducers

A transducer is a device which converts one kind of energy to the other. The conversion can be to or from electrical, electromechanical, photonic, photo-voltaic or any other form of energy. Transducers are called as the primary sensing elements. They have a vast application in the industrial measurement and control. Industry handles a large number of process which include the measurement of different process variable like flow, temperature, pressure, liquid level etc., these process variables can easily be transduced or converted in the electrical form with the help of the electrical transducers, these electrical signals which are proportional to the process variables can then be used for display and the control purposes, and the input to the transducer is termed as the information. Transducers can be of mechanical or electrical type.



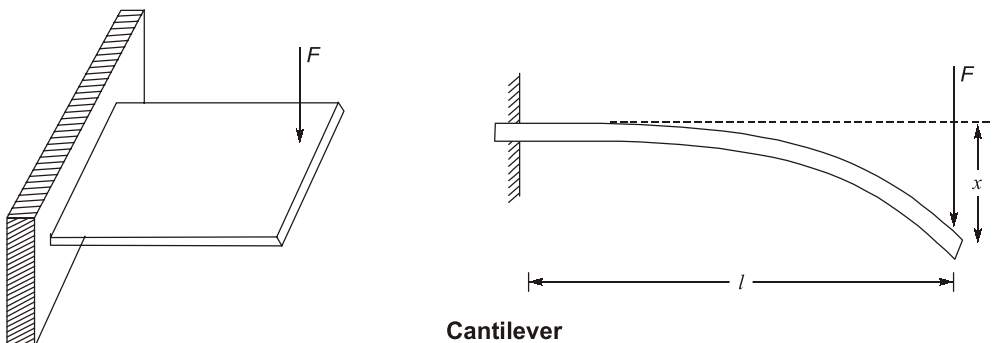
## 1.2 Mechanical Transducers

These are the transducers which convert the mechanical quantity (e.g. force or torque) into the equivalent displacement. Some of the commonly used mechanical sensing elements are :

### Spring Device

These are the mechanical transducers which convert a force or torque into displacement; These type of devices include.

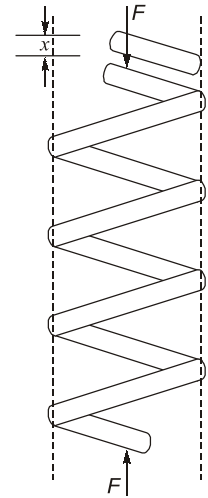
### Cantilever



When a force ' $F$ ' is applied to a cantilever beam it gets bent to some displacement ' $x$ ', the value of ' $x$ ' can be easily find and is calibrated in terms of input force applied. So knowing about the displacement ' $x$ ' we can easily find the amount of force applied to the cantilever beam.

### Helical Spring

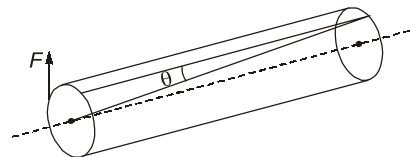
In case of Helical spring when a force ' $F$ ' is applied to both the sides of the spring then a displacement ' $x$ ' produced in the spring which is calibrated in terms of the applied force. So if we know about the displacement ' $x$ ' we can easily determine the applied force ' $F$ '.



**Helical Spring**

### Torsion Bars

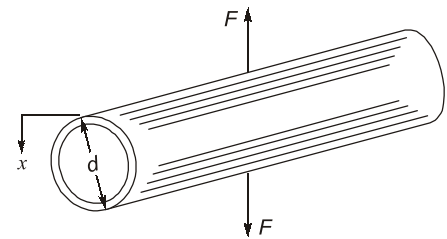
These type of transducers are used for the measurement of torque. The torsion bars are fixed to the shaft for torque measurement. When torque is produced, it produces an angular deflection/displacement into the bar, this angular displacement ( $\theta$ ) is proportional to the torque of the shaft.



**Torsion Bar**

### Proving Rings

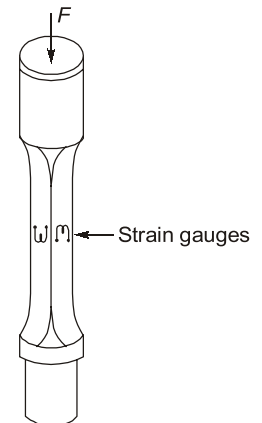
These type of transducers are used for the measurement of weight, force or load. The lateral outward force ' $F$ ' is applied to the proving ring which causes the deflection ' $x$ ' as shown in the given figure. The deflection ' $x$ ' is calibrated in terms of the force ' $F$ ' and hence can be used for the measurement of force. Proving rings are made of steel.



**Proving Ring**

### Load Cell

Load cells are used for the measurement of force. They utilise the elastic members as the primary sensing element. The strain is produced in the load cells when an external force ' $F$ ' as shown in the figure is applied to them. This strain is then sensed with the help of strain gauges. The strain measured is proportional to the applied force.



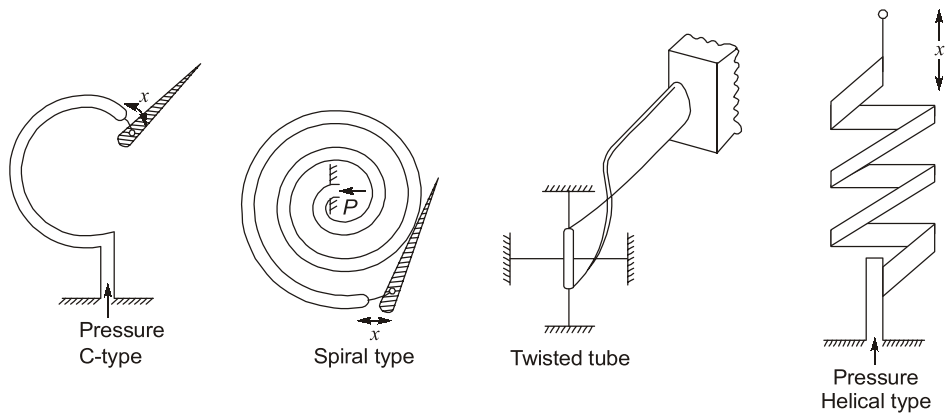
**Load Cell**

### Pressure Sensitive Devices

Pressure sensitive devices are those devices which convert the applied pressure into the displacement. The displacement is then measured with the help of the electrical transducers. The output of the electrical transducers is proportional to the displacement and hence gives the measure of applied pressure. These can be of different types:

### Bourdon Tube

Bourdon tube is a C-shaped tube which has two ends, with one end free (which is closed) and the other end kept open through which the fluid is allowed to enter. When the fluid pressure is allowed to enter the tube, the free end of the tube tries to straighten out end hence leads to the displacement of the pointer attached to it. This displacement can be amplified with the help of gear and pinion arrangement.



Bourdon tubes are preferred for high pressure measurement, as they have good accuracy at high pressure.

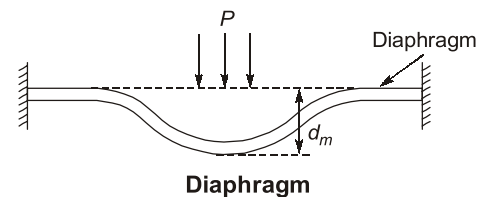
- Bourdon tubes measure gauge pressure, they are made up of brass, phosphor, bronze, beryllium copper, and steel.
- **C-type Bourdon tube** have an arc of  $250^\circ$  and hence they are called as the C-shaped Bourdon tubes.
- C-type Bourdon tube have low output displacement hence they need amplification which is provided by the gear and pinion arrangement, but the backlash error comes in due to pressure of gear and pinion arrangement.
- Spiral and Helical type have high output displacement and hence they do not need any gear pinion arrangement for amplification also they are free from backlash error.

### Accuracy order

Spiral type > Helical type > C-type

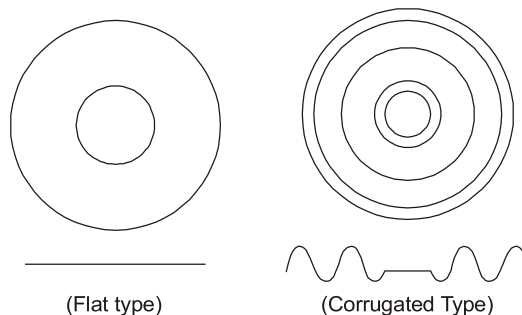
### Diaphragm

- Diaphragm are the thin circular disc which is used for the measurement of pressure. The unknown pressure is applied to one of its ends, and it tends to expand towards the opposite side. The displacement produced is proportional to the applied pressure.
- This displacement ( $d_m$ ) is proportional to the applied pressure ( $P$ ), hence knowing about this displacement we can easily find the pressure applied.



Diaphragm are used for low pressure measurement.

### Types of Diaphragm



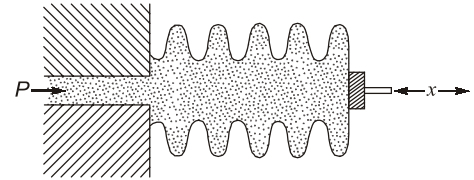
- Corrugated type diaphragm has greater surface area hence its sensitivity is higher than the flat diaphragm

**Order of sensitivity:**

Corrugated type > Flat type

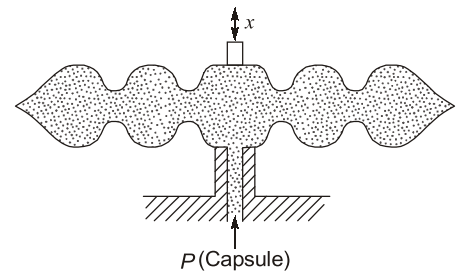
### Bellows

- Bellows are the pressure sensing elements. They consist of cylindrical metal box with corrugated walls made of spring material like phosphor bronze, brass and steel. The walls are very thin with thickness of about 0.1 mm.
- As shown in the figure above, the pressure ' $P$ ' is allowed to enter from the left side of the bellows, this tends to expand the bellows to the right side (as the right end is free), but the bellows material is springy by nature and hence it puts a backward force and tries to come back and results in a displacement ' $x$ ', proportional to the applied pressure ' $P$ '.
- If the input pressure is low then the spring material can provide sufficient backward force to control it, but if input pressure is high the displacement will go uncontrolled. Thus, **Bellows are used for low pressure measurement.**



### Capsules

- Two diaphragms are joined to each other to form a capsule. Increasing the pressure into the capsule causes it to expand while decreasing its pressure causes it to contract, so the resultant displacement produced is an indication of the pressure applied.
- Since two diaphragms are attached to each other, hence the combined displacement produced is greater than that produced by the single diaphragm. Hence, **Sensitivity of capsule > Sensitivity of diaphragm.**



#### Example - 1.1

A transducer converts

- mechanical energy into electrical energy only
- mechanical displacement into electrical signal only
- one form of energy into another form of energy only
- electrical energy into mechanical form only

**Solution : (c)**

## 1.3 Electrical Transducers

- The transducers which convert the mechanical quantity into electric energy are called as the electric transducers. The mechanical quantity to be measured is first converted into displacement which is then converted to the electric energy with the help of an electrical transducer.
  - Primary Transducers**  
Mechanical (pressure or force etc.)  $\rightarrow$  Displacement
  - Secondary Transducers**  
Displacement  $\rightarrow$  Electric Energy



- The electrical output may be either, current, voltage or frequency.  
Another name of transducer is pickup.

### Advantages of Electrical Transducers Over Mechanical Transducers

- Mechanical transducers have frictional losses while electrical transducers are free from frictional losses.
- Electrical signals can be easily amplified while it is difficult to amplify mechanical output.
- Electrical signals can be easily transmitted from one place to another hence they have vast telemetry applications.
- Mechanical transducers have mass-inertia effects, which can be minimized by electrical transducers.
- Signal conditioning can be done with the electrical transducers output, which can be brought to a desired power level, but mechanical output cannot be signal conditioned.

### Classification of Transducers

The transducers can be classified on the basis of

- Transduction form used.
- Primary and Secondary transducers.
- Active and Passive transducers.
- Digital and Analog transducers.
- Transducers and Inverse transducers.

### Classification Based on Principle of Transduction

- On the basis of principle of transduction used they can be classified as resistive, inductive or capacitive transducers, the transduction depends on how they transduce the input quantity into the resistance, capacitance or inductance which can be measured by bridge or any other electronic circuitry.

Electrical Parameter and Class of Transducer	Principle of Operation and Nature of Device	Typical Application
	<b>Passive Transducers (externally powered)</b>	
<b>1. Resistance</b>		
Potentiometer device	Positioning of the slider by an external force varies the resistance in a potentiometer or a bridge circuit.	Pressure, displacement
Resistance strain gauge	Resistance of a wire or semiconductor is changed by elongation or compression due to externally applied stress.	Force, torque, displacement.
Pirani gauge or hot wire meter	Resistance of a heating element is varied by convection cooling of a stream of gas.	Gas flow, gas pressure.
Resistance thermometer	Resistance of pure metal wire with wire with a large positive temperature co-efficient of resistance varies with temperature.	Temperature, radiant heat

Thermistor	Resistance of certain metal oxides with negative temperature coefficient of resistance varies with temperature.	Temperature, flow
Resistance hygrometer	Resistance of a conductive strip changes with moisture content.	Relative humidity
Photoconductive cell	Resistance of the cell as a circuit element varies with incident light.	Photosensitive relay.
<b>2. Capacitance</b>		
Variable capacitance pre gauge	Distance between two parallel is varied by an externally applied force.	Displacement, pressure.
Capacitor microphone	Sound pressure varies the capacitance between a fixed plate and a movable diaphragm.	Speech, music, noise
<b>3. Inductance</b>		
Magnetic circuit transducer	Self-inductance or mutual inductance of a.c. excited coil is varied by changes in the magnetic circuit.	Pressure, displacement
Reluctance pick up	Reluctance of the magnetic circuits is varied by changing the position of the iron core of coil.	Pressure, displacement, vibration, position
Differential transformer	The differential voltage of two secondary windings of a transformer is varied by positioning the magnetic core through an externally applied force.	Pressure, force, displacement, position
Eddy current gauge	Induction of a coil is varied by the proximity of an eddy current plate.	Displacement, thickness.
Magnetostriction gauge	Magnetic properties are varied by pressure and stress.	Force, pressure, sound.
<b>4. Voltage and Current</b>		
Hall effect pickup	A potential difference is generated across a semiconductor plate (germanium) when magnetic flux interacts with an applied current.	Magnetic flux, current power
Tonization chamber	Electron flow induced by ionization of gas due to radioactive radiation.	Particle counting radiation.
Photoemissive cell	Electron emission due to incident radiation upon photoemissive surface.	Light and radiation.
Photomultiplier tube	Secondary electron emission due to incident radiation on photosensitive cathode.	Light & radiation, photosensitive relays.
<b>Self-generating Transducers (No External Power)</b>		
Thermocouple and thermopile	An emf is generated across the junction of two dissimilar metals or semiconductors when that junction is heated.	Temperature, heat, flow, radiation
Moving coil generator	Motion of a coil in a magnetic field generates a voltage.	Velocity, vibrations
Piezoelectric pick-up	An emf is generated when an external force is applied to certain crystalline materials, such as quartz.	Sound, vibrations, acc., pre changes.
Photovoltaic	A voltage is generated in a semiconductor junction device when radiant energy stimulates the cell.	Light meter, solar cell

### Primary and Secondary Transducers

- The transducers which convert the physical quantity into the mechanical displacement are called as the Primary transducers. eg. Bourdon Tube converts the input pressure into the displacement hence it acts as the primary transducers.
- The transducers which convert the mechanical displacement into the electrical output are called as the Secondary transducers. eg. LVDT, they can convert the displacement into the electrical signal by transformation and hence are termed as the Secondary transducers.

Secondary transducers are followed by the Primary transducers.

### Active and Passive Transducers

- Active transducers are those transducers which do not need any external power source to produce output. They produce the electrical power output themselves. eg. Piezoelectric transducers, thermocouple.
- Passive transducers are those transducers which draws power from an external source to produce their output. eg. Potentiometer.

### Digital and Analog Transducers

- The transducers which convert the input quantity into an analog output (i.e. the output which is continuous with time) are called analog transducers. eg. LVDT, Strain gauge etc.
- The transducers which convert the input quantity into an output which is in the form of pulses are called as the digital transducers. eg. Shaft encoders.

### Transducers and Inverse Transducers

- Transducers and inverse transducers find a wide application in the feedback control loop for a process control and in servomechanism, where we need to convert the electric quantity to mechanical in the forward loop and mechanical to electrical quantity in the feedback or the backward loop.
- Transducers in the broader sense are the device which are used to convert the mechanical quantity into the electric quantity. eg. LVDT, thermocouple etc.
- Inverse Transducers are the device which convert the electric quantity into mechanical quantity. eg. Piezoelectric crystal.

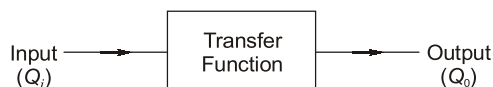
**Note:** Piezoelectric crystal acts as both transducer and inverse transducer.

## 1.4 Characteristics of Transducers

### Transfer Characteristics

The transfer characteristics of an instrument is a graphical representation of the relationship between the input and output of the transducer.

### Transfer Function



So the transfer function is the relationship between the input and the output.

**Sensitivity**

Sensitivity is defined as the ratio of the change in the output to the change in the input for a small time.

$$\text{Sensitivity} = \frac{\Delta Q_0}{\Delta Q_i}$$

- Note:** ■ Sensitivity of a transducer is not constant but it depends on the input.  
■ Sensitivity of the transducers becomes constant for the linear transfer characteristics.

**Scale factor**

Scale factor is defined as the inverse of sensitivity.

$$\text{Scale factor} = \frac{1}{S} = \frac{\Delta Q_i}{\Delta Q_0}$$

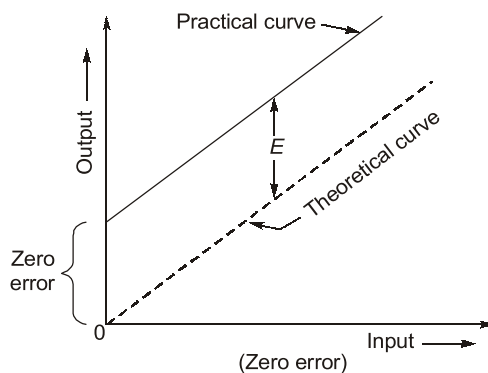
**Error**

Error in the transducers is said to occur when the input-output relationship deviates from the already defined input-output relationship for the transducer. Suppose, the output on account of the input ( $Q_i$ ) has to be ( $Q_0$ ) but practically it comes out to be ( $Q'_0$ ) then the error for the transducer is given as ( $\epsilon$ ).

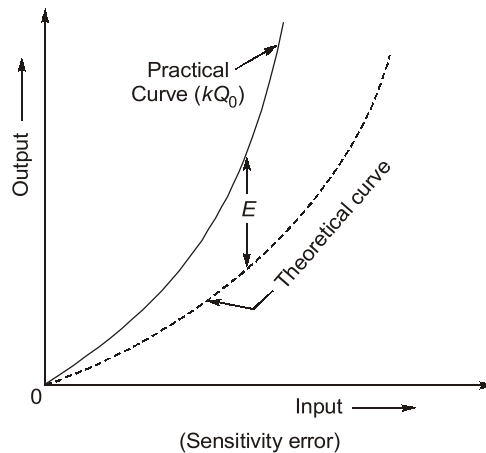
$$\epsilon = Q'_0 - Q_0$$

**Zero Error**

When the output deviates from the correct value of the output by a constant value over the entire range of the transducer, then the error is called as the zero error.

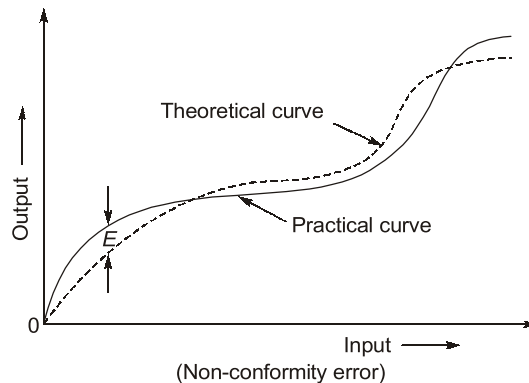
**Sensitivity Error**

This type of error occurs when there is a proportional change all along the upward scale of the instrument. Suppose, the correct output is  $Q_0$ , the output due to error would become ( $KQ_0$ ) over the entire range, where 'K' is the constant, then the error encountered is the sensitivity error as shown in the figure below:



### Non-Conformity Error

Nonconformity errors are those errors in which the experimentally obtained transfer function is different from the theoretically obtained transfer function.



### Hysteresis Error

When the transducer is used again and again or used more frequently, then the output of the transducer at a particular instant become sensitive to the previous values of inputs together with the input at that instant, hence we may get different outputs for same magnitude of inputs applied this is called as the hysteresis error.

#### Example - 1.2

In a transducer, the experimentally obtained transfer function is different from the theoretical transfer function, the errors result from this difference are called.

(a) Zero errors

(b) Sensitivity errors

(c) Nonconformity errors

(d) Dynamic errors

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**Solution : (c)**

#### Example - 1.3

In a transducer, the observed output deviates from the correct value by a constant factor the resulting error is called

(a) Zero error

(b) Sensitivity error

(c) Nonconformity error

(d) Hysteresis error

**Solution : (b)**

**Student's  
Assignments****1**

- Q.1** Inverse transducers
- (a) converts electrical energy to any other form of energy
  - (b) converts electrical energy to light energy
  - (c) converts mechanical displacement into electrical signal
  - (d) converts electrical energy to mechanical form
- Q.2** Which of the following is an active transducer?
- (a) strain gauge
  - (b) selsyn
  - (c) photovoltaic cell
  - (d) photoemissive cell

**Student's  
Assignments****2**

- Q.3** Some of the functional building blocks of a measurement system are:  
Primary Sensing Element (PSE)  
Variable Conversion Element (VCE), or Transducer  
Data Transmission Element (DTE)  
Variable Manipulation Element (VME)  
Data Presentation Element (DPE)  
The correct sequential connection of the functional building blocks for an electronic pressure gauge will be
- (a) PSE, VME, VCE, DPE, DTE
  - (b) PSE, VCE, VME, DTE, DPE
  - (c) DTE, DPE, VCE, PSE, VME
  - (d) PSE, VCE, DTE, DPE, VME

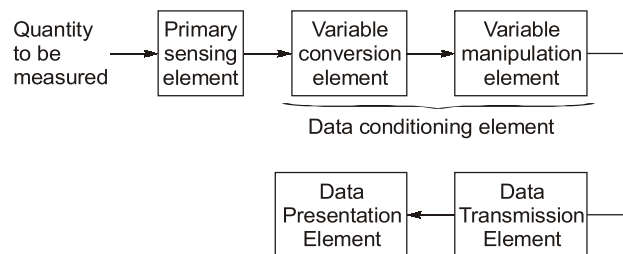
- Q.4** The transducers which convert the physical quantity into the mechanical displacement are called
- (a) Primary transducers
  - (b) Secondary transducers
  - (c) Tertiary transducers
  - (d) Inverse transducers
- Q.5** The transducers which convert the mechanical displacement into the electrical form are called the
- (a) Primary transducers
  - (b) Secondary transducers
  - (c) Tertiary transducers
  - (d) Inverse transducers

**ANSWERS**

1. (a)    2. (c)    3. (b)    4. (a)    5. (b)

**HINTS**

3. (b)  
Functional elements of a measurement system are shown below:



■■■■

## Basic Optics

### 1.1 Basic Optics

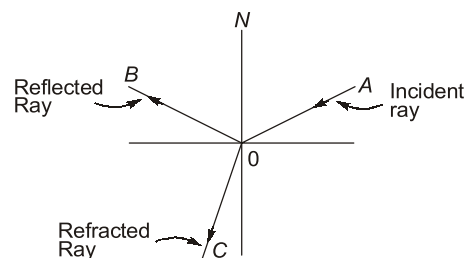
- Light or visible light is the portion of electromagnetic spectrum that is visible to human eye and is responsible for the sense of sight.
- Visible light has a wavelength in the range of about 380 or 400 nm to about 760 or 780 nm, with a frequency of 405 THz To 790 THz.

Speed of light in vacuum =  $3 \times 10^8$  m/sec.

$$\text{Refractive index of a material} = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in that medium}}$$

when a ray of light travelling in air or in any other medium strikes the surface of another medium then a part of light is reflected back into the same (that is travelling) medium and is called as the Reflected Ray.

While a part of it is refracted to the medium to which it strikes and is called as the Refracted Ray.



### 1.2 Reflection

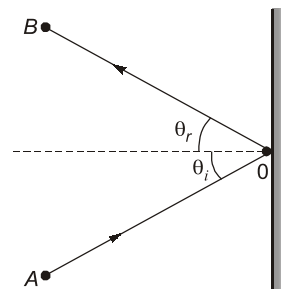
- Bending of light in the same medium in which it is travelling, when it strikes the surface of some other medium is called as the Reflection of light.

AO = Incident light ray

OB = reflected light ray

$\theta_i$  = incident angle

$\theta_r$  = reflected angle

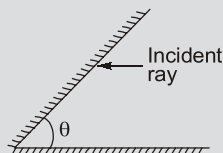


#### Law of Reflection

$$\angle \theta_i = \angle \theta_r$$

**NOTE**

No. of images formed when two mirrors are inclined at an angle  $\theta$  is:



$$n = \frac{360^\circ}{\angle \theta} - 1$$

$n$  = number of images formed.

$\theta$  = angle between the two inclined mirrors.

### 1.3 Refraction

Bending of light into the medium to which it strikes is called as the Refraction of light.

$AO$  = Incident ray

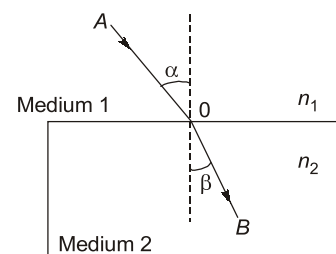
$OB$  = Refracted ray

$\alpha$  = Angle of incidence

$\beta$  = Angle of refraction

$n_1$  = RI of medium 1

$n_2$  = RI of medium 2



### 1.4 Snell's Law

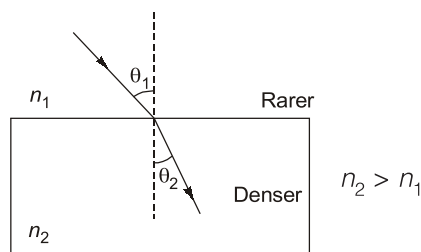
According to Snell's law

In figure (a)

$$n_1 \sin \alpha = n_2 \sin \beta$$

### Conditions of Refraction

(i) Rarer to Denser medium:

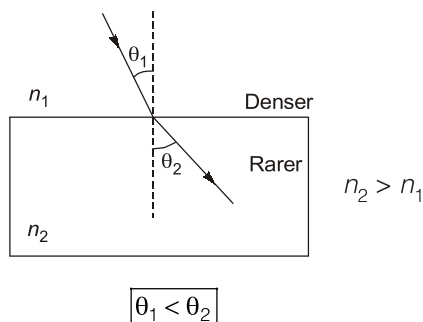


$$\theta_1 > \theta_2$$

When a ray of light travels from rarer to a denser medium it bends towards the normal.



**(ii) Denser to Rarer Medium:**



When a ray of light travels from denser to a rarer medium it bends away from the normal.

**Remember:** In TIR (Total Internal Reflection) of light the light beam travels from Denser to Rarer medium.

**Refractive Index of one Medium with Respect to other**

${}_1\mu_2$  = RI of 2nd medium with respect to 1st.

$${}_1\mu_2 = \frac{\mu_2}{\mu_1}$$

$\mu_2$  = RI of medium 2nd with respect to air

$\mu_1$  = RI of medium 1st with respect to air

**NOTE**



Light velocity change from one medium to the other.

Velocity of light in any medium =  $\frac{\text{Velocity of light in air}}{\text{RI of medium } (\mu_m)}$

$$v_m = \frac{3 \times 10^8}{\mu_m}$$

$${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_{m_1}}{v_{m_2}} = \frac{\lambda_1}{\lambda_2}$$

**Example - 1.1**

The Refractive index (RI) of the glass and water is 1.5 and 1.33 respectively.

If the glass is immersed in water, its refractive index is.

- (a) 0.89 (b) 1.13  
(c) 1.99 (d) 2.83

[GATE - 2001]

**Solution : (b)**

$$\begin{aligned} {}_{\text{water}}\mu_{\text{glass}} &= \frac{\mu_{\text{glass}}}{\mu_{\text{water}}} = \frac{1.5}{1.33} \\ &= 1.125 \text{ or } 1.13 \end{aligned}$$

**Example - 1.2**

Find the time taken by light to travel a distance of 600 m in water.

(a) 0.26 n sec

(b) 0.43 n sec

(c) 0.15 n sec

(d) 0.13 n sec

**Solution: (a)**

$$V_{\text{water}} = \frac{3 \times 10^8}{4/3} \therefore \left[ V_{\text{water}} = \frac{V_{\text{air}}}{\text{RI of water}} \right]$$

$$t = \frac{d}{V_{\text{water}}} = \frac{600}{2.25 \times 10^8} = 0.26 \text{ n sec}$$

**Example - 1.3**

A laser light with a wavelength of 633 nm is passed through 1 cm length of tissue and 2 cm length of glass. The refractive index of tissue and glass are 1.33 and 1.5 respectively. The velocities of laser light in the tissue and in the glass are in the ratio of

(a) 1.33 : 0.75

(b) 1.33 : 3.00

(c) 1.33 : 1.50

(d) 1.50 : 1.33

[GATE - 2008]

**Solution: (d)**

$$\mu_t = 1.33$$

$$\mu_g = 1.5$$

$$\lambda = 633 \times 10^{-9}$$

$$d_t = 1 \text{ cm}$$

$$d_g = 2 \text{ cm}$$

$$\frac{V_t}{V_g} = \frac{\mu_g}{\mu_t} = \frac{1.5}{1.33}$$

$$V_t : V_g = 1.5 : 1.33$$

Here the distance ( $d$ ) has no significance in this example.**Example - 1.4**

A light ray incident on air-glass interface with an angle of  $30^\circ$ . If glass refractive index is 1.5, then find the refracted angle.

(a)  $20.30^\circ$ (b)  $18.70^\circ$ (c)  $19.47^\circ$ (d)  $21.80^\circ$ **Solution: (c)**

$$n_1 \sin i = n_2 \sin r$$

$$1 \cdot \sin 30^\circ = 1.5 \sin r$$

$$\frac{1}{2} = 1.5 \sin r$$

$$\sin r = \frac{1}{3}$$

$$r = \sin^{-1}\left(\frac{1}{3}\right) = 19.47^\circ$$

Student's  
Assignments

1

- Q.1** Light shows  
 (a) wave like behaviour  
 (b) particle like behaviour  
 (c) both particle as well as wave like behaviour  
 (d) none of these
- Q.2** Brewster's law can be expressed as  
 (a)  $\mu = \tan r$  (b)  $\tan r = \frac{1}{\mu}$   
 (c)  $\mu = \tan i$  (d)  $\tan i = \frac{1}{\mu}$
- Q.3** In a light wave the plane at right angles to the plane of vibration is called  
 (a) vibrating plane (b) plane of polarization  
 (c) polarized plane (d) all the these
- Q.4** The vibrations of the light rays  
 (a) is parallel to the direction of propagation  
 (b) is perpendicular to direction of propagation  
 (c) some parallel and some perpendicular to the direction of propagation  
 (d) none of these
- Q.5** The refractive index is  
 (a) Velocity of light in vacuum  $\times$  Velocity of light in medium  
 (b)  $\frac{\text{Velocity of light in medium}}{\text{Velocity of light in vacuum}}$   
 (c)  $\frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}}$   
 (d) None of these
- Q.6** When a light passes from a medium of refractive index  $n_1$  to a medium of refractive index  $n_2$ , the reflection loss at the boundary is expressed as  
 (a)  $n_1 + n_2$  (b)  $n_1 - n_2$   
 (c)  $(n_1 - n_2)^2$  (d)  $\left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$
- Q.7** The refractive index of a material (varies with the wavelength as)

- (a)  $\mu \propto \lambda$  (b)  $\mu \propto \frac{1}{\lambda}$   
 (c)  $\mu \propto \lambda^2$  (d)  $\mu \propto \frac{1}{\lambda^2}$

- Q.8** A ray of light incident on a glass plate at an angle  $60^\circ$ . The reflected and refracted rays are found to be mutually perpendicular. The refractive index of glass is  
 (a) 1.5 (b) 1.62  
 (c)  $\frac{2}{\sqrt{3}}$  (d)  $\sqrt{3}$
- Q.9** Speed of light in air is  $3 \times 10^8$  m/sec. What will be the speed of light in diamond. (Given R.I. of diamond is 2.4).  
 (a)  $3 \times 10^8$  m/sec (b) 322 m/sec  
 (c)  $1.25 \times 10^8$  m/sec (d)  $7.2 \times 10^8$  m/sec

Student's  
Assignments

2

- Q.10** A ray of light is made to fall on a glass plate at an angle of  $58^\circ$  and the reflected ray is completely plane polarized. Find the angle of refraction  
 (a)  $58^\circ$  (b)  $28^\circ$   
 (c)  $38^\circ$  (d)  $32^\circ$
- Q.11** A ray of light is incident on the surface of a glass sheet of refractive index 1.55 at the polarizing angle. Calculate the angle of refraction  
 (a)  $36^\circ$  (b)  $38^\circ$   
 (c)  $33^\circ$  (d)  $42^\circ$
- Q.12** Find the time taken by light to travel a distance of 700 m in water. ( $\mu_{\text{water}} = 4/3$ )  
 (a) 0.26 nsec (b) 0.36 nsec  
 (c) 0.21 nsec (d) 0.31 nsec
- Q.13** A laser beam is made to pass through two separate tissues. Speed of light in tissue A is  $2 \times 10^5$  m/sec. and in tissue B is  $2.5 \times 10^5$  m/sec. If the length of both the tissues are same and equal to 2 m find the ratio of the refractive index of tissue B with respect to the tissue A.  
 (a) 4/5 (b) 5/4  
 (c) 8/5 (d) 5/8

**ANSWERS**

1. (c)    2. (a)    3. (b)    4. (b)    5. (c)  
6. (d)    7. (d)    8. (d)    9. (d)    10. (d)  
11. (c)    12. (d)    13. (a)

**HINTS**

2. (a)

$$i = \tan^{-1} \mu = \tan^{-1} 1.55$$

$$i + r = 90^\circ$$

$$\therefore r = 90^\circ - 57.17 = 32.83^\circ$$

8. (d)

$$\mu = \tan^{-1} 60^\circ = \sqrt{3}$$

9. (d)

$$\frac{n_1}{n_2} = \frac{v_2}{v_1}$$

$$\therefore v_{\text{diamond}} = \frac{1}{24} \times 3 \times 10^8$$

$$= 1.25 \times 10^8 \text{ m/sec}$$

10. (d)

$$i + r = 90^\circ$$

$$\therefore r = 90^\circ - 58 = 32^\circ$$

11. (c)

$$i = \tan^{-1} (1.55) = 57.17^\circ$$

$$r = 90^\circ - 57.17^\circ = 32.8^\circ$$

12. (d)

$$V_{\text{water}} = \frac{3 \times 10^8}{4/3} = 2.25 \times 10^8 \text{ m/sec}$$

$$t = \frac{d}{V_{\text{water}}} = \frac{700}{2.25 \times 10^8}$$

$$= 0.31 \text{ nsec}$$

13. (a)

$$RI = \frac{\text{Velocity of light in air}}{\text{Velocity of light in the medium}}$$

$$\therefore RI \propto \frac{1}{\text{Velocity of light in medium}}$$

$$\frac{\mu_B}{\mu_A} = \frac{V_A}{V_B}$$

$$= \frac{2 \times 10^5}{2.5 \times 10^5} = \frac{20}{25} = \frac{4}{5}$$

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