POSTAL Book Package 2023

Mechanical Engineering

Conventional Practice Sets

Engineering Mechanics

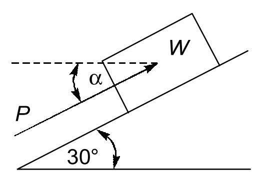
Contents

MADE EASY Publications

1 CHAPTER Equilibrium of Forces and Moment

Practice Questions

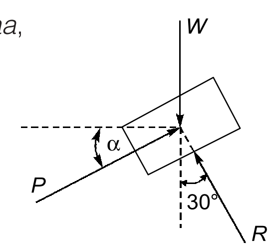
Q.1 Determine the magnitude and direction of the smallest force P, which will maintain the body of weight W = 300 N on an inclined smooth plane as shown in figure is in equilibrium.

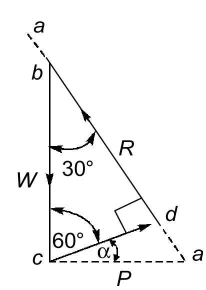


Solution:

The body is acted upon by three forces, namely the action of gravity force W, the applied force P and the reaction R. Since these three forces are in equilibrium, the vectors representing them must build a closed triangle, we begin with the known vector $\overline{bc}$ representing to a certain scale, the weight of the body, and then draw the line aa parallel to the R.

The side $\overline{cd}$ will be minimum if it is perpendicular to line aa, that is P will be minimum, if it is perpendicular to aa.





From the triangle bcd, $\angle c = 90° - 30° = 60°$

$$\therefore \quad \alpha = 90° - 60° = 30°$$

and using the triangle bcd, we obtain,

$$P = W\sin 30° = \frac{W}{2} = 150 \text{ N}$$

Alternate solution: After drawing the free-body diagram of the body of above, then applying the Lami's theorem to the free-body diagram of the body as shown in figure we get

$$\frac{W}{\sin(90° - \alpha + 30°)} = \frac{P}{\sin(\pi - 30°)} = \frac{R}{\sin(90° + \alpha)}$$

Using the first two of the equation we obtain

$$\frac{W}{\cos(30° - \alpha)} = \frac{P}{\sin 30°}$$

$$P = \frac{W\sin 30°}{\cos(30° - \alpha)}$$

From equation, P will be minimum, if the denominator is maximum, i.e.

$$\cos(30° - \alpha) = 1$$

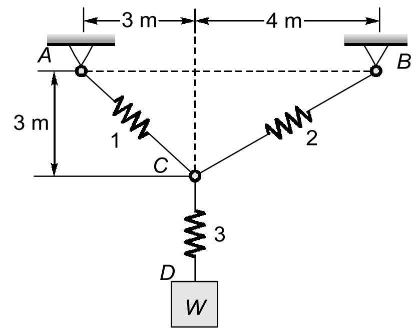
$$\Rightarrow \quad 30° - \alpha = 0$$

$$\Rightarrow \quad \alpha = 30°$$

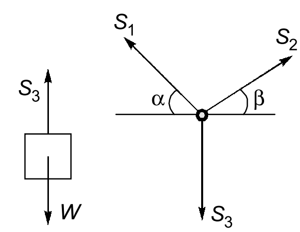
and substituting this value into equation, we get the value of

$$P = W\sin 30° = 150 \text{ N, as before}$$

Q.2 Determine the stretch in each spring for equilibrium of the weight $W = 40$ N block as shown in figure. The springs are in equilibrium position. The stiffness of each spring is given as: $k_1 = 40$ N/m, $k_2 = 50$ N/m, and $k_3 = 60$ N/m



Solution:



Draw the free-body diagram of the body as shown in figure.

Only two forces are acting on the body, gravity force W and the reactive force caused by the spring S_3. Since the body is in equilibrium, from the law of equilibrium of two forces,

$$S_3 = W$$

Now, draw the free-body diagram of the point C. At the joint, C three forces are acting all are reactive forces caused by the springs. The angles that springs S_1 and S_2 make with the horizontal are calculated as below:

$$\tan\alpha = \frac{3}{3} = 1 \Rightarrow \alpha = 45^\circ$$

$$\tan\beta = \frac{3}{4} \Rightarrow \beta = 36.87^\circ$$

Since the joint C is in equilibrium, applying Lami's theorem, we obtain

$$\frac{S_1}{\sin\left(\frac{\pi}{2}+\beta\right)} = \frac{S_2}{\sin\left(\frac{\pi}{2}+\alpha\right)} = \frac{S_3}{\sin(\pi-\alpha-\beta)}$$

From equation we get

$\Rightarrow$
$$S_1 = \frac{S_3\cos\beta}{\sin(\alpha+\beta)} = \frac{W\cos\beta}{\sin(\alpha+\beta)}$$

$$S_2 = \frac{S_3\cos\alpha}{\sin(\alpha+\beta)} = \frac{W\cos\alpha}{\sin(\alpha+\beta)}$$

$$EF = EC + CF = r_1 + r_2 = 100 + 50 = 150 \text{ mm}$$

and
$$EH = OI - OG - BI$$

$$OI = a = 200 \text{ mm}$$

and
$$OG = r_2 = 50 \text{ mm}$$

$$BI = EI\sin\frac{\alpha}{2}\left[\because EI = \frac{BE}{\cos\frac{\alpha}{2}} = \frac{r_1}{\cos 30^\circ} = \frac{100}{\cos 30^\circ} = 115.47 \text{ mm}\right]$$

$\therefore$
$$BI = 115.47\sin 30^\circ = 57.74 \text{ mm and}$$

$\therefore$
$$EH = 200 - 50 - 57.74 = 92.26 \text{ mm}$$

$$\cos\beta = \frac{EH}{EF} = \frac{92.26}{150} = 0.615$$

∴ $\beta = 52.05°$

$$R_c \cos\beta = R_d$$

$$R_c \sin\beta = Q$$

Substituting the values for β and Q in the above equations and solving for R_c and R_d, we obtain

$$R_c = \frac{Q}{\sin\beta} = \frac{800}{\sin 52.05} = 1014.52 \text{ N}$$

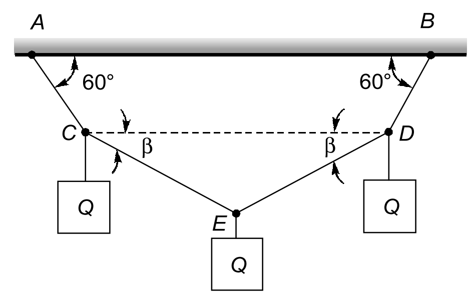
$$R_d = R_c \cos\beta = 1014.52 \times \cos 52.05° = 623.9 \text{ N}$$

$$R_a = R_c \frac{\cos\beta}{\sin\alpha} = 1014.52 \times \frac{\cos 52.05}{\sin 60} = 720.42 \text{ N}$$

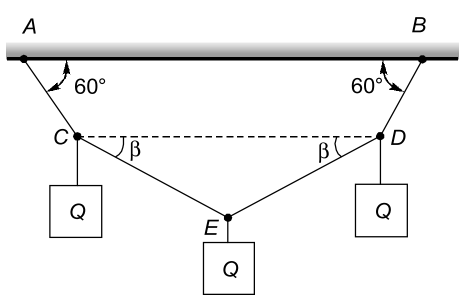
$$R_b = R_c \sin\beta + P - R_a \cos\alpha$$

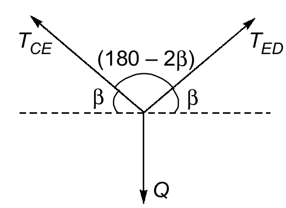
$$= 1014.52 \times \sin 52.05° + 2000 - 720.42 \cos 60° = 2439.79 \text{ N}$$

Q.3 On the string *ACEDB* are hung three equal weights *Q* symmetrically placed with respect to the vertical line through the mid-point *E*. Determine the value of the angles *b* if the other angles are as shown in the figure.



Solution:





At point *E*,

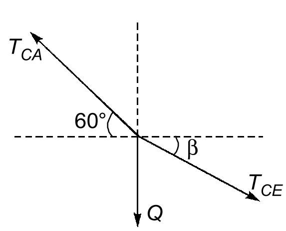
By symmetry,

$$T_{CE} = T_{ED}$$

Lami's theorem $$\frac{T_{CE}}{Sin(90+\beta)} = \frac{T_{ED}}{\sin(90+\beta)} = \frac{Q}{\sin(180-2\beta)}$$

$$T_{CE} = \frac{Q\cos\beta}{\sin 2\beta} = \frac{Q}{2\sin\beta} \qquad \text{...(i)}$$

At point *C*:



Lami's theorem $$\frac{T_{CA}}{\sin(90-\beta)} = \frac{T_{CE}}{\sin 150} = \frac{Q}{\sin(120+\beta)}$$

Now, $$T_{CE} = \frac{Q \times \sin 150}{\sin(120+\beta)} \qquad \text{...(ii)}$$

By equation (i) and (ii) $$\frac{Q}{2\sin\beta} = \frac{Q\times 1/2}{\sin(120+\beta)}$$

$$\sin(120+\beta) = \sin\beta$$

$$\sin[90+(30+\beta)] = \sin\beta$$

$$\cos\beta\cdot\cos 30 - \sin\beta\cdot\sin 30 = \sin\beta$$

$$\cos\beta\times\frac{\sqrt{3}}{2} = \sin\beta+\frac{1}{2}(\sin\beta)$$

$$\frac{\cos\beta}{\sin\beta} = \frac{\frac{3}{2}}{\left(\frac{\sqrt{3}}{2}\right)}$$

$$\left(\frac{\cos\beta}{\sin\beta}\right) = \sqrt{3}$$

$$\tan\beta = \frac{1}{\sqrt{3}}$$

$$\beta = 30^\circ$$

Alternate:

$$\sin[180-(120+\beta)] = \sin\beta$$

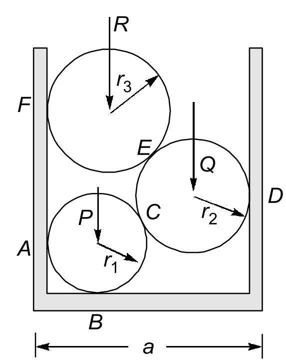
comparing on both sides

$$180-120-\beta = \beta$$

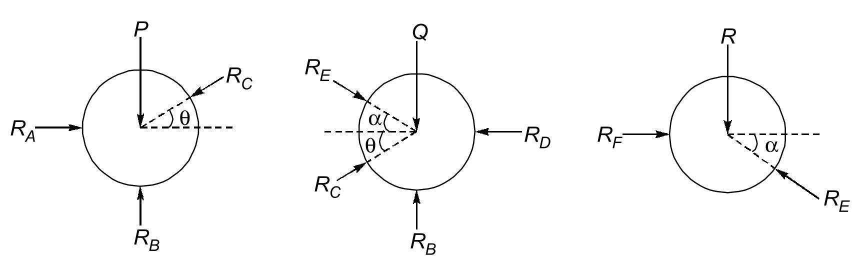
$$60 = 2\beta$$

$$\beta = 30^\circ$$

Q.4 The smooth cylinders rest in a horizontal channel having vertical walls, the distance between which is a. Find the pressures exerted on the walls and floor at the points of contact *A*, *B*, *D* and *F*. the following numerical data are given: P = 200 N, Q = 400 N, R = 300 N, r_1 = 120 mm, r_2 = 180 mm, r_3 = 150 mm and a = 540 mm.



Solution:



For cylinder 2:

$$\cos\alpha = \frac{540-180-150}{180+150}$$

$$\alpha = 50.47^\circ$$

$$\frac{R}{\sin(180-50.47)^\circ} = \frac{R_E}{\sin 90^\circ} = \frac{R_F}{\sin(90+50.47)^\circ}$$

$$R_E = \frac{R}{\sin(129.53°)} = 388.96 \text{ N}$$

$$R_F = \frac{R \times \sin 140.47°}{\sin(129.53°)} = 247.565 \text{ N}$$

For cylinder 1:
$$\cos\theta = \frac{540 - 120 - 180}{120 + 180}$$
$$\theta = 36.87°$$

Now
$$R_C \cos\theta = R_A \quad \text{...(i)}$$
$$P + R_C \sin\theta = R_B \quad \text{...(ii)}$$

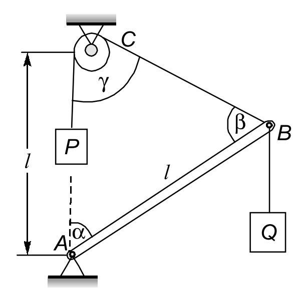
Now,
$$R_C = 1.25\, R_A$$
$$P + 0.6\, R_C = R_B \Rightarrow P + 0.75\, R_A = R_B$$

For cylinder 2:
$$Q + R_E \sin\alpha = R_C \sin\theta$$

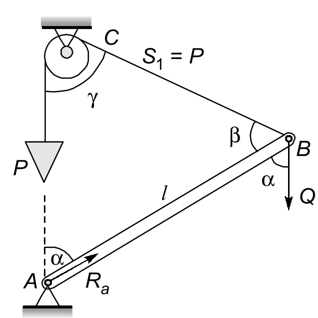
Now,
$$R_C = \frac{400 + 388.96 \times \sin 50.47°}{\sin 36.87°}$$
$$R_C = 1166.67 \text{ N}$$
$$R_D = R_E \cos\alpha + R_C \cos\theta$$
$$R_D = 388.96 \cos 50.47° + 1166.67 \cos 36.87°$$
$$R_D = 1180.9 \text{ N}$$

For cylinder 1:
$$R_A = \frac{R_C}{1.25} = \frac{1166.67}{1.25} = 933.336 \text{ N}$$
$$R_B = P + 0.75\, R_A$$
$$= 200 + 0.75 \times 933.336$$
$$R_B = 900 \text{ N}$$

Q.5 A prismatic bar *AB* of negligible weight and length *l* is hinged at *A* and supported at *B* by a string that passes over a pulley *C* and carries a load *P* at its free end. Assuming that the distance *h* between the hinge *A* and the pulley *C* is equal to the length *l* of the bar, find the angle α at which the system will be in equilibrium.



Solution:



The triangle *ABC* is isosceles

$$\beta = \gamma = \frac{\pi - \alpha}{2} = 90° - \left(\frac{\alpha}{2}\right)$$

Taking point *A* as the moment center (thus eliminating consideration of the unknown reaction at *A*), we obtain