Mechanical Engineering

Engineering Mechanics

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016
E-mail: infomep@madeeasy.in
Contact: 011-45124660, 8860378007
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Engineering Mechanics

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CHAPTER 01

Composition, Resolution and Equilibrium of Forces

1.1 Force

Force is the action of one body on another. It may be defined as an action which changes or tends to change the state of rest or of uniform motion of body. For representing the force acting on the body, the magnitude of the force, its point of action and direction of its action should be known. There are different types of forces such as gravitational, frictional, magnetic, inertia or those caused by mass and acceleration.

According to Newton's second law of motion, we can write force as

$$F = ma = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2}$$

One Newton force is defined as that which gives an acceleration of 1 m/s^2 to a body of mass of 1 kg in the direction of force.

Thus, $$1\text{ N} = 1\text{ kg} \times 1\text{ m/s}^2 = 1\text{ kg-m/s}^2$$

The action of one body and another, which changes or tends to change the state of rest or of uniform motion of body is called as force.

The three requisites for representing the force acting on the body are:

- Magnitude of force
- It points of action, and
- Direction of its action

1.2 Effects of a Force

A force may produce the following effects in a body, on which it acts:

1. It may change the motion of a body i.e. if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate or retard it.
2. It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.
3. It may give rise to the internal stresses in the body, on which it acts.

1.3 Characteristics of a Force

To know the effect of force on a body, the following elements of force should be known.

1. Magnitude (i.e. 2 N, 5 kN, 10 kN etc.)
2. Direction or line of action.

3. Sense or nature (push or pull).
4. Point of application.

1.4 Force Systems

A force system is collection of forces acting on a body in one or more planes. According to the relative position of the lines of action of the forces, the forces may be classified as follows:

1. **Collinear:** The forces whose lines of action lie on the same line are known as collinear forces.
2. **Concurrent:** The forces, which meet at one point, are known as concurrent forces. Concurrent forces may or may not be collinear.
3. **Coplanar:** The forces whose line of action lie on the same plane are known as coplanar forces.
4. **Coplanar concurrent:** The forces, which meet at one point and their line of action lie on the same plane, are known as coplanar concurrent forces.
5. **Non-coplanar concurrent:** The forces, which meet at one point but their lines of action do not lie on the same plane, are known as coplanar non-concurrent forces.
6. **Coplanar non-concurrent:** The forces, which do not meet at one point but their line of action lie on the same plane, are known as coplanar non-concurrent forces.
7. **Non-coplanar non-concurrent:** The forces, which do not meet at one point and their line of action do not lie on the same plane, are known as non-coplanar non-concurrent forces.

1.5 Resultant Force

A single force which produces same effect on the body as the system of forces is called as resultant force.

1.6 Parallelogram Law of Forces

This law is used for finding the resultant of two forces acting at a point.

If two forces F_1 and F_2 are acting at a point and are represented in magnitude and direction by two sides of a parallelogram, then their resultant is represented by the diagonal of the parallelogram both in magnitude and direction.

Consider a parallelogram *OACB* as shown in figure 1.1 where sides *OA* and *OB* represent the forces F_1 F_2 acting at a point *O*. According to the parallelogram law of forces, the resultant *R* is represented by a diagonal *OC*.

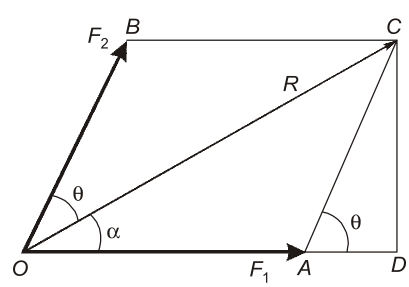


Fig. 1.1

Let θ be the angle between the forces F_1 and F_2 and α be the angle made by *R* with force F_1.

From the figure 1.1 we can write

$$BC = OA = F_1$$
$$AC = OB = F_2$$
$$\angle BOA = \theta = \angle CAD$$

and ΔODC and ΔADC are right angle triangles.

From triangle *ADC*, we can write

$$AD = AC\cos\theta = F_2\cos\theta$$
$$CD = AC\sin\theta = F_2\sin\theta$$

From triangle *ODC*, we can write

$$OC^2 = OD^2 + CD^2 = (OA + AD)^2 + CD^2$$
$$R^2 = (F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2$$

$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2\cos^2\theta + F_2^2\sin^2\theta$$

$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2(\cos^2\theta + \sin^2\theta)$$

$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2$$

$$R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2} \quad \text{... (i)}$$

From triangle *ODC*,

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta} \quad \text{... (ii)}$$

Thus
$$R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2}$$

and
$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

1.7 Triangle Law of Forces

This law states that:

If two forces acting simultaneously on a body are represented in magnitude and direction by two sides of a triangle taken in order then their third side will represent the resultant of two forces in the direction and magnitude taken in opposite order.

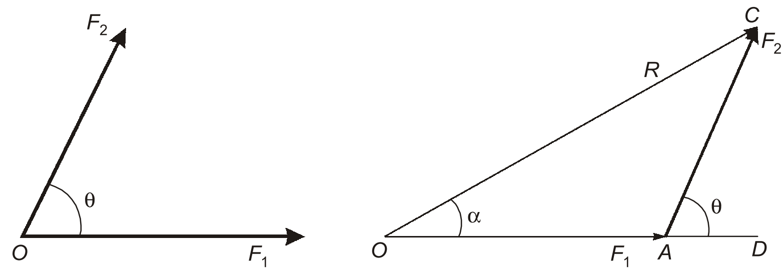


Fig. 1.2

If three forces are acting on a body and they are represented by three sides of the triangle in magnitude and direction, then the body will be in equilibrium condition.

1.8 Polygon Law of Forces

When two more forces are acting on the body, the triangle law can be extended to polygon law.

If a number coplanar concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in order, then their resultant can be represented by closing side of the polygon in magnitude and direction in the opposite order.

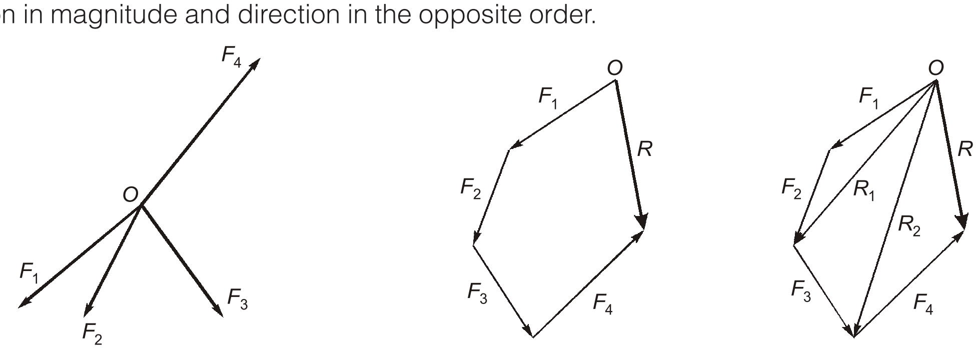


Fig. 1.3

Consider the forces F_1, F_2 and F_3 acting at a point O as shown in figure 1.3. As per the polygon law of forces the resultant force R is as shown in figure 1.3. According to parallelogram law, then the resultant of F_1 and F_2 is represented by R_1 and resultant of R_1 and F_3 is represented by R_2. The resultant R is the resultant of F_4 and R_2. This procedure can be extended to any number of forces acting at a point in a plane.

1.9 Composition of Forces

Conversion of system of forces into an equivalent single force system is known as the composition of forces. The effect of single equivalent force will be same as the effect produced by number of forces action on a body.

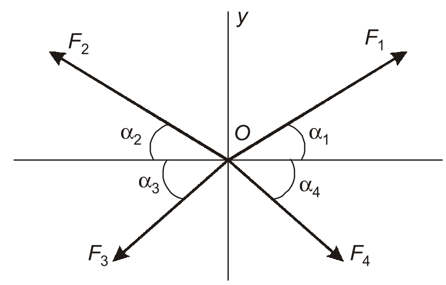


Fig. 1.4

Let the forces F_1, F_2, F_3, F_4 are acting on a body in a plane making angle α_1, α_2, α_3 and α_4 with x-axis as shown in figure 1.4. Let R be the resultant force of all the forces acting at the point making an angle θ with horizontal as shown in figure. Resolving the forces along x-axis and y-axis, we get

$$\Sigma F_x = F_1 \cos \alpha_1 - F_2 \cos \alpha_2 - F_3 \cos \alpha_3 + F_4 \cos \alpha_4$$
$$\Sigma F_y = F_1 \sin \alpha_1 + F_2 \sin \alpha_2 - F_3 \sin \alpha_3 - F_4 \sin \alpha_4$$

Component of R along x-axis = $R \cos\theta$

Component of R along y-axis = $R \sin\theta$

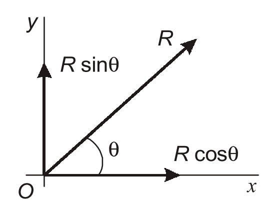


Fig. 1.5

$$R \cos \theta = \Sigma F_x$$

and $$R \sin \theta = \Sigma F_y$$

$$R^2 (\sin^2 \theta + \cos^2 \theta) = (\Sigma F_x)^2 + (\Sigma F_y)^2$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

and $$\tan \theta = \frac{\Sigma F_y}{\Sigma F_x}$$

A body which is under co-planar system of concurrent forces is in equilibrium if $R = 0$ or

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

1.10 Resolution of Forces

Replacing force F by two forces along x and y axis acting on the same body is called resolution of forces. Resolution is the reverse process of composition.

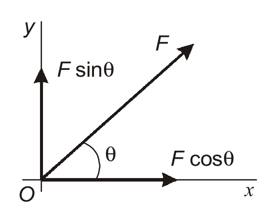


Fig. 1.6

Case I: A force F acting at a point 'O' making angle θ with horizontal as shown in figure 1.6. Then its components along x and y axis are given by

$F_x = F \cos \theta$ and $F_y = F \sin \theta$

Case II: The resolution of force W when the body is on an inclined plane. The components of the body force W are given by

$W_n = W \cos \theta$ and $W_p = W \sin \theta$

where W_n is normal component to inclined plane and W_p is parallel component to inclined plane.

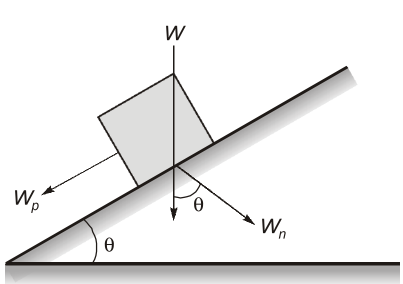
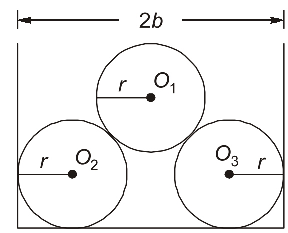


Fig. 1.7

Example 1.8 Three cylinders weighing W each and r in radius are placed in a channel of $2b$ width as shown in figure. Determine the pressure exerted by the cylinder.



(1) O_1 on cylinder O_2, (2) O_2 on the horizontal wall, (3) O_3 on the vertical wall

Determine all the values if $2b$ = 36 cm, r = 8 cm and W = 400 N.

Solution:

From the geometry of figure (a) we get,

$$\sin\alpha = \frac{O_2N}{O_1O_2} = \frac{b-r}{2r}$$

$$\cos\alpha = \frac{\sqrt{(r+b)(3r+b)}}{2r}$$

Due to symmetry of figure equal reaction forces may be observe easily. All forces acting on system is shown in figure (b). Here

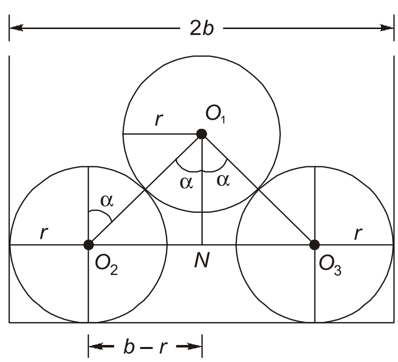


Fig. (a)

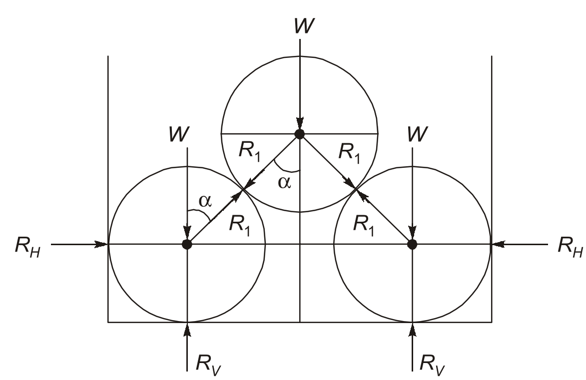


Fig. (b)

(1) R_1 is pressure exerted by the cylinder O_1 on cylinder O_2.

(2) R_V is pressure exerted by the cylinder O_2 on the horizontal wall

(3) R_H is pressure exerted by the cylinder O_3 on the vertical wall

Now consider *FBD* of O_1 as shown in figure (c),

Apply Lami's theorem,

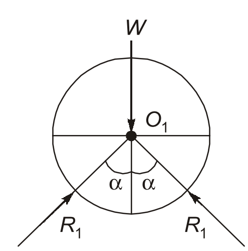


Fig. (c)

$$\frac{W}{\sin 2\alpha} = \frac{R_1}{\sin(180^\circ-\alpha)} = \frac{R_1}{\sin(180^\circ-\alpha)}$$

or
$$\frac{W}{2\sin\alpha\cos\alpha} = \frac{R_1}{\sin\alpha}$$

Thus
$$R_1 = \frac{W}{2\cos\alpha}$$

Now consider the *FBD* of O_2 as shown in figure,

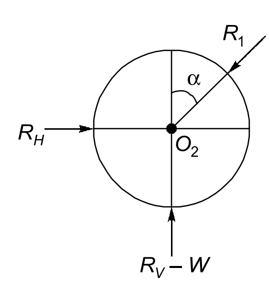


Fig. (d)

Applying Lami's theorem,
$$\frac{R_V-W}{\sin(90^\circ+\alpha)} = \frac{R_H}{\sin(180^\circ-\alpha)} = \frac{R_1}{\sin 90^\circ}$$

or
$$\frac{R_V-W}{\cos\alpha} = \frac{R_H}{\sin\alpha} = R_1 = \frac{W}{2\cos\alpha}$$

Thus, $$R_H = R_1 \sin\alpha = \frac{W\sin\alpha}{2\cos\alpha} = 0.5\ W\tan\alpha$$

$$R_v - W = R_1 \cos\alpha$$

or $$R_v = \frac{W\cos\alpha}{2\cos\alpha} + W = 1.5\ W$$

Now for $2b = 36$ cm, $r = 80$ mm, $W = 200$ N, we get

$$\sin\alpha = \frac{18-8}{16} = \frac{5}{8} = 0.625$$

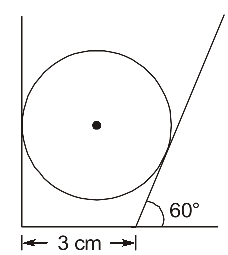
$$\cos\alpha = 0.78,\ \tan\alpha = 0.80$$

Thus $$R_1 = \frac{200}{2\times 0.78} = 128\text{ N}$$

$$R_H = \frac{200\times 0.80}{2} = 80\text{ N}$$

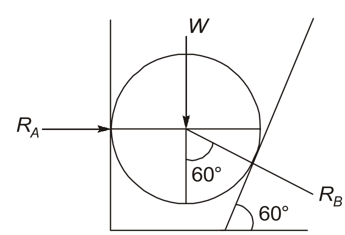
$$R_v = 1.5\times 200 = 300\text{ N}$$

Example 1.9 A smooth sphere of 2 kN weight and 2 cm radius is resting against the walls as shown in figure. Determine the reaction at the supporting points.



Solution:

Sphere is in contact with wall at two point. At these point reaction force is exerted by wall. In figure (a) all forces has been shown.



Applying Lami's theorem, we get

$$\frac{W}{\sin(90°+60°)} = \frac{R_A}{\sin(180°-60°)} = \frac{R_B}{\sin 190°}$$

or $$\frac{2}{\cos 60°} = \frac{R_A}{\sin 60°} = R_B$$

Thus $$R_A = \frac{2\sin 60°}{\cos 60°} = 2\tan 60° = 2\sqrt{3}\ \text{kN}$$

$$R_B = \frac{2}{\cos 60°} = \frac{2}{1/2} = 4\text{ kN}$$

Example 1.10 A uniform rod *AB* remains in equilibrium in a vertical plane, resting on smooth inclined place *AC* and *BC*, which are at right angles. If the plane *BC* is at α with the horizontal find the inclination θ of the rod with the plane *AC*.

Solution:

As per statement of problem the configuration is shown in figure (a). All angles and forces is shown in figure (b).

The rod is in equilibrium under action of the following forces.

1. Weight W of the rod acting vertical downward through the middle point G of the rod.
2. Reaction R_A at point of contact with plane *AC* normal to the plane *AC*.
3. Reaction R_B at point of contact with plane *BC* normal to the plane *BC*.

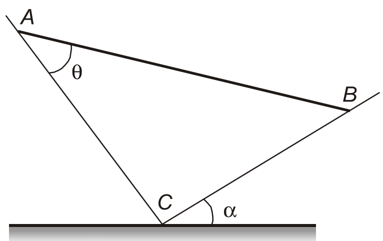


Fig. (a)

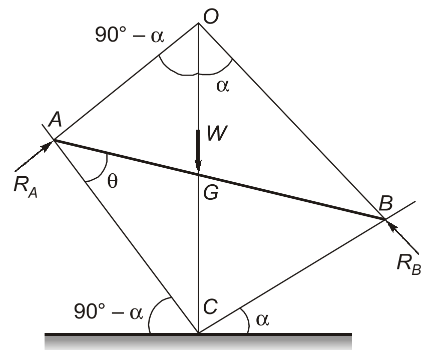


Fig. (b)

If rod is in equilibrium then, three force must be concurrent. Let these forces meet at *O*.

since $AO \perp AC$ and $BO \perp BC$

Thus $AO \,||\, BC$ and $BO \,||\, AC$

$$AOB = 90°$$

Now the figure *AOBC* is a rectangle whose diagonal *OGC* is vertical.

Also $GA = GO$

Thus $GAC = GCA$

$$\theta = \alpha \qquad \text{Hence Proved.}$$

Example 1.11 A rod whose center of gravity divide it into two portion of length *a* and *b* rests inside a smooth sphere in a position inclined to the horizontal. Show that if θ be the inclination to the horizontal and 2 α the angle that it subtends at the center of sphere

$$\tan\theta = \frac{b-a}{b+a}\tan\alpha$$

Solution:

As per statement of problem the configuration is shown in figure (a). The rod *AB* is equilibrium under these forces as shown in figure (b).

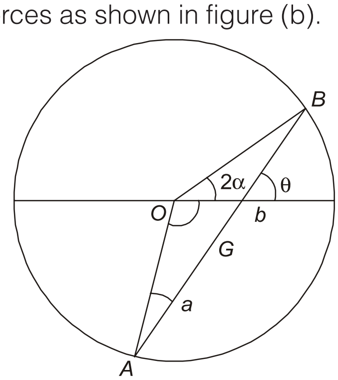


Fig. (a)

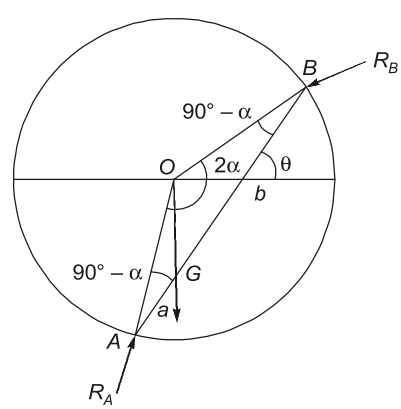


Fig. (b)

1. Weight *W* of the rod acting vertically downward.
2. Reaction R_A at point of contact *A* and normal to sphere where passes through *O*.

3. Reaction R_B at point of contact *B* and normal to sphere which passes through *O*.
For rod to be in equilibrium these forces must be concurrent. So line of acting of center of gravity will pass through *O*.
Now from geometry of figure (b)

$$BOG = 180° - (90° - \alpha) - (90° - \theta) = \alpha + \theta$$
$$AOG = 2\alpha - (\alpha + \theta) = \alpha - \theta$$

From the ΔAOG

$$\frac{AG}{\sin AOG} = \frac{AO}{\sin AGO}$$

or $$\frac{a}{\sin(\alpha - \theta)} = \frac{r}{\sin(90 + \theta)}$$

or $$\frac{a}{\sin(\alpha - \theta)} = \frac{r}{\cos\theta} \quad \text{... (i)}$$

From the ΔBOG

$$\frac{BG}{\sin BOG} = \frac{OB}{\sin OGB}$$

or $$\frac{b}{\sin(\alpha + \theta)} = \frac{r}{\cos\theta} \quad \text{... (ii)}$$

From equations (i) and (ii), we get

$$\frac{a}{\sin(\alpha - \theta)} = \frac{b}{\sin(\alpha + \theta)}$$

or $$b(\sin\alpha\cos\theta - \cos\alpha\sin\theta) = a(\sin\alpha\cos\theta - \cos\alpha\sin\theta)$$

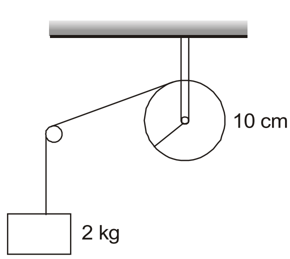
or $$(b - a)\sin\alpha\cos\theta = (a + b)\cos\alpha\sin\theta$$

or $$\frac{(b - a)\sin\alpha}{(b + a)\cos\alpha} = \frac{\sin\theta}{\cos\theta}$$

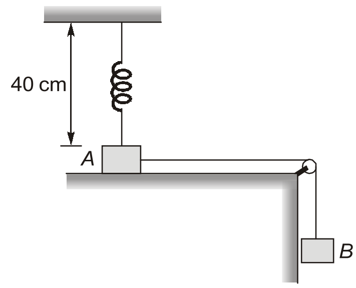
or $$\tan\theta = \frac{b - a}{b + a}\tan\alpha$$ Hence proved

Objective Brain Teasers

Q.1 A string is wrapped on a wheel of moment of inertia 0.2 kg/m^2 and radius 10 cm and goes through a light pulley to support a block of mass 2.0 kg as shown in figure. The acceleration of block is _______ m/s^2.

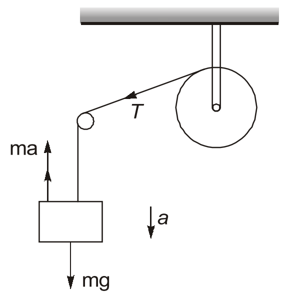


Q.2 Figure shows two blocks *A* and *B*, each having a mass of 320 g connected by a light string passing over a smooth light pulley. The horizontal surface on which block *A* slides is smooth. The block *A* is attached to a spring of spring constant 40 N/m. Initially spring is vertical and unstretched. The extension in the spring at the instant when block *A* breaks off is ___________ cm. (Take g = 10 m/s^2)



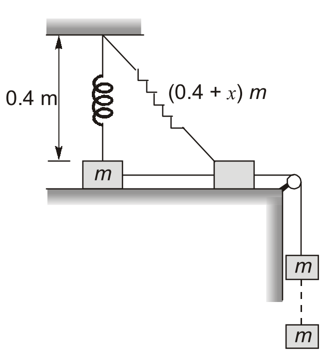
Hints & Explanation

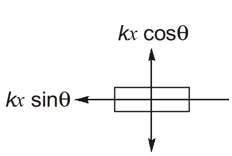
1. (0.89)(0.87 to 0.91)



$$I = 0.2 \text{ kgm}^2$$
$$r = 0.1$$
$$m = 2 \text{ kg}$$
$$\therefore \quad mg - T = ma$$
$$\text{Now} \quad T = \frac{Ia}{r^2}$$
$$\Rightarrow \quad mg = \left(m + \frac{I}{r^2}\right)a$$
$$\Rightarrow \quad 2 \times 9.81 = \left(2 + \frac{0.2}{0.01}\right)a$$
$$\Rightarrow \quad a = 0.89 \text{ m/s}^2$$

2. (10)





Given: m = 0.32 kg, k = 40 N/m, h = 0.4 m, g = 10 m/s^2

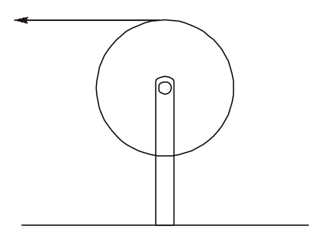
From FBD, $kx\cos\theta$ = mg

$$\Rightarrow \quad kx\cos\theta = \frac{mg}{kx}$$
$$\Rightarrow \quad \frac{0.4}{0.4+x} = \frac{mg}{kx}$$
$$\Rightarrow \quad \frac{0.4}{0.4+x} = \frac{0.32\times 10}{40x}$$

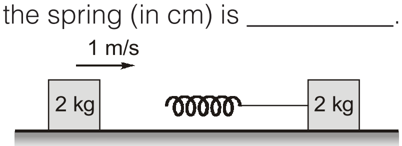
On solving x = 0.1 m = 10 cm

Student's Assignments

Q.1 A string is wrapped around the rim of a wheel of moment of Inertia 0.2 kg m^2 and radius 20 cm. The wheel is free to rotate about its axis. Initially the wheel is at rest. The string is now pulled with a force of 20 N. The angular velocity of wheel after 5 second is ____________ rad/s.



Q.2 A block of mass 2 kg is moving on a horizontal frictionless surface with velocity of 1 m/s, towards another block of equal mass kept at rest. The spring constant of the spring fixed at one end is 100 N/m. The maximum compression in the spring (in cm) is ___________.



Q.3 A particle of mass m is kept on a fixed, smooth sphere of radius 10 cm at a position, where the radius through the particle makes an angle 30° with the vertical. The particle is released from this position. The distance travelled by the particle before it leaves the contact with sphere is _____________ cm.