Mechanical Engineering

Fluid Mechanics and Fluid Machinery

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Fluid Mechanics and Fluid Machinery

First Edition: 2015
Second Edition: 2016
Third Edition: 2017
Fourth Edition: 2018
Fifth Edition: 2019
Sixth Edition: 2020
Seventh Edition: 2021
Eighth Edition: 2022

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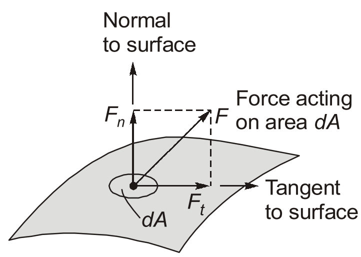
CHAPTER 02

Fluid Pressure & its Measurement

2.1 Introduction

- Stress at a point is defined as force per unit area and is determined by dividing the force by the area upon which it acts.
- The normal component of a force acting on a surface per unit area is called the normal stress, and the tangential component of the force is called shear stress (Figure 2.1).
- When fluid is confined within solid boundaries, it exerts forces against boundary surfaces. The exerted force always act in direction normal to the surface in contact. Because fluid at rest cannot sustain shear stress and hence no tangential component.
- In a fluid at rest, the normal stress is called pressure.
- In this chapter, we will discuss the fluid pressure and the various instruments available for its measurement.



Normal stress : $\sigma = \dfrac{F_n}{dA}$

Shear stress : $\tau = \dfrac{F_t}{dA}$

Figure 2.1 Components of a force

2.2 Fluid Pressure at a Point

- Pressure or intensity of pressure may be defined as the force exerted on a unit area. Thus, $p = \dfrac{dF}{dA}$, where dF = force acting on an infinitesimal area dA

2.2.1 Pascal's Law for Pressure at a Point

- According to Pascal's law, pressure at a point in a fluid system is equally distributed in all directions (Figure 2.2).
- It means that the pressure at a point in a fluid at rest, or in motion, is independent of direction as there are no shearing stresses present.
- Pressure in a fluid system has magnitude but not a specific direction and thus, it is a scalar quantity.
- It applies to a fluid at rest.

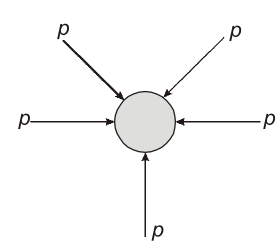


Figure 2.2 A point in a fluid system

- In case of flowing fluid, shear stresses will be set up as a result of relative motion between particles of the fluid.

  The pressure at a point is then considered to be the mean of the normal forces per unit area (stresses) on three mutually perpendicular planes. Since, these normal stresses are usually large compared to shear stresses, it is generally assumed that Pascal's law still applies.

Validation of the Law: Consider a small wedge-shaped fluid element of unit length in equilibrium as shown in Figure (2.3). The mean pressure at the three surfaces are p_1, p_2 and p_3 and the force acting on a surface is the product of mean pressure and the surface area. From Newton's second law, a force balance in the x-direction and z-direction gives

$$\Sigma F_x = ma_x = 0; \quad p_1 \Delta y \Delta z - p_3 \Delta y\, l \sin\theta = 0 \qquad ...(1)$$

$$\Sigma F_z = ma_z = 0; \quad p_2 \Delta y \Delta x - p_3 \Delta y\, l \cos\theta - \frac{1}{2}\rho g \Delta x\, \Delta y\, \Delta z = 0 \qquad ...(2)$$

Geometric relations are : $\Delta x = l\cos\theta$, $\Delta z = l \sin\theta$

Apply geometric relations in Equation (1) and (2), we get

$$p_1 - p_3 = 0$$

$$\Rightarrow \quad p_1 = p_3$$

$$p_2 - p_3 - \frac{1}{2}\rho g\, \Delta z = 0$$

For infinitesimal element, $\Delta z \to 0$

then, $p_2 = p_3$

$\therefore \quad p_1 = p_2 = p_3 = p \qquad ...(3)$

Thus, we conclude that the pressure at a point in a fluid has the same magnitude in all directions.

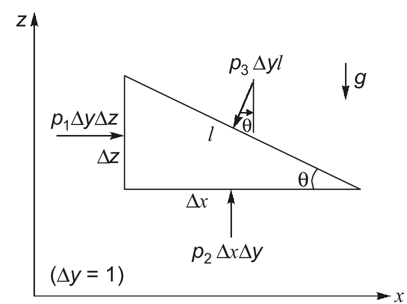


Figure 2.3 Fluid Element

2.2.2 Units of Pressure

1. 1 Pa = 1 N/m^2
2. 1 kgf/cm^2 = 9.81×10^4 N/m^2
3. 1 bar = 10^5 Pa
4. 1 atm = 101325 Pa
5. 1 psi = 6888.1 Pa, 1 atm = 14.7 psi
6. 1 torr = 1 mm Hg
7. Pressure can also be represented in terms of height of liquid columns.
   Ex.: 1 atm = 760 mm of Hg = 10.3 m of water.

2.3 Different Types of Pressure

(1) **Atmospheric Pressure :** The pressure exerted by atmospheric air normally upon all surfaces with which it is in contact, is known as atmospheric pressure.

(2) **Absolute Pressure :** The pressure measured above absolute zero (or complete vacuum) is known as absolute pressure.

(3) **Gauge Pressure :** The pressure measured above or below the atmospheric pressure is known as gauge pressure. Thus, gauge pressure can be negative or positive. Its value will be negative if value of the absolute pressure is less than atmospheric pressure & vice-versa.

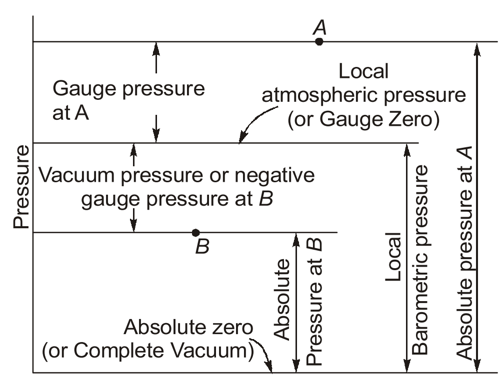


Figure 2.4 Relationship between Absolute, Gauge and Vacuum Pressures

If the pressure of a fluid is below atmospheric pressure, it is designated as vacuum pressure (or suction pressure or negative gauge pressure); and its gauge value is the amount by which it is below that of the atmospheric pressure.

$$\text{Absolute pressure} = \text{Atmospheric pressure} + \text{Gauge pressure}$$

$$\text{Absolute pressure} = \text{Atmospheric pressure} - \text{Vacuum pressure}$$

- Pressures are assumed to be gauge pressures unless specifically designated as absolute. For e.g., 100 kPa (abs) would refer to absolute pressures.

2.4 Variation of Pressure in a Fluid

- Consider a small fluid element of size $\delta x \times \delta y \times \delta z$ at any point in a static mass of fluid as shown in Figure (2.5). The forces acting on the element are the pressure forces on its faces and the self-weight of the element. Since, the element is in equilibrium under these forces, the algebraic sum of the forces acting on it in any direction must be zero.

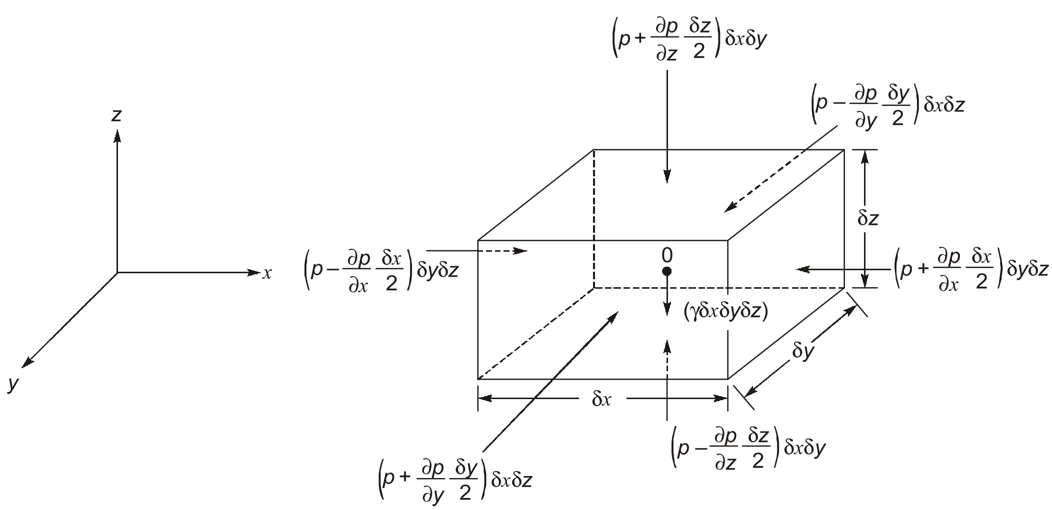


Figure 2.5 Fluid Element

i.e. $$\Sigma F_x = 0$$

or $$\left(p-\frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\cdot\delta y\delta z - \left(p+\frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\cdot\delta y\delta z = 0$$

or $$\frac{\partial p}{\partial x} = 0 \qquad \text{...(4)}$$

Also, $$\Sigma F_y = 0$$

or $$\left(p-\frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\cdot\delta x\delta z - \left(p+\frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\cdot\delta x\delta z = 0$$

or $$\frac{\partial p}{\partial y} = 0 \qquad \text{...(5)}$$

Again, $$\Sigma F_z = 0$$

or $$\left(p-\frac{\partial p}{\partial z}\frac{\delta z}{2}\right)\cdot\delta x\delta y - \left(p+\frac{\partial p}{\partial z}\frac{\delta z}{2}\right)\cdot\delta x\delta y-\gamma(\delta x\delta y\delta z) = 0$$

or $$\frac{\partial p}{\partial z} = -\gamma \quad ...(6)$$

Thus, Equations (4), (5) and (6) indicate that the pressure intensity 'p' at any point in a static fluid does not vary in x and y-directions and it varies only in z-direction. Partial derivative of Equation (6) can be reduced to total (or exact) derivative as follows

$$\frac{dp}{dz} = -\gamma = -\rho g \quad ...(7)$$

- The minus sign (–) indicates that the pressure decreases in the direction in which z increases, i.e. in the upward direction.
- The above Equation (7) holds for both compressible and incompressible fluids and indicates that within a body of fluid at rest the pressure increases in the downward direction at the rate equivalent to the specific weight 'γ' of the liquid.
- If $dz = 0$, then, dp is also equal to zero; which means that the pressure remains constant over any horizontal plane in a fluid.
- It shows that in a incompressible fluid mass pressure changes according to the change in vertical column of liquid above the considered point.

$$\therefore \quad \frac{\partial p}{\partial h} = \rho \times g = \gamma \qquad (\because \rho\times g = \gamma) \quad ...(8)$$

where γ = weight density of fluid.

Equation (8) states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is Hydrostatic Law.

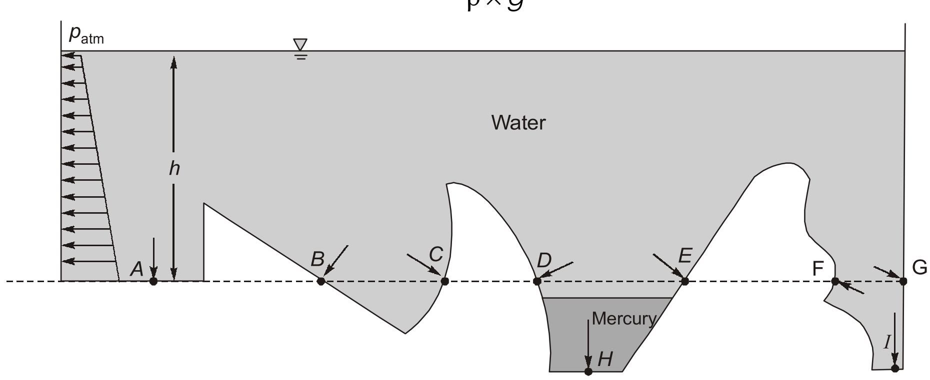
By integrating the above equation (8) for liquid, we get

$$\int dp = \int \rho g dh$$

or $$p = \rho g h \quad ...(9)$$

where p is the pressure above atmospheric pressure and h is the depth of the point from free surfaces.

From eq. (9), we have $$h = \frac{p}{\rho\times g} \quad ...(10)$$



$p_A = p_B = p_C = p_D = p_E = p_F = p_G = p_{atm} + \rho g h$

$p_H \neq p_I$

Figure 2.6 Pressure at different points lying at same depth

- In Figure (2.6), pressures at points *A*, *B*, C, *D* , *E* ,*F and G* lying on the same horizontal level and at the same vertical height *h* below the free surface of the liquid, will be same. But, the pressure at *H* and *I* are not same even if they are at same level.

2.5 Pressure Head

- The vertical height of the free surface above any point in a liquid at rest is known as pressure head for that point.

$$h = \frac{p}{\rho g} = \frac{p}{\gamma} \qquad \text{...(11)}$$

- Relationship between the heights of columns of different liquids which would develop the same pressure at any point, $p = \gamma_1 h_1 = \gamma_2 h_2$. If S_1 and S_2 are specific gravities of the two liquids then,

$$p = S_1\gamma_w h_1 = S_2\gamma_w h_2$$

$$\Rightarrow \qquad S_1 h_1 = S_2 h_2$$

Remember

Pressure in a Compressible Fluid

For a compressible fluid, the density varies with the pressure, therefore

$$\int_{p_1}^{p_2} \frac{dp}{\gamma} = -\int_{z_1}^{z_2} dz$$

where, p_1 = pressure at elevation z_1 ; p_2 = pressure at elevation z_2

2.6 Pressure Measurement Devices

TYPES OF DEVICES

- **Barometer**
- **Manometers**
  - **Simple Manometers**
    1. Piezometer
    2. U-tube manometer
    3. Single column Manometer
  - **Differential Manometers**
    1. Two Piezometer Manometer
    2. Inverted U-Tube Manometer
    3. U-Tube Differential Manometer
    4. Micromanometer
- **Mechanical Gauges**
  1. Bourdon Tube Pressure Gauge
  2. Diaphragm Pressure Gauge
  3. Bellows Pressure Gauge
  4. Dead-weight Pressure Gauge

2.6.1 Barometer

- Atmospheric pressure is measured by a device called barometer, thus, the atmospheric pressure is often referred to as the barometric pressure.
- The barometer consists of a inverted mercury-filled tube into a mercury container that is open to the atmosphere as shown in Figure (2.7).
- The pressure at point *B* is equal to the atmospheric pressure, and the pressure at *C* can be taken to be zero since there is only mercury vapour above point *C* and the pressure is very low relative to p_{atm} and can be neglected.

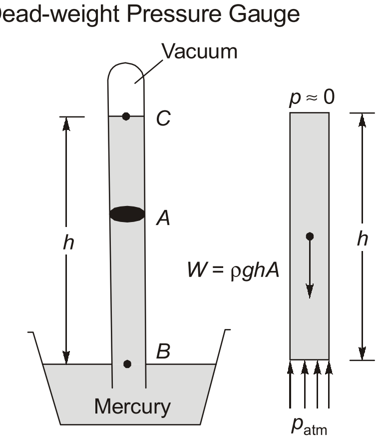


Figure 2.7 Barometer

- Writing a force balance in the vertical direction gives

$$p_{atm} = \rho g h$$

- In barometer, Hg is used because of its two important properties :
  (i) Hg is a high density fluid.
  (ii) Hg has very low vapour pressure.

Do You Know?

- The atmospheric pressure at a location is the weight of the air above that point. So, as one goes up in atmosphere, feels reduction in pressure as the air above that person continuously reduces.
- Barometer was invented by Torricelli. To honour him, pressure is represented in unit of 'torr, where 1 torr = 1 mm Hg.

2.6.2 Manometers

- Manometers are those pressure measuring devices which are based on the principle of balancing the column of liquid (whose pressure is to be found) by the same or another column of liquid.
- Manometers are classified as :
  1. Simple manometers
  2. Differential manometers

2.6.2.1 Simple Manometers

- A simple manometer consists of a glass tube having one of its ends connected to the gauge point where the pressure is to be measured and the other remains open to atmosphere. Following are the types of simple manometers:

(i) Piezometers:

- A piezometer is the simplest form of manometer which can be used for measuring moderate pressures of liquids.
- It consists of a glass tube inserted in the wall of a pipe or a vessel, containing a liquid whose pressure is to be measured. The tube extends vertically upward to such a height that liquid can freely rise in it without overflowing.
- The pressure at any point in the liquid is indicated by the height of the liquid in the tube above that point.
- The pressure measured correspond to gauge pressure. To find absolute pressure at the point, atmospheric pressure is added to the gauge pressure.
- Location of the point of insertion of a piezometer makes no difference in reading.
- To avoid the effect of capillarity, pipe-diameter of piezometric tube should be sufficiently large.

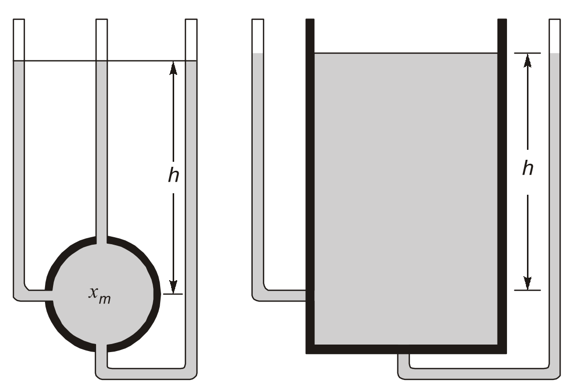


Figure 2.8 Piezometers

Limitations:

- Cannot be used when large pressure in lighter liquids is to be measured.
- Gas pressure cannot be measured, because gas forms no free atmospheric surface.

(ii) U-tube Manometer:

- A U-tube manometer consists of a glass tube in U-shape, one end of which is connected to the gauge point and the other end open to the atmosphere.
- The tube contains a liquid of specific gravity greater than that of the fluid of which the pressure is to be measured.
- Limitations imposed by piezometer are removed by use of U-tube manometers.
- The choice of the manometric liquid depends on the range of pressure to be measured. For low pressure range, liquids of lower specific gravity are used and for high range, generally mercury is employed.
- Consider a U-tube simple manometer is measuring pressure of a fluid of specific gravity S_1 (Figure 2.9). To write an equation for the pressure of the fluid following points should be kept in mind.
  - Start from one end of gauge to another.
  - Write the pressure at one end. Add the change in pressure while moving from one level to another.
  - Use positive sign if the next level of contact is lower than the first and negative if it is higher.
- For the Figure (2.9)

$$p_A - \rho_1 g h_1 - \rho_2 g h_2 = 0$$

$$\rho_1 = S_1 \rho_w$$

$$\rho_2 = S_2 \rho_w$$

where, ρ_w is density of water

Divide by $\rho_w g$

$$\frac{p_A}{\rho_w g} - S_1 h_1 - S_2 h_2 = 0$$

$$\frac{p_A}{\rho_w g} = S_1 h_1 + S_2 h_2$$

$$\frac{p_A}{\gamma_w} = S_1 h_1 + S_2 h_2$$

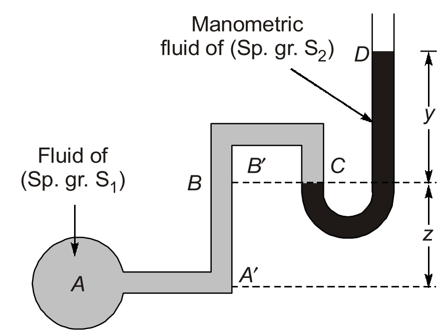


Figure 2.9 U-tube Simple Manometer

If, A contains gas, $S_1 << S_2$

$$\frac{p_A}{\gamma_w} = y S_2 \quad ...(12)$$

- A U-tube manometer can also be used to measure negative or vacuum pressure. For measurement of small negative pressure, a U-tube manometer without any manometric fluid may be used.

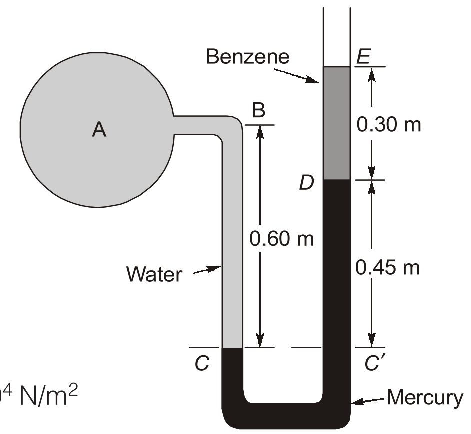
Limitations

- This method requires reading of fluid level at two or more points, since a change in pressure causes a rise of liquid in one limb of the manometer and a drop in the other.

Example 2.1 The left leg of U-tube mercury manometer is connected to a pipe-line conveying water. The level of mercury in the leg is 0.6 m below the center of pipe-line and the right leg is open to atmosphere. The level of mercury in the right leg is 0.45 m above that in the left leg and the space above mercury in the right leg contains Benzene (specific gravity 0.88) to a height of 0.3 m. Find the pressure in the pipe.

Solution :

In the accompanying figure, the pressures at C and C' are equal. Thus computing the pressure heads at C and C' from either side and equating the same, we get



$$\underset{\text{(Left Leg)}}{\frac{p_A}{\gamma_w}+0.6} = \underset{\text{(Right Leg)}}{0.45\times13.6+0.3\times0.88}$$

or $$\frac{p_A}{\gamma_w} = 5.784 \text{ m of water}$$

$\therefore$ $$p_A = (5.784\times9810) = 5.674\times10^4 \text{ N/m}^2$$

(iii) Single Column Manometer

- The limitation of U-tube manometer is removed in single column manometer.
- It is a modified form of U-tube manometer in which a shallow reservoir having a large cross-sectional area (about 100 times) as compared to the area of the tube is introduced into one limb of the manometer.
- For any change in pressure, the change in the liquid level in the reservoir will be so small that it may be neglected, and the pressure is indicated approximately by the height of the liquid in the other limb.
- Only one reading in the narrow limb of the manometer need to be taken for pressure measurement.

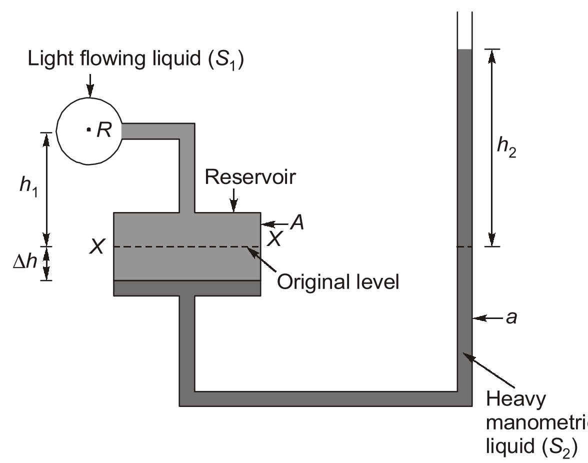


Figure 2.10 Single column manometer

- Narrow limb may be straight or inclined.
- The inclined type is useful for the measurement of small pressures as they are more sensitive than the vertical type.

$$A\Delta h = ah_2$$

or $$\Delta h = \frac{ah_2}{A}$$

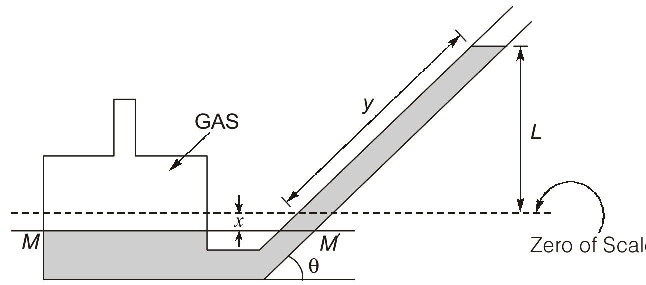
Now, $$p_R + \rho_1 g(h_1+\Delta h) = \rho_2 g(h_2+\Delta h)$$

$$p_R = \rho_2 g h_2 - \rho_1 g h_1 + \Delta h g(\rho_2 - \rho_1)$$

If Δh is very small, $$p_R = \rho_2 g h_2 - \rho_1 g h_1$$

Example 2.2 A manometer consists of an inclined glass tube which is connected to a metal cylinder standing upright. A manometric liquid fills the apparatus to a fixed zero mark on the tube when both cylinder and the tube are open to atmosphere. The upper end of the cylinder is then connected to a gas supply at a pressure p and the liquid rises in the tube.

Find an expression for the pressure *p* in cm of water when the liquid reads *y* cm in the tube, in terms of the inclination **θ** of the tube, the specific gravity of the liquid *S*, and the ratio *a* of the diameter of the cylinder to the diameter of the tube. Hence, determine the value of *a* so that the error due to disregarding the change in level in the cylinder will not exceed 0.1%, when **θ** = 30°.



Solution :

$$\text{Diameter ratio} = a$$

So,
$$\text{Area ratio} = a^2$$

Now,
$$x\left(\frac{\pi D^2}{4}\right) = y\left(\frac{\pi d^2}{4}\right)$$

$$xa^2 = y$$

$$x = \frac{y}{a^2}$$

$$p_{(\text{incorrect})} = \rho_M g\,(y \sin 30°)$$

$$p_{(\text{correct})} = \rho_M\, g\,(y \sin 30° + x)$$

$$= \rho_M g\left(y \sin 30° + \frac{y}{a^2}\right)$$

Now,
$$\%\ \text{Error} = \frac{p_{(\text{correct})} - p_{(\text{incorrect})}}{p_{(\text{correct})}} \times 100$$

$$0.1 = \frac{\frac{y}{a^2}}{y \sin 30° + \frac{y}{a^2}} \times 100$$

$\Rightarrow$ $$a^2 = 2000$$

$\Rightarrow$ $$a = 44.72$$

2.6.2.2 Differential Manometers

- For measuring the difference of pressure between any two points in a pipeline or in two pipes or containers, a differential manometer is employed.
- In general, a differential manometer consists of a bent glass tube, the two ends of which are connected to each of the two gauge points between which the pressure difference is required to be measured.
- Following are the common types of differential manometers:

(i) Two-Piezometer Manometer

- It consists of two separate piezometer which are inserted at the two gauge points between which the pressure difference is required.
- This method is useful only if the pressure at each of the two points is small.
- It can not be used to measure pressure difference in gases.

(ii) Inverted U-tube Manometer

- It consists of a glass tube bent in U-shape and held inverted as shown in Figure (2.11).Thus, it is as if two piezometers described above are connected with each other at top.
- When the two ends of the manometer are connected to the points between which the pressure difference is required to be measured, the liquid under pressure will enter the two limbs of the manometer, thereby, causing the air within the manometer to get compressed.

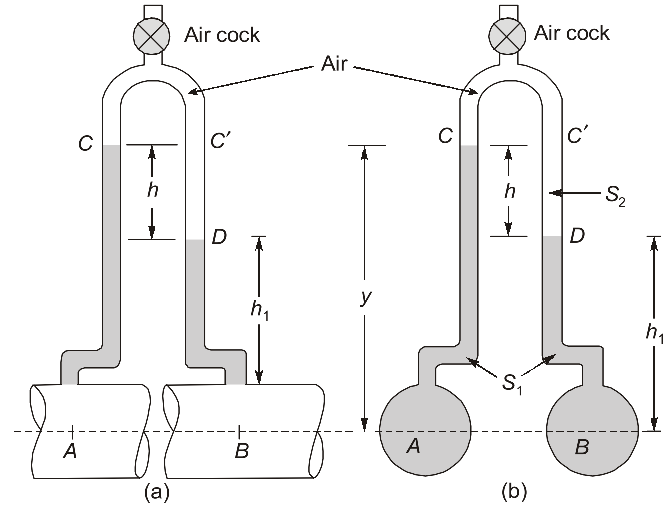


Figure 2.11 Inverted U-tube Differential Manometer

- An air cock is usually provided at the top of the inverted U–tube which facilitates the raising of the liquid columns to suitable level in both the limbs by driving out a portion of the compressed air. It also permits the expulsion of air bubbles which might have been entrapped somewhere in the pipe line.

Now, $$p_A - \rho_1 g(h_1+h)+\rho_2 gh + \rho_1 gh_1 = p_B$$
$$p_A - \rho_1 gh + \rho_2 gh = p_B$$
$$p_A - p_B = (\rho_1 - \rho_2).g.h$$

Divide by $\rho_w g$ $$\frac{p_A - p_B}{\rho_w g} = (S_1 - S_2)\, h$$

Since the density of air is very small, so

$$\frac{p_A - p_B}{\rho_w g} = S_1 h \qquad ...(13)$$

- Inverted U-tube manometers are suitable for the measurement of small pressure difference in liquids.
- Sensitivity of inverted U-tube manometer can be increased by replacing air with a manometric fluid of specific gravity such that $S_2 < S_1$. In this case, gauge equation can be written as;

$$\frac{p_A}{\gamma_w} - \frac{p_B}{\gamma_w} = h\,(S_1 - S_2) \qquad ...(14)$$

(iii) U-tube Differential Manometer

- In this type, a U-tube is connected between two gauge points.
- The lower part of the manometer tube contains a manometric fluid which is heavier than the liquid for which pressure is to be measured (i.e. $S_2 > S_1$).
- The pressure gauge equation in the case when the two points are at same level

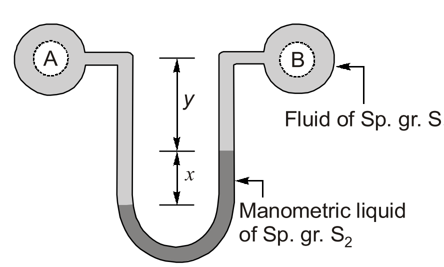


Figure 2.12 U-tube Differential Manometer

(Figure 2.12) can be written as

$$p_A + \rho_1 gy + \rho_1 gx = p_B + \rho_1 gy + \rho_2 gx$$

$$p_A = p_B = xg(\rho_2 - \rho_1)$$

Divide by $\rho_w g$ $\quad \dfrac{p_A - p_B}{\rho_w g} = (S_2 - S_1) \quad$ ...(15)

- In case, the points A and B between which the pressure difference is to be measured are not at same level (Figure 2.13) and the fluids in A and B are also of different specific gravity then, the general gauge equation can be written as

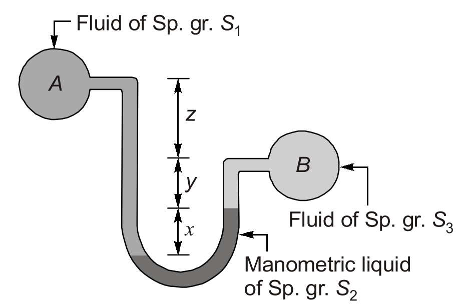


Figure 2.13 U-tube Differential Manometer

$$p_A + \rho_1 g(z + y + x) - \rho_2 gx - \rho_3 gy = p_B$$

$$p_A - p_B = \rho_2 gx + \rho_3 gy - \rho_1 g(x + y + z)$$

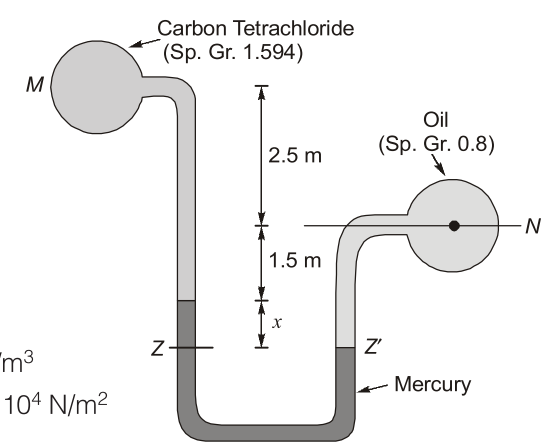
Divide by $\rho_w g$

$$\frac{p_A - p_B}{\rho_w g} = S_2 x + S_3 y - S_1(x + y + z) \quad ...(16)$$

If there is same liquid at A and B i.e. $S_1 = S_3$, then

$$\frac{p_A}{\gamma_w} - \frac{p_B}{\gamma_w} = x(S_2 - S_1) - zS_1 \quad ...(17)$$

Example 2.3 As shown in the accompanying figure, pipe M contains carbon tetrachloride of specific gravity 1.594 under a pressure of 1.05 kg(f)/cm² and pipe N contains oil of specific gravity 0.8. If the pressure in the pipe N is 1.75 kg(f)/cm² and the manometric fluid is mercury, find the difference x between the levels of mercury.



Solution :

$$S_{CT} = 1.594$$

$\therefore$ $$\rho_{CT} = 1.594 \times 1000 = 1594 \text{ kg/m}^3$$

$$p_M = 1.05 \text{ kgf/cm}^2 = 1.05 \text{ g} \times 10^4 \text{ N/m}^2$$

where $g = 9.81$ m/s²

$$S_{oil} = 0.8$$

$\therefore$ $$\rho_{oil} = 0.8 \times 1000 = 800 \text{ kg/m}^3$$

$$p_N = 1.75 \text{ kgf/cm}^2 = 1.75 \text{ g} \times 104 \text{ N/m}^2$$

Applying the pressure balance equation,

Pressure at section Z = Pressure at section Z'

$$p_M + \rho_{CT} g(2.5 + 1.5) + \rho_{Hg} gx = p_N + \rho_{oil}\, g(1.5 + x)$$

$$1.05g \times 10^4 + 1594g \times 4 + 13600g \times x = 1.75g \times 10^4 + 800g \times (1.5 + x)$$

$$1.05 \times 10^4 + 1594 \times 4 + 13600x = 1.75 \times 10^4 + 800 \times 1.5 + 800x$$

$$12800x = 1824$$

or $$x = 0.1425 \text{ m} = 14.25 \text{ cm}$$

(iv) Micromanometers

- For the measurement of very small pressure differences, or for the measurement of pressure differences with high precision, special forms of manometer is called micromanometer.
- Micromanometers either magnify the reading or permit the readings to be observed with greater accuracy.
- The manometer contains two manometric liquids of different specific gravities and immiscible with each other and with the fluid for which the pressure difference is to be measured.

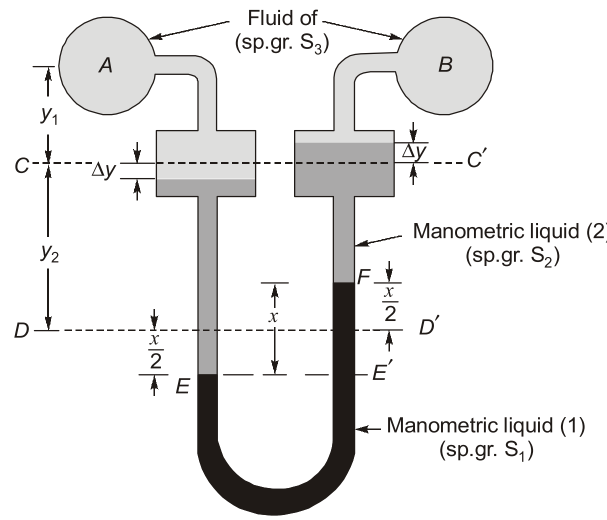


Figure 2.14 Micromanometer

- Let the pressure at A is greater than at B ($p_A > p_B$) then level of the lighter manometric liquid will fall in the left basin and rise in the right basin by the same amount Δy. Similarly the level of the heavier manometric liquid will fall in the left limb to point E and rise in the right limb to point F. If A and a are the cross - section areas of the basin and the tube respectively, then since the volume of the liquid displaced in each basin is equal to the volume of the liquid displaced in each limb of the tube, then

$$A(\Delta y) = a\left(\frac{x}{2}\right)$$

Starting from point A the following gauge equation can be obtained

$$\frac{p_A}{\gamma_w} + (y_1 + \Delta y)S_3 + \left(y_2 - \Delta y + \frac{x}{2}\right)S_2 - xS_1 - \left(y_2 - \frac{x}{2} + \Delta y\right)S_2 - (y_1 - \Delta y)S_3 = \frac{p_B}{\gamma_w}$$

Substituting the value of Δy and simplifying the above equation it becomes

$$\frac{p_A}{\gamma_w} - \frac{p_B}{\gamma_w} = x\left[S_1 - S_2\left(1 - \frac{a}{A}\right) - S_3\frac{a}{A}\right] \quad ...(18)$$

If $a << A \Rightarrow \frac{a}{A} \approx 0$; then the above equation can be written as

$$\frac{p_A}{\gamma_w} - \frac{p_B}{\gamma_w} = x(S_1 - S_2) \quad ...(19)$$

Example 2.4 **A two-liquid double column manometer with enlarged ends is used to measure a high precision pressure difference between two points of a pipeline containing gas under pressure. The basins are partly filled with methyl alcohol of specific gravity 0.78 and the lower portion of the U-tube is filled with mercury of specific gravity 13.6. The specific weight of the gas which is methane is 0.476 kg(f)/m^3, find the pressure difference if the U-tube reading is 30 mm and the diameter of the basin is 15 times that of the U-tube.**

Solution :

Let p_1 and p_2 be the pressure intensities at the two points 1 and 2 in the pipeline. Equating the pressure heads at the two points Z and Z', as shown in the accompanying figure, we get

$$\frac{p_1}{\gamma_w} + (l+y)\frac{0.476}{1000} + m(0.78) + 30(0.78) = \frac{p_2}{\gamma_w} + l\left(\frac{0.476}{1000}\right) + (m+y)0.78 + 30(13.6)$$

or $\quad \frac{p_1}{\gamma_w} - \frac{p_2}{\gamma_w} = 30(13.6 - 0.78) + y(0.78 - 0.476 \times 10^{-3}) = 384.6 + y(0.7795)$

Further $\quad A \times y = 30 \times a$

or $\quad y = 30\frac{a}{A}$ mm

But $\quad A = (15)^2 a = 225\ a$

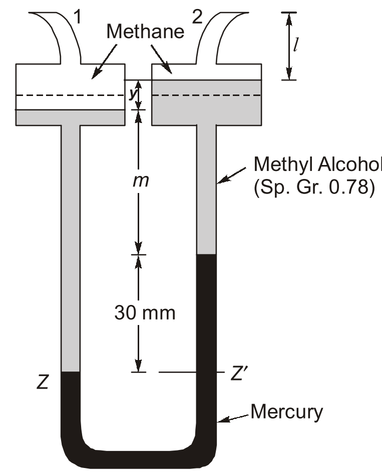
$\therefore \quad y = \frac{30}{225}$

By substituting these values, we get

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = 384.6 + \frac{30}{225}(0.7795)$$

$= 384.704$ mm of water

$= 38.4704$ cm of water



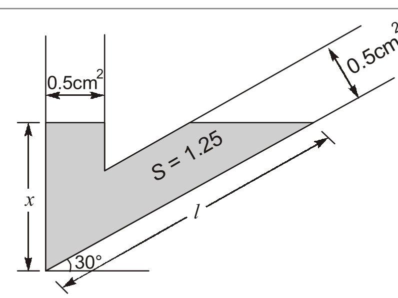
2.6.3 Mechanical Gauge

- Mechanical gauges are those pressure measuring devices, which embody an elastic element, which deflects under the action of the applied pressure p and this movement after being (mechanically magnified), operates a pointer moving against a graduated circumferential scale.
- Generally, these gauges are used for measuring high pressures and where high precision is not required.
- Following are the types of the mechanical gauges :
  (a) bourdon Tube Pressure Gauge (b) diaphragm Pressure Gauge
  (c) bellows Pressure Gauge (d) dead - weight Pressure Gauge

Do you know? Strain gauge transducers and piezoelectric transducers are also used to measure pressure. Quartz or Rochelle salt are example of materials that are used in piezoelectric transducers.

ILLUSTRATIVE EXAMPLES

Example 2.5 An equilibrium liquid level condition is shown in the given figure. If additional 10 cm^3 of water is added through the inclined limb, then, what will be the rise in the meniscus in vertical tube?



Solution :

Let $\quad x$ = Original liquid level in the vertical limb

$\quad l$ = Original liquid level in the inclined limb

From the original pressure balance

$$l \sin 30° = x$$

$\Rightarrow \quad x = (l/2) \quad$ ...(i)

From Volume Balance $\Delta x_1 = \Delta x$ ...(ii)

Length of water added $= \frac{10}{0.5} = 20$ cm

From final pressure balance

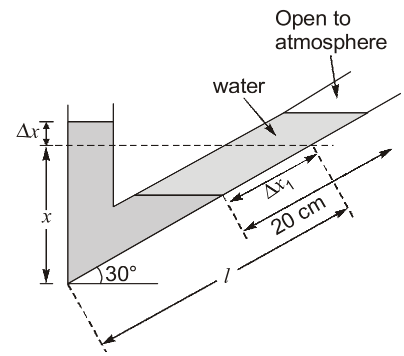
$$(x + \Delta x) \times 1.25 = (l - \Delta x)\sin 30° \times 1.25 + 20 \sin 30° \times 1$$

$$\Rightarrow \quad (x + \Delta x) \times 1.25 = \frac{l}{2} \times 1.25 - \frac{\Delta x}{2} \times 1.25 + \frac{20}{2}$$

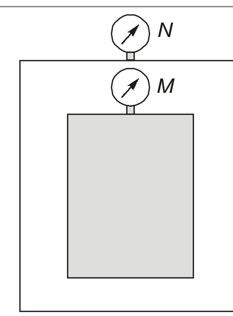
$$\Rightarrow \quad \left(\frac{l}{2} + \Delta x\right) \times 1.25 = \frac{l}{2} \times 1.25 - \frac{\Delta x}{2} \times 1.25 + 10 \quad \{\because x = l/2\}$$

$$\Rightarrow \quad \Delta x\left[1.25 + \frac{1.25}{2}\right] = 10$$

$$\Delta x = \frac{10 \times 2}{(2.5 + 1.25)} = \frac{20}{3.75} = 5.33 \text{ cm}$$



Example 2.6 Two pressure tanks are built one inside the other as shown in the figure. A Bourdon gauge *M* is connected to the inner tank reads 20 kPa. Another Bourdon gauge *N* connected to the outer tank reads 35 kPa. An aneroid barometer reads 750 mm of mercury. Calculate the absolute pressure recorded at *M* and *N* mm in of mercury.



Solution:

A Bourdon gauge records the gauge pressure relative to the pressure of the medium surrounding the tube.

Local atmospheric pressure is measured by the aneroid barometer. In the present case local atmospheric pressure outside the gauge *N* is 750 mm.

Hence absolute pressure at *N*

$$p_{N(\text{abs})} = 750 + \left(\frac{35}{13.6 \times 9.81} \times 1000\right) = 1012.34 \text{ mm of mercury}$$

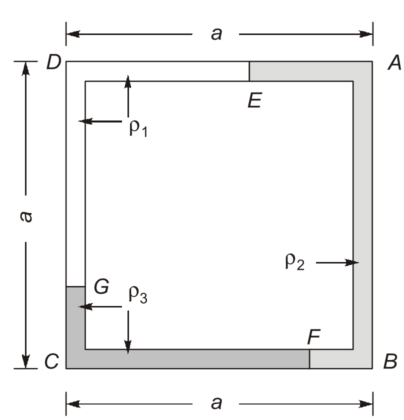
The gauge *M* reads relative to its surrounding pressure of 1012.34 mm of mercury(*abs*).

Hence, $$p_{M\,(\text{abs})} = 1012.34 + \left(\frac{20 \times 1000}{13.6 \times 9.81}\right) = 1162.25 \text{ mm of mercury}$$

Example 2.7 A glass tube of uniform bore is bent into the form of a square of sides *a* and filled with equal amounts of three immiscible liquids of densities ρ_1, ρ_2 and ρ_3. It is known that $\rho_1 < \rho_2 < \rho_3$. If the tube arrangement is placed in a vertical plane (i.e. two sides vertical) and if one of the vertical sides is completely filled with the liquid of density ρ_2.

(a) Show that $\frac{1}{3}(2\rho_3 + \rho_1) > \rho_2 > \frac{1}{3}(\rho_3 + 2\rho_1)$

(b) If the relative densities of the first and third liquids are 1.0 and 1.2 respectively, find the range of the relative densities of the second liquid which makes the above arrangement possible.



Solution:

(a) Referring to accompanying figure, let E, F and G be the Interfaces.

Let $EA = x$.

Then $DE = DA - EA = (a - x)$

$$\text{Total tubing length} = 4a$$

$$\text{Length of each liquid} = \frac{4}{3}a$$

Hence for liquid 1 :

$$EG = \frac{4}{3}a$$

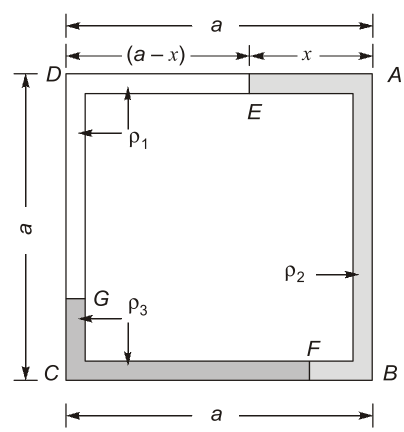
$$DG = \frac{4}{3}a - (a - x) = \frac{1}{3}a + x$$

For liquid 3 :

$$GC = a - \left(\frac{1}{3}a + x\right) = \frac{2}{3}a - x$$

$$CF = \frac{4}{3}a - \left(\frac{2}{3}a - x\right) = \frac{2}{3}a + x$$

$$FB = a - \left(\frac{2}{3}a + x\right) = \frac{1}{3}a - x$$



$$\left(\text{Check : } FB + BA + AE = \frac{1}{3}a - x + a + x = \frac{4}{3}a\right)$$

At the interface F: The pressure balance is Pressure of Column DG + Column GC = Pressure of column AB

$$\rho_1 g\left(\frac{1}{3}a + x\right) + \rho_3 g\left(\frac{2}{3}a - x\right) = \rho_2 g a$$

$$x(\rho_3 - \rho_1) = \frac{1}{3}a(2\rho_3 + \rho_1 - 3\rho_2)$$

$$x = \frac{1}{3}a\frac{(2\rho_3 + \rho_1 - 3\rho_2)}{(\rho_3 - \rho_1)}$$

It is known that $x > 0$ and also $x < \frac{1}{3}a$

Hence $$0 < x < \frac{a}{3}$$

$\therefore$ $$0 < \frac{2\rho_3 + \rho_1 - 3\rho_2}{\rho_3 - \rho_1} < 1$$

Also since $\rho_1 < \rho_2 < \rho_3$ the denominator $(\rho_3 - \rho_1)$ is positive. Hence the numerator is

$$0 < (2\rho_3 + \rho_1 - 3\rho_2) < 1$$

or $$(2\rho_3 + \rho_1) > 3\rho_2$$

or $$\rho_2 < \frac{1}{3}(2\rho_3 + \rho_1) \qquad \text{...(i)}$$

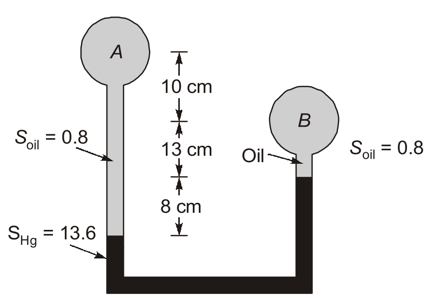
Objective Brain Teasers

Ex.1 If a Mohr circle is drawn for a fluid element inside a fluid body at rest, it would be :
(a) a circle not touching the origin
(b) a circle touching the origin
(c) a point on the normal stress axis
(d) a point on the shear stress axis

Ex.2 The pressure in meters of oil of specific gravity 0.8 equivalent to 80 m of water is :
(a) 64 m (b) 88 m
(c) 80 m (d) 100 m

Ex.3 The mass density of a liquid with variable density is given by ρ = 1000 + 0.008 $y^{3/2}$, where ρ is in kg/m^3 ; y is measured in meters. The depth at which the pressure intensity will be 900 kPa, is
(a) 91.5 m (b) 101.5 m
(c) 112.5 m (d) 114.5 m

Ex.4 The pressure difference between point A and B for the set up shown in figure in kPa is

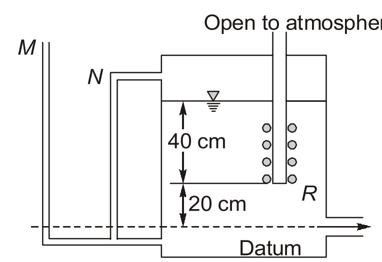


(a) 9.26 (b) 10.54
(c) 10.65 (d) 11.66

Ex.5 In a mercury column-type barometer, the correct local atmospheric pressure is obtained by considering correction due to vapour pressure of mercury as follows ; H_a =
(a) $H - h_v$ (b) $H_0 + h_v$
(c) H_0 / h_v (d) $h_v - H_0$
[where, H_a = correct local pressure in mm of mercury, H_0 = observed barometer reading in mm of mercury and h_v = vapour pressure of mercury in mm.]

Ex.6 The standard atmospheric pressure is 101.32 kPa. The local atmospheric pressure at a location was 91.52 kPa. If a pressure is recorded as 22.48 kPa (gauge), it is equivalent to
(a) 123.80 kPa (abs) (b) 88.84 kPa(*abs*)
(c) 114.00 kPa(*abs*) (d) 69.04 kPa(*abs*)

Ex.7 The tank shown in figure discharge water at constant rate for all water levels above the air inlet R. The height above datum to which water would rise in manometer tubes M and N respectively, are

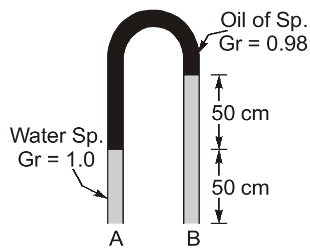


(a) (60 cm, 20 cm) (b) (40 cm, 40 cm)
(c) (20 cm, 20 cm) (d) (20 cm, 60 cm)

Ex.8 Normal stresses are of the same magnitude in all directions at a point in a fluid
(a) only when the fluid is frictionless
(b) only when the fluid is at rest
(c) only when there is no shear stress
(d) in all cases of fluid motion

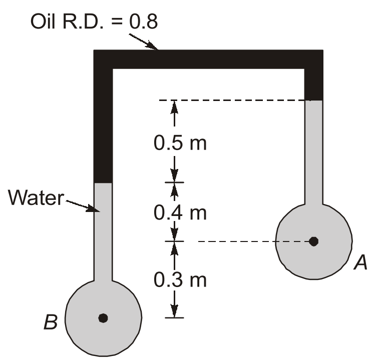
Ex.9 Identify the CORRECT statement:
(a) Local atmospheric pressure is always less than the standard atmospheric pressure
(b) Local atmospheric pressure depends only on the elevation of the place
(c) A barometer reads the difference between the local and standard atmospheric pressure
(d) Standard atmospheric pressure is 760 mm of mercury

Ex.10 In the setup shown in given figure assuming the specific weight of water as 10 kN/m^3, the pressure difference between the two points A and B will be



(a) 100 N/m^2 (b) – 100 N/m^2
(c) 200 N/m^2 (d) – 200 N/m^2

Ex.11 An inverted differential manometer is shown in given figure. The differential pressure ($p_B - p_A$) in terms of column height of oil of relative density 0.8 is



(a) 0.25 m (b) 0.5 m
(c) 0.85 m (d) None of these

ANSWERS

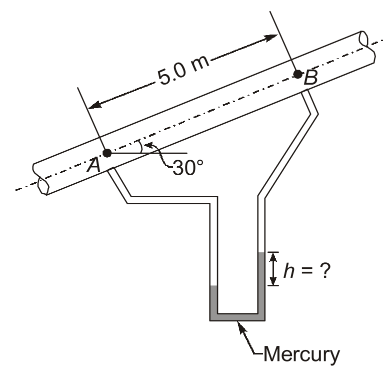
1. (c)	2. (d)	3. (a)	4. (a)	5. (b)
6. (c)	7. (d)	8. (b)	9. (d)	10. (a)
11. (a)				

Student's Assignments

Ex.1 A certain fluid of specific gravity 0.8 flows upwards through a vertical pipe. *A* and *B* are two points on the pipe, *B* being 0.3 m higher than *A*. A U-tube mercury manometer is connected at points *A* and *B*. If the difference in pressure between *A* and *B* is 5 kPa, find the difference in the heights of the mercury columns in the manometer.

Ans. h = 21.4 mm

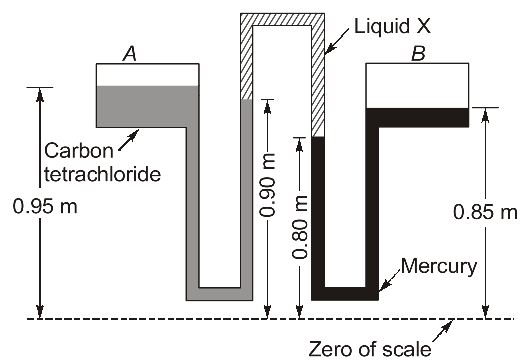
Ex.2 If the pipe in the given figure contains water and there is no flow, calculate the value of the manometer reading *h*.



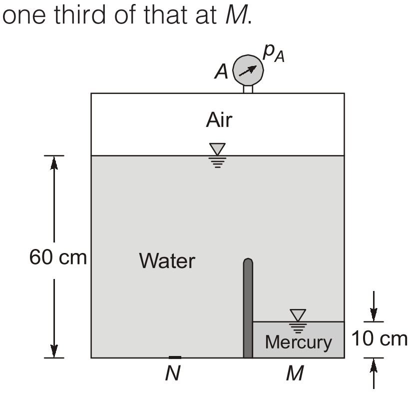
Ans. $h = 0$

Ex.3 In the manometer shown in the given figure, the liquid on the left side is carbon - tetrachloride of specific gravity 1.60 and liquid on the right side is mercury. If (p_A – p_B) is 525 kg(f)/m^2 (5150.25 N/m^2), find the specific gravity of the liquid X.

Ans. 0.75



Ex.4 For the system shown in given figure calculate the air pressure p_A to make the pressure at *N* one third of that at *M*.



Ans. p_A = 0.294 kPa