Mechanical Engineering

Heat Transfer

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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CHAPTER 5

Transient Conduction

LEARNING OBJECTIVES

After Completing this chapter, you will be able to :

1. Understand lumped capacitance method.
2. Understand the physical interpretation of Biot number and Fourier number.
3. Study the transient thermal response of different geometries.

5.1 Introduction

In general, the temperature field in any transient problem is given by

$$T = T(x, y, z, t)$$

The solution of an unsteady state problem will be more complex than that of a steady state one because of the presence of another variable 'time', t. In other words, a one-dimensional transient problem would be as complex as a two-dimensional steady state problem. In this Chapter we shall took at ways to determine temperature distributions for some transient heat conduction problems.

5.2 Lumped Heat Analysis - Systems with Negligible Internal Resistance

- The process in which the internal resistance is ignored being negligible in comparison with its surface resistance is called the Newtonian heating or cooling process.
- In a Newtonian heating or cooling process the temperature throughout the solid is considered to be uniform at a given time.
- Such an analysis is also called the lumped heat capacity analysis because the whole solid, whose energy at any times is a function of its temperature and total heat capacity is considered as one lump.

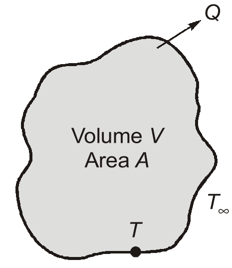


Figure 5.1 Lumped Heat Capacity System

NOTE : Small bodies with high thermal conductivities and low convection coefficients are most likely to satisfy the criterion for lumped system analysis.

Let us now consider a solid of area $A\ m^2$, whose initial temperature is T_0 throughout and which is suddenly placed in a new environment at a constant temperature T_∞ as shown in Figure 5.1. The lumped heat capacity of the solid is ρcV, where ρ = density of solid, kg/m^3, c = specific heat of solid, J/kgK and V = volume of solid, m^3.

The convective heat transfer coefficient between the solid and surroundings is h (W/m^{2}K). At any instant of time, t, the convective heat loss from the body is equal to the decrease in internal energy of the solid. Thus

$$Q = hA(T - T_\infty) = -\rho cV\frac{dT}{dt} \quad \text{...(5.1)}$$

Rewrite Equation (5.1) as

$$\frac{dT}{T - T_\infty} = \frac{-hA}{\rho cV}dt \quad \text{...(5.2)}$$

Integrating Equation (5.2), we get

$$\ln(T - T_\infty) = \frac{-hA}{\rho cV}t + C_1 \quad \text{...(5.3)}$$

The constant of integration, C_1 can be found by applying the initial condition, $T = T_0$ at $t = 0$ giving

$$C_1 = \ln(T_0 - T_\infty)$$

Substituting for C_1 in Equation (5.3)

$$\ln\left(\frac{T - T_\infty}{T_0 - T_\infty}\right) = \frac{-hA}{\rho cV}.t$$

or

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-\left(\frac{hA}{\rho cV}\right).t\right] \quad \text{...(5.4)}$$

Equation (5.4) gives the temperature distribution for a solid initially at a temperature, T_0, which is placed in a convective environment at temperature, T_∞.

NOTE : Temperature of body approaches the ambient temperature T exponentially and larger value ($hA/\rho Vc$) indicates that the body will approach the environment temperature in short duration.

- Let us now introduce a non-dimensional parameter, called the Biot number, Bi, to test the validity of the lumped heat capacity approach.

$$Bi = \frac{\text{Internal resistance}}{\text{Convective resistance}} = \frac{hL_c}{k}$$

where L_c is a characteristic length, and is equal to the volume of the body divided by its surface area. The characteristic length L_c for some common shapes can be calculated as follows :

Plane wall (thickness $2L$), $$L_c = \frac{A.2L}{2.A} = L$$

Long cylinder radius R, $$L_C = \frac{\pi R^2.L}{2\pi R.L} = \frac{R}{2}$$

Sphere radius R, $$L_C = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$$

Cube side L, $$L_C = \frac{L^3}{6L^2} = \frac{L}{6}$$

It has been observed that for simple shapes such as plates, cylinders, sphere and cubes the lumped heat capacity approach can be used to advantage if $Bi < 0.1$.

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-\left(\frac{hA}{\rho c V}\right).t\right] \qquad \text{if } Bi < 0.1 \qquad ...(5.5)$$

where $F_0 = \left(\frac{\alpha t}{L_c^2}\right)$ = Fourier number or relative time.

$$\therefore \qquad \frac{T - T_\infty}{T_0 - T_\infty} = \exp\,(-Bi.F_0) \qquad ...(5.6)$$

The heat rate Q at any time, t, can be obtained from Equation (5.1) as

$$Q = \rho c V\frac{dT}{dt} = \rho c V(T_0 - T_\infty).\left(\frac{-hA}{\rho c V}\right).\exp\left[-\left(\frac{hA}{\rho c V}\right).t\right]$$

The total quantity of heat, U, given off, during a time interval, $(0,t)$ is equal to the change in internal energy when the work done is negligible (First law of thermodynamics)

$$U = \oint_0^t Q dt = \int_0^t -hA(T_0 - T_\infty).\exp\left(\frac{-hA}{\rho c V}.t\right)dt = \rho c V(T_0 - T_\infty)\left[\exp\left(\frac{-hA}{\rho c V}.t\right) - 1\right]$$

$$U = \rho c V(T_0 - T_\infty)[\exp(-Bi\,.\,F_0) - 1] \qquad ...(5.7)$$

The change in the internal energy of the system during the time interval $(0, t)$ is also given by Equation (5.7).

NOTE : Heat transfer to or from a body reaches its maximum value when the body reaches the environment temperature.

Example 5.1 An aluminium sphere weighing 5.5 kg and initially at a temperature of 290° C is suddenly immersed in a fluid at 15°C. The convective heat transfer coefficient is 58 W/m^{2}K. Estimate the time required to cool the aluminium to 95°C, using the lumped capacity method of analysis.

Taking the properties of aluminium as ρ = 2700 kg/m^3; c = 900 J/kgK; k = 205 W/mK

Solution :

$$V = \frac{4}{3}\pi R^3 = \frac{\text{Mass}}{\rho} = \frac{5.5}{2700} = 2.037\times 10^{-3}\ \text{m}^3$$

$$\therefore \qquad R = \left(\frac{3V}{4\pi}\right)^{1/3} = 0.0786\ \text{m}$$

$$L_c = \frac{R}{3} = 0.0262\ \text{m}$$

Using Equation (5.4),
$$\frac{T_0 - T_\infty}{T_0 - T_\infty} = \exp\left[-\left(\frac{hA}{\rho c V}\right).t\right]$$

We have
$$T = 90^\circ\text{C}$$
$$T_\infty = 15^\circ\text{C}$$
$$T_0 = 290^\circ\text{C}$$

$$\frac{hA}{\rho c V} = \frac{3h}{\rho c R} = \frac{3\times 58}{2700\times 900\times 0.0786} = 9.1\times 10^{-4}/\text{s}$$

$$\frac{95-15}{290-15} = \frac{80}{275} = \exp\left(-9.1\times 10^{-4}t\right)$$

or
$$3.4375 = \exp(9.1 \times 10t)$$

Hence
$$t = 1357\ \text{s}$$

Example 5.2 Match List-I with List-II and select the correct answer using the code given below the Lists :

List-I	List-II
A. Thermal diffusivity (α)	1. $\frac{hL}{k}$
B. Biot number (Bi)	2. $\frac{\alpha\tau}{L_c^2}$
C. Fourier number (F_o)	3. $\frac{hL_c}{k}$
D. Nusselt number (Nu)	4. $\frac{k}{\rho c}$

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	1	3	2	4
(c)	4	3	2	1
(d)	4	2	3	1

Ans. : (c)

Example 5.3 The average heat transfer coefficient for flow of 100°C air over a flat plate is measured by observing the temperature time history of a 3 cm thick copper slab exposed to 100°C air. In one test run, the initial temperature of the plate was 210°C, and in 5 minutes the temperature decreased by 40°C. Calculate the heat transfer coefficient for this case.

Solution :

Taking the property values of copper. Thickness of plate = 2 L = 0.03 m; $L_c = L$ = 0.015 m

$$\frac{hA}{\rho cV} = \frac{h}{\rho c L_c} = \frac{h}{(9000)(380)(0.015)} = 1.949\times10^{-5}h$$

Now
$$\frac{T-T_\infty}{T_0-T_\infty} = \exp\left[-\left(\frac{hA}{\rho cV}\right)t\right]$$

We have
$$T_0 = 210°C; \quad T_\infty = 100°C$$
$$T = 170°C; \quad t = 300\text{ s}$$

$$\frac{170-100}{210-100} = \exp[-(1.949\times10^{-5})(300)h]$$

or
$$\frac{70}{110} = \exp(-5.848\times10^{-3}h)$$

Hence
$$h = 77.24\text{ W/m}^2\text{K}$$

5.3 Response Time of a Temperature measuring Instrument

- The quantity $\left(\frac{\rho cV}{hA}\right)$ has the units of time and is often called the time constant of the system, and its denoted by the symbol t^*. Hence at time $t = t^*$ (one time constant).

$$\frac{T-T_\infty}{T_0-T_\infty} = e^{-1} = 0.368 \quad ...(5.8)$$

- Thus at the end of a time period equal to t* the temperature difference between the body and the ambient (source) would be 0.368 of the initial temperature difference. In other words, the temperature difference would be reduced by 63.2 %.
- The time required by a thermocouple to reach 63.2 percent of the value of the initial temperature difference is called its sensitivity.

NOTE : The lower the value of the time constant, the better the response of the thermocouple. For all practical purposes a reading of the thermocouple should be taken after a period equal to 3 times periods has elapsed.

Example 5.4 A chromel-alumel thermocouple (diameter 0.71 mm) is used to measure the temperature of a gas stream for which h = 600 W/m^{2}K. Estimate the time constant of the thermocouple. What is the time period after which an acceptable reading of temperature can be recorded?

[c = 420J/kgK, ρ = 8600 kg/m^3]

Solution :

Time constant of thermocouple

$$t^* = \frac{\rho c V}{hA} = \frac{\rho c\left(\frac{\pi}{4}D^2L\right)}{h(\pi DL)} = \frac{\rho c D}{4h} = \frac{(8600)(420)(0.71\times10^{-3})}{4(600)} = 1.068 \text{ s}$$

At time $t = t^*$

$$\frac{T-T_\infty}{T_0-T_\infty} = e^{-1} = 0.368$$

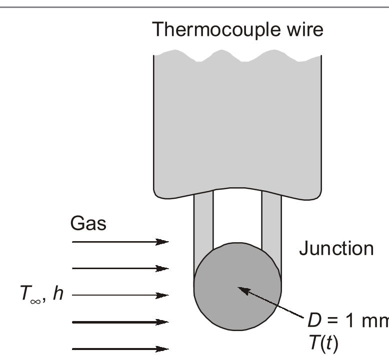
For getting a true reading of gas temperature, it should be recorded after 3t^* = 3.2 seconds after the thermocouple has been introduced into the stream.

Example 5.5 What does transient conduction mean?

(a) Heat transfer for a short time
(b) Conduction when temperature at a point varies with time
(c) Very little heat transfer
(d) Heat transfer with very small temperature difference

Ans. : (b)

Example 5.6 The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1-mm-diameter sphere, as shown in figure. The properties of the junction are k = 35 W/m°C, ρ = 8500 kg/m^3, and c_p = 320 J/kg°C, and the convection heat transfer coefficient between the junction and the gas is h = 210 W/m^2°C. Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference.



Solution :

The characteristic length of the junction,

$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = \frac{1}{6}(0.001) = 1.67\times10^{-4}\text{ m}$$

Then the Biot number,

$$Bi = \frac{hL_c}{k} = \frac{210\times(1.67\times10^{-4})}{35} = 0.001 < 0.1$$

Therefore, lumped system analysis is applicable.

In order to read 99 percent of the initial temperature difference $T_i - T_\infty$ between the junction and the gas, we must have

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = 0.01$$

$$\frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{210}{(8500)\times(320)\times(1.67\times10^{-4})} = 0.462\ s^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-\left(\frac{hA_s}{\rho c_p V}\right)t} \rightarrow 0.01 = e^{-(0.462)t}$$

which yields

$$t = 10\text{ s}$$

Therefore, we must wait at least 10 s for the temperature of the thermocouple junction to approach within 99 percent of the initial junction-gas temperature difference.

Example 5.7 A thermo couple junction is in the form of 8 mm diameter sphere, this junction is initially at 40°C and inserted in a stream of hot air of 300°C. Find the time constant of thermo couple. [c = 420 J/kg°C, ρ = 8000 kg/m^3, k = 40 W/m°C). h = 40 W/m^{2}K)

(a) 76 second (b) 89 seconds (c) 112 second (d) 128 second

Solution : (c)

$$\tau^* = \frac{\rho Vc}{hA_c} = \frac{\rho\times\left(\frac{4}{3}\pi R^3\right)\times c}{h\times 4\pi R^2} = \frac{\rho Rc}{3h}$$

$$= \frac{8000\times0.004\times420}{3\times40} = 112\text{ seconds}$$

Example 5.8 A person is found dead at 5 PM in a room whose temperature is 20°C. The temperature of the body is measured to be 25°C when found, and the heat transfer coefficient is estimated to be h = 8 W/m^2°C. Modeling the body as a 30 cm-diameter, 1.70 m long cylinder, estimate the time of death of that person.

Take properties of water at the average temperature of (37 + 25)/2 = 31°C as k = 0.617 W/m°C, ρ = 996 kg/m^3, and c_p = 4178 J/kg°C.

Solution :

The characteristic length of the body,

$$L_c = \frac{\pi r_0^2 L}{2\pi r_0 L + 2\pi r_0^2} = \frac{\pi(0.15)^2(1.7)}{2\pi(0.15)(1.7)+2\pi(0.15)^2} = 0.0689 \text{ m}$$

Then the Biot number becomes,

$$Bi = \frac{hL_c}{k} = \frac{(8)\times(0.0689)}{0.617} = 0.89 > 0.1$$

Therefore, lumped system analysis is not applicable. However, we can still use it to get a "rough" estimate of the time of death. The exponent b in this case is

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{8}{996\times 4178\times 0.0689} = 2.79\times 10^{-5}\text{ s}^{-1}$$

We now substitute these values into

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \qquad \rightarrow \qquad \frac{25-20}{37-20} = e^{-(2.79\times 10^{-5})t}$$

which yields, t = 43,860 s = 12.2 h

Therefore, as a rough estimate, the person died about 12 h before the body was found, and thus the time of death is 5 AM.

Example 5.9 A spherical steel ball of 12 mm diameter is initially at 1000 K. It is slowly cooled in a surrounding of 300 K. The heat transfer coefficient between the steel ball and the surroundings is 5 W/m²K. The thermal conductivity of steel is 20 W/mK. The temperature difference between the centre and the surface of the steel ball is

(a) large because conduction resistance is far higher than the convective resistance.
(b) large because conduction resistance is far less than the convective resistance.
(c) small because conduction resistance is far higher than the convective resistance.
(d) small because conduction resistance is far less than the convective resistance.

Solution : (d)

$$\text{Biot number} = \frac{hL}{K} = \frac{5\times 12}{20\times 1000\times 2\times 3} = 0.5\times 10^{-3}$$

Since value of biot number is very less hence conduction resistance is far less than convective resistance.

Example 5.10 The temperature distribution across a large concrete slab 500 mm thick heated from one side as measured by thermocouples approximates to the following relation

$$T = 120 - 100x + 24x^2 + 40x^3 - 30x^4$$

where T is in°C and x is in metres. Considering an area of 4 m², calculate :

(i) The heat entering and leaving the slab in unit time ;
(ii) The heat energy stored in unit time ;
(iii) The rate of temperature change at both sides of the slab;
(iv) The point where the rate of heating of cooling is maximum.

The properties for concrete are : k = 1.20 W/m°C, $\alpha = 1.77\times 10^{-3}$ m²/h

Solution :

Given : $A = 4\ m^2$, $x = 500$ mm $= 0.5$ m, $k = 1.22$ W/m°C, $\alpha = 1.77 \times 10^{-3}\ m^2/h$

$$T = 120 - 100x + 24x^2 + 40x^3 - 30x^4$$

Temperature distribution polynomial

$$\therefore \quad dT/dx = -100 + 48x + 120x^2 - 120x^3$$

$$\frac{d^2T}{dx^2} = 48 + 240x - 360x^2$$

(i) The heat entering and leaving the slab in unit time :

Heat entering the slab, $Q_{in} = -kA\left[\frac{dT}{dx}\right]_{x=0} = (-1.20 \times 5)(-100) = 600$ W

Heat leaving the slab, $Q_{out} = -kA\left(\frac{dT}{dx}\right)_{x=0.5}$

$$= (-1.20 \times 5)(-100 + 48 \times 0.5 + 120 \times 0.5^2 - 120 \times 0.5^3)$$

$$= (-6.0)(-100 + 24 + 30 - 15) = 366 \text{ W}$$

(ii) The heat energy stored in unit time :

Rate of heat storage $= Q_{in} - Q_{out} = 600 - 366 = 234$ W

(iii) The rate of temperature change at both sides of the slab ;

Rate of temperature change is given by

$$\frac{dT}{dt} = \alpha\frac{d^2T}{dx^2} = \alpha(48 + 240x - 360x^2)$$

$$\therefore \quad \left(\frac{dT}{dt}\right)_{x=0} = 1.77 \times 10^{-3}(48) = 0.08496°\text{C/h}$$

and, $$\left(\frac{dT}{dt}\right)_{x=0.5} = 1.77 \times 10^{-3}(48 + 240 \times 0.5 - 360 \times 0.5^2)$$

$$= 1.3806°\text{C/h}$$

(iv) The point where the rate of heating or cooling is maximum , x ;

For the rate of heating or cooling to be maximum

$$\frac{d}{dx}\left(\frac{dT}{dt}\right) = 0$$

or, $$\frac{d}{dx}\left[\alpha\frac{d^2T}{dx^2}\right] = 0$$

or, $$\frac{d^3T}{dx^3} = 0$$

or, $$240 - 270x = 0$$

$$\therefore \quad x = \frac{240}{720} = 0.333 \text{ m}$$

Example 5.11 An average convective heat transfer coefficient for flow of 90°C air over a flat plate is measured by observing the temperature time history of a 40 mm thick copper slab ($\rho = 9000\ kg/m^3$, $c = 0.38$ kJ/kg°C, $k = 370$ W/m°C) exposed to 90°C air. The initial temperature of plate was 200°C and in 4.5 minutes the temperature decreased by 35°C. The heat transfer coefficient will be

(a) 64.89 W/m²°C (b) 78.28 W/m²°C (c) 96.9 W/m²°C (d) 108.74 W/m²°C

Solution : (c)

$$L_C = \frac{L}{2} = \frac{0.04}{2} = 0.02 \text{ m}$$

$$\frac{hA_c}{\rho Vc} = \frac{h}{\rho\left(\frac{Vc}{A_c}\right)} = \frac{h}{\rho c\, L_c} = \frac{h}{9000 \times 0.38 \times 1000 \times 0.02} = 1.462 \times 10^{-5}\, h$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-\left(\frac{hA_c}{\rho Vc}\right)t}$$

$$\frac{165 - 90}{200 - 90} = e^{-(1.469\times10^{-5} h \times 4.5\times60)}$$

$$h = 96.9 \text{ W/m}^2{}^\circ\text{C}$$

Example 5.12 A solid sphere of diameter 10 cm is heated to 1000°C and suspended in a room whose walls are at 30°C. Compute the following:

(i) Rate of heat transfer due to radiation only neglecting other losses.

(ii) Time taken by the sphere to cool to 500°C assuming emissivity for the sphere = 0.1 and density 8.68 gm/cc. specific heat 0.098 J/kgK.

Solution :

Diameter = 10 cm, heated to 1000°C

Room temperature = 30°C = 303 K

(i) $Q = \sigma A(T^4 - T_0^4) = 5.67 \times 10^{-8} \times 4\,\pi \times (0.05)^2 \times (1273^4 - 303^4)$

$= 4.662$ kW

(ii) Performing energy balance

$$-mc\frac{dT}{dt} = \sigma \in A(T^4 - 303^4)$$

$$-\frac{mc}{\sigma \in A}\int_{1273}^{773} \frac{dT}{(T^4 - 303^4)} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{\rho \times 4/3\pi(0.05)^3 \times 0.098}{5.67\times10^{-8} \times 0.1 \times 4\pi \times (0.05)^2} \times \int_{1273}^{773}\frac{dT}{(T^4 - 303^4)} = t$$

$$-2.5 \times 10^9 \times (-5.67 \times 10^{-10}) = t$$

$$\Rightarrow \quad t = 1.418 \text{ seconds}$$

Summary

1. Conduction of heat in unsteady state refers to the transient conditions wherein heat flow and the temperature distribution at any point of the system vary continuously with time.
2. The process in which the internal resistance is assumed negligible in comparison with its surface resistance is called the Newtonian heating or cooling process.

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\theta}{\theta_i} = \exp\left[-\frac{hA_s}{\rho Vc}t\right] \quad \text{...(i)}$$

where,
ρ = Density of solid, kg/m^3
V = Volume of the body, m^3
c = Specific heat of the body, J/kg°C

$$h = \text{Unit surface conductance, W/m}^2{}^\circ\text{C}$$
$$A_s = \text{Surface area of the body, m}^2$$
$$T = \text{Temperature of the body at any time,}^\circ\text{C}$$
$$t = \text{Time, s.}$$

Biot number, $Bi = \dfrac{hL_c}{k}$

Fourier number, $F_0 = \dfrac{\alpha t}{L_c^2}$

where, L_c = Characteristic length, and

$$\alpha = \left[\frac{k}{\rho c}\right] = \text{Thermal diffusitivity of the solid.}$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_0 - T_\infty} = e^{-Bi\,F_0} \quad \text{...(ii)}$$

Instantaneous heat flow rate :

$$Q_i = -hA_s\,(t_i - t_a)e^{-Bi\,Fo} \quad \text{...(iii)}$$

Total or cumulative heat transfer :

$$Q' = \rho\,Vc(t_1 - t_a)[e^{-Bi\,Fo} - 1] \quad \text{...(iv)}$$

3. Time constant and response of temperature measuring instruments :

The quantity $\dfrac{\rho Vc}{hA_s}$ is called time constant (τ^*)

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_0 - T_\infty} = e^{-(t/t^*)}$$

The time required by a thermocouple to reach its 63.2 percent of the value of the initial temperature difference is called its sensitivity.

Objective Brain Teasers

Q.1 Copper balls (ρ = 8933 kg/m^3, k = 401 W/m°C, c_p = 385 J/kg°C, $\alpha = 1.166 \times 10^{-4}$ m^2/s) initially at 200°C are allowed to cool in air at 30°C for a period of 2 minutes. If the balls have a diameter of 2 cm and the heat transfer coefficient is 80 W/m^2°C, the center temperature of the balls at the end of cooling is

(a) 103.58° C (b) 87°C
(c) 198°C (d) 126°C

Q.2 Carbon steel balls (ρ = 7830 kg/m^3, k = 64 W/m°C. c_p = 434 J/kg°C) initially at 150°C are quenched in an oil bath at 20°C for a period of 3 minutes. If the balls have a diameter of 5 cm and the convection heat transfer coefficient is 450 W/m^2°C. The center temperature of the balls after quenching will be
(**Hint** : Check the Biot number).

(a) 27.4°C (b) 143°C
(c) 12.7°C (d) 48.2°C

Q.3 A 6 cm diameter 13 cm high canned drink (ρ = 977 kg/m^3, k = 607 W/m°C. c_p = 4180 J/kg°C) initially at 25°C is to be cooled to 5°C by dropping it into iced water at 0°C. Total surface area and volume of the drink are A_s = 301.6 cm^2 and V = 367.6 cm^3. If the heat transfer coefficient is 120 W/m^2°C. Assume the can is agitated in water and thus the temperature of the drink changes uniformly with time.

(a) 1.5 min (b) 8.7 min
(c) 11.1 min (d) 26.6 min

Q4 Lumped system analysis of transient heat conduction situations is valid when the Biot number is

(a) very small (b) approximately
(c) very large (d) any real number

Q.5 Polyvinylchloride automotive body panels (k = 0.92 W/mK, c_p = 1.05 kJ/kg, ρ = 1714 kg/m^3), 3 mm thick, emerge from an injection molder at 120°C. They need to be cooled to 40°C by exposing both sides of the panels to 20°C air before they can be handled. If the convective heat transfer coefficient is 30 W/m^{2}K and radiation is not considered, the time that the panels must be exposed to air before they can be handled is

(a) 1.6 min (b) 2.4 min
(c) 2.8 min (d) 3.5 min

Q.6 The Biot number can be thought of as the ratio of

(a) The conduction thermal resistance to the convective thermal resistance.
(b) The convective thermal resistance to be conduction thermal resistance.
(c) The thermal energy storage capacity to the conduction thermal resistance.
(d) The thermal energy storage capacity to the convection thermal resistance.

ANSWERS

1. (a) 2. (a) 3. (c) 4. (a) 5. (b) 6. (a)

Hints & Explanation

1. (a)

$$\rho = 8933 \text{ kg/m}^3$$
$$k = 401 \text{ W/m°C}$$
$$c = 385 \text{ J/kg°C}$$
$$\alpha = 1.166 \times 10^{-4} \text{ m}^2/\text{s}$$
$$T_{initial} = 200°\text{C}$$
$$T_\infty = 30°\text{C}$$
$$t = 2 \text{ min}$$
$$\text{Diameter} = 2 \text{ cm}$$
$$h = 80 \text{ W/m}^2\text{°C}$$
$$T_{final} = ?$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA\tau}{\rho v c_p}}$$

where
$$\frac{A}{V} = \frac{4\pi R^2}{\frac{4}{3}\pi R^3} = \frac{3}{R} = \frac{3}{1\times 10^{-2}}$$
$$= 300 \text{ m}^{-1}$$

$$\frac{T_{final} - 30}{200 - 300} = e^{-\frac{80 \times 300 \times (2\times 60)}{8933 \times 385}}$$
$$= 0.433$$
$$T_{final} = 30 + 0.433 \times (200 - 30)$$
$$= 103.58°\text{C}$$

2. (a)

Given data:

$$\rho = 7830 \text{ kg/m}^3$$
$$k = 64 \text{ W/m°C}$$
$$c_p = 434 \text{ J/kg°C}$$
$$T_{initial} = 150°\text{C}$$
$$T_\infty = 20°\text{C}$$
$$t = 3 \text{ min} = 3 \times 60 = 180 \text{ s}$$
$$\text{Diameter} = 5 \text{ cm} = \frac{5}{100} = 0.05 \text{ m}$$
$$h = 450 \text{ W/m}^2\text{°C}$$

$$\frac{V}{A} = L_C = \frac{4}{3}\frac{\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{d}{6}$$
$$= \frac{0.05}{6} = 8.33 \times 10^{-3} \text{ m}$$

$$\text{Biot Number} = \frac{hL_c}{k} = \frac{450 \times 8.33 \times 10^{-3}}{64}$$

$$= 0.058 < 0.1$$

Hence lumped heat analysis is valid.

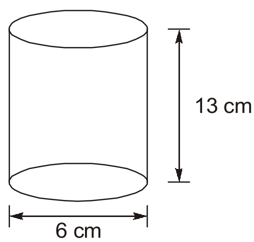
$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA\tau}{\rho V c_p}}$$

$$\frac{T - 20}{150 - 20} = e^{-\frac{450 \times 180}{7830 \times 8.33 \times 10^{-3} \times 434}}$$

$$T = 27.4°\text{C}$$

3. (c)

$\rho = 977$ kg/m^3
$k = 607$ W/m°C
$c_p = 4180$ J/kg°C
$T_{initial} = 25°$C
$T_{final} = 5°$C
$T_\infty = 0°$C
$A_s = 301.6$ cm^2
$V = 367.6$ cm^3
$h = 120$ W/m^2°C



$$\text{Biot number} = \frac{hL_c}{k} = \frac{120}{0.607} \times \frac{V}{A}$$

$$= \frac{120}{607} \times \frac{367.6 \times 10^{-6}}{301.6 \times 10^{-4}}$$

$$= 2.4 \times 10^{-3}$$

$$\frac{T_{final} - T_\infty}{T_{initial} - T_\infty} = e^{-\frac{hA_s}{\rho V c_p}}$$

$$\frac{5 - 0}{25 - 0} = e^{-\frac{120 \times c}{977 \times \frac{267.6 \times 10^{-6}}{301.6 \times 10^{-4}}} \times 4180}$$

$$\frac{5}{25} = e^{-2.4108 \times 10^{-3}\tau}$$

$$0.2 = e^{-2.4108 \times 10^{-3}\tau}$$

$$\ln 0.2 = -2.4108 \times 10^{-3}\tau$$

$$\tau = 667.595 \text{ s} = 11.1 \text{ min}$$

4. (a)

Lumped system analysis of transient heat conduction situation is valid when the Biot number is very small.
usually Bi < 0.1

5. (b)

Given:
$k = 0.92$ W/mK
$c_p = 1.05 \times 10^3$ J/kgK
$\rho = 1714$ kg/m^3
Thickness = 3 mm
$T_{initial} = 120°$C
$T_{final} = 40°$C
$T_\infty = 20°$C
$h = 30$ W/m^{2}K
$\tau = ?$

$$\text{Biot number} = \frac{h.V}{kA} = \frac{30}{0.92} \times \frac{3 \times 10^{-3}}{2}$$

$$= 0.0489$$

$$\frac{T_s - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho V c_p}\tau}$$

$$\frac{40 - 20}{120 - 20} = e^{-\frac{30 \times \tau}{1714 \times \frac{3 \times 10^{-3}}{2} \times 1.05 \times 10^3}}$$

$$\ln 0.2 = -0.0111\tau$$

$$\tau = 144.83 \text{ s}$$

$$= 2.4 \text{ min}$$

6. (a)

The biot number can be thought of as the ratio of conduction thermal resistance to the connection thermal resistance.

$$\text{Bi} = \frac{hL_c}{k}$$

$$\text{Conduction resistance} = \frac{L}{kA}$$

$$\text{Connective resistance} = \frac{1}{hA}$$

$$\text{Biot number} = \frac{\frac{L}{kA}}{\frac{1}{hA}} = \frac{hL}{k}$$

Student's Assignments

Q.1 The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1.2 mm diameter sphere. The properties of the junction are k = 35 Wm°C, ρ = 8500 kg/m^3, and c_p = 320 J/kg°C, and the heat transfer coefficient between the junction and the gas is h = 90 W/m^2°C. Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference.

Ans. 27.8 s

Q.2 A cylindrical nickel-steel billet of 0.1 m diameter and 0.5 m length, initially at 800°C, is suddenly dropped in a large vessel containing oil at 30°C. The convection heat transfer coefficient between the billet and the oil is 20 W/(m^{2}K). Calculate the time required for the billet to reach a temperature of 250°C. Take for nickel-steel, k = 20 W/(mK), ρ = 8000 kg/m^3, c_p = 0.45 kJ/(kgK).

Ans. 1.57 h

Q.3 An aluminium sphere of 0.1 m diameter and at a uniform temperature of 500°C is suddenly exposed to an environment at 20°C , with convection heat transfer coefficient 30 W (m^{2}K). Calculate the temperature of the sphere (i) 100 s, (ii) 300 s, and (iii) 500 s after it is exposed to the environment. Justify any method you use for the analysis ; take, for aluminium, k = 200 W/(mK), ρ = 2700 kg/m^3,
c_p = 0.9 kJ/(kgK).

Ans. (i) 466°C, (ii) 404°C,(iii) 350°C

Q.4 A chrome-nickel wire of 2 mm diameter, initially at 25°C, is suddenly exposed to hot gases at 725°C. If the convection heat transfer coefficient is 20 W/(m^{2}K). Calculate the time constant of the wire as a Lumped-capacity system. Take k = 20 W/(mK), ρ = 7800 kg/m^3.
c_p = 0.46 kJ/(kgK).

Ans. 179 s

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