POSTAL Book Package 2023

Mechanical Engineering

Conventional Practice Sets

Heat Transfer

Contents

MADE EASY Publications

5 CHAPTER Transient Conduction

Practice Questions : Level-I

Q.1 A steel ball of diameter 60 mm is initially in thermal equilibrium at 1030°C in a furnace. It is suddenly removed from the furnace and cooled in ambient air at 30°C, with convective heat transfer coefficient h = 20 W/m²K. The thermo-physical properties of steel are : density ρ = 7800 kg/m², conductivity k = 40 W/mK and specific heat c = 600 J/kgK. The time required in seconds to cool the steel ball in air from 1030°C to 430°C is

Solution:

Diameter of ball, d = 60 mm = 0.06 m;
Ambient temperature, T_0 = 30°C;
Density, ρ = 7800 kg/m³;
Specific heat, c = 600 J/kgK;
Time, t = ?
Initial temperature, T_i = 1030°C
Convective heat transfer coefficient, h = 20 W/m²K
Conductivity, k = 40 W/mK
Final temperature of steel ball, T = 430°C

The temperature distribution relation is given by

$$\boxed{\frac{T-T_0}{T_i-T_0} = e^{-\frac{ht}{\rho c l}}} \quad \left[\text{where } l = \frac{V}{A} \text{ characteristic length of balls}\right] \quad ...(i)$$

$$\because \quad V = \frac{\pi}{6}d^3, A = \pi d^2$$

$$\therefore \quad l = \frac{V}{A} = \frac{\pi}{6}\frac{d^3}{\pi d^2} = \frac{d}{6} = \frac{0.06}{6} = 0.01\,\text{m}$$

Substituting the values of T, T_o, T_i, h, ρ, c and l in Eq. (i), we get

$$\frac{430-30}{1030-30} = e^{\frac{-20\times t}{7800\times 600\times 0.01}}$$

$$0.4 = e^{-4.273\times 10^{-4}\,t}$$

Taking $\log_e$ both sides, we get $\quad \log_e 0.4 = -4.273\times 10^{-4}\times t\times 1$

or $$t = \frac{0.9162}{4.273\times 10^{-4}} = \mathbf{2144.16\ s}$$

Q.2 Stainless steel ball bearings having diameter of 1.2 cm are to be quenched in water. The balls leave the oven at 900°C and are exposed to air at 30°C for a while before they are dropped in water. If the temperature of the balls is not to fall below 850°C prior to quenching and the heat transfer coefficient in the air is 125 W/m²K, determine how long they can stand in the air before being dropped into the water. Following properties of stainless steel may be used:
ρ = 8085 kg/m³, k = 15.1 W/mK,
c_p = 0.480 kJ/kgK, α = 3.91 × 10⁻⁶ m²/s

Solution:

Given data: $T_i = 900°C$; $T_\infty = 30°C$; $h = 125\ W/m^2K$; $T = 850°C$; Time taken = t = ?

Since Metallic steel balls are of small size,

$$Bi = \frac{hl}{K} = \frac{hR}{3K} = \frac{125 \times 0.6 \times 10^{-2}}{3 \times 15.1} = 0.016$$

$$\frac{hl}{k} = \text{Biot No} < 0.1 \Rightarrow \text{Lumped heat analysis is valid}$$

$$\therefore \quad e^{\left(\frac{hA}{\rho V c_p}\right)t} = \frac{T_i - T_\infty}{T - T_\infty}$$

Put $$\frac{A}{V} = \frac{3}{R}$$

$$\Rightarrow \quad 125 \times \frac{3}{\left(\frac{0.6}{100}\right)} \times \frac{1}{8085} \times \frac{1}{0.480 \times 10^3} \times t = \ln\left(\frac{900-30}{850-30}\right)$$

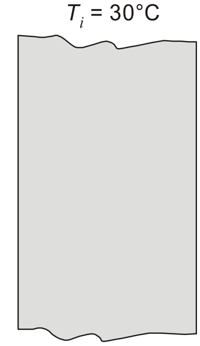
$\Rightarrow$ Time: t = **3.675 s**

Q3 A 4 mm thick panel of aluminium alloy (ρ = 2800 kg/m³, c = 880 J/kgK and k = 177 W/mK) is finished on both sides with an epoxy coating that must be cured at or above 150°C for atleast 5 min. The curing operation is performed in a large oven with air at 175°C and convection coefficient of h = 40 W/m²K. If the panel is placed in the oven at an initial temperature of 30°C, at what total elapsed time, will the cure process be completed?

Solution:

Given data: $T_\infty = 175°C$; $h = 40\ W/m^2K$; $\rho = 2800\ kg/m_3$, $c = 880\ J/kgK$

$k = 177\ W/mK$



Assumptions:

1. Uniform heating
2. Constant properties

Characteristic length of plate, $L_c = L$ = 2 mm

$$\text{Biot number, } Bi = \frac{hL_c}{k} = \frac{40 \times 2 \times 10^{-3}}{177} = 4.5198 \times 10^{-4}$$

Since, $Bi < 0.1$, the lumped heat capacitance approximation can be used.

As we know,

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{hA}{\rho V c}t\right]} = \exp\left[-\frac{h}{\rho L_c c}t\right]$$

$$\Rightarrow \quad \frac{150 - 175}{30 - 175} = \exp\left[-\frac{40}{2800 \times 0.002 \times 880}t\right]$$

$$t = 216.57\ s$$

Total time to complete the 5 min duration cure,

$$t_e = t_i + t = (5 \times 60 + 216.57)s$$

$$t_e = 516.57\,s = 8.61\ min$$

Q4 A bearing piece in the form of half of a hollow cylinder of 60 mm ID, 90 mm OD and 100 mm long is to be cooled to –100°C using a cryogenic gas at –150°C with a convective heat transfer coefficient of 70 W/m²K. Determine the time required. Take properties of bearing material as c = 444 J/kgK, ρ = 8900 kg/m³ and k = 17.2 W/mK.

Solution:

Given: A piece of bearing as half of hollow cylinder with

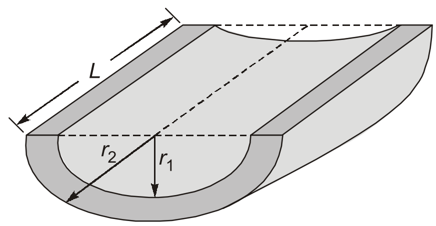
D_1 = 60 mm or r_1 = 0.03 m D_2 = 90 mm or r_2 = 0.045 m

L = 100 mm = 0.1 m, T_i = 30°C T_∞ = –150°C

T = 100°C c = 444 J/kgK, ρ = 8900 kg/m^3

k = 17.2 W/mK, h = 70 W/m^{2}K

To find: Time required to reach –100°C.



The characteristics length of the cylinder.

The volume of bearing piece,

$$V = \left(\frac{1}{2}\right)\times\pi(r_2^2 - r_1^2)L = \left(\frac{1}{2}\right)\times\pi\{(0.045)^2 - (0.03)^2\}\times 0.1$$

$$= 1.766\times 10^{-4}\ \text{m}$$

Surface area of the bearing, A_s = (Front + back + lateral + longitudinal) area

$$= 2\times(1/2)\pi\,(r_2^2 - r_1^2) + \pi L(r_1 + r_2) + 2\times L\times(r_2 - r_1)$$

$$= \pi\times(0.045^2 - 0.03^2) + \pi\times 0.1\times(0.03 + 0.045) + 2\times 0.1\times(0.045 - 0.03)$$

$$= 3.53\times 10^{-3} + 0.0235 + 3\times 10^{-3} = 0.03003\ \text{m}^2$$

$$L_c = \frac{V}{A_s} = \frac{1.766\times 10^{-4}}{0.03003} = 5.872\times 10^{-3}\ \text{m}$$

Biot number, $$B_i = \frac{hL_c}{k} = \frac{70\times 5.872\times 10^{-3}}{17.2} = 0.0239$$

Which is less than 0.1, hence the lumped heat capacity system analysis may be applied. Using for temperature distribution

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{ht}{\rho L_c C}\right]}$$

Substituting the values, $$\frac{-100-(-150)}{30-(-150)} = \exp\left\{-\frac{70t}{8900\times 5.872\times 10^{-3}\times 444}\right\}$$

or, $$t = 424.6\ \text{s}$$

Practice Questions : Level-II

Q.5 A 15 mm diameter mild steel sphere (k = 42 W/m°C) is exposed to cooling air flow at 20°C resulting in the convective coefficient h = 120 W/m^2°C. Determine the following:

(i) Time required to cool the sphere from 550°C to 90°C.

(ii) Instantaneous heat transfer rate 2 minutes after the start of cooling.

(iii) Total energy transferred from the sphere during the first 2 minutes.

For mild steel take: ρ = 7850 kg/m^3, c = 475 J/kg°C and α = 0.045 m^2/h.

Solution:

Note: As unit of 'α' in in 'hr' so time 't' is taken in hours.

Given: $R = \frac{15}{2} = 7.5$ mm = 0.0075 m; $k = 42$ W/m°C; $T_\infty = 20$°C

$T_i = 550$°C; $T = 90$°C; $h = 120$ W/m^2°C

(i) **Time required to cool the sphere from 550°C to 90°C, t:**

The characteristics length L_c is given by,

$$L_c = \frac{\frac{4}{3}R^3}{4\pi R^2} = \frac{R}{3} = \frac{0.0075}{3} = 0.0025 \text{ m}$$

$$\text{Biot number, } B_i = \frac{hL_c}{k} = \frac{120 \times 0.0025}{42} = 0.007143$$

$$\text{Fourier number, } F_0 = \frac{\alpha t}{L_c^2} = \frac{0.045 \times t}{(0.0025)^2} = 7200t \qquad \text{(where } t \text{ is in hour)}$$

Since $B_i < 0.1$, so we can use lump theory to solves this problem.

The temperature variation with time is given by

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = e^{-B_i F_o}}$$

Substituting the values, we get

$$\frac{90 - 20}{550 - 20} = e^{-0.007143 \times 7200t} = e^{-51.43t}$$

$$0.132 = e^{-51.43t} \qquad \text{or} \qquad e^{51.43t} = \frac{1}{0.132} = 7.576$$

or, $$51.4t = 2.025$$

or, $$t = \frac{2.025}{51.43} = 0.03937h \;\mathbf{= 141.7\ s}$$

(ii) **Instantaneous heat transfer rate 2 minutes (0.0333h) after the start of cooling, Q_1:**

$$\boxed{Q_i = -hA_s(T_i - T_\infty)e^{-B_i F_o}}$$

Now, $$B_i F_o = (0.007143)(7200 \times 0.0333) = 1.7126$$

∴ $$Q_i = 120 \times 4\pi \times (0.0075)^2 (550 - 20)\, e^{-1.7126} \;\mathbf{= -8.1\ W}$$

The negative sign shows that heat is given off by sphere.

(iii) **Total energy transferred from the sphere during first 2 minutes (0.0333h) Q':**

Amount of heat transfer during time interval,

$$(o, t) = \int_0^t \theta \cdot dt$$

$$= \int_0^t hA(T - T_\infty) \cdot dt = \int_0^{0.033} hA(T_i - T_\infty)e^{-B_i \cdot F_0} . dt$$

$$\Delta V = \rho Vc(T_i - T_\infty)[e^{-B_i F_o} - 1]$$

$$= 7850 \times \frac{4}{3}\pi \times (0.0075)^3 (475)(550 - 20)[e^{-1.7126} - 1]$$

$$\mathbf{= -2862.3\ J}$$