Mechanical Engineering

Theory of Machines

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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E-mail: infomep@madeeasy.in
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CHAPTER 04

CAM Design

4.1 Introduction

Cam-follower systems are frequently used in all kinds of machines. The valves in automobile engine are opened by cams. Machines used in the manufacture of many consumer goods are full of cams. Cams are easier to design to give a specific output function but they are much more difficult and expensive to make as compared to linkage. A cam is a rotating or a reciprocating element of a mechanism, which imparts rotating, reciprocating or oscillating motion to another element, called the follower. There is line contact between the cam and the follower, and thereby forms a higher pair. In this chapter we shall study the various types of planar cams from the point of view of drawing their profile and motion analysis. Cams are manufactured usually by die casting, milling or by punch-presses.

The essential components of a cam mechanism are as follows :

(i) A driver member known as the **cam**.

(ii) A driven member called the **follower**.

(iii) A **frame** which supports the cam and guides the follower.

4.2 Classification of Cams

Cams are classified according to

1. Shape
2. Follower movement, and
3. The method of constraint of the follower

4.2.1 Type of cams according to shape

1. **Wedge and Flat cams :** A wedge cam has a wedge *A* which, in general, has a translational motion [Fig. 4.1 (a) and (b)]. The follower can either translate [Fig. 4.1 (a)] or oscillate [Fig. 4.1 (b)] The follower guide *C* causes the relative motion of the cam *A* and follower *B*. Instead of using a wedge, a flat plate with a groove can also be used. Fig. 4.1 (d) shows a flat plate with a groove in which the follower is held to obtain the desired motion. Thus a positive drive is achieved without the use of a spring.

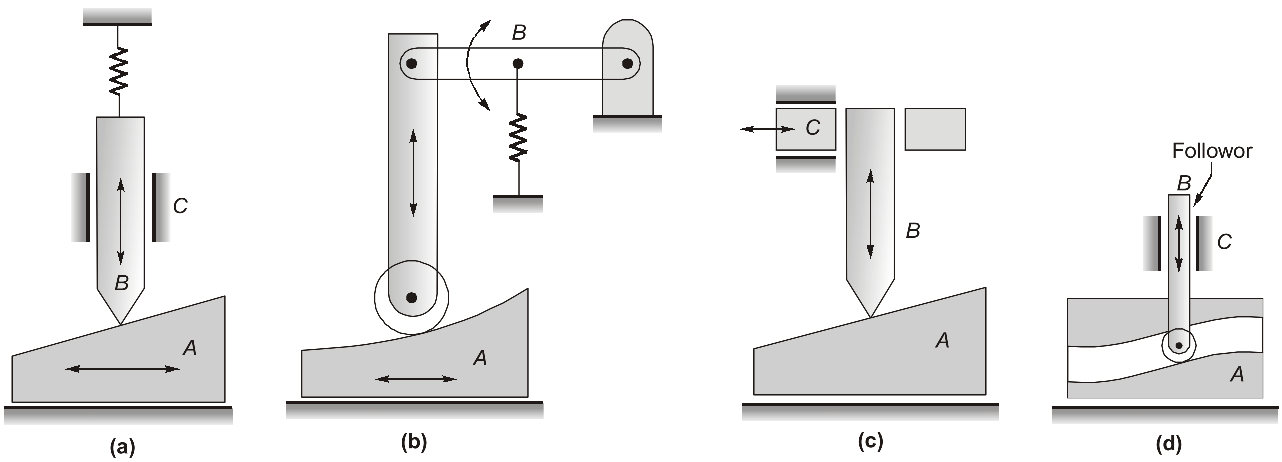


Fig. 4.1 Wedge and Flat Cams

2. **Radial and Offset cams :** A cam in which the follower moves radially from the centre of rotation of the cam is called a radial (or disc or plate) cam, as shown in Fig. 4.2 (a) and (b). The axis of the follower passes through the axis of the cam in such a cam.

   If the axis of the follower does not pass through the axis of the cam, it is called an offset cam, as shown in Fig. 4.2 (c). Radial cams are very popular due to their simplicity and compactness.

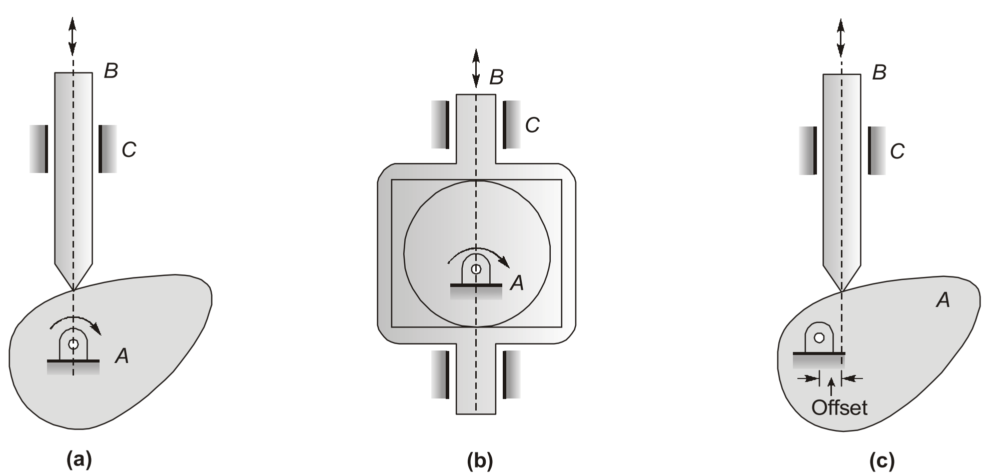


Fig. 4.2 Radial and Offset cams **(a)** Knife-edge radial **(b)** Closed cam **(c)** Offset cam

3. **Cylindrical cams :** In a cylindrical cam, a cylinder which has a circumferential groove/contour cut in the surface, rotates about its axis. The follower motion can be either oscillatory or reciprocating type, as shown in Fig. 4.3 (a) and (b). A spring-loaded follower translates along or parallel to the axis of the rotating cylinder. They are also called barrel or drum cams.

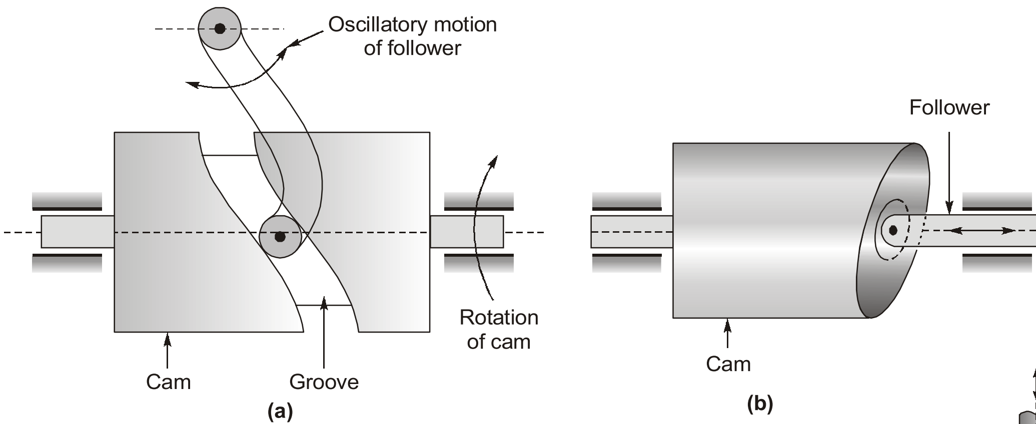


Fig. 4.3 Cylindrical Cams

4. **Spiral cams :** A spiral cam is a face cam in which a groove is cut in the form of a spiral as shown in Fig. 4.4. The spiral groove consists of teeth which mesh with a pin gear follower. The velocity of the follower is proportional to the radial distance of the groove from the axis of the cam. The use of such a cam is limited as the cam has to reverse the direction to reset the position of the follower. It finds its use in computers.

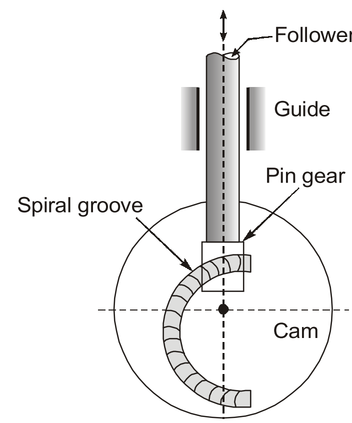


Fig. 4.4 Spiral Cams

5. **Conjugate cams :** A conjugate cam is a double-disc cam, the two disc being keyed together and are in constant touch with the two rollers of a follower, as shown in Fig. 4.5. Thus the follower has a positive constraints. Such a cam gives low wear, low noise, better control of follower, high speed and high dynamic loads, etc.

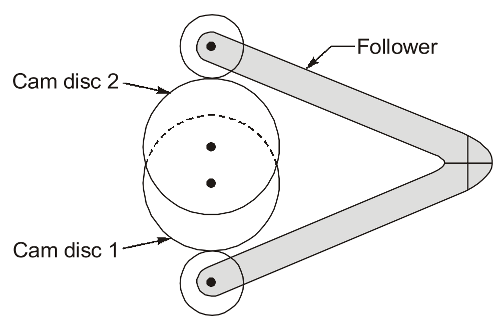


Fig. 4.5 Conjugate Cam

6. **Globoidal cams :** A globoidal cam has either a convex or concave surface.
   A circumferential groove is cut on the surface of rotation of the cam to impart motion to the follower, which has oscillatory motion, as shown in Fig. 4.6 (a) and (b). The application of such cams is limited to moderate speeds and where the angle of oscillation of the follower is large.

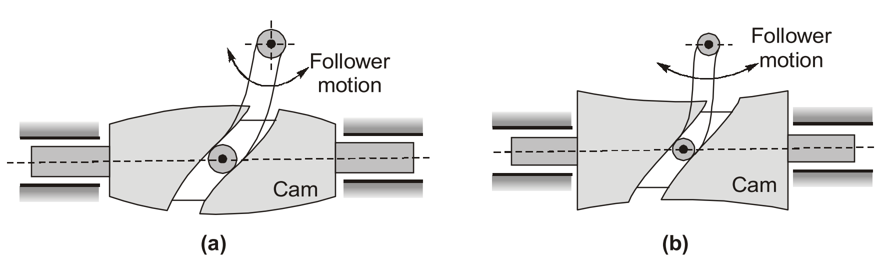


Fig. 4.6 Globoidal Cam **(a)** Convex cam **(b)** Concave cam

7. **Spherical cams :** As shown in Fig. 4.7, the spherical cam is in the form of a spherical surface which transmits motion to the follower. In spherical cam, the follower oscillates about an axis perpendicular to the axis of rotation of the cam. In a disc cam, the follower oscillates about an axis parallel to the axis of rotation of the cam (Refer Fig. 4.2).

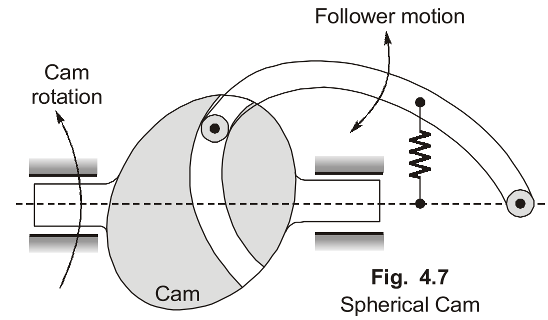


Fig. 4.7
Spherical Cam

4.2.2 Type of cams according to follower movement

The motion of the followers are distinguished from each other by the dwells they have. A dwell is the zero displacement or the absence of motion of the follower during the motion of the cam. Cams are classified according to the motions of the followers in the following ways :

1. **Rise-Return-Rise (R-R-R) :** As shown in Fig. 4.8, there is alternate rise and return of the follower with no periods of dwells. Its use is very limited in the industry. The follower has a linear or an angular displacement.

2. **Dwell-Rise-Return-Dwell (D-R-R-D) :** There is rise and return of the follower after a dwell in such type of cam, (Refer Fig. 4.9). This type is used more frequently than the R-R-R type of cam.

3. **Dwell-Rise-Dwell-Return-Dwell (D-R-D-R-D) :** It is the most widely used type of cam. As shown in Fig. 4.10, the dwelling of the cam is followed by rise and dwell and subsequently by return and dwell as shown in Fig. 4.10.

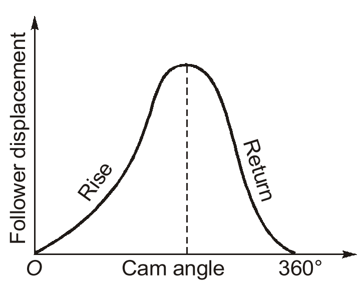


Fig. 4.8

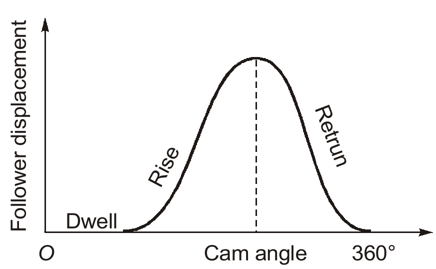


Fig. 4.9

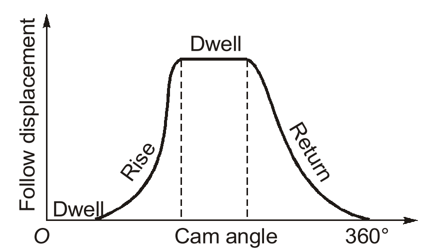


Fig. 4.10

4.2.3 Type of cams according to manner of constraint of the follower

To reproduce exactly the motion transmitted by the cam to the follower, it is necessary that the two remain to touch at all speeds and at all times. The cams can be classified according to the manner in which the reproduction of motion is achieved.

1. **Pre-loaded spring cam :** A pre-loaded compression spring is used for the purpose of keeping the contact between the cam and the follower.
   Examples are Fig. 4.1 (a), 4.1 (b), 4.7.

2. **Positive-drive cam :** The constant or positive touch between the cam and the follower is maintained by a roller follower operating in the groove of cam as shown in Fig. 4.1 (d), 4.2 (b), 4.4, 4.3 (a) and 4.6. It is insured that the follower cannot go out of this groove under the normal working operations. A constrained or positive drive is also obtained by the use of a conjugate cam as shown in Fig. 4.5.

3. **Gravity cam :** If the rise of the cam is achieved by the rising surface of the cam and the return by the force of gravity or due to the self weight of the cam, the cam is known as a gravity cam as shown in Fig. 4.1 (c). These cams are not preferred due to their uncertain behaviour.

4.3 Types of Cam-Followers

Cam follower are classified according to the

1. Shape,
2. Movement and
3. Location of line of movement.

4.3.1 Type based on the surface in contact

The follower based on the type of surface in contact can be classified as knife edge, roller, flat faced/ mushroom and spherical-faced follower as shown in Fig. 4.11.

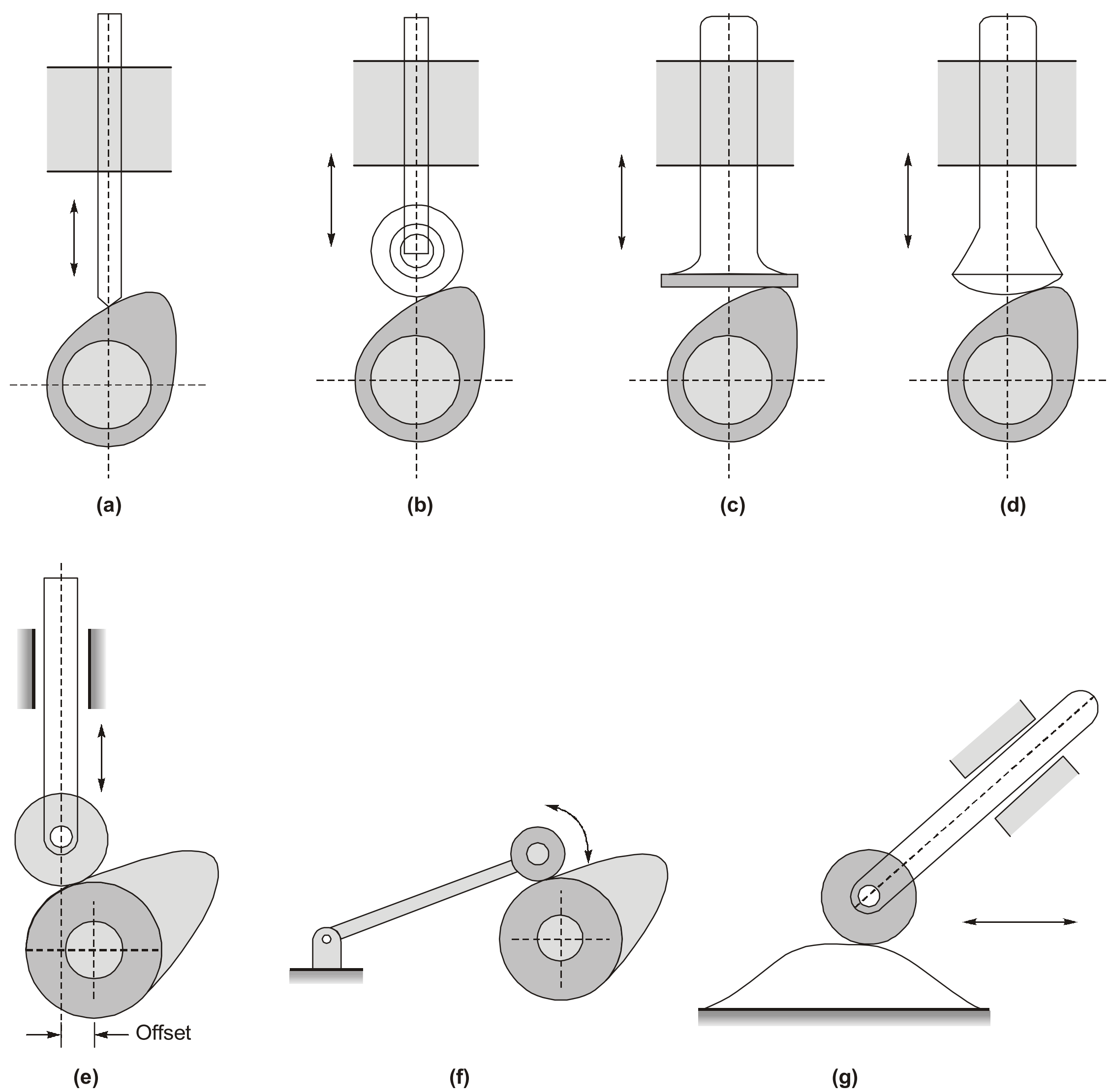


Fig. 4.11 Type of Follower **(a)** Knife-edge follower **(b)** Roller follower **(c)** Flat faced (plain/mushroom) follower **(d)** Spherical-faced follower (mushroom) **(e)** Offset follower **(f)** Oscillating follower **(g)** Translating follower

4.3.2 Type of follower according to movement

Reciprocating follower : In this type, as the cam rotates, the follower reciprocates or translates in the guided as shown in Fig. 4.1 (a).

Oscillating follower : The follower is pivoted at a suitable point on the frame and oscillates as the cam makes the rotary motion as shown in Fig. 4.1 (b).

4.3.3 Type of follower according to location of line of movement

When the axis of the follower passes through the centre of rotation of the cam, it is called a radial follower, and when the axis of the follower does not pass through the axis of the cam, it is called an offset follower.

NOTE

- Use of knife-edge follower is limited as it produces a great wear of the surface at the point of contact.
- At low speeds, roller follower has a pure rolling action but at higher speeds, some sliding also occurs.
- In case of steep rise, a roller follower jams the cam and not preferred.
- Mushroom follower does not pose the problem of jamming the cam.
- High surface stresses and wear are quite high due to deflection and misalignment if flat-faced follower is used. These disadvantages are reduced if a spherical-faced follower is used instead of a flat-faced follower.

4.4 Cam Nomenclature

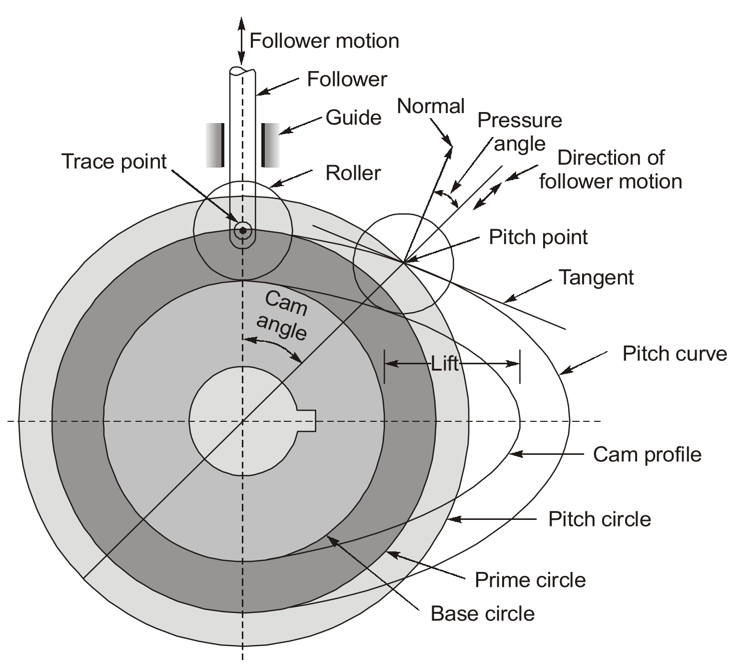


Fig. 4.12 Cam Nomenclature

A radial cam with reciprocating roller follower is shown in Fig. 4.12. The following nomenclature is used in reference to planar cam mechanisms :

Prime circle : It is the smallest circle drawn tangent to the pitch curve from the centre of rotation of the cam.

Pitch point : It is a point on the pitch curve having the maximum pressure angle.

Pitch circle : It is the circle drawn through the centre and pitch point.

Trace point : It is a reference point on the follower and is used to generate the pitch curve. In the case of a knife edge follower, it is the knife edge, and in the case of a roller follower, it is the centre of the roller.

Pitch curve : It is the curve generated by the trace point as the follower move relative to the cam. Pitch curve can be assumed to be drawn by the trace point assuming that the cam is fixed, and the trace point of the follower rotates around the cam.

Pressure angle : The angle of any point between the normal to the pitch curve and the instantaneous direction of the follower motion. This angle is important in cam design because it represents the steepness of the cam profile. It varies in magnitude at all instants of the follower motion. A high value of the maximum pressure angle is not desired as it might jam the follower in the bearings.

Stroke or Throw : The greatest distance or angle through which the follower moves or rotates. It is also called as lift of the cam.

Cam profile : The surface in contact with the follower is known as the cam profile.

Angle of ascent : It is the angle of rotation of the cam during which the follower rises up.

Angle of dwell : The angle of dwell the is angle through which the cam turns while the follower remains stationary at the highest or the lowest position.

Angle of descent : It is the angle of rotation of cam during which the follower returns to its initial position.

Angle of action : The angle of action is the total angle moved by the cam during the time, between the beginning of rise and the end of the return of the follower.

4.5 Motion Events

As a cam rotates about the axis, it imparts a specific motion to the follower which is repeated with each revolution of the cam. Thus it is enough to know the motion of the follower for only one revolution. During rotation of the cam through one revolution, the follower is made to execute a series of events such as rises, dwells and returns. Rise is the motion of the follower away from the cam centre, dwell is the motion during which the follower is at rest, and return is the motion of the follower toward the cam centre.

Motion events for the basic motion/follower movement can be understood with the help of Fig. 4.13.

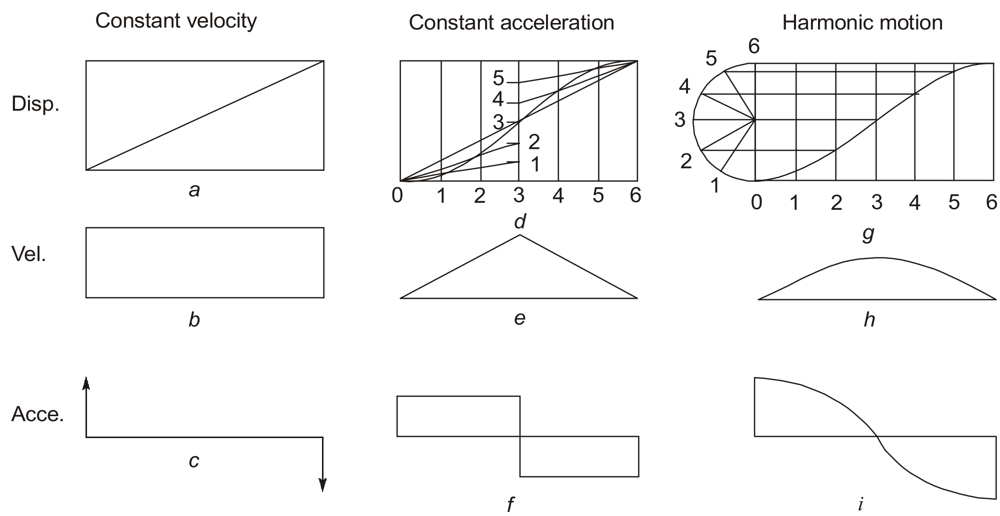


Fig. 4.13 Motion Events

NOTE

- The dynamic effects of acceleration limits the speed of the cams.
- Effects of jerks $\left(\frac{da}{dt} \text{ or } \frac{d^3x}{dt^3}\right)$ in case of high-speed mechanisms produce vibrations of the system, which is not desirable for a follower motion.
- Though it is very difficult to completely eliminate jerk but designer's effort is always to keep the jerk within tolerable limits.
- The size of the base circle controls the pressure angle.

4.6 High Speed Cams and Undercutting

A follower in cam system has some mass and accompanied acceleration. Inertia force of follower is obtained by multiplying mass with acceleration and it is felt / experienced at the contact point of the follower with cam surface and at the camshaft bearings. An abruptly changing acceleration curve exerts abrupt stresses developed by the inertia force of the follower on the cam surfaces and at the bearings. Surface wear, noise etc. are the determinantal effects due to contact stresses. Thus, it is very important to give due consideration to velocity and acceleration curves while choosing a displacement diagram.

They should not have any steep changes. At higher speeds, cams are certainly bound to show discontinuous acceleration characteristics. The higher the speed, the higher is the need for smooth curves. At very high speeds, even the jerks (da/dt or d^3x/dt^3) is made continuous as well. In section 4.7, the standard cam-follower motions have been discussed for suitable selection.

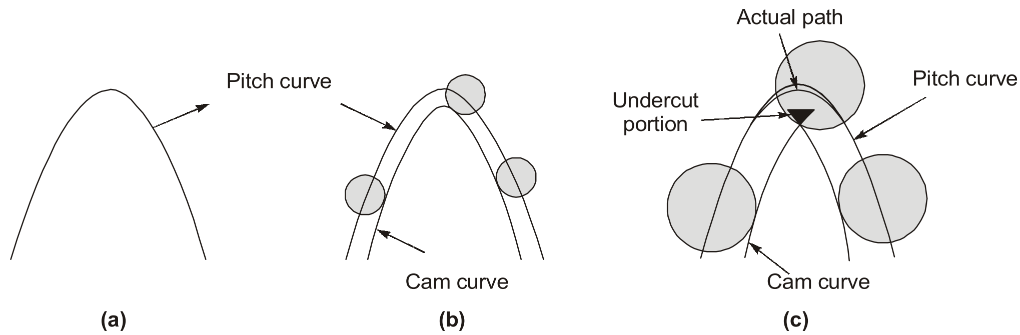


Fig. 4.14 Undercutting in Cams

Fig. 4.14 (a) shows the pitch curve of a cam. In Fig. 4.14 (b), a roller follower is shown generating this curve. In Fig. 4.14 (c), a larger roller is shown trying to generate this curve. It can easily be observed that the cam curve loops over itself in order to realize the profile of the pitch curve. As it is impossible to produce such a cam profile, the result is that the cam will be undercut and become a pointed cam. Now when the roller follower will be made to move over this cam, it will not be producing the desired motion.

4.7 Motions of The Follower

The follower can have following type of motions :

1. Simple harmonic motion (SHM)
2. Uniform acceleration and deceleration
3. Uniform velocity
4. Parabolic motion
5. Cycloidal motion

4.7.1 Simple Harmonic Motion (SHM)

Consider a particle at *A* rotating in a circle about point *O* with uniform angular speed as shown in Fig. 4.15 (a), and executing simple harmonic motion (SHM). The displacement curve shown in Fig. 4.15 (b) can be constructed as follows :

1. Draw a semicircle with follower lift as the vertical diameter.
2. Divide this semicircle into an equal parts. (30° each into 6 parts)

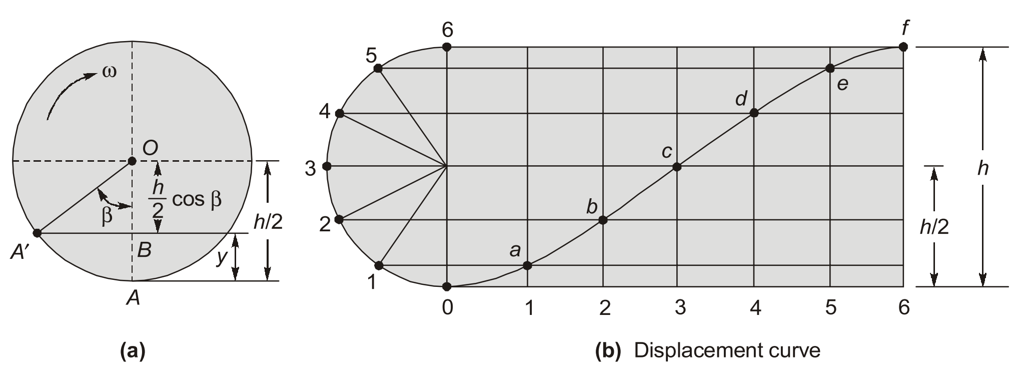


Fig. 4.15 Simple harmonic motion of follower

3. Draw cam rotation angle along x-axis. Mark the angles of ascent, dwell, descent and dwell on this line.
4. Divide the angles of ascent and descent into same equal number of parts.
5. Draw vertical lines at these points.
6. Draw horizontal lines from the points on the circumference of the semicircle to intersect the vertical lines.
7. Mark the points of intersection and join them by a smooth curve to obtain the displacement diagram.

Motion Analysis

Let,
y = displacement of the follower
h = horizontal lift/rise/displacement of the follower
θ = angle turned through by the crank OA from given datum
ϕ = cam rotation angle for the maximum follower displacement
β = angle on harmonic circle

Then from Fig. 4.15 (a), $y = OA - OB$

$$y = \frac{h}{2}\times(1-\cos\beta) \qquad \text{...(4.1)}$$

For the ascent or descent h of the follower displacement, the cam is rotated through an angle ϕ, whereas a point on the harmonic semicircle traverses an angle π radians. Thus the cam rotation is proportional to the angle turned through the point on the harmonic semicircle, i.e.,

$$\beta = \frac{\pi\theta}{\phi}$$

Now, Equation (4.1) becomes,

$$y = \left(\frac{h}{2}\right)\left[1-\cos\left(\frac{\pi\theta}{\phi}\right)\right] \qquad \text{...(4.2)}$$

Now, $\theta = \omega t$

$$y = \left(\frac{h}{2}\right)\left[1-\cos\left(\frac{\pi\omega t}{\phi}\right)\right] \qquad \text{...(4.3)}$$

Velocity, $V = \frac{dy}{dt}$ and differentiating Equation (4.3) w.r.t. time

$$\therefore \quad V = \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\phi}\right)\sin\left(\frac{\pi\omega t}{\phi}\right)$$

$$V = \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\phi}\right)\sin\left(\frac{\pi\theta}{\phi}\right) \qquad \text{...(4.4)}$$

Let,
θ_1 = angle of ascent
θ_2 = angle of dwell
θ_3 = angle of descent

Then, velocity during ascent,

$$V_a = \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\theta_1}\right)\sin\left(\frac{\pi\theta}{\theta_1}\right) \qquad \text{...(4.5(a))}$$

Velocity during ascent, $V_b = \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\theta_3}\right)\sin\left(\frac{\pi\theta}{\theta_3}\right)$ ...(4.5(b))

Velocity is maximum when $\theta = \frac{\phi}{2}$

$$V_{max} = \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\phi}\right) \quad ...(4.6)$$

Maximum velocity of follower during ascent $= \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\theta_1}\right)$ ...(4.7(a))

Maximum velocity of follower during descent $= \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\theta_3}\right)$ ...(4.7(b))

Acceleration, $a = \frac{dv}{dt}$

Now, differentiating (4.4), we get

$$a = \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\phi}\right)^2\cos\left(\frac{\pi\theta}{\phi}\right) \quad ...(4.8)$$

Acceleration during ascent,

$$a_a = \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\theta_1}\right)^2\cos\left(\frac{\pi\theta}{\theta_1}\right) \quad ...(4.9\ (a))$$

Acceleration during descent,

$$a_d = \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\theta_3}\right)^2\cos\left(\frac{\pi\theta}{\theta_3}\right) \quad ...(4.9\ (b))$$

The acceleration is maximum when $\theta = 0°$,

$$a_{max} = \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\phi}\right)^2 \quad ...(4.10)$$

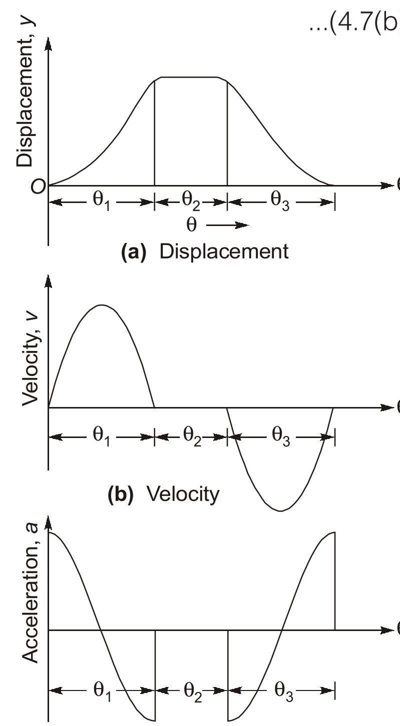


Figure 4.16 Displacement, Velocity and Acceleration Curve for SHM of Follower

Maximum acceleration of follower during ascent $= \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\theta_1}\right)^2$ ...(4.11 (a))

Maximum acceleration of follower during descent $= \left(\frac{h}{2}\right)\left(\frac{\pi\omega}{\theta_3}\right)^2$ ...(4.11 (b))

The motion of the follower is shown in Fig. 4.16 and it can be observed that there is an abrupt change of acceleration from zero to maximum at the beginning of the follower motion and also from maximum (negative) to zero at the end of the follower motion. The same pattern is repeated during descent. This leads to jerk, vibration and noise etc. Therefore SHM should be adopted only for low to moderate cam speeds.

Example 4.4 It is required to set out the profile of a cam to give motion to the follower in such a way that it rises through 31.4 mm during 180° of cam rotation with cycloidal motion and returns with cycloidal motion during 180° of cam rotation. Determine the maximum velocity and acceleration of the follower during the outstroke when the cam rotates at 1800 rpm clockwise.

Solution :

Given,
$$h = 31.4 \text{ mm} = 0.0314 \text{ m}$$
$$\beta = 180° = \pi \text{ radian}$$
$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1800}{60} = 188.495 \text{ rad/s}$$

Radius of circle generating the cycloid, $R = \dfrac{h}{2\pi} = \dfrac{31.4}{2 \times 3.14} = 5 \text{ mm}$

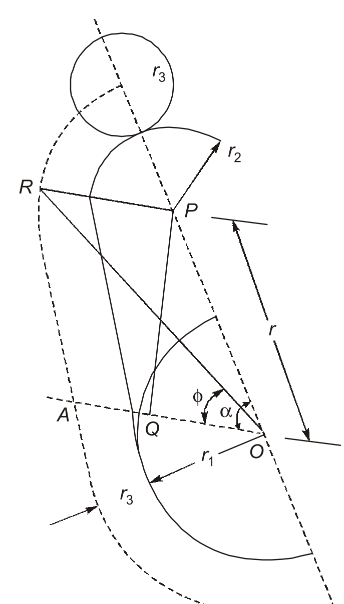
$$V_{max} = \frac{2\omega h}{\beta} = \frac{2 \times 188.495 \times 31.4 \times 10^{-3}}{3.14} = 3.77 \text{ m/s}$$
$$a_{max} = \frac{2\pi\omega^2 h}{\beta^2} = \frac{2 \times 3.14 \times 188.495^2 \times 31.4 \times 10^{-3}}{(3.14)^2} = 710.247 \text{ m/s}^2$$

Example 4.5 A symmetrical tangent cam operating a roller follower has the following particular :

Radius of base circle of cam = 40 mm
Roller radius = 20 mm
Angle of ascent = 75°
Total lift = 20 mm
Speed of cam shaft = 300 rpm. Find:
(i) the principal dimensions of the cam
(ii) the equation of the displacement curve when the follower is in contact with the straight flank.
(iii) the acceleration of the follower, when it is in contact with the straight flank where it merges into the circular nose.

Solution :

Refer figure



Given,
$$r_1 = 40 \text{ mm}$$
$$r_3 = 20 \text{ mm}$$
$$h = 20 \text{ mm}$$
$$N = 300 \text{ rpm}$$
$$\alpha = 75°$$
$$OP + r_2 = r_1 + h = 40 + 20$$
$$OP = 60 - r_2$$
$$OQ + r_2 = r_1 = 40$$
$$OQ = 40 - r_2$$
$$\cos\alpha = \frac{OQ}{OP} = \frac{40 - r_2}{60 - r_2}$$
$$0.25882 = \frac{40 - r_2}{60 - r_2}$$

or $$15.529 - 0.2588\, r_2 = 40 - r_2$$

or Nose radius, $$r_2 = \frac{40 - 15.529}{1 - 0.2588} = \frac{24.471}{0.7412}$$

$$= 33 \text{ mm}$$

Distance between cam and nose centre, $r = OP = 60 - 33 = 27$ mm

$$\tan\phi = \frac{RA}{OA} = \frac{PQ}{OA} = \frac{OP\sin\alpha}{OA} = \frac{27\sin 75^\circ}{60} = 0.43467$$

∴ $$\phi = 23.49^\circ$$

Equation of displacement curve :

1. When roller is in contact with straight flank

$$x = (r_1 + r_3)\left(\frac{1}{\cos\theta} - 1\right)$$

$$x = (40 + 20)\left(\frac{1}{\cos\theta} - 1\right) = 60\left(\frac{1}{\cos\theta} - 1\right) \text{ mm}$$

2. When roller is in contact with circular nose

$$x = r(1 - \cos\theta_1) + (r_2 + r_3) - (l^2 - r^2\sin^2\theta_1)^{0.5}$$

$$= 27(1 - \cos\theta_1) + (33 + 20) - (53^2 - 27^2\sin^2\theta_1)^{0.5}$$

$$= 27(1 - \cos\theta_1) + 53 - (2809 - 729\sin^2\theta_1)^{0.5}$$

where θ_1 is measured from apex position

Acceleration of follower :

When roller is in contact with straight flank,

$$a = \omega^2(r_1 + r_3)\left(\frac{2 - \cos^2\theta}{\cos^3\theta}\right)$$

where, $$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 300}{60} = 31.4 \text{ rad/s}$$

At, $$\theta = 0^\circ$$

$$a = (31.4)^2(0.04 + 0.02)\left[\frac{2-1}{1}\right] = 59.16 \text{ m/s}^2$$

When roller is in contact with straight flank, $\theta = \phi$

$$a = \omega^2(r_1 + r_3)\left(\frac{2 - \cos^2\phi}{\cos^3\phi}\right)$$

$$= (31.4)^2(0.04 + 0.02)\left[\frac{2 - \cos^2 23.49^\circ}{\cos^3 23.49^\circ}\right] = 88.86 \text{ m/s}^2$$

Example 4.6 The follower of a tangent cam is operated through a roller of a 50 mm diameter and its line of stroke passes through the axis of the cam. The minimum radius of the cam is 40 mm and the nose radius 15 mm. The lift is 25 mm. If the speed of the camshaft is 480 rpm, calculate the velocity and acceleration of the follower at the instant when the cam is

(a) in full lift position (b) 20° from full lift position

Solution :

Refer Fig. 4.24

Given: $r_3 = 25$ mm, $r_1 = 40$ mm, $r_2 = 15$ mm, $h = 25$ mm, $N = 480$ rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\times 3.14\times 480}{60} = 50.27 \text{ rad/s}$$

(a)
$$\cos\alpha = \frac{r_1 - r_2}{r_1 + h - r_2} = \frac{40-15}{40+25-15} = 0.5$$

So,
$$\alpha = 60°$$

At full lift position, $\theta = \alpha$

$$v = \omega(r_1 + r_3)\frac{\sin\alpha}{\cos^2\alpha}$$

$$= 50.24(40+25)\times 10^{-3}\frac{\sin 60°}{\cos^2 60°} = 11.312 \text{ m/s}$$

$$a = \omega^2(r_1+r_3)\left[\frac{2-\cos^2\alpha}{\cos^3\alpha}\right]$$

$$a = (50.24)^2(40+25)\left[\frac{2-\cos^2 60°}{\cos^3 60°}\right]\times 10^{-3}$$

$$a = 2296.89 \text{ m/s}^2$$

(b)
$$\theta = 60° - 20° = 40°$$

$$v = \omega(r_1+r_3)\frac{\sin\theta}{\cos^2\theta} = 3.577 \text{ m/s}$$

$$= 50.241(40 + 25) \times 10^{-3} \times \frac{\sin 40}{\cos^2 40}$$

$$a = (50.24)^2(40+25)\left[\frac{2-\cos^2 40°}{\cos^3 40°}\right]\times 10^{-3}$$

$$a = 515.759 \text{ m/s}^2$$

Example 4.7 The following data refers to a circular arc cam working with a flat faced reciprocating follower:

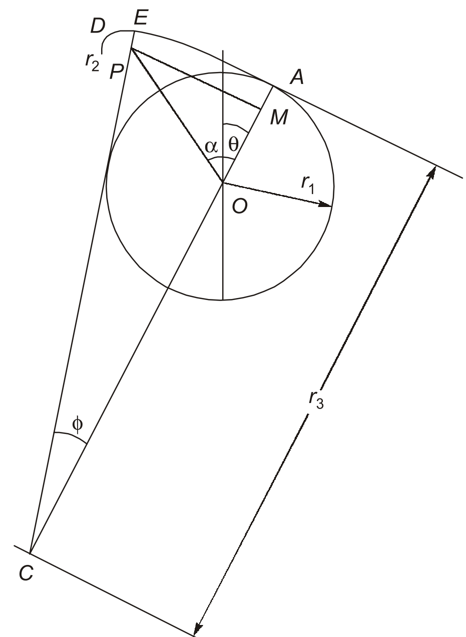
Minimum radius of cam = 30 mm, total angle of cam action = 120°, radius of circular arc = 80 mm and nose radius = 10 mm.

Find (a) the distance of the centre of the nose circle from the cam axis (b) the angle through which the cam turns when the point of contact moves from the junction of minimum radius arc and circular arc to the junction of nose arc and circular arc, and (c) velocity and acceleration of the follower when the cam has turned through an angle of 20°. The angular velocity of the cam is 20 rad/s.

Solution :

Refer Figure

Given,
$$r_1 = 30 \text{ mm}$$
$$r_2 = 10 \text{ mm}$$
$$r_3 = 80 \text{ mm}$$



$$2\alpha = 120°$$
$$\omega = 20 \text{ rad/s}$$
$$\text{lift} = 10 \text{ mm}$$

(a)
$$OP + r_2 = r_1 + \text{lift}$$
$$OP = r_1 + \text{lift} - r_2$$
$$= (30 - 10) + \text{lift} = 20 + \text{lift}$$
$$x = (r_3 - r_1)(1 - \cos\theta)$$
$$OP = 20 + 25 = 45 \text{ mm}$$

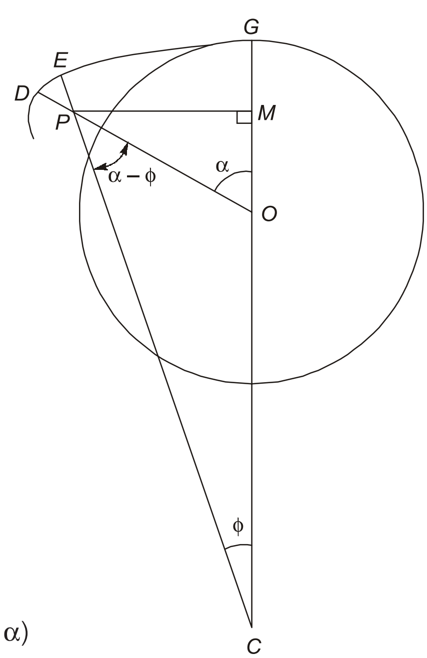
At $\theta = \alpha = 60°$, lift,
$$x = (80 - 30)(1 - \cos\theta) = 25 \text{ mm}$$

(b)
$$\tan\phi = \frac{PM}{CM} = \frac{OP\sin\alpha}{OC + OM} = \frac{OP\sin\alpha}{OC + OP\cos\alpha}$$
$$\tan\phi = \frac{45\sin 60°}{50 + 45\cos 60°} = 0.5375$$
$$\therefore \quad \phi = 28.26° \qquad (\because OC = AC - OA)$$
$$r_3 - r_1 = 80 - 30 = 50 \text{ mm}$$

(c) Velocity,
$$v = \omega(r_3 - r_1)\sin\theta = 20\,(80 - 30) \times 10^{-3} \sin 20°$$
$$= 0.342 \text{ m/s}$$

Acceleration, $a = \omega^2(r_3 - r_1)\cos\theta = (20)^2\,(80 - 30) \times 10^{-3} \cos 20° = 18.794 \text{ m/s}^2$

Example 4.8 The suction valve of a four stroke petrol engine is operated by a circular arc cam with a flat-faced follower. The lift of the follower is 10 mm, base circle diameter of the cam is 40 mm and the nose radius is 2.5 mm. The crank angle when suction valve opens is 4° after top dead centre and when the suction valve closes, the crank angle is 50° after bottom dead centre. If the cam shaft rotates at 300 rpm determine (a) maximum velocity of the valve, and (b) maximum acceleration and retardation of the valve.



Solution :

Given,
$$h = 10 \text{ mm}$$
$$r_1 = 20 \text{ mm}$$
$$r_2 = 2.5 \text{ mm}$$
$$N = 300 \text{ rpm}$$
$$2\alpha = \frac{(180° - 4° + 50°)}{2} = \frac{226}{2} = 113°$$
$$\Rightarrow \quad \alpha = 56.5°$$

In given figure,
$$r_3 = CP + r_2 = OC + r_1$$
$$CP = OC + r_1 - r_2$$
$$CP = OC + 20 - 2.5 = OC + 17.5$$
$$OP + r_2 = r_1 + h$$
$$OP = r = 20 + 10 - 2.5 = 27.5 \text{ mm}$$

In ΔCOP, we have
$$CP^2 = OC^2 + OP^2 - 2\,.\,OC\,OP\cos(180° - \alpha)$$

$$\text{or} \quad (OC + 17.5)^2 = OC^2 + (27.5)^2 - 2\,.\,OC \times 27.5 \cos(123.5°)$$

$$\Rightarrow \quad OC = 96.91 \text{ mm}$$

$$CP = 114.41 \text{ mm}$$

$$r_3 = OC + r_1 = 96.91 + 20 = 116.91 \text{ mm}$$

$$\frac{OP}{\sin\phi} = \frac{CP}{\sin(180° - \alpha)}$$

$$\sin\phi = \left(\frac{27.5}{114.4}\right)\sin 123.5° = 0.20043$$

$$\phi = 11.56°$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.41 \text{ rad/s}$$

Maximum velocity, $v_{max} = \omega(r_3 - r_1)\sin\phi$

$$= 31.41\,(116.91 - 20)\sin 11.56° = 609.987 \text{ mm/s} = 0.61 \text{ m/s}$$

$$a_{max} = -\omega^2(r_3 - r_1) = -(31.41)^2\,(96.91) = 95610.252 \text{ mm/s}^2 = -95.61 \text{ m/s}^2$$

NOTE : In this chapter, emphasis has been given only on basics of cam design and analytical method of motion analysis from the competitive examination point of view.

Summary

1. A cam is a mechanical member used to impart desired motion to a follower by direct contact.
2. The essential components of a cam mechanism are : cam, follower and frame.
3. Complicated output motions which are otherwise difficult to achieve can easily be produced with the help of cams.
4. Cams are classified according to shape, follower movement and the manner of constraint of the follower.
5. Cam followers are classified according to shape, movement, and the location of line of movement.
6. Base circle is the smallest circle tangent to the cam profile (contour) drawn from the centre of rotation of a radial cam.
7. Pressure angle of a cam is the angle between the direction of follower motion and a normal to the pitch curve.
8. Lift is the maximum travel of the follower from the lowest position to the topmost position.
9. The cam size is defined by the following parameters:
   - Pressure angle
   - Radius of curvature of cam profile
   - Hub size
10. Cycloidal motion is the most ideal for high speed follower motion.
11. Pitch curve is the curve drawn by the trace point assuming the cam to be fixed and rotating the trace point of the follower around the cam.
12. A tangent cam is symmetrical about the centre line. It has straight flanks with circular nose.
13. A circular arc cam is made up of three areas of different radii. In such cams, the acceleration may change abruptly at the blending points due to instantaneous change in the radius of curvature.

Objective Brain Teasers

1. The pitch point on a cam is
   (a) any point on the cam profile
   (b) any point on base circle
   (c) the point on pitch curve with maximum pressure angle
   (d) a point at a distance equal to pitch circle radius from the centre

2. The jerk in cam motion is
   (a) rate of change of displacement
   (b) rate of change of velocity
   (c) rate of change of acceleration
   (d) rate of change of pressure angle

3. Follower motion best suitable for high speed application
   (a) SHM (b) cycloidal
   (c) parabolic (d) depends on the data

4. Which of the following is constant acceleration cam?
   (a) polynomial (b) circular arc
   (c) cycloidal (d) parabolic

5. Throw of a cam is the maximum distance of the follower from
   (a) inner circle (b) prime circle
   (c) root circle (d) base circle

6. The locus of the trace point if the follower is moved around the cam is known as
   (a) prime circle (b) pitch curve
   (c) base circle (d) cam circle

7. The size of the cam does not depend on
   (a) pitch circle
   (b) base circle
   (c) pressure angle
   (d) radius of curvature of cam profile

8. In its simplest or equivalent form, a cam mechanism consists of following number of links
   (a) 4 (b) 3
   (c) 2 (d) 1

9. The equation of jerk of simple harmonic follower motion for a rise motion is
   (a) $\frac{h}{2}\left[1-\cos\left(\pi\frac{\theta}{\beta}\right)\right]$ (b) $\frac{\pi^2}{\beta^2}\frac{h}{2}\cos\left(\pi\frac{\theta}{\beta}\right)$
   (c) $-\frac{\pi^3}{\beta^3}\frac{h}{2}\sin\left(\pi\frac{\theta}{\beta}\right)$ (d) $\frac{\pi}{\beta}\frac{h}{2}\sin\left(\pi\frac{\theta}{\beta}\right)$

   where h is the total size, or lift, θ is the camshaft angle, and β is the total angle of the rise interval.

10. Which of the following cam follower motion gives good compromise for acceleration and jerk?
    (a) Harmonic displacement
    (b) 3-4-5 polynomial displacement
    (c) 4-5-6-7 polynomical displacement
    (d) Cycloidal displacement

11. Select the correct statements pertaining to the fundamental law of cam design :
    P. The cam profile functions must have second-order continuity.
    Q. In order to obey the fundamental law of cam design, one must start with at least a 5-degree polynomial as the displacement function for a double-dwell cam.
    R. The jerk function must be finite across the entire cam rotation.
    (a) P and Q (b) Q and R
    (c) R only (d) P, Q and R

12. The angle of any point on the pitch curve of the cam included between the normal to that point on the curve and line of motion of the follower at that instant is known as
    (a) pitch angle (b) profile angle
    (c) cam angle (d) pressure angle

13. The maximum value of the pressure angle in case of cam is usually kept as
    (a) 20° (b) 30°
    (c) 25° (d) 14°

9. (c)

The equations of SHM for a rise motion are :

Displacement, $s = \frac{h}{2}\left[1 - \cos\left(\pi\frac{\theta}{\beta}\right)\right]$

Velocity, $v = \frac{\pi}{\beta}\frac{h}{2}\sin\left(\pi\frac{\theta}{\beta}\right)$

Acceleration, $a = \frac{\pi^2}{\beta^2}\frac{h}{2}\cos\left(\pi\frac{\theta}{\beta}\right)$

Jerk, $j = \frac{da}{dt} = -\frac{\pi^3}{\beta^3}\frac{h}{2}\sin\left(\pi\frac{\theta}{\beta}\right)$

10. (b)

SHM $\rightarrow \infty$ jerk

3-4-5 poly displacement $\rightarrow$ good compromise for jerk and acceleration

Cycloidal displacement $\rightarrow$ smooth acceleration and jerk

4-5-6-7 poly displacement $\rightarrow$ smooth jerk - high acceleration

11. (b)

Cam motion functions must have third-order continuity i.e. the function plus two derivatives at all boundaries, so that jerk should be finite.

17. (b)

The pressure angle can be reduced by increasing the angle of rotation of the cam, thereby lengthening the pitch curve for the specified follower displacement. The cam profile becomes flatter and the pressure angle becomes smaller.

18. (a)

The equation for cycloidal motion is

$$y = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right)$$

L = 1 cm, $\beta = 180° = \pi$, $\theta = \pi/3$

$$y = 10\left[\frac{\pi}{3\pi} - \frac{1}{2\pi}\sin\left(\frac{2\pi}{3}\right)\right]$$

$$= 10\left(\frac{1}{3} - \frac{1}{2\pi}\times 0.866\right)$$

$$= 1.95 \text{ mm}$$

19. (d)

The equation for SHM is

$$y = \frac{L}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right)$$

L = 20 mm, $\beta = 180° = \pi$, $\theta = \pi/4$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60}$$

$$= 10.472 \text{ rad/s}$$

$$y = \frac{20}{2}(1 - \cos\theta) = 10(1 - \cos\theta)$$

$$\dot{y} = 10\,\omega \sin\theta$$

$$\ddot{y} = 10\,\omega^2 \cos\theta$$

$$\ddot{y}\big|_{\theta=\frac{\pi}{4}} = 10 \times (10.472)^2 \cos 45°$$

$$= 775.433 \text{ mm/s}^2$$

Student's Assignments

1. Draw the profile of a cam operating a knife-edge follower when the axis of the follower passes through the axis of the cam shaft. The following data is given; Lift = 40 mm, angle of ascent = 60°, dwell = 45° angle of descent = 90° and dwell for the remaining period of cam rotation. The motion of the cam is simple harmonic during both ascent and descent. The least radius of cam is 50 mm. If the cam rotates at 300 rpm, determine the maximum velocity and acceleration of the follower during ascent and descent.

Ans. 1.88 m/s, 1.26 m/s; 177.47 m/s^2, 78.87 m/s^2

2. A cam with 30 mm as minimum diameter is rotating clockwise at a uniform speed of 1200 rpm and operates a roller follower of 10 mm diameter as given below:

(i) Outward stroke of 30 mm during 120° of cam rotation with equal uniform acceleration and retardation.

(ii) Follower is to dwell for 50° of cam rotation.

(iii) Inward stroke during 90° of cam rotation with equal uniform acceleration and retardation.

(iv) Follower is to dwell for the remaining period of cam rotation.

Draw the cam profile if the axis of follower passes through the axis of the cam. Determine the maximum velocity and acceleration during outward and inward strokes.

Ans. 3.6 m/s, 4,8 m/s; 432 m/s^2, 768 m/s^2

3. A cam with 30 mm as minimum diameter is rotating clockwise at a uniform speed of 1200 rpm and has to give the following motion to a roller follower 10 mm in diameter:

(a) Follower to complete outward stroke of 25 mm during 120° of cam rotation with equal uniform acceleration and retardation.

(b) Follower to dwell for 60° of cam rotation.

(c) Follower to return to its initial position during 90° of cam rotation with equal uniform acceleration and retardation

(d) Follower to dwell for the remaining 90° of cam rotation.

Draw the cam profile if the axis of the roller follower passes through the axis of the cam. Determine the maximum velocity of the follower during the outstroke and return stroke and also the uniform acceleration of the follower on the out stroke and the return stoke.

Ans. 3 m/s, 4 m/s, 360.2 m/s^2, 640.34 m/s^2

4. A cam profile consists of two circular arcs of radii 24 mm and 12 mm joined by straight lines, giving the follower a lift of 12 mm. The follower is a roller of 24 mm radius and its line of action is a straight line passing through the cam shaft axis. When the cam shaft has a uniform speed of 500 rev/min, find the maximum velocity and acceleration of the follower while in contact with the straight flank of the cam.

Ans. 1.2 m/s; 198 m/s^2

5. The suction valve of a four stroke petrol engine is operated by a circular arc cam with a flat faced follower. The lift of the follower is 10 mm; base circle diameter of the cam is 40 mm and the nose radius is 2.5 mm. The crank angle when suction valve opens is 4° after top dead centre and when the suction valve closes, the crank angle is 50° after bottom dead centre. If the cam shaft rotates at 600 r.p.m., determine:

1. maximum velocity of the valve, and
2. maximum acceleration and retardation of the valve.

Ans. 1.22 m/s ; 383 m/s^2, 108.6 m/s^2

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