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**Engineering Mathematics**

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## Laplace Transform

Q.1 Evaluate :  $L \left\{ \int_0^t \int_0^t \int_0^t (t \sin t) dt dt dt \right\}$

- (a)  $\frac{2}{s(s^2+1)^2}$  (b)  $\frac{2}{s^2(s^2+1)}$   
 (c)  $\frac{2}{s^2(s^2+1)^2}$  (d)  $\frac{2}{(s^2+1)^2}$

Q.2 Evaluate :  $L \left\{ \int_0^t \frac{e^{-t} \sin t}{t} dt \right\}$

- (a)  $\cot^{-1}(s-1)$  (b)  $\frac{1}{s} \cot^{-1}(s-1)$   
 (c)  $\frac{1}{s} \cot(s-1)$  (d)  $\cot(s-1)$

Q.3 Find the inverse transform of  $\frac{s+2}{s^2-4s+13}$ .

- (a)  $e^{2t} \cos 3t$  (b)  $\frac{4}{3} \sin 3t$   
 (c)  $\frac{4}{3} e^{2t} \sin 3t$  (d)  $e^{2t} \cos 3t + \frac{4}{3} e^{2t} \sin 3t$

Q.4 Laplace transform of  $\sin^3 2t$  is

- (a)  $\frac{24}{(s^2+4)(s^2+36)}$  (b)  $\frac{1}{(s^2+4)(s^2+64)}$   
 (c)  $\frac{48}{(s^2+4)(s^2+36)}$  (d)  $\frac{64}{(s^2+4)(s^2+36)}$

Q.5 Laplace transform of  $t^2 + 2t + 3$  is

- (a)  $\frac{-2}{s^3} - \frac{2}{s^2} - \frac{3}{s}$  (b)  $\frac{2}{s^3} + \frac{2}{s^2} - \frac{3}{s}$   
 (c)  $\frac{2}{s^3} + \frac{2}{s^2} + \frac{3}{s}$  (d)  $\frac{-2}{s^3} + \frac{2}{s^2} - \frac{3}{s}$

Q.6 The Laplace transform of the function  $e^{-4t} \frac{\sin 3t}{t}$ , for  $t > 0$ , is

- (a)  $\tan^{-1}\left(\frac{3}{s+4}\right)$  (b)  $\sec(3s+4)$   
 (c)  $\cot^{-1}\left(\frac{3}{s+4}\right)$  (d)  $\cos\left(\frac{s}{s+4}\right)$

Q.7 The Laplace transform of the function " $t^2 \cos(at)$ " for  $t > 0$  is

- (a)  $\frac{2s(3a^2 - s^2)}{(s^2 + a^2)^3}$  (b)  $\frac{2s(s^2 - 3a^2)}{(s^2 + a^2)^3}$   
 (c)  $\frac{2a(3s^2 - a^2)}{(s^2 + a^2)^3}$  (d)  $\frac{2a(a^2 - 3s^2)}{(s^2 + a^2)^3}$

Q.8 A differential equation is given by,  $y'' + 25y = a \cos 5t$  with initial conditions,  $y(0) = 2$  and  $y'(0) = 0$ . The Laplace transform of  $y(t)$  is given by,

$\frac{2s(s^2 + 30)}{(s^2 + 25)^2}$ . The value of " $a$ " is \_\_\_\_\_.



### Answers Laplace Transform

1. (c) 2. (b) 3. (d) 4. (c) 5. (c) 6. (a) 7. (b) 8. (10)

### Explanations Laplace Transform

1. (c)

Since  $L(\sin t) = \frac{1}{s^2+1}$

$\therefore L(t \sin t) = -\frac{d}{ds} \frac{1}{(s^2+1)} = \frac{2s}{(s^2+1)^2}$

Thus,  $L \left\{ \int_0^t \int_0^t \int_0^t (t \sin t) dt dt dt \right\} = \frac{1}{s^3} L(t \sin t)$

$= \frac{1}{s^3} \cdot \frac{2s}{(s^2+1)^2} = \frac{2}{s^2(s^2+1)^2}$

**2. (b)**

$$\begin{aligned} \text{Since } L\left(\frac{\sin t}{t}\right) &= \int_s^\infty \frac{ds}{s^2+1} \\ &= \tan^{-1} s = \frac{\pi}{2} - \tan^{-1} s \\ &= \cot^{-1} s \\ \therefore L\left\{e^t\left(\frac{\sin t}{t}\right)\right\} &= \cot^{-1}(s-1), \\ &\text{(by shifting property)} \end{aligned}$$

$$\begin{aligned} \text{Thus, } L\left[\int_0^t \left\{e^t\left(\frac{\sin t}{t}\right)\right\} dt\right] \\ = \frac{1}{s} \cot^{-1}(s-1) \end{aligned}$$

**3. (d)**

$$\begin{aligned} L^{-1}\left(\frac{s+2}{s^2-4s+13}\right) &= L^{-1}\left[\frac{s+2}{(s-2)^2+9}\right] \\ &= L^{-1}\left[\frac{s-2+4}{(s-2)^2+3^2}\right] \\ &= L^{-1}\left[\frac{s-2}{(s-2)^2+3^2}\right] + 4L^{-1}\left[\frac{1}{(s-2)^2+3^2}\right] \\ &= e^{2t} \cos 3t + \frac{4}{3} e^{2t} \sin 3t \end{aligned}$$

**4. (c)**

$$\begin{aligned} \sin^3 2t &= \frac{1}{4} (3 \sin 2t - \sin 6t) \\ (\because \sin 3\theta &= 3 \sin \theta - 4 \sin^3 \theta) \end{aligned}$$

Take Laplace transform

$$\begin{aligned} L\{\sin^3(2t)\} &= \frac{1}{4} \left( \frac{3 \times 2}{s^2+4} - \frac{6}{s^2+36} \right) \\ &= \frac{6}{4} \left( \frac{1}{s^2+4} - \frac{1}{s^2+36} \right) = \frac{6}{4} \left( \frac{s^2+36-s^2-4}{(s^2+4)(s^2+36)} \right) \\ &= \frac{6}{4} \times \frac{32}{(s^2+4)(s^2+36)} = \frac{48}{(s^2+4)(s^2+36)} \end{aligned}$$

**5. (c)**

$$\begin{aligned} \text{Laplace transform of } x^n &= \frac{n!}{s^{n+1}} \\ t^2 + 2t + 3 &= \frac{2!}{s^3} + \frac{2 \cdot 1!}{s^2} + \frac{3 \cdot 0!}{s} \\ &= \frac{2}{s^3} + \frac{2}{s^2} + \frac{3}{s} \end{aligned}$$

**6. (a)**

$$\begin{aligned} L(\sin 3t) &= \frac{3}{s^2+3^2} \\ L\left(\frac{\sin 3t}{t}\right) &= \int_s^\infty \frac{3}{s^2+9} ds = \tan^{-1}\left(\frac{s}{3}\right) \Big|_s^\infty \\ &= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{3}\right) = \cot^{-1}\left(\frac{s}{3}\right) \\ L\left[e^{-4t}\left(\frac{\sin 3t}{t}\right)\right] &= \cot^{-1}\left(\frac{s+4}{3}\right) = \tan^{-1}\left(\frac{3}{s+4}\right) \end{aligned}$$

**7. (b)**

$$\begin{aligned} L(\cos at) &= \frac{s}{s^2+a^2} \\ L(t^2 \cos at) &= (-1)^2 \frac{d^2}{ds^2} \left( \frac{s}{s^2+a^2} \right) \\ &= \frac{d}{ds} \left[ \frac{1}{s^2+a^2} - \frac{s \times 2s}{(s^2+a^2)^2} \right] \\ &= \frac{d}{ds} \left[ \frac{s^2+a^2-2s^2}{(s^2+a^2)^2} \right] \\ &= \frac{d}{ds} \left[ \frac{a^2-s^2}{(s^2+a^2)^2} \right] \\ &= \frac{-2s}{(s^2+a^2)^2} - 2 \frac{(a^2-s^2) \times 2s}{(s^2+a^2)^3} \\ &= \frac{-2s^3-2a^2s-4a^2s+4s^3}{(s^2+a^2)^3} \\ &= \frac{2s(s^2-3a^2)}{(s^2+a^2)^3} \end{aligned}$$

**8. (10)**

By applying Laplace transform to the given differential equation, we get,

$$\begin{aligned} [s^2 Y(s) - sy(0) - y'(0)] + 25 Y(s) \\ = \frac{as}{s^2+25} \\ (s^2+25)Y(s) = 2s + \frac{as}{s^2+25} \\ Y(s) = \frac{2s^3+50s+as}{(s^2+25)^2} \\ = \frac{2s^3+(50+a)s}{(s^2+25)^2} \end{aligned}$$

By comparing the above equation with the given  $Y(s)$ , we get,

$$50 + a = 60 \Rightarrow a = 10$$

