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PRODUCTION AND INDUSTRIAL ENGINEERING

Objective Practice Sets

Engineering Mathematics *Contents*

MADE EASY Publications

8 CHAPTER Laplace Transform

Q.1 Evaluate : $L\left\{\int_0^t\int_0^t\int_0^t (t\sin t)\,dt\,dt\,dt\right\}$

(a) $\frac{2}{s(s^2+1)^2}$ (b) $\frac{2}{s^2(s^2+1)}$

(c) $\frac{2}{s^2(s^2+1)^2}$ (d) $\frac{2}{(s^2+1)^2}$

Q.2 Evaluate : $L\left\{\int_0^t \frac{e^{-t}\sin t}{t}dt\right\}$

(a) $\cot^{-1}(s-1)$ (b) $\frac{1}{s}\cot^{-1}(s-1)$

(c) $\frac{1}{s}\cot(s-1)$ (d) $\cot(s-1)$

Q.3 Find the inverse transform of $\frac{s+2}{s^2-4s+13}$.

(a) $e^{2t}\cos 3t$ (b) $\frac{4}{3}\sin 3t$

(c) $\frac{4}{3}e^{2t}\sin 3t$ (d) $e^{2t}\cos 3t+\frac{4}{3}e^{2t}\sin 3t$

Q.4 Laplace transform of $\sin^3 2t$ is

(a) $\frac{24}{(s^2+4)(s^2+36)}$ (b) $\frac{1}{(s^2+4)(s^2+64)}$

(c) $\frac{48}{(s^2+4)(s^2+36)}$ (d) $\frac{64}{(s^2+4)(s^2+36)}$

Q.5 Laplace transform of $t^2 + 2t + 3$ is

(a) $\frac{-2}{s^3}-\frac{2}{s^2}-\frac{3}{s}$ (b) $\frac{2}{s^3}+\frac{2}{s^2}-\frac{3}{s}$

(c) $\frac{2}{s^3}+\frac{2}{s^2}+\frac{3}{s}$ (d) $\frac{-2}{s^3}+\frac{2}{s^2}-\frac{3}{s}$

Q.6 The Laplace transform of the function $e^{-4t}\frac{\sin 3t}{t}$, for $t > 0$, is

(a) $\tan^{-1}\left(\frac{3}{s+4}\right)$ (b) sec $(3s + 4)$

(c) $\cot^{-1}\left(\frac{3}{s+4}\right)$ (d) $\cos\left(\frac{s}{s}+4\right)$

Q.7 The Laplace transform of the function "$t^2\cos(at)$" for $t > 0$ is

(a) $\frac{2s(3a^2-s^2)}{(s^2+a^2)^3}$ (b) $\frac{2s(s^2-3a^2)}{(s^2+a^2)^3}$

(c) $\frac{2a(3s^2-a^2)}{(s^2+a^2)^3}$ (d) $\frac{2a(a^2-3s^2)}{(s^2+a^2)^3}$

Q.8 A differential equation is given by, $y'' + 25y = a\cos 5t$ with initial conditions, $y(0) = 2$ and $y'(0) = 0$. The Laplace transform of $y(t)$ is given by, $\frac{2s(s^2+30)}{(s^2+25)^2}$. The value of "$a$" is ________.

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Answers Laplace Transform

1. (c) 2. (b) 3. (d) 4. (c) 5. (c) 6. (a) 7. (b) 8. (10)

Explanations Laplace Transform

1. (c)

Since $L(\sin t) = \frac{1}{s^2+1}$

$\therefore$ $L(t\sin t) = -\frac{d}{ds}\frac{1}{(s^2+1)} = \frac{2s}{(s^2+1)^2}$

Thus, $L\left\{\int_0^t\int_0^t\int_0^t (t\sin t)\,dt\,dt\,dt\right\} = \frac{1}{s^3}L(t\sin t)$

$= \frac{1}{s^3}\cdot\frac{2s}{(s^2+1)^2} = \frac{2}{s^2(s^2+1)^2}$

2. (b)

Since $L\left(\frac{\sin t}{t}\right) = \int_s^\infty \frac{ds}{s^2+1}$

$= \tan^{-1} s = \frac{\pi}{2} - \tan^{-1} s$

$= \cot^{-1} s$

$\therefore L\left\{e^t\left(\frac{\sin t}{t}\right)\right\} = \cot^{-1}(s-1),$

(by shifting property)

Thus, $L\left[\int_0^t \left\{e^t\left(\frac{\sin t}{t}\right)\right\} dt\right]$

$= \frac{1}{s}\cot^{-1}(s-1)$

3. (d)

$$L^{-1}\left(\frac{s+2}{s^2-4s+13}\right) = L^{-1}\left[\frac{s+2}{(s-2)^2+9}\right]$$

$$= L^{-1}\left[\frac{s-2+4}{(s-2)^2+3^2}\right]$$

$$= L^{-1}\left[\frac{s-2}{(s-2)^2+3^2}\right] + 4L^{-1}\left[\frac{1}{(s-2)^2+3^2}\right]$$

$$= e^{2t}\cos 3t + \frac{4}{3}e^{2t}\sin 3t$$

4. (c)

$$\sin^3 2t = \frac{1}{4}(3\sin 2t - \sin 6t)$$

$(\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta)$

Take Laplace transform

$$L\{\sin^3(2t)\} = \frac{1}{4}\left(\frac{3\times 2}{s^2+4} - \frac{6}{s^2+36}\right)$$

$$= \frac{6}{4}\left(\frac{1}{s^2+4} - \frac{1}{s^2+36}\right) = \frac{6}{4}\left(\frac{s^2+36-s^2-4}{(s^2+4)(s^2+36)}\right)$$

$$= \frac{6}{4}\times\frac{32}{(s^2+4)(s^2+36)} = \frac{48}{(s^2+4)(s^2+36)}$$

5. (c)

Laplace transform of $x^n = \frac{n!}{s^{n+1}}$

$$t^2 + 2t + 3 = \frac{2!}{s^3} + \frac{2.1!}{s^2} + \frac{3.0!}{s}$$

$$= \frac{2}{s^3} + \frac{2}{s^2} + \frac{3}{s}$$

6. (a)

$$L(\sin 3t) = \frac{3}{s^2+3^2}$$

$$L\left(\frac{\sin 3t}{t}\right) = \int_s^\infty \frac{3}{s^2+9}ds = \tan^{-1}\left(\frac{s}{3}\right)\Big|_s^\infty$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{3}\right) = \cot^{-1}\left(\frac{s}{3}\right)$$

$$L\left[e^{-4t}\left(\frac{\sin 3t}{t}\right)\right] = \cot^{-1}\left(\frac{s+4}{3}\right) = \tan^{-1}\left(\frac{3}{s+4}\right)$$

7. (b)

$$L(\cos at) = \frac{s}{s^2+a^2}$$

$$L(t^2\cos at) = (-1)^2\frac{d^2}{ds^2}\left(\frac{s}{s^2+a^2}\right)$$

$$= \frac{d}{ds}\left[\frac{1}{s^2+a^2} - \frac{s\times 2s}{(s^2+a^2)^2}\right]$$

$$= \frac{d}{ds}\left[\frac{s^2+a^2-2s^2}{(s^2+a^2)^2}\right]$$

$$= \frac{d}{ds}\left[\frac{a^2-s^2}{(s^2+a^2)^2}\right]$$

$$= \frac{-2s}{(s^2+a^2)^2} - 2\frac{(a^2-s^2)\times 2s}{(s^2+a^2)^3}$$

$$= \frac{-2s^3-2a^2s-4a^2s+4s^3}{(s^2+a^2)^3}$$

$$= \frac{2s(s^2-3a^2)}{(s^2+a^2)^3}$$

8. (10)

By applying Laplace transform to the given differential equation, we get,

$[s^2Y(s) - sy(0) - y'(0)] + 25Y(s)$

$$= \frac{as}{s^2+25}$$

$$(s^2+25)Y(s) = 2s + \frac{as}{s^2+25}$$

$$Y(s) = \frac{2s^3+50s+as}{(s^2+25)^2}$$

$$= \frac{2s^3+(50+a)s}{(s^2+25)^2}$$

By comparing the above equation with the given $Y(s)$, we get,

$50 + a = 60 \Rightarrow a = 10$

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