2017

MADE EASY WORKBOOK

Detailed Explanations of Try Yourself *Questions*

Mechanical Engineering

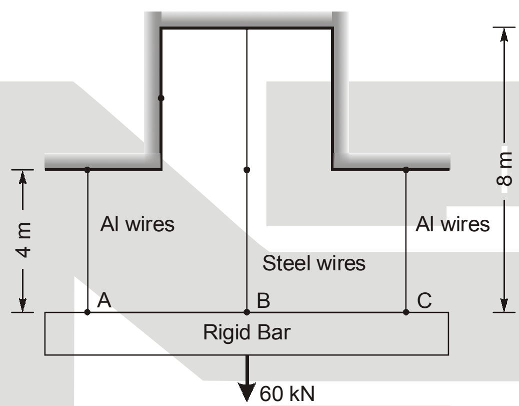
Strength of Materials

MADE EASY Publications

1 Simple Stress-Strain and Elastic Constants

Detailed Explanation of Try Yourself Questions

T1 : Solution



Let suffix 1 be used for Aluminium and 2 for steel

$A_1 = 300\ \text{mm}^2$ $\qquad E_1 = 0.667 \times 10^5\ \text{N/mm}^2$

$A_2 = 200\ \text{mm}^2$ $\qquad E_2 = 2 \times 10^5\ \text{N/mm}^2$

$l_1 = 4\ \text{m}$

$l_2 = 8\ \text{m}$

Now, $\quad \delta l_1 = \delta l_2$, change in depth, line ABC will remain straight

$$\frac{P_1 l_1}{A_1 E_1} = \frac{P_2 l_2}{A_2 E_2}$$

$$P_2 = P_1\left(\frac{l_1}{l_2}\right)\left(\frac{A_2 E_2}{A_1 E_1}\right)$$

$$P_2 = P_1\left(\frac{4}{8}\right)\left(\frac{200 \times 2 \times 10^5}{300 \times 0.667 \times 10^5}\right)$$

$$P_2 = P_1$$

Also $\quad 2P_1 + P_2 = 60$

Since $\quad P_1 = P_2$

$$2P_1 + P_1 = 60$$

$$3P_1 = 60$$

P_1 = 20 kN, in aluminium wires

P_2 = 20 kN, in steel wire

T2 : Solution

(a) Fall in temperature $(t) = 80 - 22 = 58°C$

Strain $(\epsilon) = \alpha t = 11 \times 10^{-6} \times 58 = 638 \times 10^{-6}$

Stress $(\sigma) = E\epsilon = 200 \times 10^9 \times 638 \times 10^{-6} = 127.6 \text{ MN / m}^2$

Pull exerted (P) $= \sigma A = 127.6 \times 10^6 \times \pi/4 \times 6.25 \times 10^{-4} = 62.635$ kN

(b) Length at 22° C $= l(1 - \alpha t) = 6(1 - 11 \times 10^{-6} \times 58) = 5.996172$ m

Decrease in length $= 6 - 5.996172 = 0.003828$ m

Due to yielding of walls by 1.5 mm, the actual decrease in length

$$= 0.003828 - 0.0015 = 0.002328 \text{ m} = 2328 \times 10^{-6} \text{ m}$$

$$\text{Strain } (\epsilon) = \frac{2328 \times 10^{-6}}{6} = 388 \times 10^{-6}$$

Pull exerted, $P = 200 \times 10^9 \times 388 \times 10^{-6} \times \pi/4 \times 6.25 \times 10^{-6}$

$= 38.092$ kN

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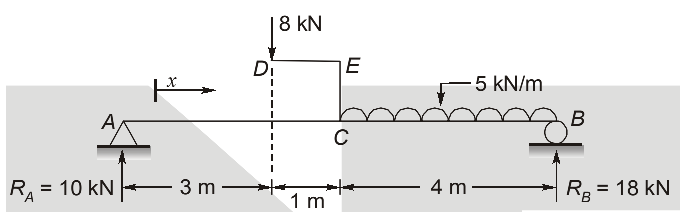
3 Shear Force and Bending Moment

Detailed Explanation of Try Yourself Questions

T1 : Solution



$$R_A \times 8 = 8 + 8 \times 4 + 5 \times 4 \times 2$$

$$R_A = 10 \text{ kN}$$

$$R_B = 18 \text{ kN}$$

SF diagram $\quad V > x > 0$

$$V = +10 \text{ kN}$$

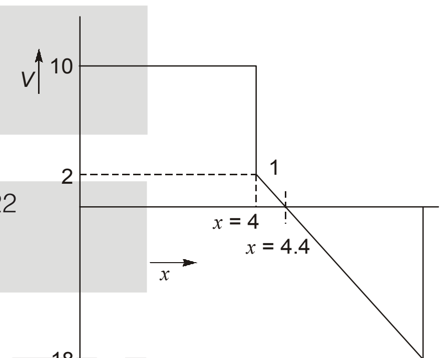
$$x > 4$$

SF $\quad V = (10 - 8) - 5(x - 4) = -5x + 22$

at $x = 8$ $\quad V = -18$

For $\quad V = 0 = -5x + 22$

$$x = 4.4$$



Bending moment diagram for $0 < x < 4$

BM $\quad M = R_A x = 10x$

$$4 \le x < 8$$

BM $\quad M = R_A x - 8(x-4) - 8 - 5\dfrac{(x-4)^2}{2} = 10x - 8x + 32 - 8 - 5\dfrac{(x-4)^2}{2}$

$$= 2x + 24 - \frac{5}{2}x^2 - 40 + 20x = 22x - 16 - \frac{5}{2}x^2$$

at $x = 4.4$ $\quad M = 32.4$ Nm

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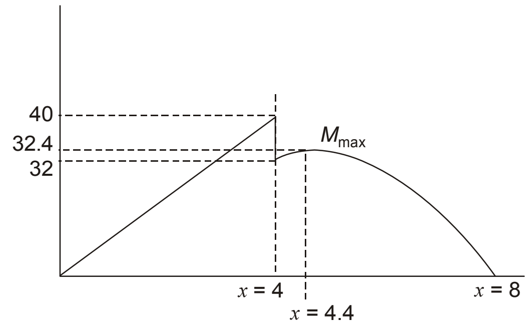
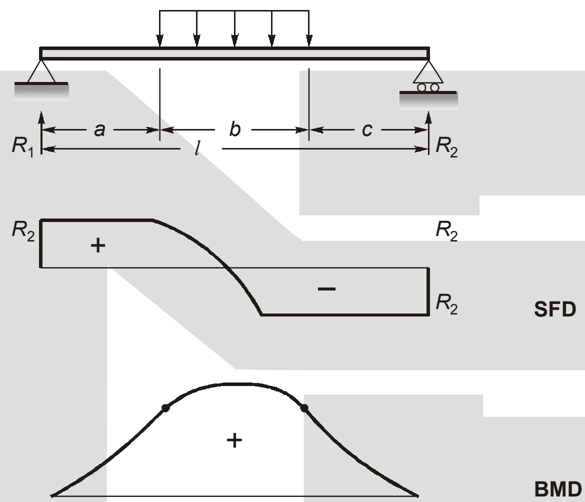
BMD



T2 : Solution



$$R_1 \times l = qb\left(c+\frac{b}{2}\right)$$

$$R_1 = \frac{qb}{l}(c+0.5b)$$

$$R_2 = qb-\frac{qb}{l}(c+0.5b) = \frac{qb}{l}[l-(c+0.5b)]$$

$$= \frac{qb}{l}\left[a+b+c-c-\frac{b}{2}\right] = \frac{qb}{l}\left(a+\frac{b}{2}\right)$$

SFD :

$AC : F_x = R_1$

$CD : F_x = R_1 - q\,(x-a)$

For $F_x = 0$

or $$x = \frac{1}{q}\left(\frac{qb}{l}(c+0.5b)qa\right) = \frac{b}{q}(c+0.5b)+a$$

$FD = R_1 - qb$

$DB : F_x = R_1 - qb$

BMD :

$$AC : M_x = R_1 x$$

$$CD : M_x = R_1 x - \frac{q(x-a)^2}{2}$$

$$DB : M_x = R_1 x - qb\,(x - a - 0.5b)$$

$$M_{max} = \frac{qb}{l}(c+0.5b)\left\{\frac{b}{l}(c+0.5b)+a\right\} - qb\left\{\frac{b}{l}(c+0.5b)+a-a-0.5b\right\}$$

$$= \frac{qb}{l}(c+0.5b)\left\{\frac{b}{l}(c+0.5b)+a\right\} - \frac{qb^2}{l}\{(c+0.5b)-0.5b\}$$

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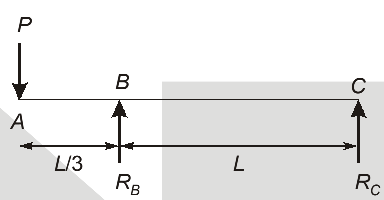
4 Deflection of Beams

Detailed Explanation of Try Yourself Questions

T1 : Solution



Calculating reactions at B and C

$$R_B + R_C = P \qquad \text{...(1)}$$

$$\underbrace{R_B \cdot L}_{c.w.} = \underbrace{\frac{P \times 4L}{3}}_{a.c.w.}$$

$$\therefore \quad R_B = \frac{4P}{3}$$

$$M_{AB} = -P \times x \;\; \forall\, x \in \left(0, \frac{L}{3}\right)$$

$$M_{BC} = -Px + \frac{4P}{3}\left(x - \frac{L}{3}\right) \quad \forall\, x \in \left(\frac{L}{3}, \frac{4L}{3}\right)$$

$$= -Px + \frac{4Px}{3} - \frac{4PL}{3}$$

$$M_{BC} = \frac{Px}{3} - \frac{4PL}{9}$$

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$$y_A = \frac{\partial U}{\partial P} = \int_0^{4L/3} \frac{M}{EI}\cdot\frac{\partial M}{\partial P}dx$$

$$y_A = \underbrace{\int_0^{L/3} \frac{M_{AB}}{EI}\cdot\frac{\partial M_{AB}dx}{\partial P}}_{I_1} + \underbrace{\int_{L/3}^{4L/3} \frac{M_{BC}}{EI}\cdot\frac{\partial M_{BC}dx}{\partial P}}_{I_2}$$

$$I_1 = \int_0^{L/3}\left[-\frac{Px}{EI}\times -x\cdot dx\right]$$

$$= \frac{P}{EI}\cdot\left[\frac{x^3}{3}\right]_0^{L/3} = \frac{P}{3EI}\times\frac{L^3}{27} = \frac{PL^3}{81EI}$$

$$I_2 = \int_{L/3}^{4L/3}\frac{1}{EI}\left[\frac{Px}{3}-\frac{4PL}{9}\right]\times\left[\frac{x}{3}-\frac{4L}{9}\right]dx$$

$$I_2 = \int_{L/3}^{4L/3}\frac{P}{EI}\left[\frac{x}{3}-\frac{4L}{9}\right]^2 dx$$

Let $\frac{x}{3}-\frac{4L}{9} = \alpha$

at $x = \frac{L}{3}$

$$\alpha = \frac{L}{9}-\frac{4L}{9} = \frac{-L}{3}$$

at $x = \frac{4L}{3}$

$$\alpha = \frac{4L}{9}-\frac{4L}{9} = 0$$

$$\frac{dx}{3} = d\alpha$$

∴ $dx = 3d\alpha$

$$I_2 = \int_{-L/3}^{0}\frac{P}{EI}\times\alpha^2\times 3d\alpha$$

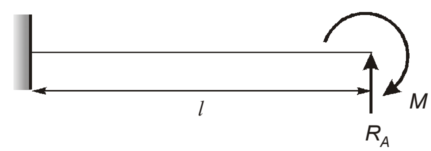
$$= \frac{3P}{EI}\int_{-L/3}^{0}\alpha^2 d\alpha = \frac{3P}{3EI}[\alpha^3]_{-L/3}^{0}$$

$$= \frac{P}{EI}\left[0-\left(-\frac{L}{3}\right)^3\right] = \frac{PL^3}{27EI}$$

$$y_A = \frac{PL^3}{81EI} + \frac{PL^3}{27EI} = \frac{PL^3 + 3PL^3}{81EI}$$

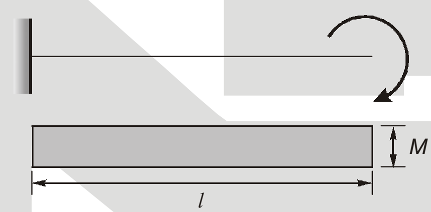
$$y_A = \frac{4PL^3}{81EI}$$

T2 : Solution



Calculating reaction R_A

Deflection due to moment at free end in downward direcion = Deflection due to load R_A at free end in upward direction



$$y_B - y_A = \frac{Ml \times l}{2EI} = \frac{Ml^2}{2EI}$$

Deflection due to point load at free end in upward direction

$$= \frac{R_A l^3}{3EI}$$

Hence $$\frac{R_A l^3}{3EI} = \frac{Ml^3}{2EI}$$

$\Rightarrow$ $$R_A = \frac{3M}{2l}$$

Moment at fixed end where ($x = l$)

$$= -M + (R_A \; x\, x) \;\; \forall \; x \in (0, l)$$

$$= -M + \frac{3M}{2l} x l = -M + \frac{3M}{2} = \frac{M}{2}$$ Hence proved

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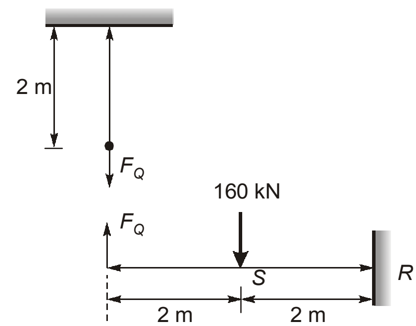
T3 : Solution



Moment of all forces about *R*

$$(F_Q \times 4) - (160 \times 2) = 0$$

$$\therefore \quad F_Q = \frac{160 \times 2}{4} = 80 \text{ kN}$$

Axial load is 80 kN on wore *PQ*.

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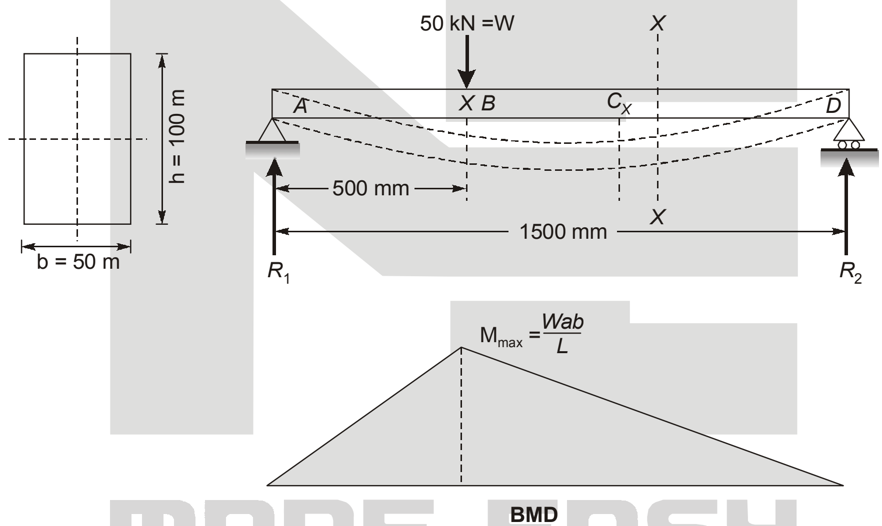
5 Bending and Shear Stresses in Beams

Detailed Explanation of Try Yourself Questions

T1 : Solution

Reactions, $R_1 = \frac{2}{3} \times 50 = \frac{100}{3}$ kN

$R_2 = \frac{50}{3}$ kN



Bending moment is maximum when shear force is zero or changes sing i..e at 500 mm from left support.

$$\therefore \quad M_{max} = R_1 \times 0.5 = \frac{100 \times 10^3}{3} \times 0.5 = 16666.67 \text{ N.m}$$

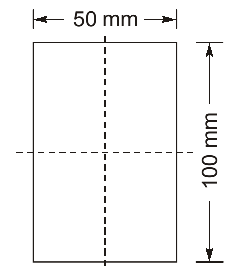
Moment of area $I = \frac{1}{12} bh^3 = \frac{1}{12} \times 50 \times (100)^3$

$I = 4.1667 \times 10^6 \text{ mm}^4$

$I = 4.1667 \times 10^{-6} \text{ m}^4$

$$\text{Maximum stress } (\sigma_{max}) = \frac{My}{I} = \frac{16666.67 \times 0.05}{4.1667 \times 10^{-6}}$$

$$\sigma_{max} = 200 \text{ N/mm}^2$$



Cross-Section

$$y_c = \frac{Wa^2b^2}{3EIL}$$

$$a = 0.5 \text{ m}, b = 1 \text{ m}, L = 1.5 \text{ m}$$

$$EI = \frac{2 \times 10^{11} \times 4.1667 \times 10^{-6}}{10^3} \text{ kNm}^2$$

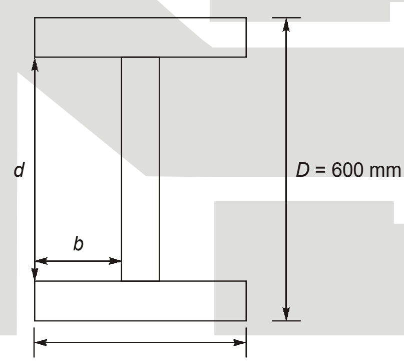
$$= 833.34 \text{ kNm}^2$$

$$y_c = \frac{50 \times 0.5^2 \times 1^2}{3 \times 833.34 \times 1.5}$$

$$= 3.33 \times 10^{-3} \text{ m}$$

$$= 3.33 \text{ mm}$$

T2 : Solution



$$I = \frac{BD^3}{12} - \frac{2 \times bd^3}{12}$$

$$= \frac{300 \times 600^3}{12} - \frac{2 \times \left\{\frac{300-16}{2}\right\} \times \{600-40\}^3}{12}$$

$$= 1243.755 \times 10^6 \text{ mm}^4$$

$$y = \frac{600}{2} = 300 \text{ mm}$$

$$\sigma = \frac{M}{I/y} = \frac{M}{41.46 \times 10^5}$$

$$\text{Area} = BD - 2bd$$

$$= (600 \times 300) - \left(2 \times \left(\frac{300 \times 16}{2}\right) \times (600-40)\right)$$

$$= 20960 \text{ mm}^2$$

Area of rectangular beam

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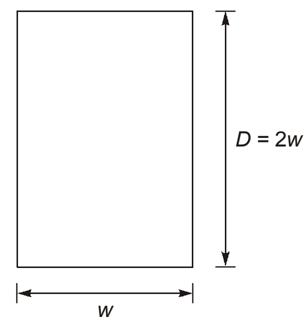

$\therefore$ $$\text{Area of } I_{\text{section}} = \text{Area of rectangular section}$$

$$20960 = 2w \times w$$

or $$w = \sqrt{\frac{20960}{2}} = 102.37 \text{ mm}$$

$$I = \frac{wD^3}{12} = \frac{102.37 \times (204.74)^3}{12}$$

$$= 732.189 \times 10^5 \text{ mm}^4$$

$$y = \frac{2w}{2} = w = 102.37 \text{ mm}$$

$$Z = \frac{I}{y} = 7.15 \times 10^5 \text{ mm}^3$$

$$\sigma_R = \frac{M}{Z} = \frac{M}{7.15 \times 10^5} \text{ mm}^3$$

$$\text{Area of } I_{\text{section}} = \text{Area of rectangular section}$$

$$= \text{Area of circular section}$$

$$20960 = \frac{\pi}{4} \times D^2$$

$\therefore$ $$D = \left\{\frac{20960 \times 4}{\pi}\right\}^{1/2} = 163.362 \text{ mm}$$

$$I_{\text{circular}} = \frac{\pi}{64} D^4 = \frac{\pi}{64} \times (163.362)^4$$

$$= 349.602 \times 105 \text{ mm}^4$$

$$Y = \frac{D}{2} = \frac{163.362}{2} = 81.681 \text{ mm}$$

$$Z = \frac{I}{Y} = \frac{349.602 \times 10^5}{81.681} = 4.3 \times 10^5$$

$$\sigma_c = \frac{M}{4.3 \times 10^5}$$

$$\sigma_I : \sigma_R : \sigma_C = ?$$

$$\frac{\sigma_I}{\sigma_R} = \frac{7.16 \times 10^5}{41.46 \times 10^5} = \frac{7.16}{41.46}$$

$$\frac{\sigma_R}{\sigma_C} = \frac{4.31 \times 10^5}{7.16 \times 10^5} = \frac{4.31}{7.16}$$

$$\frac{\sigma_I}{\sigma_R} = \frac{7.16}{41.46} \times \frac{4.31}{4.31} = \frac{30.86}{178.69}$$

$$\frac{\sigma_R}{\sigma_C} = \frac{4.31}{7.16} \times \frac{41.46}{41.46} = \frac{178.69}{296.85}$$

$$\sigma_I : \sigma_R : \sigma_C = 30.86 : 178.69 : 296.85$$

$$= 1 : 5.8 : 9.6$$

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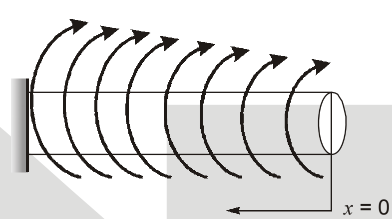
6 Torsion of Circular Shafts

Detailed Explanation of Try Yourself Questions

T1 : Solution



$$\text{Total torque} = \frac{1}{2} \times l \times t$$

Let the variation of torque be

$$t(x) = a + bx$$

Hence applying boundary condition are will obtain the values of *a* and *b*.

At $x = 0$, $t(x) = 0$, $0 = a + 0 \therefore a = 0$

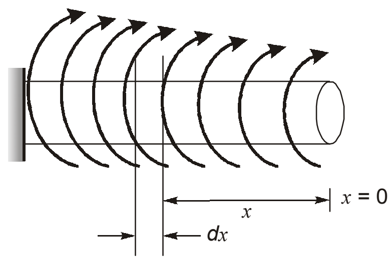
At $x = l$, $t(x) = t$

$\therefore$ $bl = t$

or $b = \frac{t}{l}$

$$t(x) = \frac{t}{l}x$$

Shear stress at section X-X at a distance x from free end of elemental length "dx".



$$T(x) = \frac{1}{2} \times x \times \frac{t}{l} \times x$$

$$= \frac{t}{2l} \times x^2$$

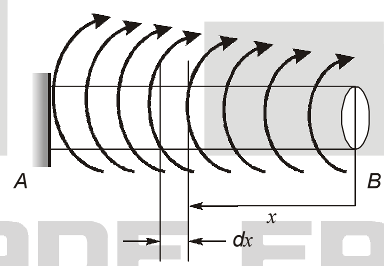
$$\text{Shear stress} = \frac{16T}{\pi d^3} = \frac{16}{\pi d^3} T(x)$$

$$= \frac{16T}{\pi d^3} \times \frac{tx^2}{2l} = \frac{16T}{\pi d^3} \times \frac{t}{2l} \times x^2$$

Shear stress will be maximum when x is maximum ($x_{max} = l$)

$$(\tau)_{max.} = \frac{16t \times l^2}{\pi d^3 \times 2l} = \frac{8tl}{\pi d^3}$$

Angle of twist at free end.



$$\theta_B - \theta_A = \int_0^l \frac{T(x)dx}{GJ}$$

$$= \frac{1}{GJ} \times \int_0^l \frac{1}{2} t(x) \times x \times dx$$

$$= \frac{1}{GJ} \times \frac{1}{2} \times \int_0^l \frac{tx^2}{l} dx$$

$$\theta_B - \theta_A = \frac{1}{2GJl} \times \left(\frac{x^3}{3}\right)_0^l = \frac{tl^2}{6GJ}$$

$$\theta_A = 0$$

$$\therefore \quad \theta_B = \frac{tl^2}{6GJ}$$

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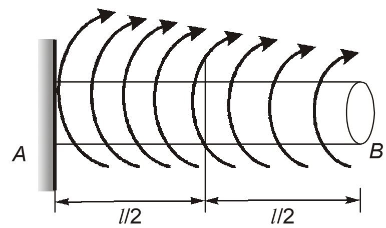


Angle of twist at $l/2$ from the free end.

$$\theta_B - \theta_{l/2} = \int_0^{l/2} \frac{T(x)dx}{GJ} = \int_0^{l/2} \left(\frac{1}{2} \times \frac{t \times x^2}{l}\right) \frac{dx}{GJ}$$

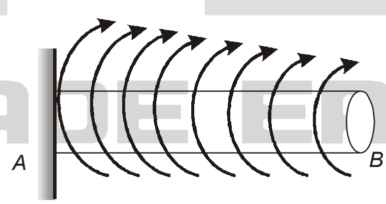
$$= \frac{t}{2lGJ} \cdot \left[\frac{x^3}{3}\right]_0^{l/2}$$

$$= \frac{t}{2lGJ} \times \frac{1}{3} \times \frac{l^3}{8} = \frac{tl^2}{48GJ}$$

$$\theta_{l/2} = Q_B - \frac{tl^2}{48GJ} = \frac{tl^2}{6GJ} - \frac{tl^2}{48GJ} = \frac{8tl^2 - tl^2}{48GJ}$$

$$\theta_{l/2} = \frac{7tl^2}{48GJ}$$

(IV) Strain energy of the shaft.



Total strain energy of the shaft $\int_0^l \frac{1}{2} \times T \times \theta$

$$= \int_0^l \frac{1}{2} \times T(x) \times \frac{T(x)dx}{GJ} = \int_0^l \frac{1}{2GJ} \times \left(\frac{1}{2} \times \frac{tx^2}{l}\right)^2 dx$$

$$= \frac{t^2}{8l^2GJ} \cdot \left(\frac{x^5}{5}\right)_0^l = \frac{t^2l^3}{40GJ}$$

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7 Principal Stresses and Strains

Detailed Explanation of Try Yourself Questions

T1 : Solution

Moment of area

$$J = \frac{\pi}{32}(110^4 - 100^4)$$

$$J = 4.5563 \times 10^6 \text{ mm}^4$$

Shear stress

$$\tau = \frac{T \times 55}{4.5563 \times 10^6} = \frac{1000 \times 10^3 \times 55}{4.5563 \times 10^6}$$

$$\tau_{XY} = 12.07 \text{ N/mm}^2$$

Principal stress

$\therefore$ $$\sigma = \frac{1}{2}\left[(\sigma_x + \sigma_y) + (\sigma_x - \sigma_y)\cos 2\theta\right] + \tau_{xy}\sin 2\theta \qquad \sigma_x = \sigma_y = 0$$

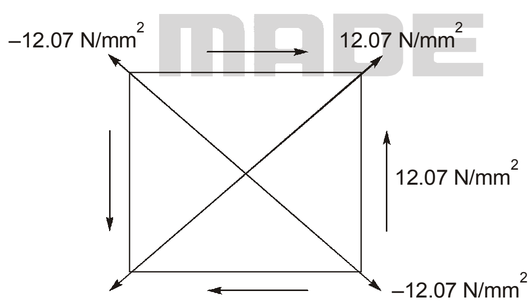
$\Rightarrow$ $$\sigma = \tau_{XY} \sin 2\theta$$

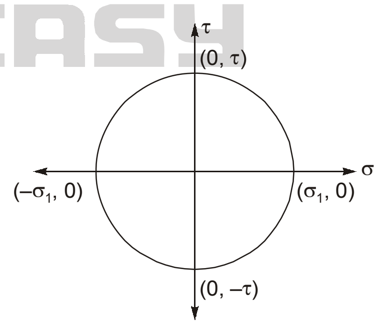
$$\sigma_{1,2} = 12.07 \sin 2\theta$$

$$\sigma_1 = 12.07 \text{ N/mm}^2, \theta = 45°$$

$$\sigma_2 = -12.07 \text{ N/mm}^2, \theta = 135°$$

θ's are with direction of torsion moment.





$$\sigma_1 = -\sigma_2 = \tau = 12.07 \text{ MPa}$$

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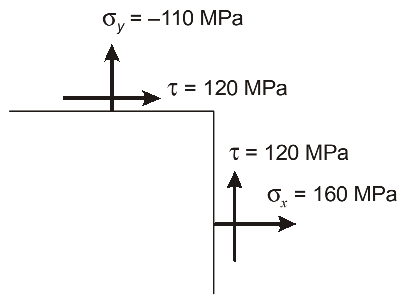
T2 : Solution



Given:

$$\sigma_x = 160 \text{ MPa}$$
$$\sigma_y = -110 \text{ MPa}$$
$$\tau = 120 \text{ MPa}$$

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$= \frac{1}{2}\left[(160 - 110) \pm \sqrt{(270)^2 + 4 \times 120^2}\right]$$

$$= \frac{1}{2}[50 \pm 361.25]$$

$$\sigma_1 = 205.625 \text{ MPa}, \quad \sigma_2 = -155.625 \text{ MPa} \qquad \text{Ans. (i)}$$

$$\tan 2\theta_P = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2 \times 120}{270} = 0.889$$

$$2\theta_P = 41.63° \text{ and } 221.63°,$$

$$\theta_P = 20.815° \text{ and } 110.815° \qquad \text{Ans. (i)}$$

For finding angle of planes as which there is no normal stress,

$$(\sigma_n)_\theta = 0 = \frac{1}{2}\left[(\sigma_x + \sigma_y) + (\sigma_x - \sigma_y)\cos 2\theta\right] + \tau_{xy}\sin 2\theta$$

$$\Rightarrow \qquad \theta = -24.79°, \ 65.21° \qquad \text{Ans. (ii)}$$

T3 : Solution

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= \frac{1}{2}(80 + 20) \pm \frac{1}{2}\sqrt{(80 - 20)^2 + 4 \times 40^2}$$

$$= \frac{1}{2}(80 + 20) \pm \frac{1}{2}\sqrt{(80 - 20)^2 + 4 \times 40^2}$$

$$= 50 \pm 50 = 100 \text{ N/mm}^2, 0 \text{ N/mm}^2$$

$$\varepsilon_1 = \frac{\sigma_1}{E} = \frac{100 \times 10^6}{200 \times 10^9} = 0.5 \times 10^{-3}$$

$$\varepsilon_2 = \frac{\nu\sigma_1}{E} = 0.25 \times 0.5 \times 10^{-3} = -0.125 \times 10^{-3}$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 40}{80 - 20} = \frac{4}{3}$$

$$\theta_1 = 26.565°,$$

$$\theta_2 = 116.565°$$

Change in diameter along $\sigma_1 = \varepsilon_1 d = 0.5 \times 10^{-3} \times 100 = 0.05$ mm

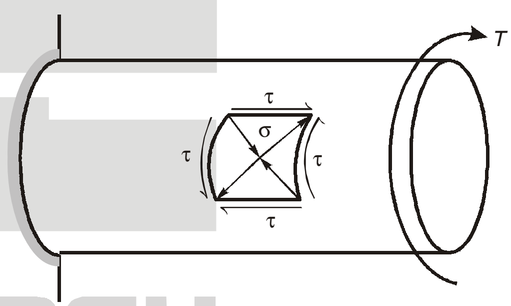
Change in diameter along $\sigma_2 = \varepsilon_2 d = -0.125 \times 10^{-3} \times 100 = 0.0125$ mm

Major diameter of ellipse $= 100 + 0.05 = 100.05$ mm

Minor diameter of ellipse $= 100 - 0.0125 = 99.9875$ mm

T4 : Solution

Only a single strain gauge is mounted on the surface. This means it will be along maximum normal stress because strain gauges are sensitive to only normal strain and not shear strain.



∴ Normal strain at the surface

$$\epsilon = 3.98 \times 10^{-4}$$

$$E = 105 \text{ GN/m}^2 \quad \text{(Given)}$$

$$\epsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E}$$

For pure shear case $\sigma_1 = -\sigma_2 = \tau$

$$3.98 \times 10^{-4} = \frac{\tau(1+0.3)}{105 \times 10^3}$$

$$\tau = 32.14 \text{ MPa}$$

From torsion equation we have

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\frac{T}{\frac{\pi}{32} \times (60)^4 \times 10^{-12}} = \frac{32.14 \times 10^6}{30 \times 10^{-3}}$$

$$T = 1363.10 \text{ N-m}$$

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 800 \times 1363.10}{60} = 114.19 \text{ kW}$$

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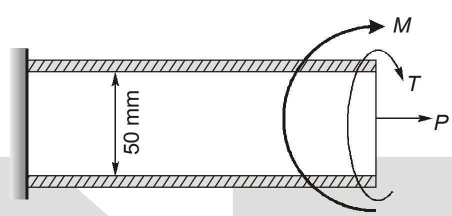
8 Theories of Failure

Detailed Explanation of Try Yourself Questions

T1 : Solution



Let the thickness of the tube is (t)

∴ Polar moment of area

$$J = \frac{\pi}{32}[d^4 - (d-2t)^4] = \frac{\pi}{32}\cdot d^4\left[1-\left(1-\frac{2t}{d}\right)^4\right]$$

$$= \frac{\pi}{32}.d^4\left[1-1+\frac{8t}{d}\right] = \frac{\pi}{4}\cdot d^3 t = \frac{\pi}{4}(50)^3 t$$

$$J = 98174.8 \text{ t mm}^4$$

$$\therefore \quad I = J/2 = 49087.4 \text{ t mm}^4$$

Area of cross-section

$$A = \pi d.t = 50 \times \pi t; \quad A = 157.08 \text{ t mm}^2$$

∴ Axial stress

$$\sigma_a = \frac{P}{A} = \frac{9000}{157.08\,t} = \frac{57.3}{t}\ \text{N/mm}^2$$

Bending stress

$$\sigma_b = \frac{Fey}{I} = \frac{1750\times120\times25}{49087.4\,t}$$

$$\sigma_b = \frac{4.28\times25}{t} = \frac{107}{t}\ \text{N/mm}^2$$

Torsional shear stress

$$\tau = \frac{T.r}{J} = \frac{72000 \times 25}{98174.8\,t}$$

$$\tau = \frac{18.335}{t}\,N/mm^2$$

Maximum possible tensile stress

$$\sigma = \frac{57.3}{t} + \frac{107}{t} = \frac{164.3}{t}\ \ N/mm^2$$

$$\therefore \quad \sigma_{1,\,2} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\frac{267}{4} = \frac{82.15}{t} + \frac{84.2}{t}$$

$$\frac{267}{4} = \frac{166.35}{t}$$

$$\Rightarrow \quad t = \frac{4 \times 166.35}{267} = 2.5\ mm$$

T2 : Solution

Principal stress = p, 0.5p, 0

ν = 0.30

(i) **Maximum shear stress theory:** The maximum shear is equal to half the difference between the maximum and minimum principal stress and since the maximum shear in simple tension is equal to the half of the tensile stress, so we have

$$\tau_{max} = \left(\frac{\sigma_1 - \sigma_3}{2}\right) = \tau_{per} = \frac{\sigma_y}{2} \qquad [\sigma_y = \text{failure stress in tension}]$$

$$\frac{p-0}{2} = \frac{\sigma}{2}$$

$$p = \sigma$$

(ii) **Strain energy theory:** Here one of principal stress has zero value

$$\therefore \quad \sigma_1^2 + \sigma_2^2 - \frac{2}{m}\sigma_1\sigma_2 = \sigma_y^{\,2}$$

$$p^2 + (0.5p)^2 - (2 \times p \times 0.5p \times 0.30) = \sigma_y^{\,2}$$

$$p^2 + 0.25p^2 - 0.3p^2 = \sigma_y^{\,2}$$

$$0.95p^2 = \sigma_y^{\,2}$$

$$0.97p = \sigma_y$$

$$p = 1.03\,\sigma$$

(iii) **Distortion energy theory:**

$$\sigma_1^{\,2} + \sigma_2^{\,2} - \sigma_1\sigma_2 = \sigma_y^{\,2}$$

$$p^2 + (0.5p)^2 - p \times 0.5p = \sigma_y^{\,2}$$

$$0.75\,p^2 = \sigma_y^{\,2}$$

$$0.866p = \sigma_y$$

$$p = 1.15\,\sigma_y$$

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9 Thin and Thick Pressure Vessels

Detailed Explanation of Try Yourself Questions

T1 : Solution

Given:
$$P = 500 \text{ atm} = 49035 \text{ kPa} = 49.035 \text{ MPa}$$
$$D = 100 \text{ mm}$$
$$\sigma_y = 500 \text{ MPa, FOS} = 2$$

For safety;

Using maximum principal stress theory

$$(\sigma_{max})_{induced} \leq \sigma_{per}$$

$$\frac{pD}{2t} \leq \frac{\sigma_y}{N}$$

$$\Rightarrow \quad t \geq 9.807 \text{ mm}$$

$$\Rightarrow \quad t = 10 \text{ mm}$$

Alternate Solution:

Using maximum shear stress theory

$$\tau_{allowable} = \frac{\sigma_y}{2\text{FOS}} = \frac{500}{2 \times 2} = 125 \text{ MPa}$$

$$\tau_{max} = \frac{PD}{4t} \text{ (for this cylindrical pressure vessel)}$$

$$\frac{PD}{4t} = 125$$

or
$$t = \frac{PD}{500} = \frac{49.035 \times 100}{500} = 9.807 \text{ mm say } 10 \text{ mm}$$

$\therefore$ Required thickness, $t =$ **10 mm** **Ans.**

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10 Columns

Detailed Explanation of Try Yourself Questions

T1 : Solution

For both ends hinged $P_{cr} = \dfrac{\pi^2 EI}{L^2}$

For both ends fixed $P_{cr} = \dfrac{4\pi^2 EI}{L^2}$

$$\frac{d_0}{d_i} = 1.25 \text{ and } \frac{d_i}{d_0} = 0.8$$

Now if P is load for both ends hinged and P + 300 is load for both ends fixed

$$P = \frac{\pi^2 EI}{L^2} \text{ and } P + 300 = \frac{4\pi^2 EI}{I^2}$$

we get, $$\frac{\pi^2 EI}{L^2} + (300 \times 10^3) = \frac{4\pi^2 EI}{L^2}$$

$$\frac{3\pi^2 EI}{L^2} = 300 \times 10^3$$

$$I = \frac{300 \times 10^3 \times 9}{3 \times \pi^2 \times 100 \times 10^9} = 9.1189 \times 10^{-7} \quad \left(\therefore \text{ Where } k = \frac{d_i}{d_0} = 0.8\right)$$

$$I = 9.12 \times 10^{-7} \text{ m}^4$$

Now $$I = \frac{\pi}{64} d_0^4 (1 - k^4) \text{ for hollow column.}$$

$$\frac{\pi}{64} d_0^4 (1 - k^4) = 9.12 \times 10^{-7}$$

$$d_0 = 74.9 \text{ mm}$$

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