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**ESE 2024 : Mains Test Series**  
UPSC ENGINEERING SERVICES EXAMINATION

**Electrical Engineering**  
**Test-1 : Electrical Circuits [All Topics]**  
**Control Systems [All Topics]**

Name : Satyam Khosrasta

Roll No : 

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<b>Test Centres</b>	<b>Student's Signature</b>
Delhi <input type="checkbox"/> Bhopal <input type="checkbox"/> Jaipur <input type="checkbox"/> Pune <input type="checkbox"/> Kolkata <input type="checkbox"/> Hyderabad <input type="checkbox"/>	

- Instructions for Candidates**
1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
  2. There are Eight questions divided in TWO sections.
  3. Candidate has to attempt FIVE questions in all in English only.
  4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
  5. Use only black/blue pen.
  6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
  7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
  8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE	
Question No.	Marks Obtained
Section-A	
Q.1	
Q.2	
Q.3	
Q.4	
Section-B	
Q.5	
Q.6	
Q.7	
Q.8	
<b>Total Marks Obtained</b>	
Signature of Evaluator	Cross Checked by
.....	.....

## IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY

### DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

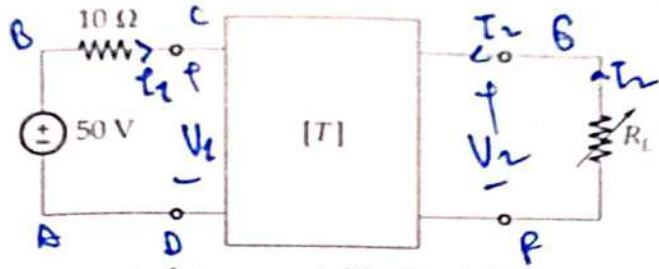
### DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.



Section A : Electrical Circuits

(a) The ABCD parameter of the two-port network in figure are  $\begin{bmatrix} 4 & 20\Omega \\ 0.1 & 2 \end{bmatrix}$ .



The output port is connected to a variable load for maximum power transfer. Find  $R_L$  and the maximum power transferred.

[12 marks]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 4 & 20\Omega \\ 0.1 & 2 \end{pmatrix}$$

$$V_1 = AV_2 - BI_2$$

$$\Rightarrow V_1 = 4V_2 - 20I_2 \quad \text{--- (i)}$$

$$\Rightarrow I_1 = 0.1V_2 - 2I_2 \quad \text{--- (ii)}$$

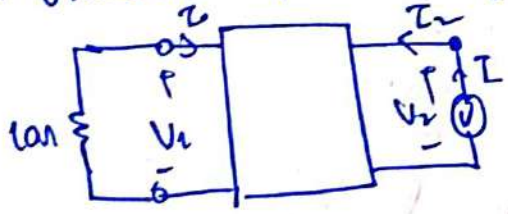
we have,

$$V_1 = 50 - 10I_1 \quad \text{(By KVL between A B C D) --- (iii)}$$

Similarly, by ohm's law between E & F, we have

$$V_2 = -I_2 R_L \quad \text{--- (iv)}$$

Now, for maximum power transfer,  $R_L = R_{th}$ .



$$V_1 = -10I_1$$

$$\Rightarrow V_1 = 4V_2 - 20I_2$$

$$\Rightarrow -10I_1 = 4V_2 - 20I_2$$

$$\Rightarrow I_1 = 0.1V_2 - 2I_2$$

Putting eq (ii) in (i), we have

$$-10I_1 = 4V_2 - 20I_2$$

$$-10(0.1V_2 - 2I_2) = 4V_2 - 20I_2$$

$$-V_2 + 20I_2 = 4V_2 - 20I_2$$

$$5V_2 = 40I_2$$

$$\frac{V_2}{I_2} = 8 \Omega$$

$R_{th} = 8 \Omega \Rightarrow R_L = 8 \Omega$  for maximum power transfer.

Now,  $R_L = 8 \Omega$ .

Putting in (ii), we have

~~$$V_2 = 8I_2$$~~

~~$$V_2 = 4V_2 - 20I_2 = 50 - 10I_1$$~~

~~$$I_1 = 0.1V_2 - 2I_2$$~~

~~$$V_1 = 50 - 10(0.1V_2 - 2I_2)$$~~

~~$$V_1 = 50 - V_2 + 20I_2$$~~

$$\Rightarrow V_1 = 32.5V$$

$$I_1 = 1.875A$$

$$V_2 = 5V$$

$$I_2 = -\frac{5}{8}A$$

$\therefore$  we have  $P = \frac{V_2^2}{R_L}$

$$P = \frac{(5)^2}{8}$$

$$P = 3.125W$$

~~$$V_2 + 8I_2 = 0$$~~

$$V_2 = -8I_2$$

$$V_2 + 8I_2 = 0 \quad \text{--- (i)}$$

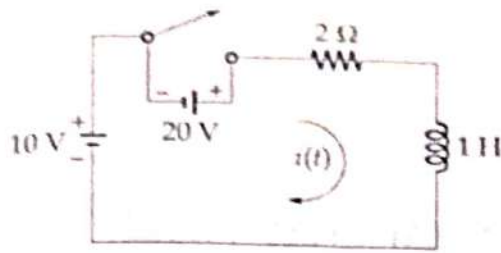
$$V_2 - 4V_2 + 20I_2 = 0 \quad \text{--- (ii)}$$

$$V_1 + 10I_1 = 50 \quad \text{--- (iii)}$$

$$I_1 - 0.1V_2 + 2I_2 = 0 \quad \text{--- (iv)}$$

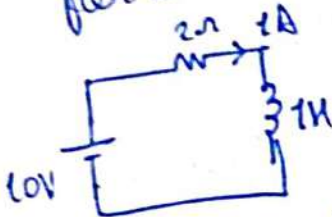


b) Determine the current  $i(t)$  in the circuit shown in figure at an instant  $t$ , after opening the switch at  $t = 0$ , if a current of 1 A had been passed through the circuit at the instant of opening.

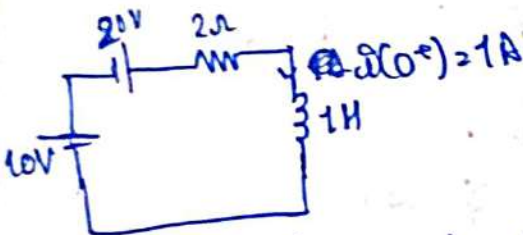


[12 marks]

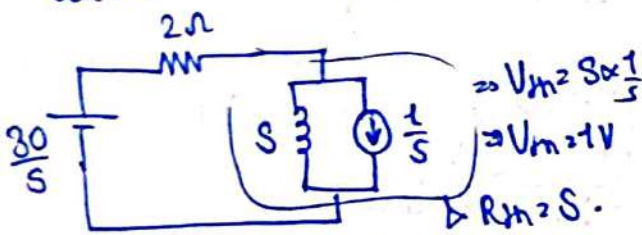
for  $t = 0^-$ , we have



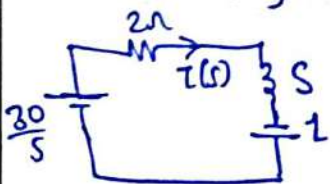
For  $t > 0$ , we have



Now, convert into Laplace, we have



Convert the Norton source to Thevenin, we have



By KVL, we have

$$\frac{30}{s} + 4 = Z(s) \quad (2e1)$$

$$Z(s) = \frac{s+30}{s(s+2)}$$

$$Z(s) = \frac{15}{s} - \frac{14}{s+2}$$

$$i(t) = 15u(t) - 14e^{-2t}u(t)$$

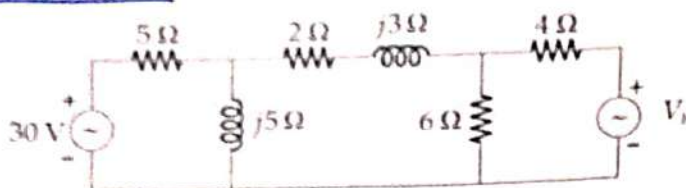
$$\Rightarrow i(t) = (15 - 14e^{-2t})u(t)$$

we have,

$$i(0^+) = 1A$$

$$i(\infty) = 15A$$

Q.1 (c) For the circuit shown below:



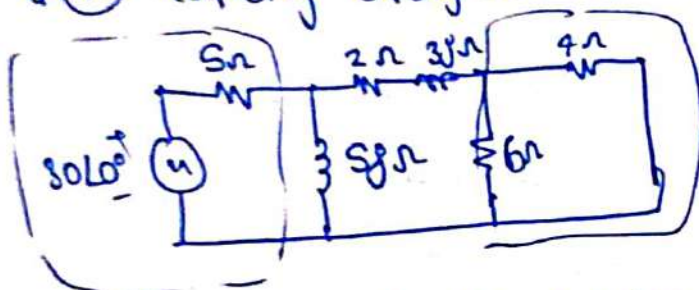
Determine the voltage  $V_x$  which results in a zero current through the  $(2 + j3)\Omega$  impedance branch. Using superposition theorem.

[12 marks]

(Ans.)

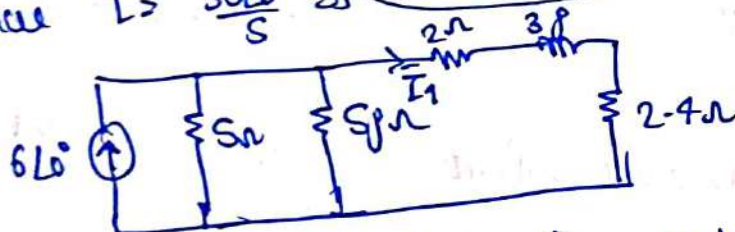
By superposition theorem, we have

① Taking only 30V source.



$$\begin{aligned} R_{eq} &= R_1 \parallel R_2 \\ &= 4 \parallel 6 \\ &= \frac{4 \times 6}{4 + 6} \Rightarrow R_{eq} = 2.4n \end{aligned}$$

Transferring the green source onto circuit source, we have  $\bar{I} = \frac{30\angle 0^\circ}{5} \Rightarrow \bar{I} = 6\angle 0^\circ \text{ A}$  &  $R = 5n$ .



∴ Current through  $(2 + j3)n$  impedance is given by  $\bar{I}_1 = 6 \times \left( \frac{1}{(2 + j3) + 2.4} \right)$  (By current division rule)

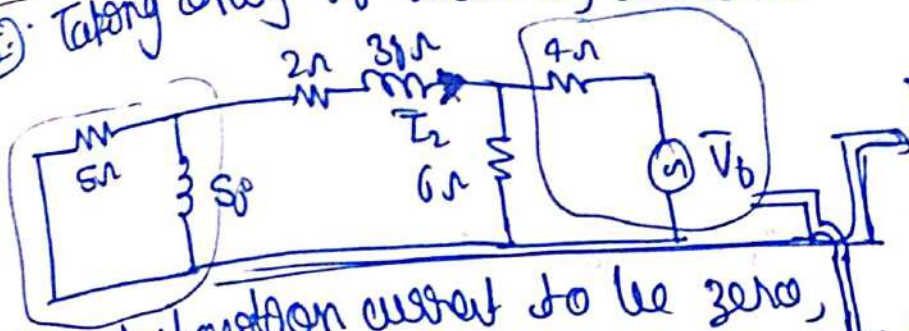
$$\left( \frac{1}{5} \right) + \left( \frac{1}{j5} \right) + \frac{1}{(2 + j3) + 2.4}$$

$$\begin{aligned} \bar{I}_1 &= 6 \times \frac{1}{(4.4 + j3)} = 6 \times (0.1827 \angle -34.286^\circ) \\ &= \frac{1}{\frac{1}{5} + \frac{1}{j5} + \frac{1}{(4.4 + j3)}} \quad (0.4686 \angle -40.228^\circ) \end{aligned}$$

$$\bar{I}_1 = 2.404 \angle 6.442^\circ \text{ A}$$



(ii) Taking only  $V_b$  source, we have



$$Z_{eq} = \frac{5 \times 5j}{5 + 5j}$$

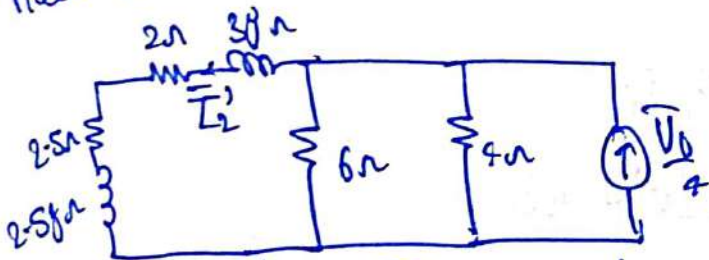
$$\Rightarrow Z_{eq} = 2.5 + j2.5 \Omega$$

For superposition current to be zero,

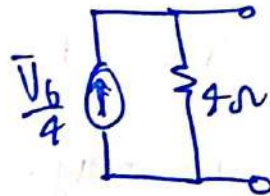
$$I_1 + I_2 = 0 \Rightarrow I_2 = -I_1$$

$$\therefore I_2 = -2.404 \angle 6.442^\circ \text{ A}$$

We have the modified circuit as.



Using source transformation, we have



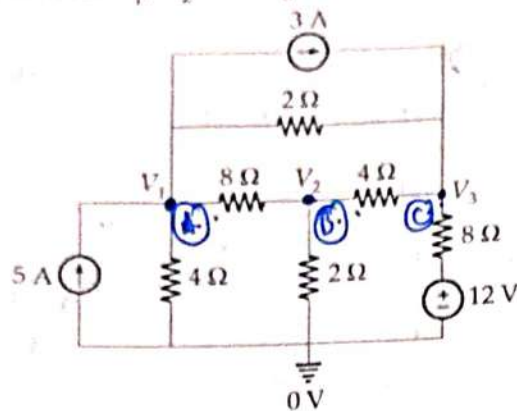
$$\text{we have, } I_2' = \left( \frac{V_b}{4} \right) \frac{1}{\left( \frac{1}{4.5 + 5.5j} \right) + \left( \frac{1}{6} \right) + \left( \frac{1}{4} \right)}$$

$$I_2' = \frac{V_b}{4} (0.222 \angle -38.558^\circ)$$

$$\Rightarrow 2.404 \angle 6.442^\circ = \frac{V_b}{4} (0.222 \angle -38.558^\circ)$$

$$\Rightarrow V_b = 35.354 \angle 45^\circ$$

Q.1 (d) Use nodal analysis to find  $V_1$ ,  $V_2$  and  $V_3$  in the circuit of figure.



[12 marks]

(Ans.)

At node (A), we have

$$I_{\text{incoming}} = I_{\text{outgoing}}$$

$$5 = \frac{V_1}{4} + \frac{V_1 - V_2}{8} + \frac{V_1 - V_3}{2} + 3$$

$$\Rightarrow 16 = 2V_1 + V_1 - V_2 + 4V_1 - 4V_3$$

$$\underline{7V_1 - V_2 - 4V_3 = 16} \quad \text{--- (1)}$$

Similarly at node (B), we have

$$\frac{V_2 - V_1}{8} + \frac{V_2}{2} + \frac{V_2 - V_3}{4} = 0$$

$$\Rightarrow V_2 - V_1 + 4V_2 + 2V_2 - 2V_3 = 0$$

$$\underline{3V_1 + 3V_2 - 2V_3 = 0} \quad \text{--- (2)}$$

At node (C), we have

$$\frac{V_3 - V_2}{4} + \frac{V_3 - 12}{8} + \frac{V_3 - V_1}{2} = 3$$



$$2V_3 - 2V_2 + V_3 - 12 + 4V_3 - 4V_1 = 24$$

$$\underline{\underline{-4V_1 - 2V_2 + 7V_3 = 36}} \quad \text{--- (iii)}$$

Now, by nodal analysis, we have equation

$$\text{(i), (ii) \& (iii)}$$

On solving equations (i), (ii) & (iii),

we have

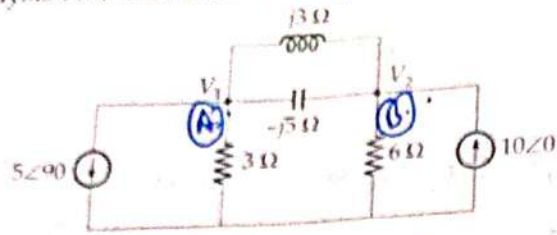
~~$$V_1 = 7.185V$$~~

$$V_1 = 7.185V$$

$$V_2 = -1.259V$$

$$V_3 = 8.888V$$

Q.1 (c) Use nodal analysis on the circuit to find  $V_2$



[12 marks]

(Ans.) By nodal analysis, we have,  
at node (A),

$$I_{in} = I_{out}$$

$$\Rightarrow 0 = 5\angle 90^\circ + \frac{V_1}{3} + \frac{V_1 - V_2}{3j} + \frac{V_1 - V_2}{-5j}$$

$$\Rightarrow 0 = (15j)(5j) + 5jV_1 + 5V_1 - 5V_2 + (15j)(V_2 - V_1)$$

$$\Rightarrow 75 = 5jV_1 + 5V_1 - 5V_2 + 3V_2 - 3V_2$$

$$\text{(i)} \quad (2 + 5j)V_1 - 2V_2 = 75$$

Similarly at node (B), we have

$$10\angle 0^\circ = \frac{V_2}{6} + \frac{V_2 - V_1}{-5j} + \frac{V_2 - V_1}{3j}$$

$$300j = 5jV_2 + 6(V_1 - V_2) + 10(V_2 - V_1)$$

$$300j = 5jV_2 + 6V_1 - 6V_2 + 10V_2 - 10V_1$$

$$300j = -4V_1 + (4 + 5j)V_2$$

$$-4V_1 + (4 + 5j)V_2 = 300j \quad \text{(ii)}$$

From equations (i) & (ii) we have

$$\begin{bmatrix} 2 + 5j & -2 \\ -4 & 4 + 5j \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 75 \\ 300j \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 + 5j & -2 \\ -4 & 4 + 5j \end{bmatrix}^{-1} \begin{bmatrix} 75 \\ 300j \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 5j & 2 \\ 4 & 2 + 5j \end{bmatrix} \begin{bmatrix} 75 \\ 300j \end{bmatrix}$$

$$(-17 + 30j)$$

$$\begin{bmatrix} 300 + 375j + 600j \\ 300 + 600j - 1500 \end{bmatrix}$$

$$(-17 + 30j)$$

$$V_1 = 29.583 \angle -46.641^\circ \text{ V}$$

$$V_2 = 38.908 \angle 33.89^\circ \text{ V}$$

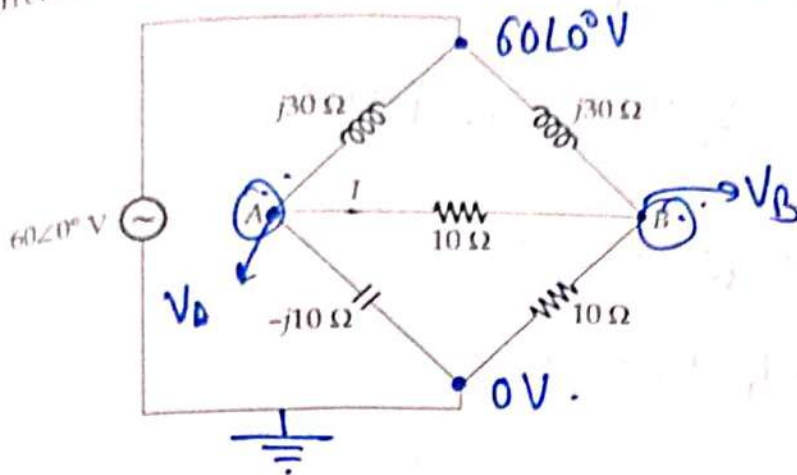
$$\Rightarrow = \begin{bmatrix} 1020.11 \angle 32.89^\circ \\ 1341.64 \angle 53.48^\circ \end{bmatrix}$$

$$37.48 \angle 119.538^\circ$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 29.583 \angle -46.641^\circ \\ 38.908 \angle 33.89^\circ \end{bmatrix}$$



Determine the current  $I$  through the terminal AB of the network shown below:



[20 marks]

Now, we have nodes A & B & assume ground, by nodal analysis, at (A).

$$\frac{V_A - 60\angle 0^\circ}{30j} + \frac{(V_A - 0)}{-10j} + \frac{(V_A - V_B)}{10} = 0$$

$$(V_A - 60) + \cancel{3j}(-V_A) + 3j(V_A - V_B) = 0$$

$$\Rightarrow \underline{V_A - 60 - 3V_A + 3jV_A - 3jV_B = 0}$$

$$\Rightarrow \cancel{V_A} V_A(-2 + 3j) + V_B(-3j) = 60 \quad \text{--- (i)}$$

Now, by nodal analysis at B, we have

$$\frac{V_B - 60}{30j} + \frac{V_B - V_A}{10} + \frac{V_B}{10} = 0$$

$$V_B - 60 + 3j(V_B - V_A) + 3jV_B = 0$$

$$\cancel{V_B} V_B - 60 + \underline{3jV_B - 3jV_A + 3jV_B} = 0$$

$$(-3j)V_A + (1+6j)V_B = 60 \quad \text{--- (ii)}$$

From eqn (i) & (ii), we have

$$V_A(-2+3j) + V_B(-3j) = 60$$

$$V_A(-3j) + V_B(1+6j) = 60$$

$$\Rightarrow \begin{bmatrix} -2+3j & -3j \\ -3j & 1+6j \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 60 \\ 60 \end{bmatrix}$$

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -2+3j & -3j \\ -3j & 1+6j \end{bmatrix}^{-1} \begin{bmatrix} 60 \\ 60 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6j & 3j \\ 3j & -2+3j \end{bmatrix} \begin{bmatrix} 60 \\ 60 \end{bmatrix}$$

$$\frac{(-2+3j)(1+6j) - (-3j)(-3j)}{(-2+3j)(1+6j) - (-3j)(-3j)}$$

$$= \frac{\begin{bmatrix} 60+360j+180j \\ 180j-120+180j \end{bmatrix}}{-2-9j-18-9}$$

$$\frac{\begin{bmatrix} 60+540j \\ -120+360j \end{bmatrix}}{-29-9j}$$

$$= \frac{\begin{bmatrix} 60+540j \\ -120+360j \end{bmatrix}}{-29-9j}$$



$$\begin{bmatrix} \bar{V}_A \\ \bar{V}_B \end{bmatrix} = \begin{bmatrix} 12.893 \angle -113.581^\circ \text{ V} \\ 12.9972 \angle -88.806^\circ \text{ V} \end{bmatrix}$$

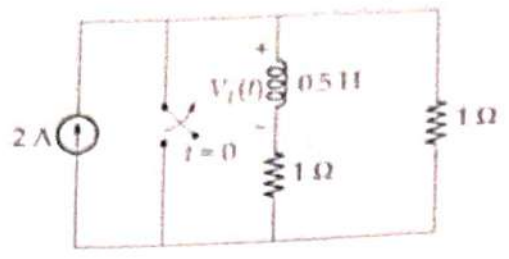
Now, By Ohm's Law, we have

$$V_A - V_B = 10 \times \bar{I}$$

$$\Rightarrow (12.893 \angle -113.581^\circ) - (12.9972 \angle -88.806^\circ) = 10 \times \bar{I}$$

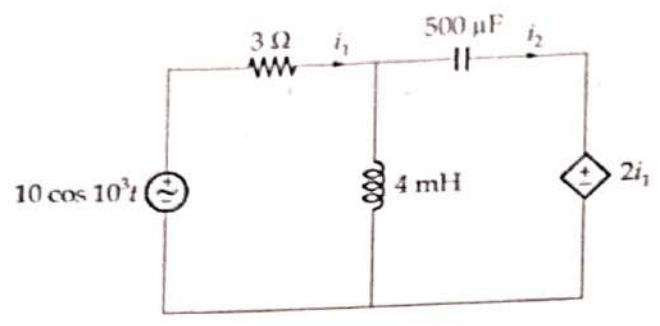
$$\Rightarrow \bar{I} = 0.838 \angle -152.291^\circ \text{ A}$$

Q.2 (b) (i) For the network shown in figure below, the switch is closed for a long time and at  $t = 0$ , the switch is opened.



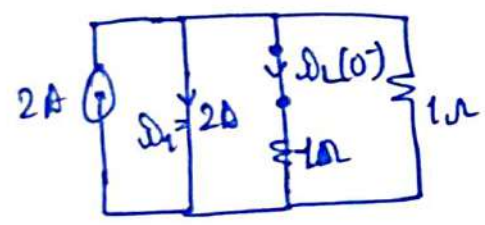
Determine the voltage across inductor for  $t > 0$ .

(ii) Obtain expressions for the time domain currents  $i_1$  and  $i_2$  in the circuit given as figure.



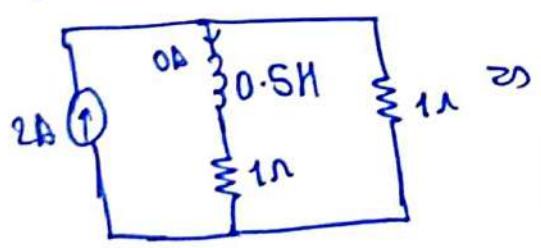
[10 + 10 marks]

(Ans.) (i) For  $t = 0^-$ , we have

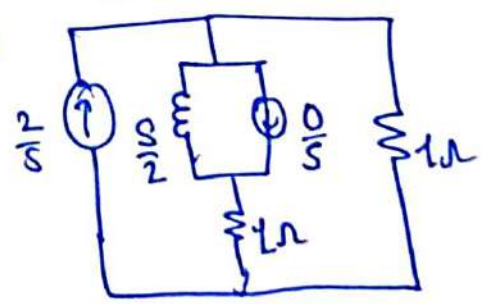


∴ of short circuit, we have  $i_L = 2A$   
 ∴  $i_L(0^-) = 2A$

Now, at  $t = 0^+$ , we have

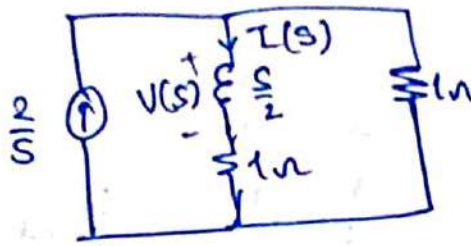


Laplace domain





∴ we have,



$$I(s) = \frac{2}{s} \frac{(1)}{(1 + \frac{s}{2} + 1)} \quad (\text{By current division rule})$$

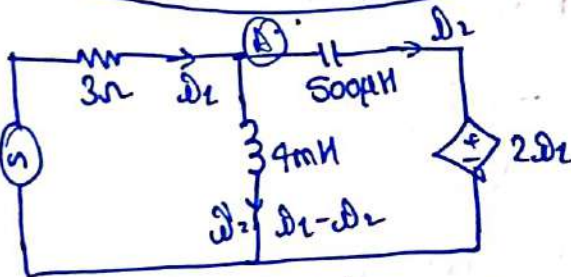
$$V(s) = \frac{2}{s} I(s) = \frac{2}{s} \left(\frac{2}{s}\right) \left(\frac{1}{2 + \frac{s}{2}}\right)$$

$$V(s) = \frac{2}{s+4}$$

$$\Rightarrow v(t) = 2e^{-4t} u(t)$$

⊙

AC source =  $\bar{V}$



By KCL at (A),

we have

$$I_1 = I' + I_2$$

$$\Rightarrow I' = I_1 - I_2$$

taking voltage as reference,

$$\bar{V} = \frac{10}{\sqrt{2}} \angle 0^\circ = 7.07 \angle 0^\circ \text{ V}$$

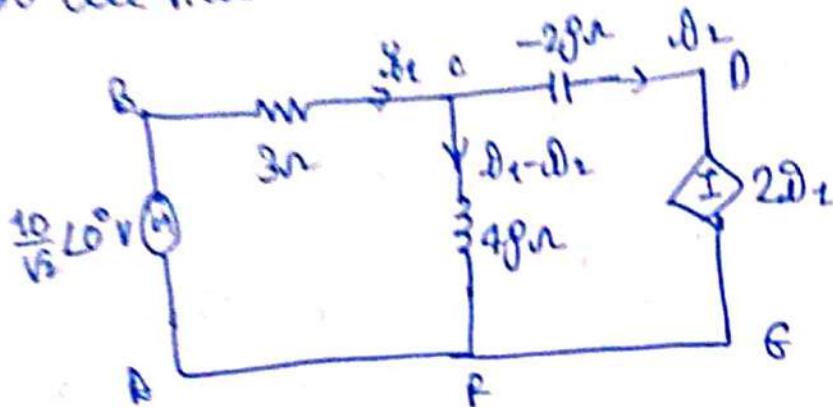
~~$\omega L = 2\pi \times 1000 \times 9 \text{ m} = 8 \mu$~~   
 ~~$\omega L = 928.133 \Omega$~~

~~$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1000}$~~

$$X_L = \omega L = 9 \times 1000 \times 9 \text{ m} = 4 \mu \Rightarrow X_L = 4 \mu \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{-j}{1000 \times 500 \mu} \Rightarrow X_C = -2 \mu \Omega$$

∴ we have



By KVL on loop ABCFD

$$V_R = 7.07 \angle 0^\circ - 3i_1 - 4j i_1 + 4j i_2 = V_R$$

$$7.07 = (3 + 4j) i_1 + (-4j) i_2 \quad \text{--- (1)}$$

By KVL on loop ABCDBA, we have

$$7.07 \angle 0^\circ - 3i_1 - (-2j)i_2 - 2i_1 = 0$$

$$7.07 = 5i_1 - 2j i_2 \quad \text{--- (2)}$$

∴ from (1) & (2), we have

$$\begin{bmatrix} 3+4j & -4j \\ 5 & -2j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 7.07 \\ 7.07 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 3+4j & -4j \\ 5 & -2j \end{bmatrix}^{-1} \begin{bmatrix} 7.07 \\ 7.07 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{\begin{bmatrix} -2j & 4j \\ -5 & 3+4j \end{bmatrix} \begin{bmatrix} 7.07 \\ 7.07 \end{bmatrix}}{(-6j + 8 + 20j)}$$



$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \frac{\begin{bmatrix} 10\sqrt{2} \angle 0^\circ \\ -5\left(\frac{10}{\sqrt{2}}\right) + (8+4j)\left(\frac{10}{\sqrt{2}}\right) \end{bmatrix}}{(8+14j)} = \frac{\begin{bmatrix} 10\sqrt{2} \angle 90^\circ \\ (-1+2j)(10\sqrt{2}) \end{bmatrix}}{(8+14j)}$$

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \frac{\begin{bmatrix} 14.14 \angle 90^\circ \\ 31.622 \angle 116.565^\circ \end{bmatrix}}{8+14j}$$

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} \frac{14.14 \angle 90^\circ}{8+14j} \\ \frac{31.622 \angle 116.565^\circ}{8+14j} \end{bmatrix} = \begin{bmatrix} 0.877 \angle 29.749^\circ \text{ A} \\ 1.9611 \angle 56.3099^\circ \text{ A} \end{bmatrix}$$

$$i_1(t) = \sqrt{2} \times 0.877 \cos(1000t + 29.749^\circ)$$

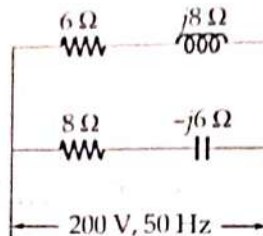
$$i_1(t) = 1.24 \cos(1000t + 29.749^\circ) \text{ A}$$

$$i_2(t) = 1.9611 \angle 56.3099^\circ \text{ A}$$

$$i_2(t) = \sqrt{2} \times 1.9611 \cos(1000t + 56.3099^\circ)$$

$$i_2(t) = 2.773 \cos(1000t + 56.3099^\circ) \text{ A}$$

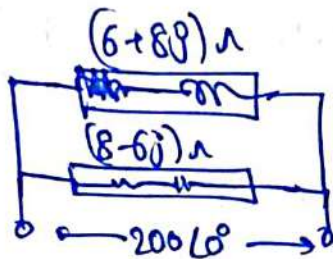
- Q.2 (c) For the circuit shown below, calculate,
- Total admittance, total conductance and total susceptance.
  - Total current and total power factor (pf).
  - The value of pure capacitance to be connected in parallel with the above combination to make the total power factor (pf) unity.



[20 marks]

(Ans)

9.



Taking voltage as reference,  
we have

$$Z_{eq} = \frac{(6 + j8)(8 - j6)}{(6 + j8) + (8 - j6)}$$

$$Z_{eq} = 7 + j2 \quad \text{--- (i)}$$

$$\Rightarrow Y_{eq} = \frac{1}{Z_{eq}} = \frac{1}{7 + j2}$$

$$\Rightarrow Y_{eq} = 0.14 - j0.02 \text{ S} = G + jB$$

$$G_{eq} = 0.14 \text{ S}$$

$$B_{eq} = -0.02 \text{ S}$$

~~$$B_{eq} = -0.02 \text{ S}$$~~

10. Now, for total current, we have

$$\bar{I} = \frac{200 \angle 0^\circ}{Z_{eq}} = \frac{200 \angle 0^\circ}{7 + j2}$$



$$\bar{I} = \frac{200 \angle 0^\circ}{7 + j9}$$

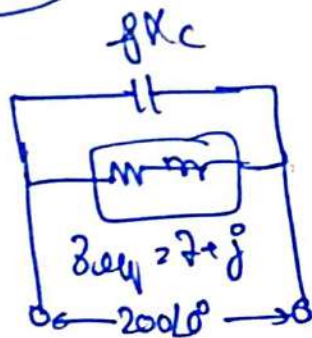
$$\Rightarrow \bar{I} = 28.284 \angle -8.130^\circ \text{ A}$$

∴ Total current = 28.284 A

$$\text{p.f.} = \cos(8.130^\circ)$$

$$\Rightarrow \text{p.f.} = 0.9899 \text{ lag.}$$

(iii) We have,



$$\therefore Y_{eq} = \frac{1}{7 + j9} + \frac{1}{j\omega C} = \frac{7}{50} - \frac{j9}{50} + \frac{j}{\omega C}$$

∴ for power factor to be unity, imaginary part = 0

$$\Rightarrow \frac{j}{50} + \frac{j}{\omega C} = 0 \Rightarrow \frac{1}{50} = \frac{1}{\omega C}$$

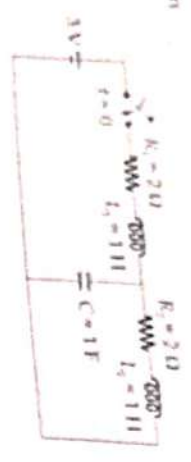
$$\Rightarrow \omega C = 50$$

$$\Rightarrow \frac{1}{2\pi \times 50 \times C} = 50$$

$$\Rightarrow C = 63.66 \mu\text{F}$$

Q.3 (a)

In the network shown in figure the switch is closed at time  $t = 0$ . Assuming all the initial currents and voltages as zero, find the current through the inductor  $I_2$  by the use of Norton's theorem



[20 marks]



Show that the dominant frequency  $\omega_d$  of a series R-L-C circuit is geometric mean of  $\omega_c$  and  $\omega_{cl}$ , the upper and lower half power frequencies respectively.

[20 marks]

MADE ERSU Question Cum Answer Booklet

Calculate the power supplied to the 10 Ω resistor in the ideal transformer circuit shown in the figure below.

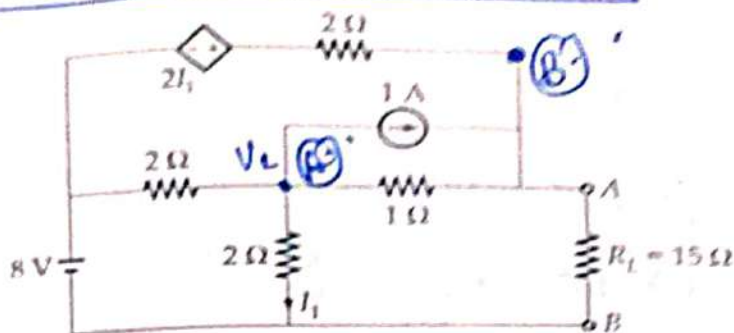


[20 marks]





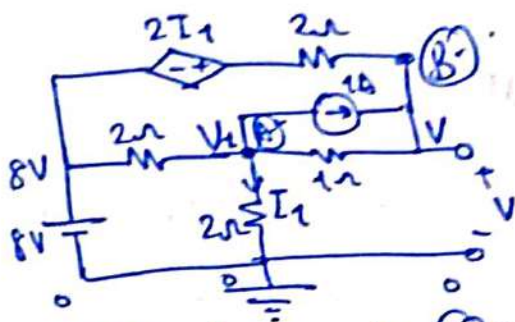
Determine the current through the load resistance  $R_L = 15 \Omega$  across the terminal A-B of the circuit shown in figure below, using Thevenin's theorem. Also find the maximum power that can be transferred to the load resistance  $R_L$ .



For Thevenin's theorem, we find the  $V_{th}$  first

[20 marks]

$V_{th}$



Applying KCL at node A, we have

$$\frac{V_1 - 8}{2} + I_1 + \frac{V_1 - V}{1} + 1 = 0$$

$$\Rightarrow \underline{V_1 - 8} + 2I_1 + \underline{2V_1 - 2V} + 2 = 0$$

$$\Rightarrow 3V_1 - 2V + 2I_1 = 6 \quad \text{--- (i)}$$

By ohm's law between node A & ground, we have

$$V_1 = 2I_1 \quad \text{--- (ii)}$$

By KCL at node B, we have

$$\frac{V - (8 + 2I_1)}{2} + \frac{V - V_1}{1} = 1$$

$$\Rightarrow \underline{V - 8 - 2I_1} + \underline{2V - 2V_1} = 2$$

$$3V - 2V_1 - 2I_1 = 6 \quad \text{--- (iii)}$$



Now, we have

$$3V_1 - 2V + 2I_1 = 6 \quad \text{--- (i)}$$

~~$$V_1 - 2I_1 = 0$$~~

$$V_1 + 0 \cdot V + (-2)I_1 = 0 \quad \text{--- (ii)}$$

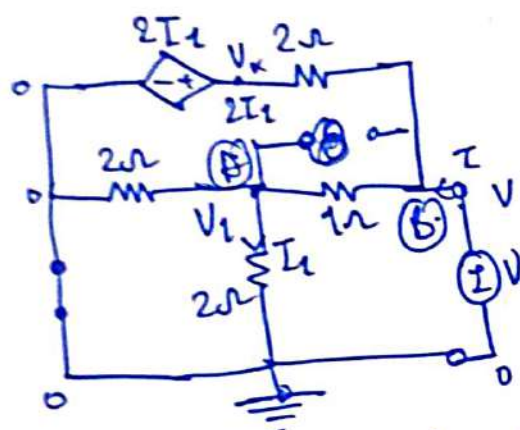
$$(-2)V_1 + (3)V + (-2)I_1 = 6 \quad \text{--- (iii)}$$

∴, we have on solving (i), (ii) & (iii)

$$V_1 = 5V, \quad V_2 = 7V, \quad I_1 = 2.5A$$

∴, we have  $V_{th} = 7V$ .

Now, for  $R_{th}$ , we have



$$V_x = 0 + 2I_1 \quad (\text{By KVL})$$

$$\Rightarrow V_x = 2I_1$$

Let voltage at (A) =  $V_1$ . By KCA at node A, we have

$$\frac{V_1}{2} + \frac{V_1}{2} + \frac{V_1 - V}{1} = 0$$

$$\Rightarrow 2V_1 = V$$

$$\Rightarrow V_1 = \frac{V}{2} \quad \text{--- (iv)}$$

By Ohm's law between (A) & ground, we have

$$V_1 = 2I_1 \quad \text{--- (v)}$$

By KCA at node (B), we have

$$I = \frac{V - V_x}{2} + \frac{V - V_1}{1}$$

$$\Rightarrow I = \frac{V - 2I_1}{2} + V - V_1$$

$$\Rightarrow I = \frac{V}{2} - I_1 + V - V_1$$

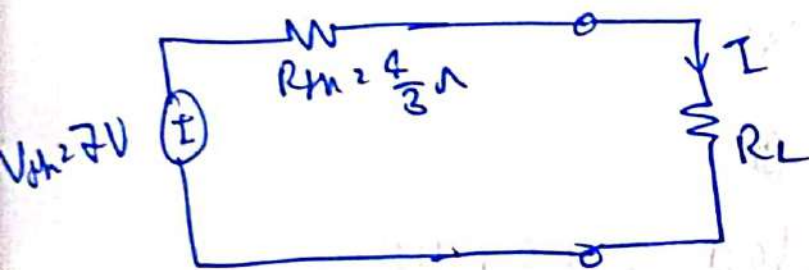
$$I = \frac{3V}{2} - V_1 - I_1$$

$$I = \frac{3V}{2} - \frac{V}{2} - \frac{V_1}{2}$$

$$I = V - \frac{V_1}{2} = V - \frac{V}{4} = \frac{3V}{4}$$

$$\Rightarrow \frac{V}{2} = \frac{4}{3} \Rightarrow R_{th} = \frac{4}{3} \Omega$$

\(\therefore\) we have Thevenin equivalent circuit as



\(\therefore\) If  $R_L = 1 \Omega$ , then

$$I = \frac{7}{\frac{4}{3} + R_L} = \frac{7}{\frac{4}{3} + 1} = \frac{7 \times 3}{4 + 3}$$

$$I = 0.4285 \text{ A} \Rightarrow \underline{\text{Current through } 1 \Omega \text{ load}}$$

Now, for maximum power transfer,  $R_L = R_{th}$

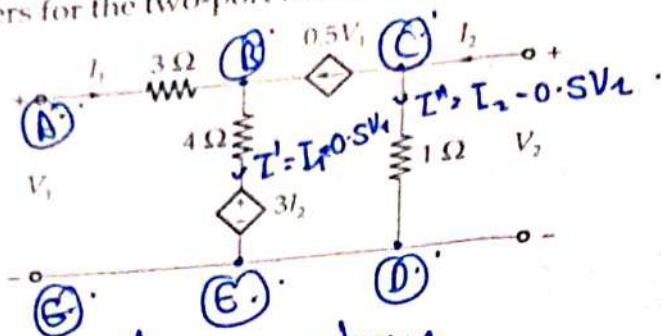
\(\Rightarrow R\_L = \frac{4}{3} \Omega \Rightarrow\) for maximum power transfer.

$$\& P_{max} = \frac{V^2}{4R_{th}} = \frac{(7)^2}{4 \times (\frac{4}{3})} = \frac{49}{(\frac{16}{3})}$$

$$P_{max} = 9.1875 \text{ W} \Rightarrow \underline{\text{Maximum power}}$$



Q.4 (b) Find the  $h$ -parameters for the two-port network shown



[20 marks]

(Ans.)

For  $h$ -parameters, we have

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

By KVL on loop EA BE, we have

$$V_1 + V_1 - 3I_1 - 4I' - 3I_2 = 0$$

$$V_1 = 3I_1 + 3I_2 + 4I' \quad \text{--- (1)}$$

By KCL at node (B), we have

$$I' = I_1 + 0.5V_1 \quad \text{--- (2)}$$

Putting (2) in (1), we have

$$V_1 = 3I_1 + 3I_2 + 4I_1 + 2V_1$$

$$\underline{V_1 = -7I_1 - 3I_2} \quad \text{--- (3)}$$

By Ohm's law between (C) & (D), we have

$$I_2 = \frac{V_2}{1} \Rightarrow I_2 = V_2 \quad \text{--- (4)}$$

Similarly, by KCL at (C), we have



$$I_2 = I_1 + 0.5V_1$$

$$\Rightarrow I_1 = I_2 - 0.5V_1 \quad \text{--- (i)}$$

$$\therefore \textcircled{a} = \textcircled{d}$$

$$\therefore V_2 = I_2 - 0.5V_1 \quad \text{--- (ii)}$$

Now, from (i) & (ii), we have

$$V_1 = -2I_1 - 3I_2 \quad \text{--- (iii)}$$

$$V_2 = I_2 - 0.5V_1 \quad \text{--- (ii)} \Rightarrow I_2 = V_2 + 0.5V_1 \quad \text{--- (iv)}$$

putting (iv) in (iii), we have

$$V_1 = -2I_1 - 3V_2 - 1.5V_1$$

$$2.5V_1 = -2I_1 - 3V_2$$

$$\boxed{V_1 = -2.8I_1 - 1.2V_2} \quad \text{--- (v)}$$

Now, putting (v) in (ii), we have

$$V_2 = I_2 - 0.5(-2I_1 - 3V_2)$$

$$V_2 = I_2 + 3.5I_1 + 1.5V_2$$

$$V_2 = 2.5I_2 + 3.5I_1$$

$$2.5I_2 = -3.5I_1 + V_2$$

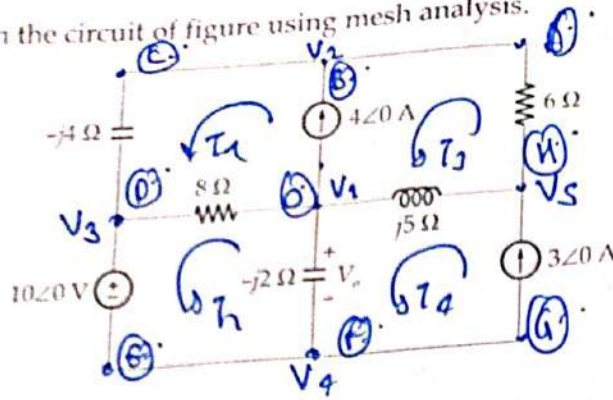
$$I_2 = -\frac{3.5}{2.5}I_1 + \frac{1}{2.5}V_2$$

$$\boxed{I_2 = -1.4I_1 + 0.4V_2} \quad \text{--- (vi)}$$

From (v) & (vi), we have

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} -2.8 & -1.2 \\ -1.4 & 0.4 \end{bmatrix}$$

Q.4 (c) Solve for  $V_o$  in the circuit of figure using mesh analysis.



[20 marks]

n-)

all have,

By KVL on ABCD O H A

$$V_o - (-4j)I_1 - 8(I_1 - I_2) - 5j(I_3 - I_4) - 6I_3 = 0$$

$$4jI_1 - 8I_1 + 8I_2 - 5jI_3 + 5jI_4 - 6I_3 = 0$$

$$(-4j + 8)I_1 + 8I_2 + (-6 - 5j)I_3 + (5j)I_4 = 0$$

all have,  $I_4 = 3\angle 0^\circ A$

all have,  $I_1 - I_3 = 4\angle 0^\circ A$

By KVL on loop FODGF, all have

$$V_o - (-2j)(I_2 - I_4) - 8(I_2 - I_1) - 10 = 0$$

$$10 = 2jI_2 - 2jI_4 - 8I_2 + 8I_1$$

$$8I_1 + 2j(8 + 2j)I_2 - 2jI_4 = 10$$

$$8I_1 + (8 + 2j)I_2 = 10 + 6j$$



Partly (P) only

$$(8-4j)I_1 + (-8)I_2 + (6+5j)I_3 = 15j$$

$$(8-4j)I_1 + (-8)I_2 + (6+5j)(I_1-4) = 15j$$

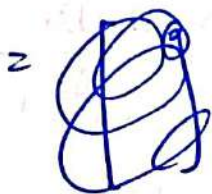
$$(14+j)I_1 + (-8)I_2 = 39+20j$$

$$8I_1 + (8+2j)I_2 = 10+6j$$

$$\begin{bmatrix} 14+j & -8 \\ 8 & 8+2j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 39+20j \\ 10+6j \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} 8+2j & 8 \\ -8 & 14+j \end{bmatrix} \begin{bmatrix} 39+20j \\ 10+6j \end{bmatrix}}{(14+j)(8+2j) - 64}$$

$$= \begin{bmatrix} 2.5525 \angle 27.904^\circ \\ 1.0684 \angle -171.348^\circ \end{bmatrix}$$



$$V_o = (-2j)(I_4 - I_2)$$

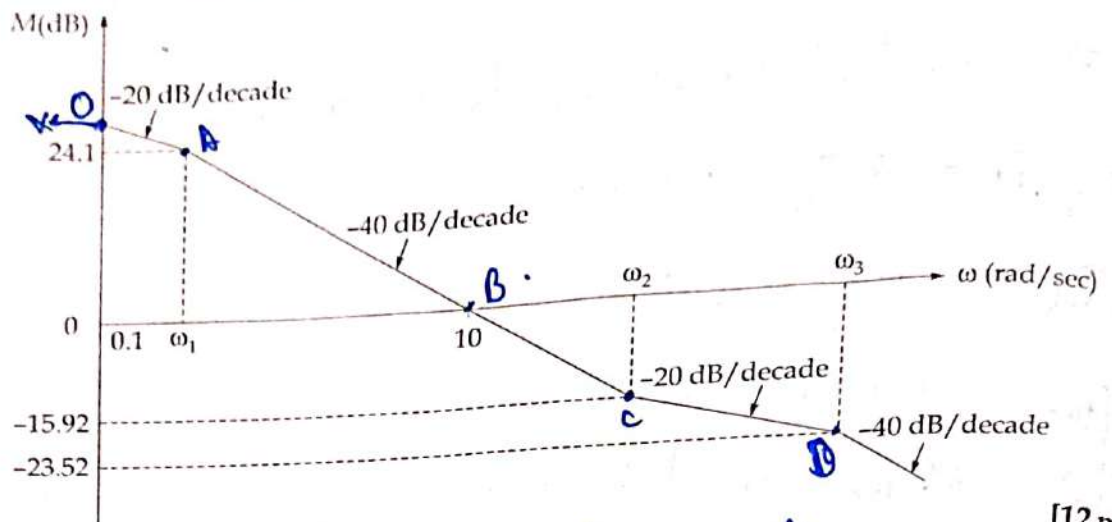
$$V_o = (-2j)(3 - 1.0684 \angle -171.348^\circ)$$

$$V_o = 8.1188 \angle -87.73^\circ \text{ V}$$



## Section B : Control System

Q.5 (a) Obtain the open loop transfer function for a unity negative feedback system whose bode magnitude plot is shown below:



[12 marks]

(Ans.)

We have ~~three~~ corner frequencies at  $\omega_1$ ,  $\omega_2$  &  $\omega_3$ .

∴ slope decreases by 20 dB/dec ∴ we have 1 pole at  $\omega_1$ . Similarly ∴ slope increases by 20 dB/dec. ∴ we have 1 zero at  $\omega_2$  & similarly we have 1 pole at  $\omega_3$ . Initial slope = -20 dB/dec.

∴ we have initial 1 pole at origin.

Now, taking the line segment between A & B,

we have

$$\frac{24.1 - 0}{\log\left(\frac{10}{\omega_1}\right)} = 40 \Rightarrow 4 = \frac{10}{\omega_1} \Rightarrow \omega_1 = 2.5 \text{ rad/sec}$$

∴ Now, taking the line segment between O & A, we have

Gain at 0 =  $\alpha$  (asy-)

$$\therefore \frac{\alpha - 29.1}{\log\left(\frac{\omega_1}{0.1}\right)} = 20 \Rightarrow \frac{\alpha - 29.1}{\log(25)} = 20$$

$$\Rightarrow \alpha = 52.058 \text{ dB}$$

$$20 \log_{10} k = 52.058 \Rightarrow k = 400.811$$

Now, for B, C we have

$$\frac{0 - (-15.92)}{\log\left(\frac{\omega_2}{10}\right)} = 40 \Rightarrow \frac{15.92}{40} = \log\left(\frac{\omega_2}{10}\right)$$

$$\Rightarrow \omega_2 = 25 \text{ rad/sec}$$

Now, for C, we have

$$\frac{-15.92 - (-23.52)}{\log\left(\frac{\omega_3}{25}\right)} = 20$$

$$\omega_3 = 60 \text{ rad/sec}$$

Now, from above, we can conclude O.L.T.F.,

$$G(s) = \frac{(400.811) \left(1 + \frac{s}{25}\right)}{s \left(1 + \frac{s}{25}\right) \left(1 + \frac{s}{60}\right)}$$



Q.5 (b) A servo mechanism is represented by the equation :

$$\frac{d^2y}{dt^2} + 4.8 \frac{dy}{dt} = 144E$$

where  $E = C - 0.5y$  is the actuating signal. Find the value of damping ratio, damped, undamped frequency of oscillation. Draw the block diagram of the system described by the above equation.

[12 marks]

(Ans.)

$$\frac{d^2y}{dt^2} + 4.8 \frac{dy}{dt} = 144E \quad \text{--- (1)}$$

$$E = C - 0.5y \quad \text{--- (2)}$$

Putting (2) in (1), we have

$$\ddot{y} + 4.8\dot{y} = 144(C - 0.5y) = 144C - 72y$$

$$\ddot{y} + 4.8\dot{y} + 72y = 144C$$

⊗ Taking Laplace Transform, we have

$$s^2 Y(s) + 4.8sY(s) + 72Y(s) = 144C(s)$$

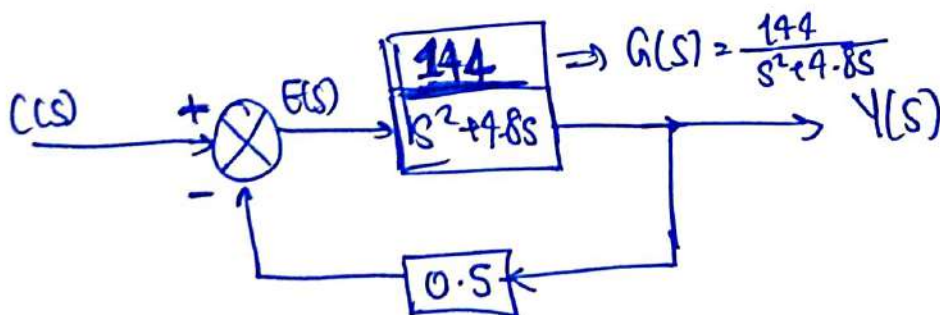
$$\frac{Y(s)}{C(s)} = \frac{144}{s^2 + 4.8s + 72} = \frac{k}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Now, we have on comparing,

$$\omega_n^2 = 72 \Rightarrow \omega_n = 6\sqrt{2} \text{ rad/sec.}$$

$$\Rightarrow 2\zeta(6\sqrt{2}) = 4.8 \Rightarrow \zeta = 0.2828.$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 6\sqrt{2} \sqrt{1 - (0.2828)^2} \Rightarrow \omega_d = 5.1386 \text{ rad/sec}$$





- c) Closed loop system with unity feedback has the forward loop transfer function as :

$$G(s) = \frac{28.8}{s(1 + 0.2s)}$$

Modify the design using cascaded compensation to satisfy the optimum performance criterion, so that the transient response to unit step input reaches its final steady state value in minimum time without having any overshoot. Take gain of proportional controller equal to 5.

[12 marks]



(d) A unity negative feedback system has open loop transfer function,  $G(s) = \frac{K}{s(1+sT)}$ , where  $K$  and  $T$  are positive constants. Determine the factor by which the amplifier gain  $K$  be reduced so that peak overshoot of the unit step response is reduced from 80% to 50%? [12 marks]

We have,  $G(s) = \frac{K}{s^2T + s}$

Now,  $1 + G(s)H(s) = 1 + \frac{K}{s^2T + s} = \frac{s^2T + s + K}{s^2T + s} = 0$

$\Rightarrow s^2 + \left(\frac{1}{T}\right)s + \left(\frac{K}{T}\right) = 0$

Comparing with  $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$ , we have  $\omega_n = \sqrt{\frac{K}{T}}$

$2\zeta\sqrt{\frac{K}{T}} = \frac{1}{T}$

$\Rightarrow \zeta = \frac{1}{2\sqrt{KT}}$

Now, we have initial overshoot = 80%

$\Rightarrow 0.8 = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$

$\Rightarrow \ln(0.8) = -\frac{\zeta\pi}{\sqrt{1-\zeta^2}} \Rightarrow 0.021 = \frac{\zeta}{\sqrt{1-\zeta^2}}$

$\Rightarrow 14.028\zeta = \sqrt{1-\zeta^2}$

$199.2125\zeta^2 = 1$

$\zeta = 0.02208$  (i)

Now, for overshoot to be 50%, we have

$e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.5$

$\Rightarrow \frac{\zeta\pi}{\sqrt{1-\zeta^2}} = \ln(0.5) \Rightarrow \frac{\zeta}{\sqrt{1-\zeta^2}} = 0.2206 \Rightarrow 4.532\zeta = \sqrt{1-\zeta^2}$

$20.5423\zeta^2 = 1 - \zeta^2$

$\zeta = 0.2154$  (ii)



Now, from (i), we have

$$r_1 = 0.0208 = \frac{1}{2\sqrt{k_1 T}} \quad \text{--- (i)}$$

from (ii), we have

$$r_2 = 0.2154 = \frac{1}{2\sqrt{k_2 T}} \quad \text{--- (ii)}$$

(i)  $\div$  (ii), we have

$$\frac{0.0208}{0.2154} = \sqrt{\frac{k_2}{k_1}}$$

$$\Rightarrow 0.108 = \frac{k_2}{k_1}$$

$$\Rightarrow k_2 = 0.108 k_1$$

$$\begin{aligned} \therefore \% \text{ reduction} &= \frac{|k_2 - k_1|}{k_1} \times 100\% \\ &= \frac{0.108 k_1 - k_1}{k_1} \times 100\% \end{aligned}$$

$\therefore$  there should be 89.196% reduction in  $k_1$ .

e) The open loop transfer function of a unity negative feedback system is given as,

$$G(s) = \frac{K}{2s(1+0.1s)(1+s)}$$

Determine the value of 'K' for which the gain margin of the system is 14 dB.

We have,  $G(j\omega) = \frac{k}{2j\omega(1+0.1j\omega)(1+j\omega)}$

[12 marks]

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(0.1\omega) - \tan^{-1}(\omega)$$

for  $\omega_{pc}$ ,

$$\angle G(j\omega)|_{\omega=\omega_{pc}} = -180^\circ$$

$$-90^\circ - \tan^{-1}(0.1\omega) - \tan^{-1}\omega = -180^\circ$$

$$\tan^{-1}(0.1\omega) = 90^\circ - \tan^{-1}\omega$$

$$\tan^{-1}(0.1\omega) = \tan^{-1}\left(\frac{1}{\omega}\right)$$

$$\Rightarrow \omega^2 = 10$$

$$\Rightarrow \omega = \sqrt{10}$$

$$\Rightarrow \omega_{pc} = 3.1623 \text{ rad/sec}$$

Now, for gain margin, we first find

$$|G(j\omega)|_{\omega=\omega_{pc}}$$

$$|G(j\omega)| = \frac{k}{2\omega \sqrt{1+0.01\omega^2} \sqrt{1+\omega^2}}$$

at  $\omega = \sqrt{10}$  rad/sec, we have

$$|G(j\omega)|_{\omega=\omega_c} = \frac{k}{2\sqrt{10} \sqrt{1+0.01k\omega} \times \sqrt{1+10}} \\ = \frac{k}{2\sqrt{10} \sqrt{1.1} \sqrt{11}} = \frac{k}{22}$$

$$|G(j\omega)|_{\omega=\omega_c} = K = \frac{k}{22}$$

$$\text{Now, G.M.} = 20 \log\left(\frac{1}{K}\right) \text{ (in dB)}$$

$$\Rightarrow 14 \text{ dB} = 20 \log\left(\frac{1}{K}\right)$$

$$\Rightarrow 0.7 = \log\left(\frac{1}{K}\right)$$

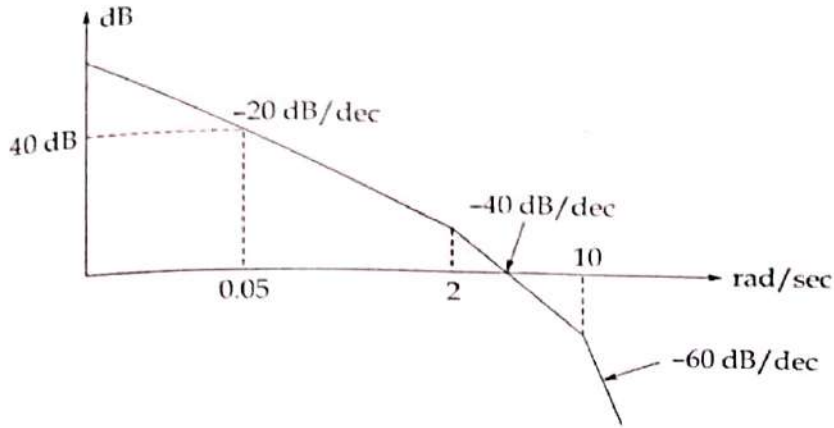
$$0.7 = \log\left(\frac{22}{k}\right)$$

$$\frac{22}{k} = 10^{0.7}$$

$$\Rightarrow k = 4.3895$$



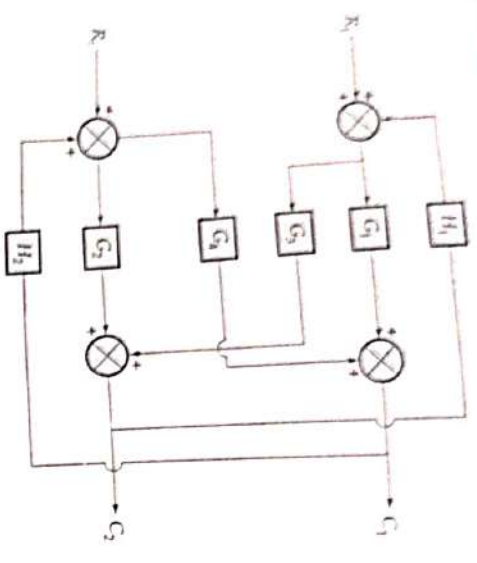
a) The open loop transfer function of a unity feedback system is given by  $G(s)H(s) = e^{-T}G_1(s)$ , where  $G_1(s)$  is minimum phase system. The approximate bode magnitude plot of the open loop transfer function is shown in the figure below. If the phase margin of the system is  $-18.19^\circ$ , determine the transportation lag  $T$ .



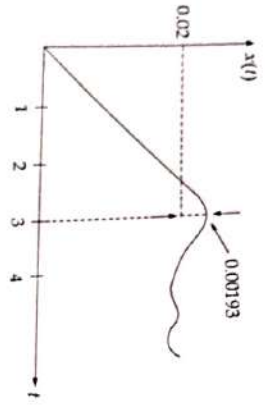
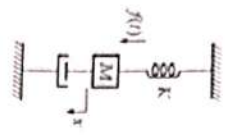
[20 marks]



Q6(b) (i) Evaluate  $\frac{C_2}{R_1}$  for the system whose block diagram representation is shown in Figure below. (Use block diagram reduction technique to solve).



(ii) Figure below shows a mechanical system and the response when 10 N of force is applied to the system. Determine the values of M, F, K. The dimension 'x' is in meter.



[10 + 10 marks]







Q 6 (c)

Derive the expression for the transfer function of an RC servomotor and obtain the same in respect of a servomotor having following data

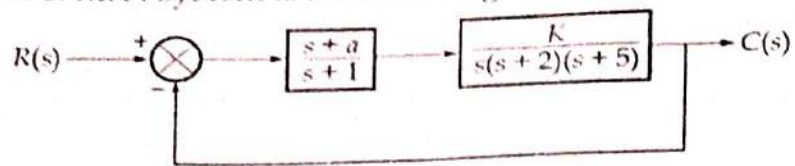
- (i) starting torque =  $0.146 \text{ N}\cdot\text{m}$
- (ii) Moment of inertia,  $J = 1 \times 10^{-4} \text{ kgm}^2$
- (iii) supply voltage =  $115 \text{ Volts}$
- (iv) No load speed =  $2004 \text{ rpm}$
- (Assume friction to be zero)

[15 + 5 = 20 marks]





- Q.7 (a) (i) A position control system is shown in figure below :



$K$  and  $a$  are the parameters of the system. Determine the range of  $K$  and  $a$  for which the system is stable.

- (ii) Sketch the root-locus of  $G(s) = \frac{K(s+1)}{s^2(s+2)}$ .

[10 + 10 marks]

(Ans.) (ii) We have,  $G(s) = \frac{K(s+1)}{s^2(s+2)}$

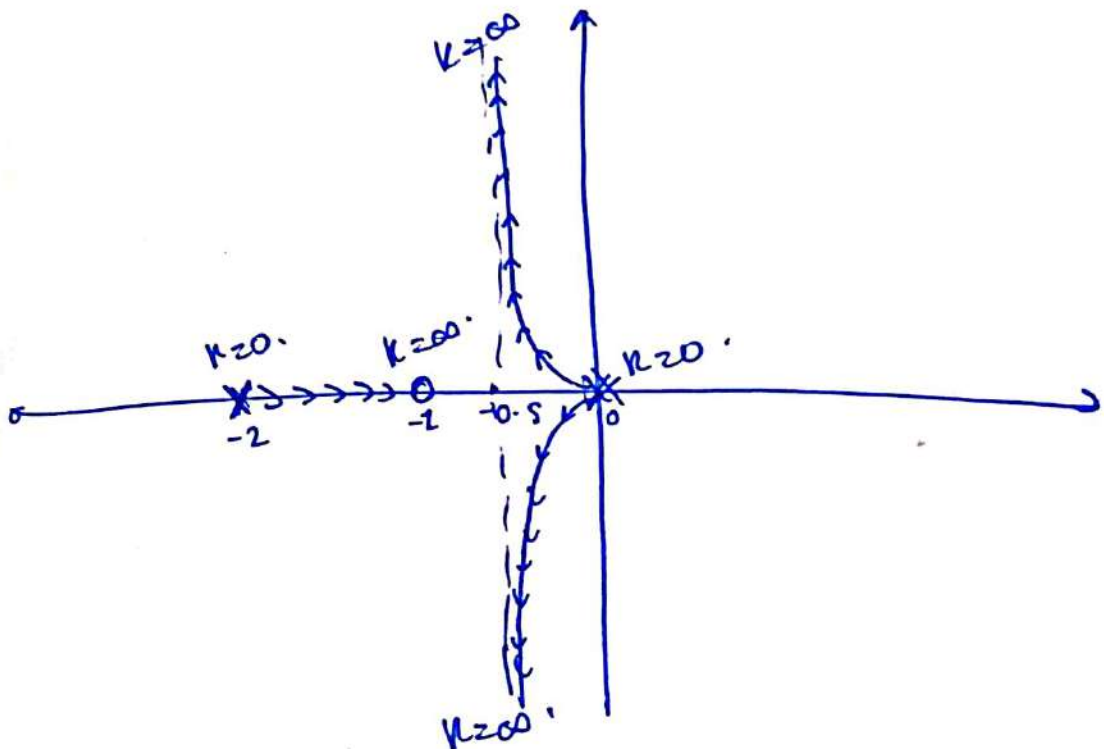
$$P \Rightarrow s = 0, 0, -2 \Rightarrow K = 0$$

$$Z \Rightarrow s = -1 \Rightarrow K = \infty$$

$P - Z = 2 \Rightarrow 2$  asymptotes.

$$\angle \phi_A = \frac{(2q+1) \cdot 180^\circ}{2} = 90^\circ, 270^\circ$$

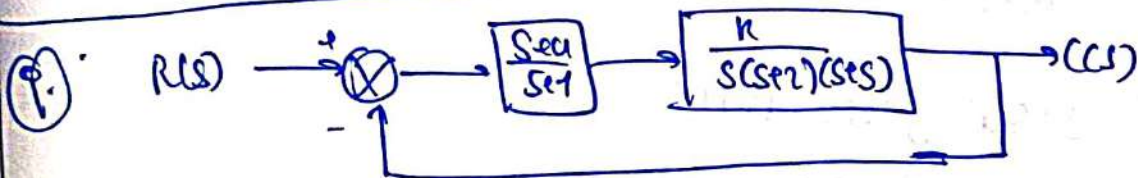
$$\text{Centroid} = \frac{\sum P - \sum Z}{P - Z} = \frac{(-2 - (-1))}{2} = -0.5$$



Root locus exists between  $-1$  &  $-2$  on real axis  $\infty$  of which  $\sum (P+Z)$  on Right side of  $\sigma$

Now, the root locus would depart to infinity from origin  $\infty$  of two real at origin  $\infty$

Centroid would be at  $-0.5$  & angles of asymptote would be  $90^\circ$  &  $270^\circ$ .



$$G(s) = \frac{(s+1)k}{s(s+1)(s+2)(s+5)}$$

$$1 + G(s)H(s) = 1 + \frac{(s+1)k}{s(s+1)(s+2)(s+5)}$$

$$= (s^2+5s)(s^2+3s+2) + k + ak$$

$$= s^4 + 3s^3 + 2s^2 + 5s^3 + 15s^2 + 10s + k + ak$$

Characteristic eq<sup>n</sup>  $= s^4 + 8s^3 + 17s^2 + (k+10)s + ak$



$$s^4 + 8s^3 + 17s^2 + (k+10)s + ak = 0$$

$s^4$	1	17	ak	
$s^3$	8	$(k+10)$	0	
$s^2$	$\frac{136-k-10}{8}$ $= \frac{126-k}{8}$	ak	0	
$s^1$	$\frac{(\frac{126-k}{8})(k+10) - 8ak}{(\frac{126-k}{8})}$	0	0	
$s^0$	ak			

From  $s^2$  row, we have

$$\frac{126-k}{8} > 0 \Rightarrow k < 126$$

From  $s^1$  row, we have

$$\frac{(\frac{126-k}{8})(k+10) - 8ak}{(\frac{126-k}{8})} > 0$$

$$\Rightarrow (126-k)(k+10) - 8ak > 0$$

$$126k + 1260 - k^2 - 10k - 8ak > 0$$

$$k^2 - 116k + 1260 < 0$$

$$k^2 + k(-116) + (-1260) < 0 \quad \text{--- (i)}$$

$$ak > 0 \Rightarrow \left. \begin{array}{l} k > 0 \ \& \ a > 0 \\ \text{or} \\ k < 0 \ \& \ a < 0 \end{array} \right\} \text{--- (ii)}$$



From (ii), we have

$$k^2 + k(a-116) + (-1260) > 0$$

∴  $D < 0$  (for system to be always positive)

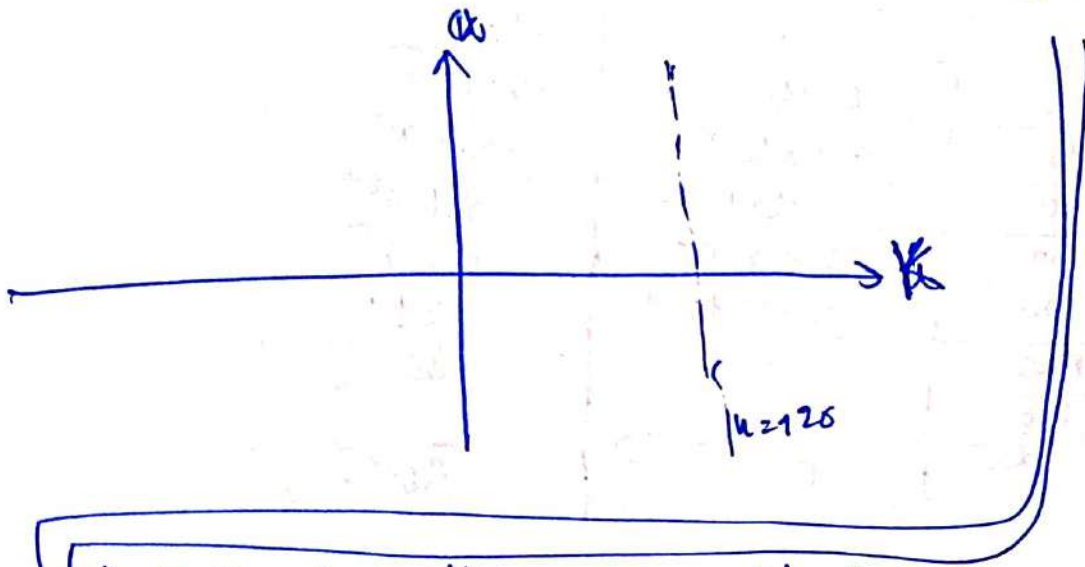
$$(a-116)^2 - 4(1)(-1260) < 0$$

$$(a-116)^2 + 5040 < 0$$

$$a^2 - 232a + 18496 < 0$$

~~True for no value of a.~~

∴  $4096a^2 - 14848a + 18496 < 0 \Rightarrow$  Not true for any value of a.



∴ The roots would always be negative hence the system can never be stable.

Q.7 (b) Sketch the polar plot of the transfer function given below. Determine whether the plot crosses the real axis. If so, determine the frequency at which the plot crosses the real axis and the corresponding magnitude  $|G(j\omega)|$ .

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

(Ans.)

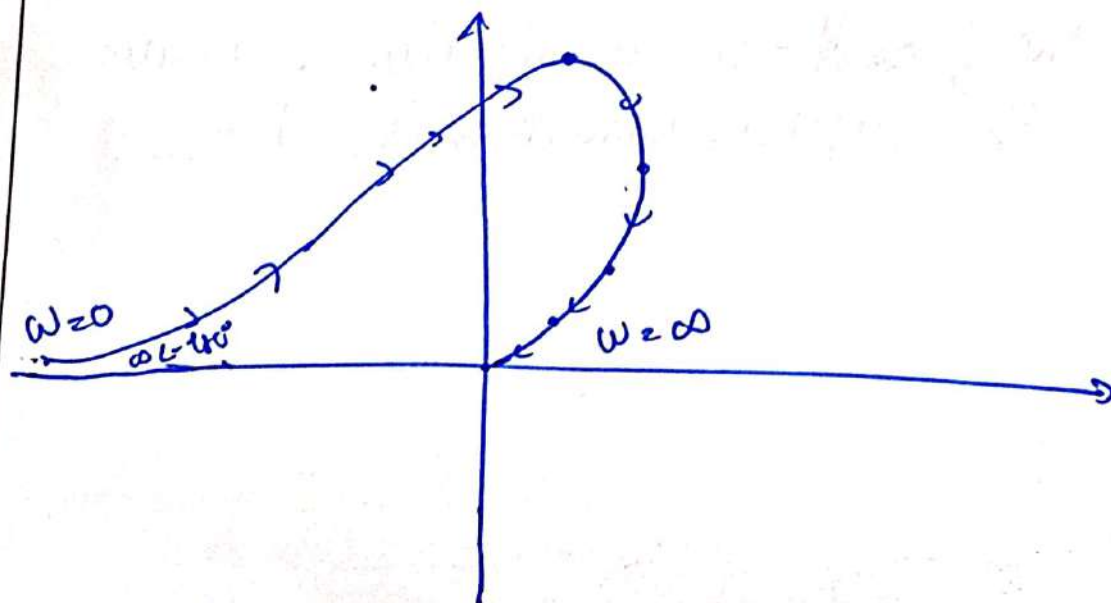
we have,  $G(s) = \frac{1}{s^2(1+s)(1+2s)}$

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+2j\omega)}$$

$$|G(j\omega)| = \frac{1}{\omega^2 \sqrt{1+\omega^2} \sqrt{1+4\omega^2}}$$

$$\angle G(j\omega) = -180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

$\omega$	$ G(j\omega) $	$\angle G(j\omega)$
0	$\infty$	$-180^\circ$
1	0.3162	$-288.4^\circ$
5	$2.8 \times 10^{-4}$	$-392.9^\circ$
10	$4.9 \times 10^{-5}$	$-381.4^\circ$
50	$2.9 \times 10^{-8}$	$-388.2^\circ$
100	$4.9 \times 10^{-9}$	$-359.1^\circ$
<del>500</del>	0	$-360^\circ$
<del>1000</del>	<del>0</del>	<del><math>360^\circ</math></del>





$$R(j\omega) = \frac{-1}{\omega^2(1-2\omega^2+3j\omega)} = \frac{-1(1-2\omega^2-3j\omega)}{\omega^2(1-2\omega^2+3j\omega)(1-2\omega^2-3j\omega)}$$

$$C(j\omega) = \frac{-1(1-2\omega^2-3j\omega)}{\omega^2((1-2\omega^2)^2+9\omega^2)}$$

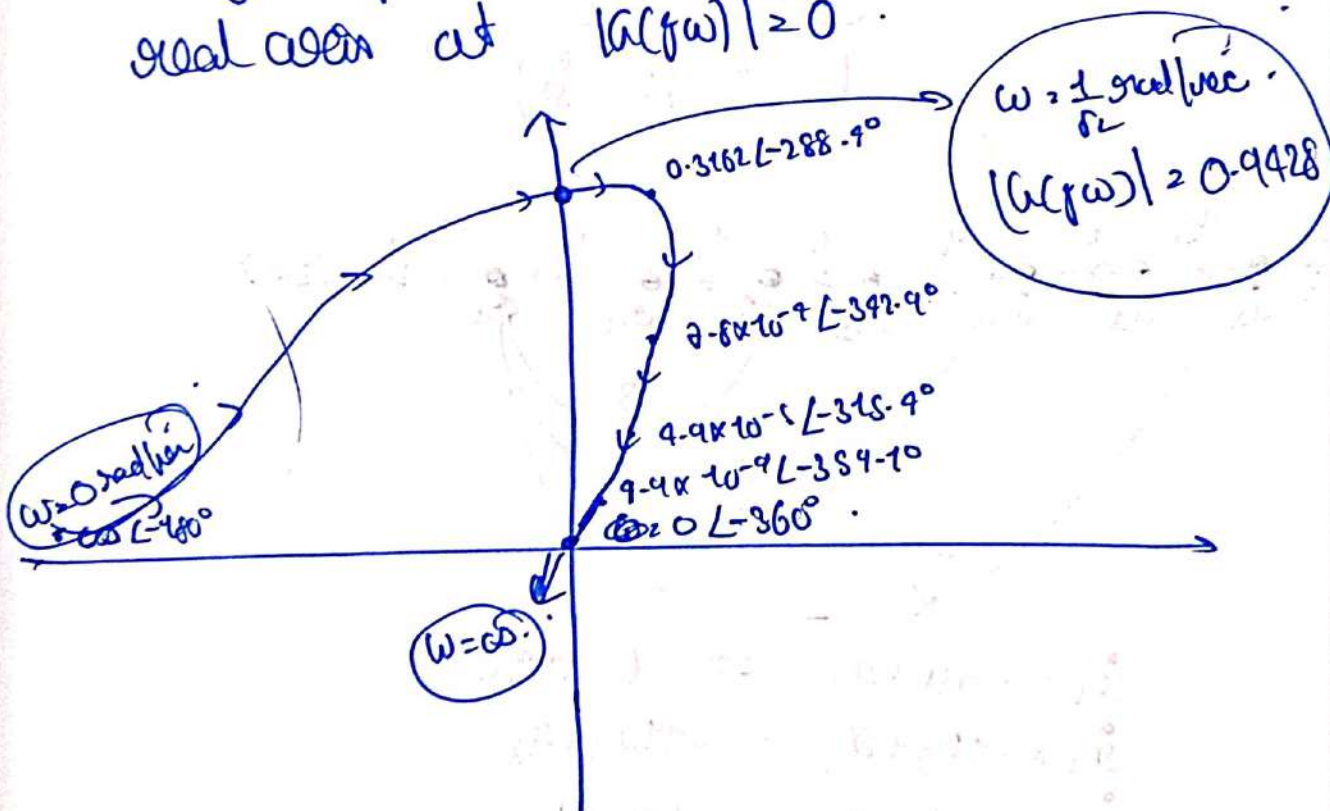
$$= \frac{(2\omega^2-1)+j(3\omega)}{\omega^2(4\omega^4+8\omega^2+1)}$$

Now, into cross real axis, imaginary part is

$$\Rightarrow \frac{3\omega}{\omega^2(4\omega^4+8\omega^2+1)} = 0$$

$$\Rightarrow \frac{3}{\omega(4\omega^4+8\omega^2+1)} = 0$$

$\therefore$  it is possible to cross real axis at  $\omega = \infty$  only as per equation. hence it meets real axis at  $|C(j\omega)| \geq 0$ .





Q.7 (c) Construct the state model for a system characterised by the differential equation :

$$\frac{d^3 y}{dt^3} + \frac{6d^2 y}{dt^2} + \frac{11dy}{dt} + 6y = u$$

Give the block diagram representation of the state model.

[15 + 5 = 20 mar

(Ans.)

We have,  $\ddot{y} + 6\dot{y} + 11y + 6y = u$

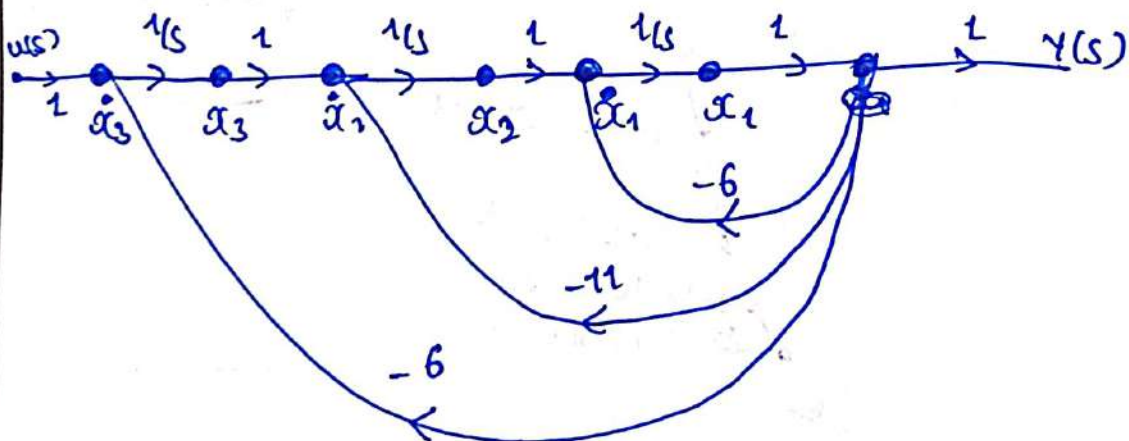
Now taking Laplace transform, we have

$$(s^3 + 6s^2 + 11s + 6)Y(s) = U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^3 + 6s^2 + 11s + 6}$$

$$= \frac{1/s^3}{1 + \frac{6}{s} + \frac{11}{s^2} + \frac{6}{s^3}} = \frac{1/s^3}{1 - \left(\frac{6}{s} - \frac{11}{s^2} - \frac{6}{s^3}\right)}$$

$$= \frac{\left(\frac{1}{s^3}\right)}{1 - \left(\frac{6}{s} - \frac{11}{s^2} - \frac{6}{s^3}\right)}$$



$$\dot{x}_1 = -6y + x_2 \Rightarrow -6x_1 + x_2$$

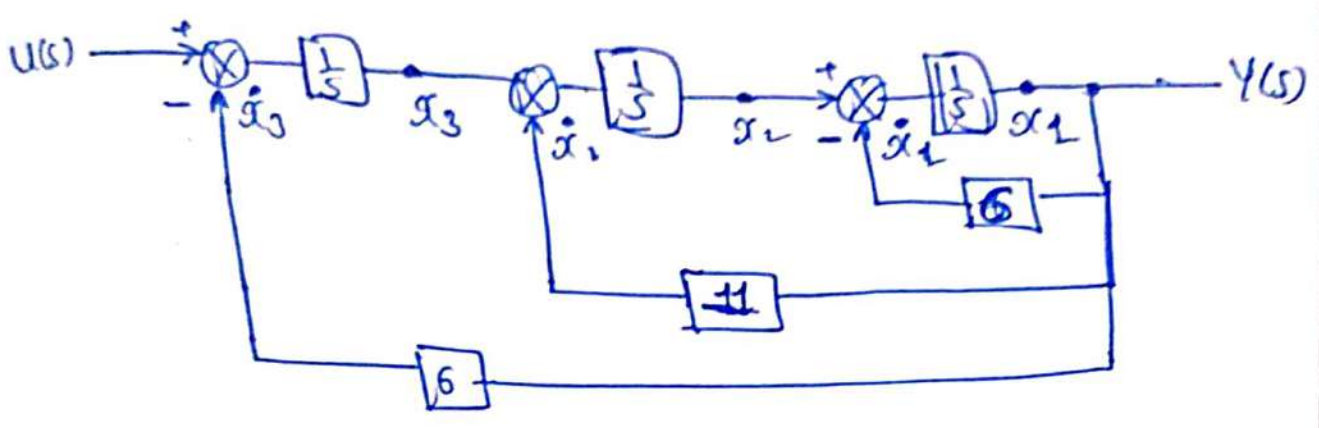
$$\dot{x}_2 = -11y + x_3 = -11x_1 + x_3$$

$$\dot{x}_3 = -6y + u = -6x_1 + u$$

$$y = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -6 & 1 & 0 \\ -11 & 0 & 1 \\ -6 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



Q.10 (a) The square has a side length of 4 units. Find the area of the square.

Area =  $4 \times 4 = 16$

(b) The side length of the square is 5 units. Find the area of the square.

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The open loop transfer function of a feedback control system is

$$G(s)H(s) = \frac{K(s+2)}{(s+1)(s+2+j)(s+2-j)}$$

Find the range of gain  $K$  for stability. Find the value of  $K$  for the system to have a gain margin of 3 dB. With this value of  $K$ , find the gain cross over frequency and phase margin. Use Nyquist Approach.

[20 marks]



(a) The state space model of a second order system given below is designed using feedback control system.

$$\dot{x} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

- (i) What are the conditions for the desired response? Also check whether desired response is possible or not.
- (ii) Design an observer system such that the above system has settling time of 0.5 sec and damping frequency of 6 rad/sec.

[3 + 12 marks]



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Space for Rough Work

Space for Rough Work

Space for Rough Work