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ESE 2022

Main Exam Detailed Solutions

Electrical Engineering

PAPER-I

EXAM DATE : 26-06-2022 | 9:00 AM to 12:00 PM

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ANALYSIS

Electrical Engineering Paper-I
ESE 2022 Main Examination

Sl.	Subjects	Marks
1.	Electric Circuits	44
2.	Electromagnetic Fields	52
3.	Electrical Materials	64
4.	Engineering Mathematics	72
5.	Basic Electronics Engineering	82
6.	Computer Fundamental	64
7.	Electrical and Electronic Measurements	102
	Total	**480**

Scroll down for detailed solutions

Section-A

Q.1 (a) Find the eigen values and the corresponding eigen vectors of the matrix

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & 2 \\ -2 & 2 & 6 \end{bmatrix}$$. Is this matrix diagonalizable? Justify your answer.

[12 marks : 2022]

Solution:

Let $$A = \begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & 2 \\ -2 & 2 & 6 \end{bmatrix}$$

The characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & -1 & -2 \\ -1 & 3-\lambda & 2 \\ -2 & 2 & 6-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)[(3-\lambda)(6-\lambda)-4] - (-1)[-1(6-\lambda) - 2\times(-2)] - 2[-1\times 2 - (-2)\times(3-\lambda)] = 0$$

$$(3-\lambda)[18 - 3\lambda - 6\lambda + \lambda^2 - 4] + [\lambda - 6 + 4] - 2[-2 + 6 - 2\lambda] = 0$$

$$(3-\lambda)[\lambda^2 - 9\lambda + 14] + \lambda - 2 + 4\lambda - 8 = 0$$

$$3\lambda^2 - 27\lambda + 42 - \lambda^3 + 9\lambda^2 - 14\lambda + 5\lambda - 10 = 0$$

$$-\lambda^3 + 12\lambda^2 - 36\lambda + 32 = 0$$

$$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$$

$$\lambda = 8, 2, 2$$

Hence, eigen values are 8, 2, 2.

If X, Y, Z be the components of an eigen vector corresponding to the eigen value λ, we have

$$[A - \lambda I]\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = 0$$

For $\lambda = 8$,

$$\begin{bmatrix} 3-8 & -1 & -2 \\ -1 & 3-8 & 2 \\ -2 & 2 & 6-8 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = 0$$

$$\begin{bmatrix} -5 & -1 & -2 \\ -1 & -5 & 2 \\ -2 & 2 & -2 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = 0$$

$$-5X - Y - 2Z = 0$$

$$-2X + 2Y - 2Z = 0$$

$$\frac{X}{2+4} = \frac{Y}{-6} = \frac{Z}{-12} = K$$

$$\frac{X}{6} = \frac{Y}{-6} = \frac{Z}{-12} = K$$

$$\frac{X}{1} = \frac{Y}{-1} = \frac{Z}{-2} = K$$

$$\text{Eigen vector} \equiv \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

For $\lambda = 2$

$$\begin{bmatrix} 3-2 & -1 & -2 \\ -1 & 3-2 & 2 \\ -2 & 2 & 6-2 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ -2 & 2 & 4 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = 0$$

$$X - Y - 2Z = 0$$

$$-X + Y + 2Z = 0$$

$$X - Y - 2Z = 0 \Rightarrow \text{Only one independent equation}$$

Let $Y = K_1$ and $Z = K_2$

$$\text{Vector} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} K_1 + 2K_2 \\ K_1 \\ K_2 \end{bmatrix}$$

$$= K_1\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + K_2\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

Hence, $$\text{Eigen vector} \equiv \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

Necessary and sufficient condition for square matrix A to be diagonalized is that all the eigen vectors of A must be linearly independent.

Since, $$\begin{vmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{vmatrix} \neq 0$$

Here, all eigen vectors are linear independent. Hence, matrix are diagonalized.

End of Solution

Q.1 (b) Classify the ways in which materials respond to magnetic fields and also specify their value of susceptibility.

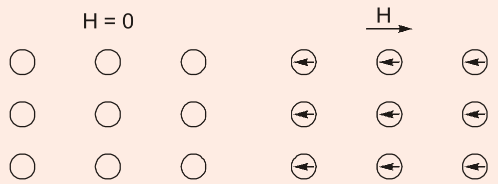
[12 marks : 2022]

Solution:

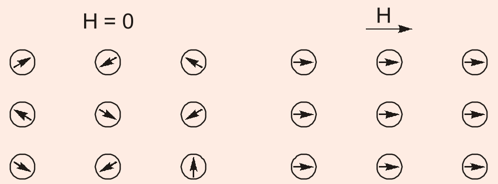
Magnetic materials are classified into following categories on the basis of their response of magnetic field applied externally.

- Diamagnetic materials
- Paramagnetic materials
- Ferromagetic materials
- Antiferromagnetic materials
- Ferrimagnetic materials

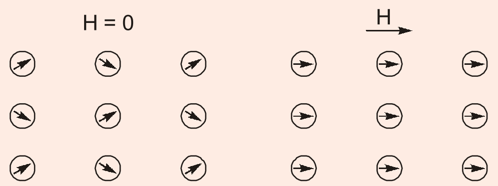
Diamagnetic Materials : They get feebly magnetised opposite to externally applied magnetic field as shown below. The value of magnetic susceptibility, χ_m for diamagnetic material is small negative of order of 10^{-8} and it is independent of temerature.



Paramagnetic Materials : These materials get feebly magnetised in the same direction of applied external magnetic field as shown below. The value of magnetic susceptibility is small positive of order of 10^{-5} to 10^{-2}. Magnetic susceptibility is inversely proportional to temperature.



Ferromagnetic Materials : These materials get strongly magnetised in the applied external field direction as shown below. The value of magnetic susceptibility is positive and very high.



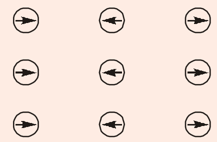
The variation of magnetic susceptibility with temperature for these material is represented in below :

$$\chi_m = \frac{C}{T - T_C}$$

where T_C is critical temperature.

Above T_C ferromagnetic materials become paramagnetic.

Antiferromagnetic Materials : In these materials magnetic moments are aligned antiparallel to each other resulting in a net zero magnetic dipole as shown below.



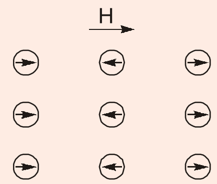
Magnetic susceptibility of antiferromagnetic material varies according to below relation

$$\chi = \frac{C}{T + T_N}$$

where T_N is Neel temperature.

Above Neel temperature antiferromagnetic material become paramagnetic.

Ferrimagnetic Materials : These materials consists of ferromagnetic parallel spins combined with some antiparallel spins such that net magnetic dipole is not cancelled to zero as in case of antiferromagnetic material which is shown below.



Magnetic susceptibility of ferrimagnetic material is positive, but its magnitude varies with kind of ordering and temperature.

End of Solution

Q.1 (c) The material parameters of a certain food item are given by σ = 2.2 s/m, $\epsilon = 48\epsilon_0$ and $\mu = \mu_0$ at the operating frequency, f = 2.54 GHz of a microwave oven. Determine α, β, γ, v_p and $\bar{\eta}$. Take $\epsilon_0 = \frac{10^{-9}}{36\pi}$ and $\mu_0 = 4\pi \times 10^{-7}$.

[12 marks : 2022]

Solution:

Now for any material

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

Given the values :

$$\gamma = \sqrt{j(2\pi \times 2.54 \times 10^9)(4\pi \times 10^{-7})(2.2 + j \times (2\pi \times 2.54 \times 10^9)(48)\left(\frac{10^{-9}}{36\pi}\right)}$$

$$\gamma = \sqrt{142825.15\angle 162.00}$$

$$\gamma = 377.922\angle 81° = 59.10 + j373.27 = \alpha + j\beta$$

So, α = 59.10 Nep/m, β = 373.27 rad/m

Now, $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{373.27} = 0.0168$ m

$$V_P = \frac{1}{\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu_o \times 48\varepsilon_o}}$$

$$V_P = \frac{1}{\sqrt{4\pi\times 10^{-7} \times 48 \times \frac{10^{-9}}{36\pi}}} = 4.33 \times 10^7 \text{ m/s}$$

Now, for $\bar{\eta} = \left|\sqrt{\frac{j\omega\mu}{(\sigma + j\omega \in)}}\right|$

$$\bar{\eta} = \left|\sqrt{\frac{200.55\times 10^2 j}{(2.2+6.773)}}\right| = \left|\sqrt{2817.31\angle 18.00}\right|$$

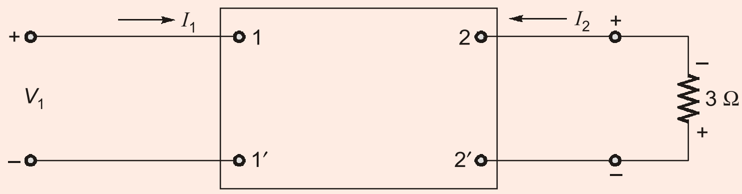
$$\bar{\eta} = \sqrt{2817.31} = 53.08$$

End of Solution

Q.1 (d) The following equations refer to a two-port network

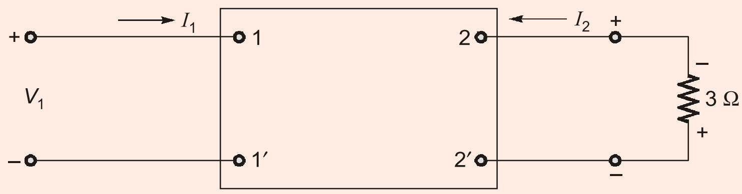
$$V_1 = 5I_1 + 2I_2$$
$$V_2 = 2I_1 + I_2$$

A load resistance of 3 ohms is connected across port-2 terminals as shown in figure. Calculate the input Impedance.



[12 marks : 2022]

Solution:



$$V_1 = 5I_1 + 2I_2 \quad ...(i)$$
$$V_2 = 2I_1 + I_2 \quad ...(ii)$$

As a load resistance of 3 Ω is connected across port-2

$$V_2 = -V_L = 3I_2$$
$$V_2 = -3I_2$$

Put the value of V_2 in eqn. (ii)

$$-3I_2 = 2I_1 + I_2$$

$$-4I_2 = 2I_1$$

$$I_2 = -\frac{1}{2}I_1$$

Now put the value of I_2 in eqn. (i)

$$V_1 = 5I_1 + 2 \times \left(-\frac{1}{2}I_1\right)$$

$$= 5I_1 - I_1 = 4I_1$$

Input impedance, $\frac{V_1}{I_1} = 4\ \Omega$

Hence, Input impedance $= 4\ \Omega$

End of Solution

Q.1 (e) **Write a program in any programming language to compute the money accumulated in a cumulative fixed deposit of a bank for the principal *P* after *n* years. The rate of interest is *r* percent per year compounded annually. The program should read the values of *P*, *r* and *n*. It should display the accumulated money as *F*.**

[12 marks : 2022]

Solution:

The bank compounds the money anually, i.e., the fixed deposit principal amount, i.e., P will be compounded at a interest rate *r* for *n* years to accumulate the amount *F*.

Hence, the formula used, $F = P\left(1 + \frac{r}{100}\right)^n$

The C-program to calculate *F* is given below :

```
# include <stdio.h>
# include <math.h> // for using pow, i.e., power function
void main ( )
{
float P, r, n;
float F,
    printf {"\n Enter the principal amount");
    scanf("%f", &P)
    printf("\n Enter the interest rate");
    scanf("%f", %r);
    printf("\n Eter the number of years for which the amount will be compounded");
    scanf("%f", &n);
    F = P*C pow((1 + r/100), n))
    printf ("\n The accumulated moey is % f", F);
}
```

End of Solution

Q.2 (a) (i) Use the residue theorem to evaluate the integral $\int_C \frac{e^z - 1}{z(z-1)^2(z-i)} dz$, where C is the circle $|z| = 3$.

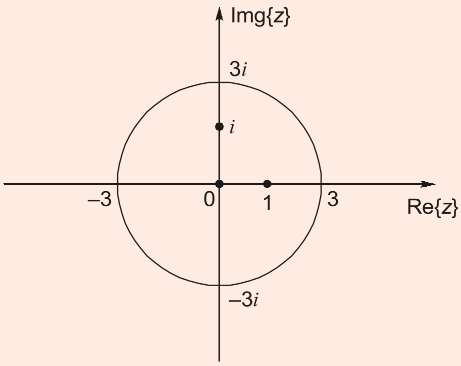
(ii) Find the volume of the solid bounded by the sphere $x^2 + y^2 + z^2 = 20$ and the paraboloid $x^2 + y^2 = z$.

[10 + 10 marks : 2022]

Solution:

(i) Given : Integral $\quad I = \int_C \frac{e^z - 1}{z(z-1)^2(z-i)}, \ |z| = 3$

Singularity points are at $z = 0, 1, i$ and all lie inside the contour.



Residue at $z = 0$, i.e., R_1

$$R_1 = \lim_{z\to 0} z f(z)$$

where, $\quad f(z) = \frac{e^z - 1}{z(z-1)^2(z-i)}$

$$\Rightarrow \quad R_1 = \lim_{z\to 0} z \cdot \frac{e^z - 1}{z(z-1)^2(z-i)}$$

$$\Rightarrow \quad R_1 = 0$$

Residue at $z = 1$, i.e., R_2

$$R_2 = \frac{1}{(m-1)!} \lim_{z\to z_0} \frac{d^{m-1}}{dz^{m-1}} (z - z_0)^m f(z)$$

where m is the order of singularity.

$$\Rightarrow \quad R_2 = \frac{1}{(2-1)!} \lim_{z\to 1} \frac{d^{2-1}}{dz^{2-1}} (z-1)^2 \frac{(e^z - 1)}{z(z-1)^2(z-i)}$$

$$\Rightarrow \quad R_2 = \lim_{z\to 1} \frac{d}{dz}\left[\frac{e^z - 1}{z^2 - iz}\right]$$

$$= \frac{e^z(z^2 - iz) - (e^z - 1)(2z - i)}{(z^2 - iz)^2}$$

$$R_2 = \frac{e(1-i) - (e-1)(2-i)}{(1-i)^2}$$

$$\Rightarrow \quad R_2 = 0.5 - 0.359i$$

Residue for $z = i$, i.e., R_3

$$R_3 = \lim_{z \to i}(z - i)f(z)$$

$$R_3 = \lim_{z \to i}(z - i)\frac{e^z - 1}{z(z-1)^2(z-i)}$$

$$\Rightarrow \quad R_3 = \frac{1}{2}[e^i - 1]$$

So, from Cauchy's Integral formula,

$$I = 2\pi i[R_1 + R_2 + R_3]$$

$$\Rightarrow \quad I = 2\pi i[0.5 - 0.359i + 0.5e^i - 0.5]$$

$$\Rightarrow \quad I = 2\pi i[0.5 \cos 1 + i\, 0.5 \sin 1 - 0.359i]$$

$$I = -0.388 + 1.6974i$$

(ii)
$$V = \iiint dz dy dz$$

Domain bounded by $x^2 + y^2 + z^2 = 20$ and $x^2 + y^2 = z$

In cylindrical coordinates, there are

$$\rho^2 = z \text{ and } \rho^2 + z^2 = 20$$

Solving we get, $z = 4 \Rightarrow \rho^2 = 4 \Rightarrow \rho = 2$

So,
$$V = \iiint \rho\, d\rho\, d\phi\, dz$$

$$V = \int_0^{2\pi} d\phi \int_0^2 \rho \left(\int_{\rho^2}^{\sqrt{20-\rho^2}} dz \right) d\rho$$

$$V = 2\pi \int_0^2 \rho\left(\sqrt{20 - \rho^2} - \rho^2\right) d\rho$$

On solving, $V = 28.15$ cubic unit

End of Solution

Q.2 (b) (i) Explain the variation of transition temperature of a superconductor with its isotopic mass *M*.

(ii) The transition temperature of Mercury with an average atomic mass of 200.6 is 4.2 K. Evaluate the transition temperature of one of its isotopes, ${}_{80}Hg^{204}$.

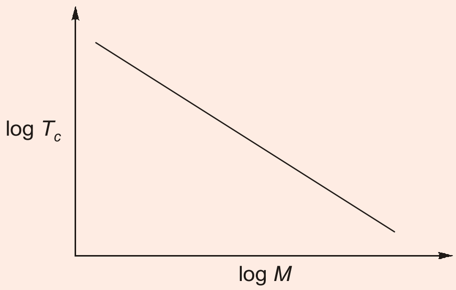
[8 + 12 marks : 2022]

Solution:

(i) According to isotopic effect, "The critical temperature T_c of a superconductor varies inversely with the square root of the isotpic mass, i.e., M. The relation can be given by

$$T_c M^{1/2} = \text{Constant}$$

$$\Rightarrow \quad T_c \propto \frac{1}{M^{1/2}}$$



This isotopic effect was discovered in 1950 by Maxwell and Reynolds while they were obtaining critical temperature of different isotoes of Mercury (Hg).

(ii) Given : Average atomic mass, $M_1 = 200.6$
Critical temperature, $T_{c1} = 4.2$ K
Isotopic mass of mercury, $M_2 = 204$
Let the critical temperature for $_{80}Hg^{204}$ be T_{c_2} in K, then

$$T_c \propto \frac{1}{M^{1/2}}$$

where T_c is critical temperlature, M is isotopic mass.

$$\Rightarrow \quad \frac{T_{c_1}}{T_{c_2}} = \sqrt{\frac{M_2}{M_1}}$$

$$\Rightarrow \quad T_{c_2} = T_{c_1}\sqrt{\frac{M_1}{M_2}}$$

$$\Rightarrow \quad T_{c_2} = 4.2\sqrt{\frac{200.6}{204}}$$

$$\Rightarrow \quad T_{c_2} = 4.165 \text{ K}$$

End of Solution

Q.2 (c) (i) A 10 HP induction motor runs from a 3-phase 400 volts supply on no-load. The motor takes a line current of 4 A at a power factor of 0.208 (lag). On full load it operates at power factor of 0.88 (lag) and with an efficiency of 89%. Determine the readings on each of the two watt-meters connected to read total power (1) on no-load and (2) on full-load.

(ii) What is a filter? State the ideal characteristics of a filter.

[10 + 10 marks : 2022]

Solution:

(i) Given : $V_L = 400\text{ V}$

(1) At no load condition :

$$\phi = \cos^{-1}(0.208) = 78°$$

Motor input power, $P_{in} = \sqrt{3}V_L I_L \cos\phi$

$$= \sqrt{3} \times 400 \times 4 \times 0.208$$

$$= 576.42\text{ Watt}$$

In two wattmeter method,

Power measured by wattmeter W_1 is,

$$P_1 = V_L I_L \cos(30 - \phi)$$

$$P_1 = 400 \times 4 \times \cos(30° - 78°)$$

$$P_1 = 1070.60\text{ Watt}$$

Similarly reading of second wattmeter (W_2)

$$P_2 = V_L I_L \cos(30° + \phi)$$

$$= 400 \times 4 \times \cos(30° + 78°)$$

$$= -494.42\text{ Watt}$$

Therefore, $P_1 = 1070.60\text{ Watts}$

$$P_2 = -494.42\text{ Watts}$$

(2) At full load condition :

$$P_{in} = \frac{P_o}{\eta} = \frac{10 \times 746}{0.89} = 8382\text{ Watts}$$

As we know, $\sqrt{3}V_L I_L \cos\phi = P_{in}$

Line current, $I_L = \dfrac{8382}{400\sqrt{3} \times 0.88} = 13.74\text{ A}$

Full load power factor angle,

$$\phi = \cos^{-1}(0.88) = 28.35°$$

Reading of first wattmeter (W_1)

$$P_1 = V_L I_L \cos(30° - \phi)$$

$$= 400 \times 13.74 \times \cos(30° - 28.35°)$$

$$P_1 = 5493.72\text{ Watts}$$

Similarly reading of second wattmeter (W_2)

$$P_2 = V_L I_L \cos(30° + \phi)$$

$$= 400 \times 13.74 \times \cos(30° + 28.35°)$$

$$P_2 = 2883.91\text{ Watts}$$

Therefore, $P_1 = 5493.72\text{ Watts}$

$$P_2 = 2883.91\text{ Watts}$$

(ii) A filter is a electronic circuit that removes any unwanted component or features from a signal. In other words, it is a circuit that rejects certain band of frequencies and allows others to pass through it. Filters are mainly classified into two categories :

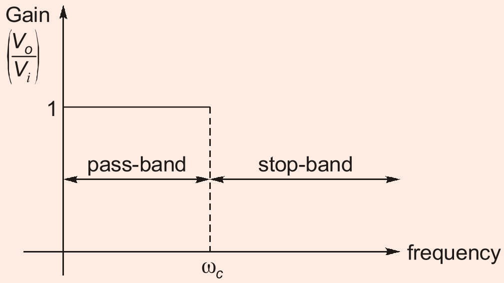
(1) Active filter; (2) Passive filter

Active Filters : This filter circuit consists of active components like transistors and op-amps in addition to RLC components.

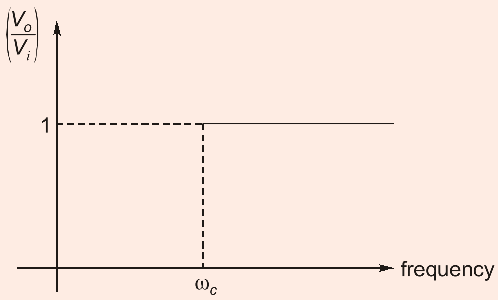
Passive Filters : These filters only consists of passive element (RLC).

Filters are further categorized based on the operating frequencies of a particular circuit.

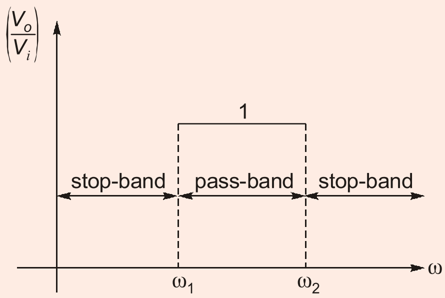
(1) **Low Pass Filter :** This filter attenuates all the frequencies above the cut-off frequencies. Ideal low pass filter provides a constant gain from zero to cut-off frequency.



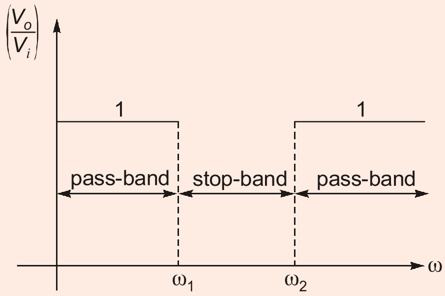
(2) **High Pass Filter :** Filter which attenuates all the frequency components below the cut-off frequency.



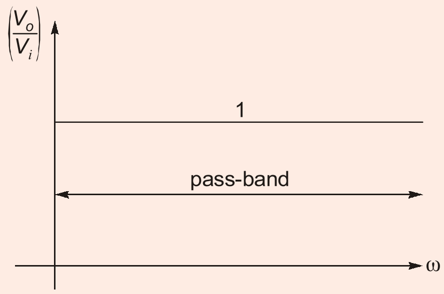
(3) **Band Pass Filters :** This filter pass all the frequency components in a particular band of frequencies and reject all the frequency components outside the pass band.



(4) **Band Reject Filters :** This filter reject all the frequency components lies in a particular band of frequencies and pass all other frequency components.



(5) **All Pass Filters :** This filter pass all frequency components of a signal. It is also known as phase-shifter, time-delay filter.



End of Solution

Q.3 (a) (i) Obtain the equations of regression lines and the correlation coefficient for the following data. Justify the sign of correlation coefficient.

x	1	2	3	4	5
y	2	5	3	8	7

(ii) The probability density function of a random variable x is

$$f(x) = \begin{cases} \alpha x(2-x), & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

for some real number α. Find the value of α and the variance of x.

[10 + 10 marks : 2022]

Solution:

(i) Given data :

X	Y	$(X-\bar{X})^2$	$(Y-\bar{Y})^2$	$(X-\bar{X})(Y-\bar{Y})$
1	2	4	9	6
2	5	1	0	0
3	3	0	4	0
4	8	1	9	3
5	7	4	4	4
$\Sigma X_i = 15$	$\Sigma Y_i = 25$	$\Sigma(X-\bar{X})^2 = 10$	$\Sigma(Y-\bar{Y})^2 = 26$	13

$$\bar{X} = \frac{\Sigma x_i}{n} = \frac{15}{5} = 3$$

$$\bar{Y} = \frac{\Sigma y_i}{n} = \frac{25}{5} = 5$$

$$\sigma_X^2 = \frac{\Sigma(X-\bar{X})^2}{n} = \frac{10}{5} = 2$$

$$\sigma_Y^2 = \frac{\Sigma(Y-\bar{Y})^2}{n} = \frac{26}{5} = 5.2$$

Co-variance, $$(X, Y) = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\eta} = \frac{13}{5} = 2.6$$

$\therefore$ $$\gamma = \frac{\text{Co-variance}(X,Y)}{\sigma_X \sigma_Y} = \frac{2.6}{\sqrt{2}\sqrt{5.2}} = 0.806$$

Regression coefficient of Y on X

$$= b_{yx} = \frac{\gamma\sigma_y}{\sigma_x}$$

$$= \frac{0.806 \times \sqrt{5.2}}{\sqrt{2}} = 1.3$$

$\therefore$ Line of regression of Y on X is

$$(y-\bar{y}) = b_{yx}(x-\bar{x})$$

$$(y-5) = 1.3(x-3)$$

$\Rightarrow$ $$1.3x - y + 1.1 = 0 \quad ...(1)$$

and line of regression of X on Y is

$$(x-\bar{x}) = b_{xy}(y-\bar{y})$$

$$(x-3) = \frac{\gamma\sigma_x}{\sigma_y}(y-5)$$

$$(x-3) = \frac{0.806 \times \sqrt{2}}{\sqrt{5.2}}(y-5)$$

$$(x-3) = 0.5(y-5)$$

$\Rightarrow$ $$x - \frac{y}{2} - \frac{1}{2} = 0 \quad ...(2)$$

Both (1), (2) are of regression lines.

$\because$ Co-variance(X, Y) is positive, so that γ is also positive.

(ii) $$f(x) = \begin{cases} \alpha x(2-x), & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

As $f(x)$ is probability density function

$$\int_{-\alpha}^{\alpha} f(x)dx = 1$$

$$\text{Now,} \quad \int_{x=0}^{x=2} f(x)dx = 1$$

$$\int_{x=0}^{x=2} \alpha x(2-x)dx = 1$$

$$\int_{x=0}^{x=2} (2\alpha x - \alpha x^2) = 1$$

$$\left[2\alpha\frac{x^2}{2} - \alpha\frac{x^3}{3}\right]_{x=0}^{x=2} = 1$$

$$4\alpha - \frac{8\alpha}{3} = 1$$

$$\alpha = \frac{3}{4} = 0.75$$

$$f(x) = \frac{3}{4}x(2-x) = \frac{3}{2}x - \frac{3}{4}x^2$$

$$\text{Mean} = E(x) = \int_{x=0}^{x=2} xf(x)dx$$

$$= \int_{x=0}^{x=2} x\left(\frac{3}{2}x - \frac{3}{4}x^2\right)dx = \int_{x=0}^{x=2}\left(\frac{3}{2}x^2 - \frac{3}{4}x^3\right)dx$$

$$= \left[\frac{3}{2}\frac{x^3}{3} - \frac{3}{4}\times\frac{x^4}{4}\right]_0^2$$

$$= 1$$

$$E(x^2) = \int_{x=0}^{x=2} x^2 f(x)dx$$

$$= \int_{x=0}^{x=2} x^2\left(\frac{3}{2}x - \frac{3}{4}x^2\right)dx$$

$$= \int_{x=0}^{x=2}\left(\frac{3}{2}x^3 - \frac{3}{4}x^4\right)dx$$

$$= \left[\frac{3}{2}\times\frac{x^4}{4} - \frac{3}{4}\times\frac{x^5}{5}\right]_{x=0}^{x=2}$$

$$= 1.2$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= 1.2 - (1)^2 = 0.2$$

End of Solution

Q.3 (b) (i) Evaluate value of frequency at which conduction current density and displacement current density are equal

(a) distilled water, where, $\sigma = 2.0 \times 10^{-4}$ s/m and $\epsilon_r = 81$.

(b) sea water where $\sigma = 4.0$ s/m and $\epsilon_r = 1$.

[10 marks : 2022]

(ii) In spherical co-ordinates $V = -35$ V on a conductor at $r = 3$ cm and $V = 150$ V at $r = 40$ cm. The space between conductors is a dielectric for which $\epsilon_r = 3.12$. Evaluate surface charge densities on the conductors.

[10 marks : 2022]

Solution:

(i) Conduction current density, $J_C = \sigma E$

Displacement current density

$$J_D = \varepsilon_o \frac{\partial D}{\partial t}$$

$$= \varepsilon_o \varepsilon_r \omega E$$

(a) For distilled water, $\sigma = 2 \times 10^{-4}$ s/m, $\varepsilon_r = 81$

$$J_C = J_D$$

$$\sigma E = \omega \epsilon_r \epsilon_o E$$

$$\sigma = \omega \epsilon_r \epsilon_o$$

$$\omega = \frac{\sigma}{\epsilon_r \epsilon_o} = \frac{2 \times 10^{-4}}{81 \times 8.88 \times 10^{-12}}$$

$$= 2.78 \times 10^5 \text{ rad/sec}$$

$$f = \frac{\omega}{2\pi} = 44.27 \text{ kHz}$$

(b) Sea water, $\sigma = 4$ s/m, $\epsilon_r = 1$

$$\omega = \frac{\sigma}{\epsilon_r \epsilon_o} = \frac{4}{1 \times 8.88 \times 10^{-12}}$$

$$= 4.504 \times 10^{11} \text{ rad/sec}$$

$$f = 7.17 \times 10^{10} \text{ Hz}$$

(ii) $V = -35$ V, $r = 3$ cm

$V = 150$ V, $r = 40$ cm

Spherical surface are equipotential.

The potential is constant with θ and ϕ.

The Laplace equation gets reduced to

$$\nabla^2 V = 0$$

$$\frac{\partial}{\partial r} r^2 \frac{\partial V}{\partial r} = 0$$

$$r^2 \frac{\partial V}{\partial r} = A$$

$$\frac{\partial V}{\partial r} = \frac{A}{r^2}$$

$$\int \partial V = \int \frac{A}{r^2} \partial r$$

$$V = -\frac{A}{r} + B$$

Put the given value to find out the value of A and B.

$$-35 = -\frac{A}{3 \times 10^{-2}} + B \quad ...(1)$$

$$150 = -\frac{A}{40 \times 10^{-2}} + B \quad ...(2)$$

By solving equation (1) and (2), we get

$$A = 6, B = 165$$

$$V(r) = -\frac{6}{r} + 165$$

Now, $\vec{E} = -\nabla V$

$$= -\frac{\partial V}{\partial r}\hat{a}_r = -\frac{6}{r^2}\hat{a}_r$$

$$\vec{D} = \varepsilon_r \varepsilon_o E = -\frac{6\varepsilon_r \varepsilon_o}{r^2}\hat{a}_r \text{ c/m}^2$$

Surface charge densities, $\rho_s = |\vec{D}| = \frac{6\varepsilon_r\varepsilon_o}{r^2} = \frac{18.72\varepsilon_o}{r^2} \text{ C/m}^2$

Surface charge density on inner sphere ($r = 3$ cm) is

$$\rho_s(r = 3 \text{ cm}) = \frac{6 \times 8.854 \times 10^{-12} \times 3.12}{(3 \times 10^{-2})^2} = 184.16 \text{ nC/m}^2$$

Surface charge density on the outer sphere ($r = 40$ cm) ($\varepsilon_r = 1$)

$$\rho_s(r = 40 \text{ cm}) = \frac{6\,\varepsilon_0(1)}{(0.40)^2} = 0.332 \text{ nC/m}^2$$

End of Solution

Q.3 (c) (i) Distinguish between a power transformer and current transformer (CT) and discuss the effect of secondary CT open circuit, while primary winding is energized.

[10 marks : 2022]

(ii) A 1000/5, 50 Hz CT has a bar primary and a resistive burden of 12.5 VA at rated current. The secondary winding has a leakage inductance of 0.96 mH. The core requires a magnetising mmf of 16 AT and a loss component of 12 AT, when rated current flows in the secondary. Calculate the secondary turns at which the ratio error is eliminated.

[10 marks : 2022]

Solution:

(i) Differences between a Power Transformer and Current Transformer :

Power Transformer	Current Transformer
Power transformer is used to convert voltage using the number of windings in the two coils as ratio of turns in both winding.	Current transformer have only one winding placed around a wire to measure current in that winding.
More emphasis is on efficiency of transformer then accuracy.	There should be minimum phase and ratio error hence measurement can be accurate.
Power transformer used for both purposes, i.e, stepdown and step up voltage and current.	Current transformer used for only step down current hence it can be measured by ordinary measuring instruments.
Power transformer can be used for open and close circuit of secondary both.	Current transformer can not be used at secondary open circuit while primary is energised.
Resistance of coil of both primary and secondary is higher than current transformer is winding.	Resistance of coil with which current transformer is secondary winding is connected is small.
Power transformer used for high power rating.	While current transformer is used for low power (VA) rating.
Eddy and hysteresis losses are higher than current transformer.	Eddy and hysteresis losses are lower than power transformer.

Effect of Current Transformers secondary open circuit while primary is energised.

If the secondary is open circuit while primary is energised, the primary winding mmf remains same. Because current in secondary winding producing counter mmf that reducing primary mmf. If secondary is O.C., then there is no secondary current, hence, no counter mmf. Hence, there will be very large primary mmf which saturated core flux hence very high voltage induced on secondary side. This high voltage causing failure of insulation on secondary side which will dangerous to the person working nearer to current transformer.

(ii) Given :

$$\frac{I_P}{I_S} = \frac{1000}{5} = 200 = n$$

Since primary has only one turn.

So,

$$I_m = 16 \text{ Amp}$$

$$I_e = 12 \text{ Amp}$$

$$I_o = \sqrt{16^2 + 12^2} = 20 \text{ Amp}$$

$$\alpha = \tan^{-1}\left(\frac{I_e}{I_m}\right) = \tan^{-1}\left(\frac{12}{16}\right) = 36.87°$$

The secondary is purely resistive so

$$(R_l + R_s) = \frac{12.5}{5^2} = 0.5 \text{ ohm}$$

and

$$X_l + X_s = 2\pi \times 50 \times 0.96 \times 10^{-3} = 0.301 \text{ ohm}$$

So, $$\delta = \tan^{-1}\left(\frac{X_s + X_l}{R_s + R_l}\right) = \tan^{-1}\left(\frac{0.301}{0.5}\right) = 31.05°$$

So, to eliminate ratio error, the actual ratio must be as nominal ratio.

$$R = n + \frac{I_o \sin(\alpha + \delta)}{I_s}$$

$$200 = \frac{N_s}{1} + \frac{20\sin(36.87° + 31.05°)}{5}$$

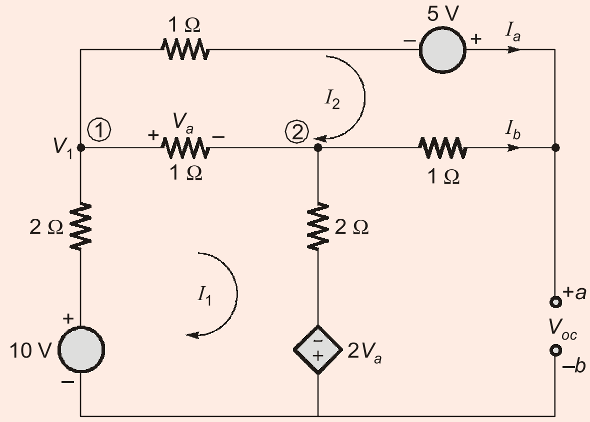
$$= N_s + 3.71$$

$$N_s = 196.29 \simeq 196$$

So, secondary turn should be equal to 196 turns, in order to eliminate the ratio error.

End of Solution

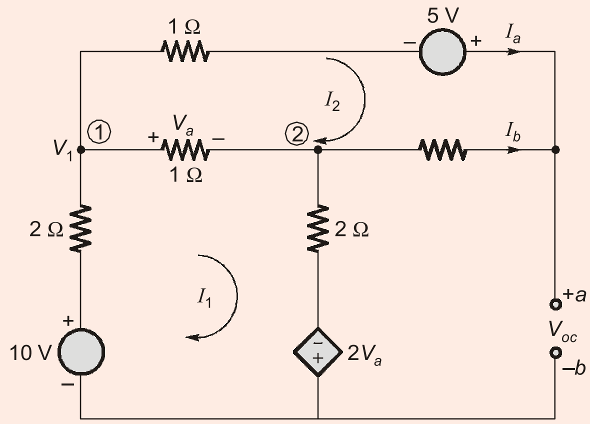
Q.4 (a) Obtain the Thevenin's equivalent at the terminals *a*, *b* for the figure shown. What will be the maximum power supplied by the circuit to a load resistance?



[20 marks : 2022]

Solution:

To Thevenin's, we find the V_{OC} across *ab* first



So writing KCL at V_2 first

$$\frac{V_2 - 5 - V_1}{2} + \frac{V_2 + 2V_a}{2} + \frac{V_2 - V_1}{1} = 0$$

$$V_2 - 5 - V_1 + V_2 + 2(V_1 - V_2) + 2V_2 - 2V_1 = 0$$

$$2V_2 - V_1 = 5 \quad \text{...(i)}$$

Writing KCL at V_1, we get

$$\frac{V_1 + 5 - V_2}{2} + \frac{V_1 - 10}{2} + \frac{V_1 - V_2}{1} = 0$$

$$V_1 + 5 - V_2 + V_1 - 10 + 2V_1 - 2V_2 = 0$$

$$4V_1 - 3V_2 = 5 \quad \text{...(ii)}$$

Using (i) and (ii), we get

$$V_1 = 5\text{ V},\ V_2 = 5\text{ V}$$

So, I_1 in figure is

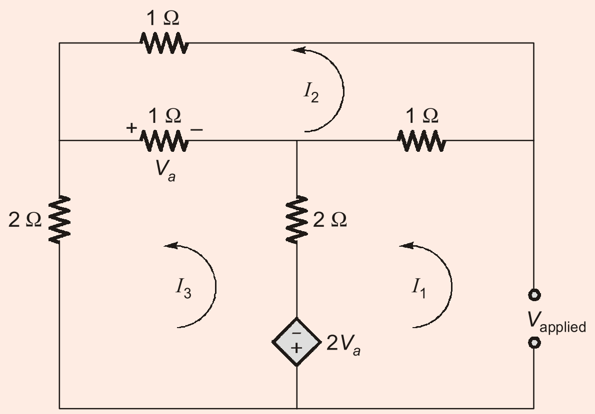
$$\frac{V_2 - 5 - V_1}{2} = -2.5\text{ A}$$

So, $V_a = V_2 - I_1 \times 1\ \Omega = 5 - [-2.5] = 7.5\text{ V}$

So, $V_{OC} = 7.5\text{ V}$

Now, for R_{int} are switched off independent sources.

Now, for R_{th}, all independent sources are replaced by their internal resistance.



Doing Mesh Analysis :

$$V - (I_1 - I_2)1 - (I_1 - I_3)2 + 2(I_2 - I_3) = 0$$

$$V - I_1 + I_2 - 2I_1 + 2I_3 + 2I_2 - 2I_3 = 0$$

$$V - 3I_1 + 3I_2 = 0 \quad \text{...(i)}$$

For Loop I_2 :

$$I_2 - I_3 + I_2 - I_1 + I_2 = 0$$

$$3I_2 = I_3 + I_1 \quad \text{...(ii)}$$

For Loop 3 :

$$2(I_2 - I_3) + 2(I_3 - I_1) + I_3 - I_2 + 2I_3 = 0$$

$$2I_2 - 2I_3 + 2I_3 - 2I_1 + I_3 - I_2 + 2I_3 = 0$$

$$I_2 - 2I_1 + 3I_3 = 0 \quad \text{...(iii)}$$

$$I_2 = 2I_1 - 3I_3$$
$$3I_2 = I_3 + I_1$$

So,
$$6I_1 - 9I_3 = I_3 + I_1$$
$$5I_1 = 10I_3$$

$\left(I_3 = \dfrac{I_1}{2}\right)$ ...(iv)

Using (iv) in (ii)

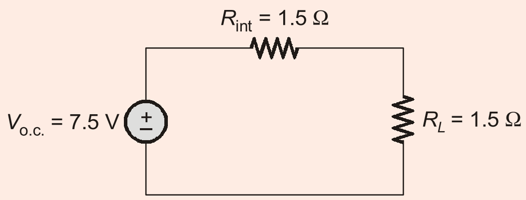
$$3I_2 = \frac{3I_1}{2}$$

$\left(I_2 = \dfrac{I_1}{2}\right)$ ...(v)

Using (v) in (i)

$$V = 3I_1 - \frac{3I_1}{2}$$
$$V = \frac{3I_1}{2}$$
$$\frac{V}{I_1} = \frac{3}{2} = R_{int}$$

So, Thevenin's equivalent circuit is



So, maximum power transfer when

$$R_{load} = 1.5\ \Omega$$

So,
$$P_{max} = \left(\frac{7.5}{3}\right)^2 \times 1.5\ \Omega$$
$$= 9.375\ \text{W}$$

End of Solution

Q.4 (b) (i) Compare the phenomenon of Piezoelectricity and Ferroelectricity.
[8 marks : 2022]

(ii) Compute the packing efficiency and density of diamond. Consider diamond as diamond cubic (DC) structure and lattice parameter, a = 3.57 Å and mass of atom $m = 1.66 \times 10^{-27}$ kg.
[12 marks : 2022]

Solution:

(i) The phenomenon of Piezoelectricity and Ferroelectricity are compared as follows :

Piezoelectricity	Ferroelectricity
(i) Material generates potential when mechanical stress is applied across it.	(i) Material exhibits spontaneous polarisation in presence of elastic field.
(ii) Larger class of material.	(i) Ferroelectronics are subclass of piezoelectrics.
(iii) All piezoelectric materials are not ferroelectirc.	(iii) All ferroelectric materials are piezoelectrics.
(iv) Used in applications like pressure-transducer.	(iv) Used in application like transformer core.
(v) e.g., Rochelle salt quartz	(v) e.g., Lead titanate $PbTiO_3$

(ii) Packing Efficiency of Diamond :

$$\text{Atoms present at CCP} = 4$$

$$\text{Atoms present at alternate } T_oV_o = 4$$

So, total number of atoms in diamond,

$$Z = 8$$

So,
$$\text{pf} = \frac{Z \times \frac{4}{3} \times \pi r^3}{a^3} \quad \text{...(i)}$$

Now to find relation between *a* and *r*.

We know atoms touch corner with TV.

So,
$$\frac{\sqrt{3}a}{4} = 2r$$

$$a = \frac{8r}{\sqrt{3}} \quad \text{...(ii)}$$

Using (ii) in (i), we get,

$$\text{pf} = \frac{\frac{4}{3} \times \pi \times 8 \times r^3}{\left(\frac{8r}{\sqrt{3}}\right)^3} = 0.34 \text{ or } 34\%$$

To find density :

$$\rho = \frac{Z \times m}{a^3}$$

Using the values given,
$$m = 1.66 \times 10^{-27} \text{ kg}$$
$$a = 3.57 \times 10^{-10} \text{ m}$$

We get,
$$\rho = \frac{8 \times 1.66 \times 10^{-27}}{(3.57 \times 10^{-10})^3} = 0.291 \times 10^3$$

$$\rho = 291 \text{ kg/m}^3$$

End of Solution

Q.4 (c) An AC bridge with terminals A, B, C, D (consecutively marked) has in arm AB, a pure resistance; arm BC a resistance of 800 ohms in parallel with a capacitor of 0.5 μF; arm CD, a resistance of 400 ohms in series with a capacitor of 1 μF; and arm DA a resistance of 1000 ohms

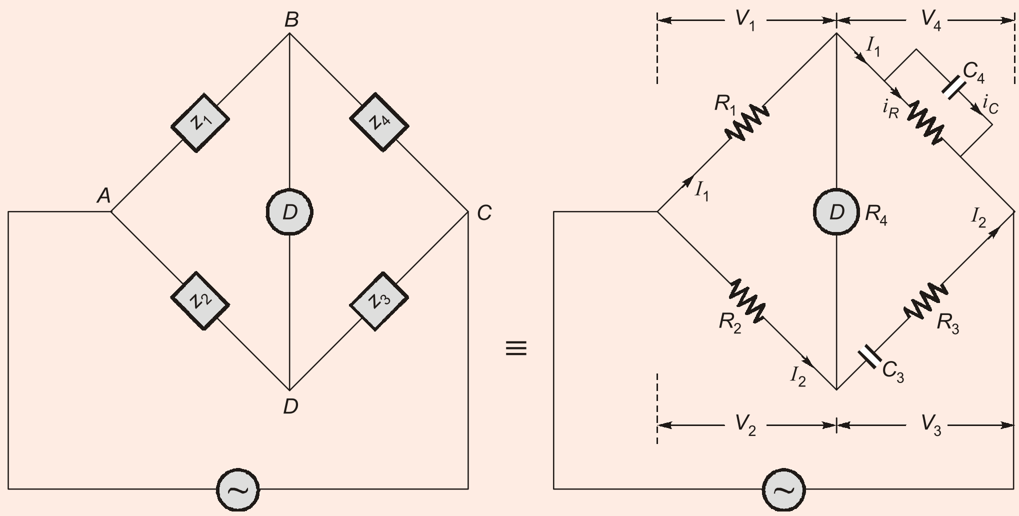
(i) obtain the value of the frequency for which the bridge can be balanced by first deriving the balance equations connecting the branch impedance, and draw phasor diagram, and

(ii) calculate the value of the resistance in arm AB to produce balance.

[20 marks : 2022]

Solution:

This is Wein's bridge :



For balance equation, $$Z_1Z_3 = Z_2Z_4$$

$$R_1\left(R_3 + \frac{1}{j\omega C_3}\right) = R_2\left(\frac{R_4 \cdot \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}}\right)$$

$$R_1\left(\frac{1 + j\omega C_3 R_3}{j\omega C_3}\right) = R_2\left(\frac{R_4}{1 + j\omega R_4 C_4}\right)$$

$$R_1(1 + j\omega C_3 R_3)(1 + j\omega R_4 C_4) = R_2 R_4 (j\omega C_3)$$

$$R_1(1 + j\omega C_3 R_3 + j\omega R_4 C_4 - \omega^2 C_3 R_3 C_4 R_4) = R_2 R_4 (j\omega C_3)$$

By separating Img and Real on sixth side and comparing

$$R_1 - \omega^2 C_3 C_4 R_3 R_4 R_1 = 0$$

$$1 - \omega^2 C_3 C_4 R_3 R_4 = 0 \Rightarrow \omega^2 = \frac{1}{R_3 R_4 C_3 R_4}$$

Hence, $$\omega = \frac{1}{\sqrt{C_3 C_4 R_3 R_4}} \quad ...(1)$$

By comparing Imaginary part

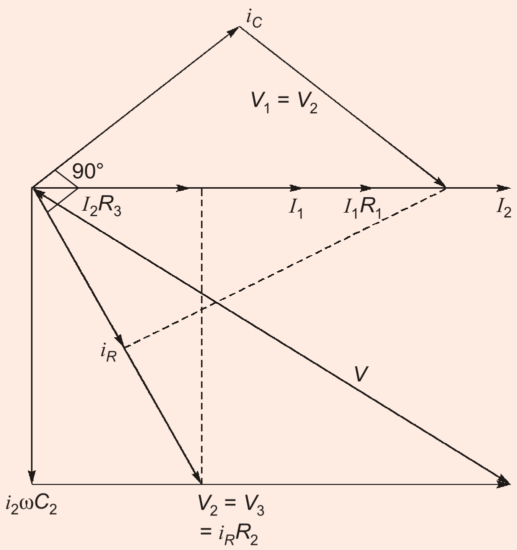
$$R_1[\omega C_3 R_3 + \omega C_4 R_4)] = \omega C_3 R_2 R_4$$

$$R_1[R_3 C_3 + C_4 R_4)] = C_3 R_2 R_4$$

$$\frac{R_2}{R_1} = \frac{R_3}{R_4} + \frac{C_4}{C_3} \quad ...(2)$$

Phasor :

$$V_1 = V_2 = I_1 R_1 = I_2 R_2$$



(i)

$$\omega^2 = \frac{1}{R_3 R_4 C_3 R_4} \Rightarrow \omega^2 = \frac{1}{R_3 R_4 C_3 C_4}$$

$$R_3 = 400\ \Omega,\ R_4 = 800\ \Omega,\ C_4 = 0.5\ \mu\text{F}$$

$$C_3 = 1\ \mu\text{F},\ R_2 = 1000\ \Omega$$

$$\omega^2 = \frac{1}{400 \times 800 \times 10^{-6} \times 0.5 \times 10^{-6}}$$

$$\omega^2 = \frac{1}{16 \times 10^{-8}} \Rightarrow \omega = \frac{1}{4} \times 10^4 \text{ rad/sec}$$

$$2\pi f = 0.25 \times 10^4 \text{ rad/sec}$$

Frequency, $f = 398.08$ Hz

(ii) From eqn. (2)

$$\frac{R_2}{R_1} = \frac{R_3}{R_4} + \frac{C_4}{C_3} \quad \text{(By putting values)}$$

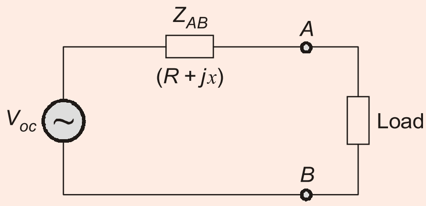
$$\frac{1000}{R_1} = \frac{400}{800} + \frac{0.5}{1}$$

$$\frac{1000}{R_1} = 1 \Rightarrow R_1 = 1\ \text{k}\Omega$$

End of Solution

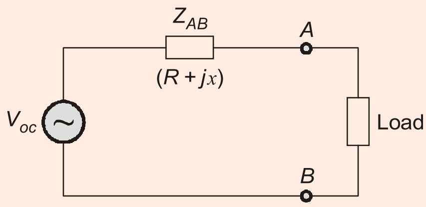
Section-B

Q.5 (a) Two terminals shown in figure coming out of a network show a voltage of 150 V on open circuit and give a current of 3 A on short-circuit. If a resistor of 10 Ω is connected as a load between the two terminals the current falls to 2.65 A. Determine the equivalent voltage generator circuit and find whether the circuit is capacitive or inductive.



[12 marks : 2022]

Solution:

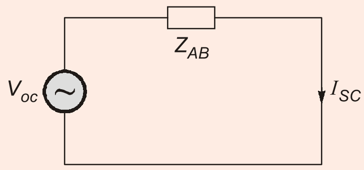


Given, Open circuit $= 150 = |V_{OC}|$

$$V_{OC} = 150\angle 0° \text{ V}$$

Now when the circuit is short circuited around points *AB* we get

$$I_{SC} = \frac{V_{OC}}{R + jx}$$



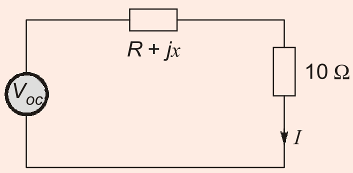
$$\Rightarrow \quad |I_{SC}| = \frac{|V_{OC}|}{\sqrt{R^2 + X^2}}$$

So,

$$\frac{150}{\sqrt{R^2 + X^2}} = 3$$

$$\Rightarrow \quad R^2 + X^2 = 2500 \qquad \text{...(i)}$$

Now when load $R = 10\ \Omega$ applied across *AB*



$$I = \frac{V_{OC}}{(R + 10) + jX}$$

$$|I| = \frac{|V_{OC}|}{\sqrt{(R + 10)^2 + X^2}} = \frac{150}{2.65} = 56.6$$

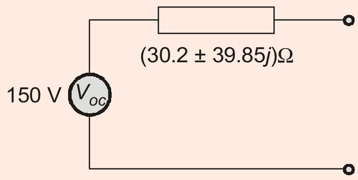
$$(R + 10)^2 + X^2 = 3203.987 \qquad \text{...(ii)}$$

Using equation (i) in (ii),

$$(R + 10)^2 + 2500 - R^2 = 3203.98$$
$$R^2 + 100 + 20R + 2500 - R^2 = 3203.98$$
$$20R = 3203.98 - 2500 - 100$$
$$20R = 603.98$$
$$R = 30.199\ \Omega$$

Now in (i),
$$R^2 + X^2 = 2500$$
$$X^2 = 1587.998$$
$\Rightarrow$
$$X = \pm 39.85\ \Omega$$

So equivalent circuit is



According to mathematical equation 'X' may be negative (or) positive then 'X' may be capacitive or inductive but for generator 'X' is always positive (inductive).

End of Solution

Q.5 (b) Zinc has HCP structure. The height of the unit cell is 0.494 Å. The nearest neighbour distance is 0.27 nm. The atomic weight of zinc is 65.37. Evaluate,
(i) The volume of the unit cell.
(ii) The density of zinc
Take NA = 6.0238 × 10^26 Molecules/Kg Mole.

[12 marks : 2022]

Solution:

Given, Nearest neighbour distance, $a = 2r = 0.27 \text{ nm} = 0.27 \times 10^{-9}$ m

Atomic weight of Z_n = 65.37

The height of the unit cell, h = 0.494 Å

(i) Volume of unit cell,

The volume of the HCP unit cell is given by

$$V = \frac{3\sqrt{3}\,a^2 h}{2} = \frac{3\sqrt{3}\,a^2 h}{2}$$

Substituting the values we get

$$V = \frac{3\sqrt{3}(0.27 \times 10^{-9})^2 \times 0.494 \times 10^{-10}}{2} = 9.35 \times 10^{-30}\ \text{m}^3$$

(ii) The density of zinc,

Effective number of atom's in unit cell of HCP,

$$n = 6$$

Using the relation, $\rho = \dfrac{nm}{N_0 \times V}$ we have,

[Where $n = 6$, $m = 65.37$ (given) $N_A = 6.0238 \times 10^{26}$]

$$\rho = \frac{6 \times 65.37}{6.0238 \times 10^{26} \times 9.35 \times 10^{-30}} \approx 69647\ \text{kg/m}^3$$

End of Solution

Q.5 (c) Distinguish between

(i) High level language and low level language.

(ii) Machine cycle and instruction cycle.

(iii) Memory mapped Input/Output and Input/Output mapped Input/Output.

[4 × 3 = 12 marks : 2022]

Solution:

(i) High level language and low level language,

The difference between high level and low level languages:

S. No.	High Level language	Low Level language
1.	It is programmer friendly language.	It is machine friendly language.
2.	High level language is less memory efficient.	Low level language is high memory efficient.
3.	It is easy to understand.	It is tough to understand.
4.	It is simple to debug.	It is complex to debug.
5.	It is simple to maintain	It is complex to maintain comparatively.
6.	It is portable.	It is non-portable.
7.	It can run on any platform.	It is machine dependent.
8.	Compiler or interpreter for translation.	It need's assembler for translation.
9.	It is used widely for programming.	It is not commonly used now a days in programming.

(ii) Machine cycle and instruction cycle

The difference between machine cycle and instruction cycle is as follows:

Machine Cycle	Instruction Cycle
The steps that get performed by the processor getting employed in a device and all the instruction's that get implemented.	A process by which a computer takes an instruction given by a program then understands it and executes if from memory.
Processes	
Fetch, decode, execute and store	Fetch, decode, execute and run
Component's	
Memory unit and central processing unit.	Arithmetic logic unit, registers, data and memory.
Value	
The steps required by CPU to fetch and execute an instruction is called an instruction cycle.	The time required by the microprocessor to complete the operation of accessing memory or I/O device is called machine cycle.

(iii) Difference between memory mapped I/O and I/O mapped I/O.

The microprocessor cannot do anything by itself therefore, it needs to be linked with memory, extra peripherals or I/O device. This linking is called interfacing.

The interfacing of the I/O device in 8085 can don in two mass:

1. Memory mapped I/O interfacing:
   In this kind of interfacing, we assign a memory address that can be used in the same manner as we use a normal memory location.
2. I/O mapped I/O interfacing:
   A kind of interfacing in which we assign an 8-bit address value of the input/output devices which can be access using IN and OUT interaction is called I/O mapped IO interfacing.

Difference between memory-mapped I/O interfacing and I/O mapped I/O interfacing.

	Features	Memory mapped I/O	I/O mapped I/O
1.	Addressing	I/O devices are accessed like any other memory location.	They cannot be accessed like any other memory location.
2.	Address size	16-bit	8-bit
3.	Instruction used	The instruction used are LDA and STA etc.	The instruction used are IN and OUT.
4.	Cycles	Cycles involved during operation are memory read, memory write.	Cycles involved during operation are IO read and IO write in the case of IO mapped IO.
5.	Registers communicating	Any register can communicate with I/O device.	Only accumulator can communicate with IO devices.
6.	Space involved	2^{16} I/O parts are possible to be used for interfacing.	any 2^8 i.e. 256 IO parts are available.
7.	$IO/\overline{M}$ signal	During writing for read cycle ($IO/\overline{M} = 0$).	During writing or read cycle ($IO/\overline{M} = 1$).
8.	Control signal	No separate control signal required since we have unified memory space.	Special control signal's are used.

End of Solution

Q.5 (d) A coil of 300 V Moving Iron Voltmeter has a resistance of 500 ohms and an inductance of 0.8 H. The instrument reads correctly at 50 Hz AC supply and takes 100 mA at full scale deflection. What is the percentage error in the instrument reading, when it is connected to 200 V DC supply ?

[12 marks : 2022]

Solution:

Let series resistance is R_s.

So, $$I = 0.10 = \frac{300}{\sqrt{(500+R_s)^2 + (2\pi \times 50 \times 0.8)^2}}$$

Solving this, $$R_s = 2489.45 \text{ ohm}$$

The current at 200 V, 50 Hz
$$I_T = (0.1)\left(\frac{200}{300}\right)$$
$$I_T = 0.0667 \text{ Amp}$$

The current at 200 V, DC
$$I_M = \frac{V_{dc}}{R_m + R_s} = \frac{200}{500 + 2489.45}$$
$$I_M = 0.0669 \text{ Amp}$$

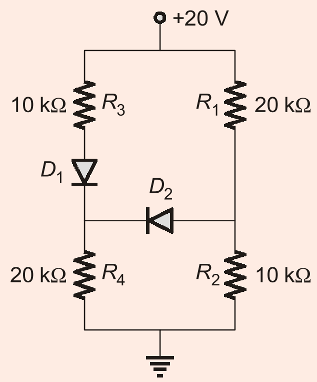
The voltage reading will be proportional to the current value.

So, error in measurement

$$\therefore \quad \text{Error} = \frac{I_M - I_T}{I_T} \times 100$$
$$= \frac{0.0669 - 0.0667}{0.0667} \times 100$$
$$\text{Error} = 0.30\%$$

End of Solution

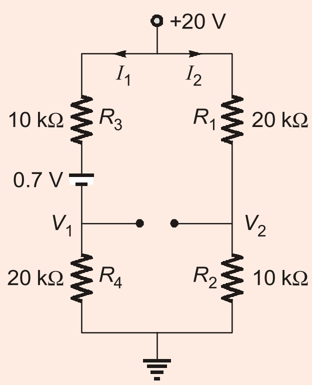
Q.5 (e) Find the currents through diodes D_1 and D_2 and the voltage across resistor R_4 in the figure shown. Assume forward voltage drop of 0.7 V for each diode.



[12 marks : 2022]

Solution:

Consider diode D_1 on and diode D_2 off, hence the circuit diagram becomes,



Current I_1 flows through diode D_1, equation becomes,

$$-20 + 10k \times I_1 + 0.7 + 20k \times I_1 = 0$$

$$I_1 = 0.64 \text{ mA} \quad \text{...(i)}$$

Current I_2 flows through R_1 and R_2

$$-20 + I_2 \times (20k + 10k) = 0$$

$$I_2 = \frac{20}{30k} = 0.67 \text{ mA} \quad \text{..(iii)}$$

Voltage, $V_1 = I_1 \times 20 \text{ k}$

$= 12.87$ V

Voltage, $V_2 = I_2 \times 10 \text{ k} = 6.67 \ V_1$

As $V_1 > V_2$, hence our assumption is correct, D_1 is on and D_2 is off

Current through D_1, $I_1 = 0.64$ mA

Current through D_2, $= 0$

Voltage drop across resistor $R_4 = I_1 \times 20$ k

$= 12.87$ V

End of Solution

Q.6 (a) Calculate the value of contact potential and the total width of the depletion region under unbiased condition of an abrupt silicon P-N junction at 300° K. The intrinsic carrier concentration of silicon is 1.7×10^{16} atoms/m³ and its relative permittivity is 11.8. The junction has acceptor density $N_A = 10^{22}$ atoms/m³ on *P*-side and donor density of $N_D = 10^{20}$ atoms/m³ on *n*-side.

Derive the equation used for calculation.

Boltzmann's constant = 1.381×10^{-23} J/°K

Electron charge = 1.602×10^{-19} C

[20 marks : 2022]

Solution:

The contact potential of the P-N junction is

$$V_{contact} = \frac{KT}{q} \ln\left(\frac{N_A N_D}{nq^2}\right)$$

$$V_{contact} = \frac{1.381 \times 10^{-23} \times 300}{1.602 \times 10^{-19}} \ln\left(\frac{10^{22} \times 10^{20}}{(1.7 \times 10^{16})^2}\right)$$

$$V_{contact} = \frac{1.381 \times 10^{-23} \times 300}{1.602 \times 10^{-19}} \ln\left(\frac{10^{42}}{(1.7)^2 10^{32}}\right)$$

$$= 258.6 \times 10^{-4} \ln\left(\frac{10^{10}}{(1.7)^2}\right)$$

$$= 21.96 \times 258.6 \times 10^{-4} = 0.568 \text{ V}$$

Width of deplection region,
$$W = \sqrt{\frac{2 \epsilon_r \epsilon_0 V_{contact}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

$$= \sqrt{\frac{2 \times 11 \times 8.85 \times 10^{-12} \times 0.568}{1.6 \times 10^{-19}}\left(\frac{1}{10^{22}} + \frac{1}{10^{20}}\right)}$$

$$= \sqrt{69.1185 \times 10^{7}\left(\frac{1.01}{10^{20}}\right)}$$

$$= \sqrt{69.8096 \times 10^{-13}}$$

$$W = 2.642 \times 10^{-6} \text{ m}$$

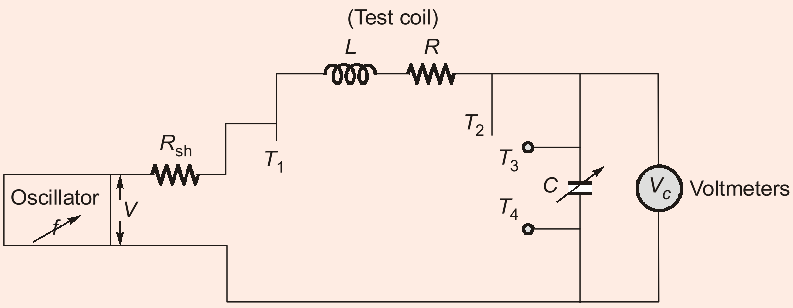
End of Solution

Q.6 (b) Draw the circuit diagram and explain the operation of a practical *Q*-meter and write its applications.

[20 marks : 2022]

Solution:

Circuit diagram:



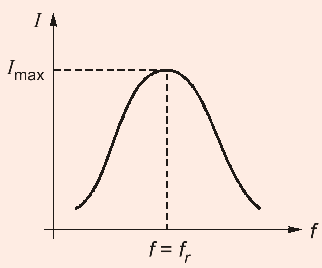
Working principle:

This meter is based on the principle of series resonance. Under series resonance,

$$\text{Inductive reactance, } X_L = \text{capacitive reactance, } X_C$$

$$I = \frac{V}{R}$$

Quality factor, $$Q = \frac{X_L}{R} = \frac{X_C}{R}$$



Voltage across capacitor, $$V_C = IX_C$$

$$= \frac{V}{R}X_C = V \cdot \frac{X_C}{R} = V \cdot Q$$

$$V_C = V \cdot Q$$
$$V_C \propto Q$$

Voltage across capacitor is directly proportional to quality factor of the meter. Hence this meter is also called voltage magnifier. By varying the capacitance 'C' the circuit is resonated and the voltage across capacitor is measured by voltmeter which is calibrated in terms of quality factor of the coil.

Application:

1. **Measurement of quality factor Q of test coil :**

   Test coil is connected between T_1 and T_2 and by varying 'C' circuit is resonated. For reducing error, R_{sh} must be of low value (in mΩ range)

$$Q_T \text{(true value)} = \frac{\omega L}{R}$$

$$Q_m \text{(measured value)} = \frac{\omega L}{R + R_{sh}} = \frac{Q_T}{\left(1 + \frac{R_{sh}}{R}\right)}$$

$$\because \quad Q_T = Q_m\left(1 + \frac{R_{sh}}{R}\right)$$

   $\because$ For $Q_T \approx Q_m \Rightarrow R_{sh}$ should be low

2. **Measurement of unknown capacitance :**

   Test capacitance, C_T is connected between terminals T_3 and T_4. By varying C to value C_1 , the circuit is resonated.

   Again varying C to a new value C_2 , new value of $V_C = V_1$ is calculated

   For $C = C_1$,
$$f = \frac{1}{2\pi\sqrt{L(C_1 + C_T)}}$$

   For $C = C_2$,
$$f = \frac{1}{2\pi\sqrt{LC_2}}$$

   Both frequency are same,

$$\frac{1}{2\pi\sqrt{L(C_1 + C_T)}} = \frac{1}{2\pi\sqrt{LC_2}}$$

$$\because \quad C_2 = C_1 + C_T$$
$$C_T = C_2 - C_1$$
$$\because \quad C_1 = \text{with } C_T$$
$$C_2 = \text{without } C_T$$
$$C_T = \text{test capacitance}$$

3. Q-meter is used to calculated self or distributed capacitance of the coil

$$\text{Distributed capacitance, } C_d = \frac{C_1 - n^2 C_2}{(n^2 - 1)}$$

$$\because \text{where, } \quad n = \frac{f_2}{f_1}$$

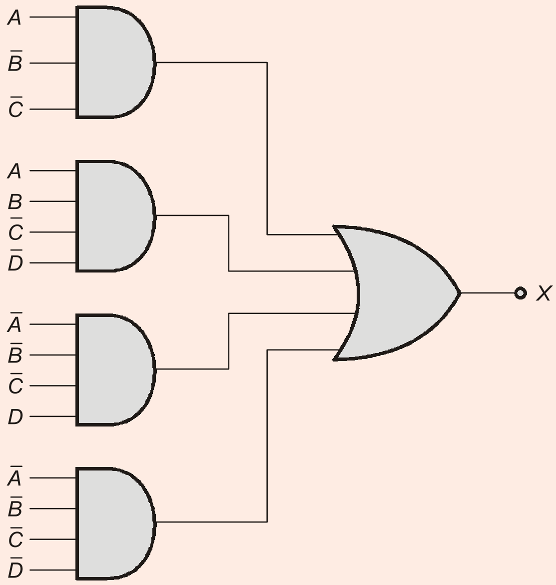
$f_2 \rightarrow$ frequency calculated when $C \rightarrow C_2$

$f_1 \rightarrow$ frequency calculated with $C \rightarrow C_1$

4. Q-meter is used in communication application and self inductance of the coil.

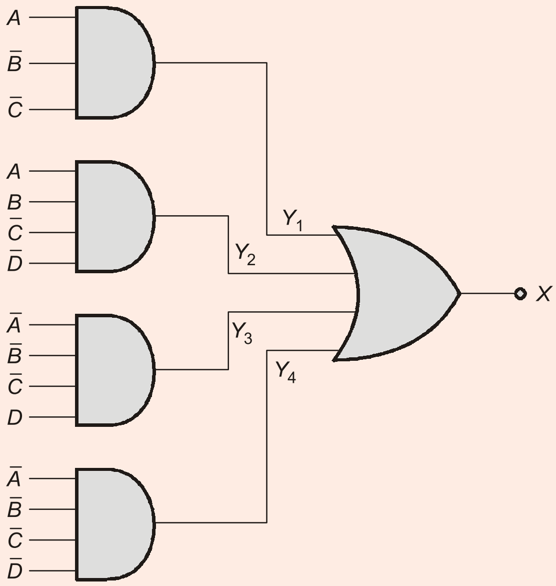
End of Solution

Q.6 (c) (i) Minimize the combinational logic shown in figure.

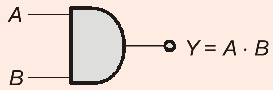


[10 marks : 2022]

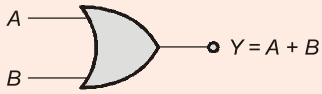
Solution:



For AND gate



For OR gate



For diagram,

$$Y_1 = A\bar{B}\bar{C},$$
$$Y_2 = AB\bar{C}\bar{D},$$
$$Y_3 = \bar{A}\bar{B}\bar{C}D$$
$$Y_4 = \bar{A}\bar{B}\bar{C}\bar{D}$$
$$X = Y_1 + Y_2 + Y_3 + Y_4$$
$$X = A\bar{B}\bar{C} + AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

Realizing K-map

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1		
$\bar{A}B$				
AB	1			
$A\bar{B}$	1	1		

$$X = \bar{B}\bar{C} + A\bar{C}\bar{D}$$

End of Solution

Q.6 (c) (ii) A machine has 32 bit Architecture, with 1 word long instruction. It has 64 registers each of which is 32 bits long. It needs to support 48 instructions which have an immediate operand in addition to two register operands. If the immediate operand is an unsigned integer, find the maximum value of the immediate operand.

[10 marks : 2022]

Solution:

Word size = 32 bit

Instruction size = 1 W = 32 bit

Instruction format

32-bit

Opcode	Reg	Reg	Immediate
$\log_2 48$	$\log_2 64$	$\log_2 64$	14 bit
↓	↓	↓	
6 bit	6 bit	6 bit	

Immediate field size = 14 bit

So unsigned data range is

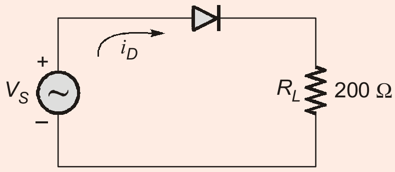
{0 to ($2^{14} - 1$)}

{0 to 16383}

So, max value is 16383

End of Solution

Q.7 (a) (i) Find the circulating current in the circuit shown below operated at a room temperature of 27° C, if the source voltage $V_S = 1.2 + 0.5 \sin 500t$ Volts. Assume $\eta V_T = 50$ mV and $V_{ON} = 0.7$ volts. Neglect contact resistance.

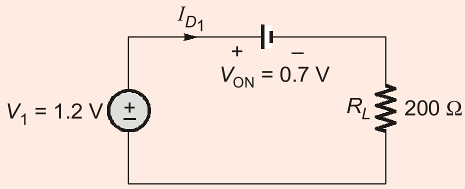


[8 marks : 2022]

Solution:

Given,
$$V_s = \underbrace{1.2}_{\text{DC component}} + \underbrace{0.5\sin 500t}_{\text{AC component of supply}} \text{ Volts}$$

For DC component,
$$V_1 = 1.2 \text{ Volt}$$

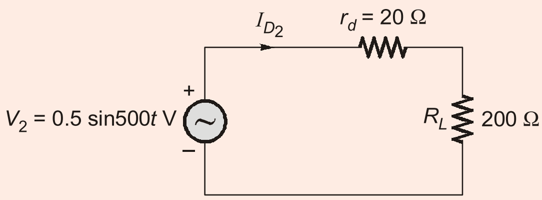


Current due to DC component of supply,
$$I_{D1} = \frac{V_1 - V_{ON}}{R_L} = \frac{1.2 - 0.7}{200} = 2.5 \text{ mA}$$

For AC component of supply,
$$V_2 = 0.5 \sin 500t \text{ Volt}$$

Dynamic AC resistance of diode is given as,
$$r_d = \frac{nV_T}{I_{D_1}} = \frac{50 \times 10^{-3}}{2.5 \times 10^{-3}} = 20\ \Omega$$

Here, we will replace the diode with its equivalent resistance for AC supply



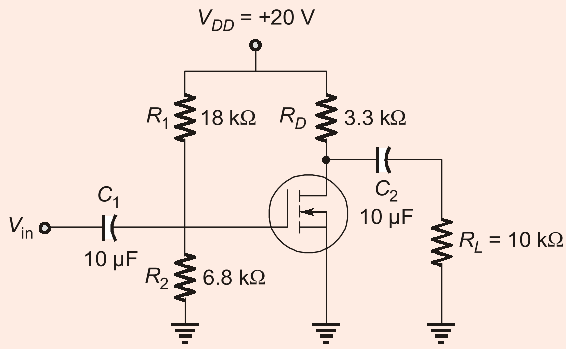
$$I_{D_2} = \frac{V_2}{r_d + R_L} = \frac{0.5\sin 500t}{220} = 2.27 \sin 500t \text{ mA}$$

Therefore, circulating current i_D is given as

$$i_D = I_{D_1} + I_{D_2} = (2.5 + 2.27\sin 500t) \text{ mA}$$

End of Solution

Q.7 (a) (ii) Find V_{GS} , I_D , V_{DS} and the AC output voltage of the E-MOSFET amplifier shown in the figure for an input of 25 m volts. The device parameters are I_{DON} = 18 mA at V_{GS} = 10 Volts, $V_{GS(th)}$ = 2.5 Volts and g_m = 23 mS.



[12 marks : 2022]

Solution:

From the given conditions:

$$I_{DON} = 18 \text{ mA at } V_{GS} = 10 \text{ V and } V_{GS(th)} = 2.5 \text{ V}$$

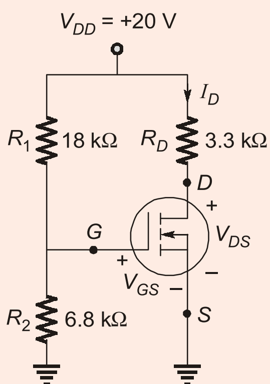
As we know,

$$I_{DON} = K_n\left[V_{GS} - V_{GS(th)}\right]^2, \qquad \text{When } K_n = \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)$$

$$18 = K_n[10 - 2.5]^2$$

$$K_n = 0.25 \text{ mA/V}$$

Now, DC equivalent circuit of given amplifier



Apply voltage division at gate terminal

$$V_G = \frac{R_2}{R_1 + R_2} V_{DD} = \frac{6.8}{18 + 6.8} \times 20 = 5.48 \text{ Volt}$$

Therefore, $V_{GS} = V_G - V_S = 5.48 - 0 = 5.48$ Volt

Now, drain current (I_D) can be obtained as

$$I_D = K_n\left[V_{GS} - V_{GS(\text{th})}\right]^2$$

$$I_D = 0.32\,[5.48 - 2.5]^2 = 2.84 \text{ mA}$$

Therefore, $V_{DS} = V_{DD} - I_D R_D$

$$V_{DS} = 20 - 2.84 \times 3.3 = 10.628 \text{ Volt}$$

Now, AC analysis:

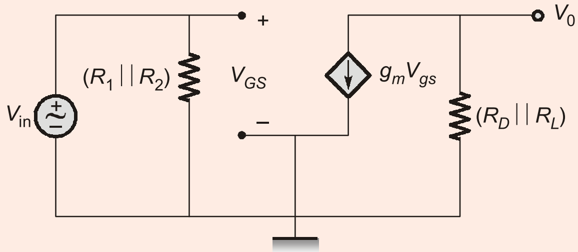
Replacing the MOSFET by its AC equivalent circuit,

Capacitance $\rightarrow$ Short circuit

DC supply $\rightarrow$ Disabled

Since, $V_A = \infty$

$\Rightarrow$ $r_0 = \infty$



$$V_0 = -g_m V_{gs}(R_D \parallel R_L)$$

$$\frac{V_0}{V_i} = -g_m(R_D \parallel R_L) \qquad [\because V_{in} = V_{gs}]$$

Therefore AC output voltage,

$$V_0 = -23 \times 10^{-3} \times [3.3 \parallel 10] \times 10^3 \times 25 \times 10^{-3}$$

$$= -1.426 \text{ Volt}$$

End of Solution

Q.7 (b) (i) A Standard Cell of 1.0186 volts used with a conventional slide wire potentiometer balances at 45 cm. Calculate

(*a*) the emf of the cell which balances at 65 cm and

(*b*) the percentage error in voltmeter which balances at 57 cm when reading 1.27 volts.

[10 marks : 2022]

Solution:

Given, $E_1 = 1.0186$ V and $l_1 = 45$ cm

(*a*) Let the emf of cell is E_2 which balances at $l_2 = 65$ cm,

As we know, from the potentiometer relation,

$$E \propto l$$

Therefore, $$\frac{E_2}{E_1} = \frac{l_2}{l_1}$$

$$E_2 = \frac{65}{45} \times 1.0186 = 1.4713 \text{ Volt}$$

Therefore emf of cell is 1.4713 volt.

(*b*) Again let the cell emf is E_3 which balances at $l_3 = 57$ cm

Therefore, $$E_3 = \frac{l_3}{l_1} \times E_1$$

$$E_3 = \frac{57}{45} \times 1.0186 = 1.2902 \text{ Volt}$$

Percentage error in voltmeter $= \dfrac{1.27 - 1.2902}{1.2902} \times 100 = -1.5677\%$

End of Solution

Q.7 (b) (ii) A moving coil instrument is designed to give a full scale reading of 30 mA and the corresponding drop across is 72 mV. Calculate the shunt resistance required for full scale deflection corresponding to 10 Amps and the series resistance for full scale reading with 400 V.

[10 marks : 2022]

Solution:

Given, $I_{fSD} = 30$ mA,

Corresponding voltage drop, $V = 72$ mV

Internal resistance of meter, $$R_m = \frac{V}{I_{fSD}}$$

$$R_m = \frac{72}{30} = 2.4\,\Omega$$

Now given, $I = 10$ A

Therefore, multiplying factor, $$M = \frac{I}{I_{fSD}} = \frac{10 \times 10^3}{30} = \frac{1000}{3}$$

To read the given meter $I = 10$ A at full scale deflection, the shunt resistance to be added in parallel is given as

$$R_{sh} = \frac{R_m}{m-1} = \frac{2.4}{\frac{1000}{3} - 1} = 7.2216\text{ M}\Omega$$

Similarly to read the given meter, $V = 400$ V

At full scale deflection,

Multiplication factor, $M = \frac{V}{V_{fSD}} = \frac{400}{72 \times 10^{-3}} = 5555.555$

To read the given meter, $V = 400$ V at full scale deflection,

The required series multiplier resistance is given as

$$R_{se} = R_m(m-1)$$
$$= 2.4[5555.55 - 1] = 13.33 \text{ k}\Omega$$

End of Solution

Q.7 (c) (i) What are the functions of Data link layer and Network layer according to ISO-OSI-7 layer Network Architecture.

[12 marks : 2022]

Solution:

- The ISO-OSI is a 7 layer architecture for implementing the network system.
- The 7 layer are application, presentation, session, transport, network, data link, physical.

The functions of data link layer and network layer are as follows:

1. Data link layer:
   - ensures **node to node** delivery of messages
   - **error** correction
   - division of data **into frames** on sending side and recombination of frames on receiving side.
   - **access control** through MAC ID identification of device.
   - **Physical addressing** after creating frames, data link layer adds physical address to header.
2. Network layer:
   - **Routing**: determines the route for data from sender to receiver.
   - Logical addressing : in order to identify various devices on the networks.
   - Ensures transmission of data across networks.
   - Sender and receivers IP address added to header by network layer.

End of Solution

Q.7 (c) (ii) If the signal speed in the cable is 2×10^5 km/sec find the maximum length of cable required to transmit the data at a rate of 600 Mbps in an Ethernet LAN with frame size of 12000 bits.

[8 marks : 2022]

Solution:

For ethernet cable if V is velocity of signal, L is length of frame and BW is bandwidth and d is length of cable.

They are related as follows:

$$d \leq \frac{L \times V}{2 \times BW}$$

So, $$d \leq \frac{12000 \times 2 \times 10^5 \text{ km/s}}{2 \times 600 \times 10^6} \leq 2 \text{ km}$$

So maximum cable length can be 2 km.

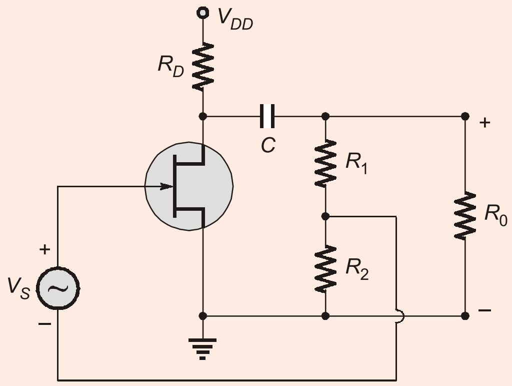
End of Solution

Q.8 (a) The JFET circuit shown in the figure is operating in the mid frequency range. Identify the type of feedback.

(i) Draw the mid frequency small signal model indicating the output, input, feedback points and the closed loop feedback block diagram indicating the forward gain and feedback factor.

(ii) Write the equation for gain without feedback and with feedback.

(iii) Calculate the amplifier gain with and without feedback if R_1 = 80 kΩ, R_2 = 20 kΩ, R_0 = 25 kΩ and R_D = 5 kΩ. Assume FET g_m = 5.5 mS.

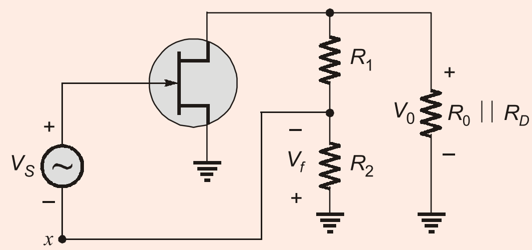


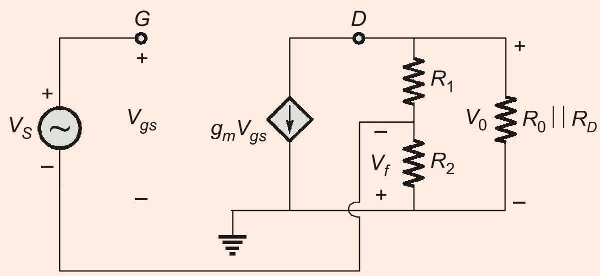
[2 + 6 + 6 + 6 = 20 marks : 2022]

Solution:

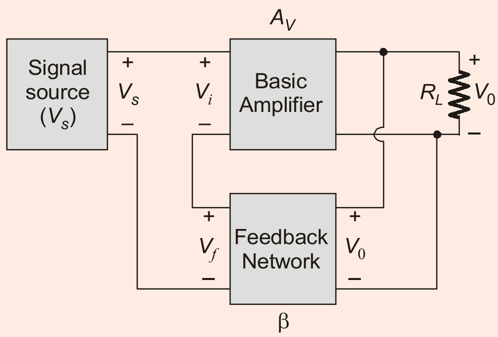
Voltage divider network of R_1 and R_2 result in feedback
If feedback network is connected to O/P node then consider Voltage sampling
If it is not connected to I/P node then consider Series mixing
∴ Voltage series feedback is present

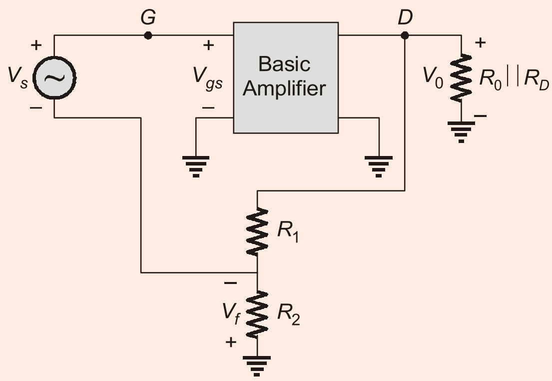
(i)
$$-V_f = \frac{V_0 R_2}{R_1 + R_2}$$
$$\frac{V_f}{V_0} = \beta = \frac{-R_2}{R_1 + R_2}$$





Block diagram for voltage series feedback,



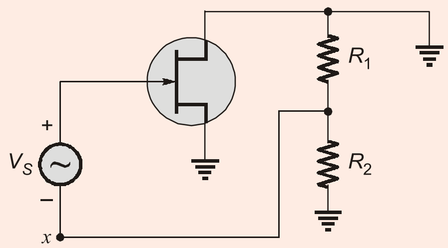


(ii) Gain without feedback

Drawing circuit without feedback

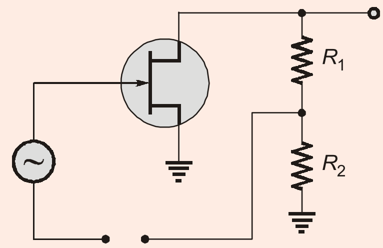
(*a*) To draw input circuit, ground O/P node

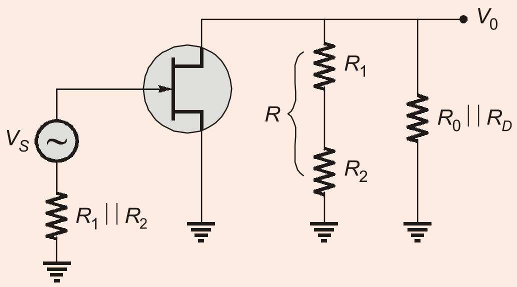
$\Rightarrow R_1$ and R_2 appear in parallel between node x and ground

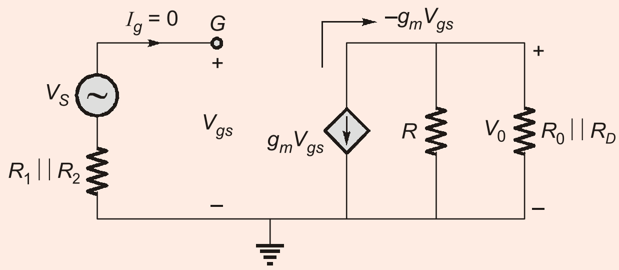


(*b*) To draw output circuit, break input loop

$\Rightarrow R_1$ and R_2 appear in series between drain and ground







Gain without feedback, $A_v = \dfrac{V_0}{V_s} = \dfrac{-g_m V_{gs}(R \| R_0 \| R_D)}{V_{gs}}$

$$A_v = -g_m V_{gs}(R \| R_0 \| R_D),$$

$$R = R_1 + R_2$$

Voltage gain with feedback, $A_{vf} = \dfrac{A_v}{1+\beta A_v}$

$$\beta = \frac{-R_2}{R_1 + R_2}$$

(iii) Gain with feedback, $\beta = \dfrac{-R_2}{R_1+R_2} = \dfrac{-20}{80+20} = -0.2$

$$A_v = -g_m(R \| R_0 \| R_D),$$

$$R = R_1 + R_2 = 100 \text{ k}\Omega$$

$$A_v = -5.5(100 || 25 || 5)$$

$$A_v = -22$$

$$A_{vf} = \frac{A_v}{1+\beta A_v} = \frac{-22}{1+(-0.2)(-22)}$$

$$A_{vf} = -4.07$$

and gain without feedback, $A_v = -22$

End of Solution

Q.8 (b) (i) A solenoid of length l and radius a consists of N turns wire carrying current I. Derive an expression for H at point P along its axis. Also find expression for H if $l >> a$.

(ii) Show that the capacitor of figure (a) has capacitance

$$C_{eq} = \frac{\epsilon_0 \epsilon_{r_1} A_1}{d} + \frac{\epsilon_0 \epsilon_{r_2} A_2}{d}$$ and capacitor of figure (b) has reciprocal capacitance $$\frac{1}{C_{eq}} = \frac{1}{\frac{\epsilon_0 \epsilon_{r_1} A}{d_1}} + \frac{1}{\frac{\epsilon_0 \epsilon_{r_2} A}{d_2}} = \frac{1}{C_1} + \frac{1}{C_2}.$$

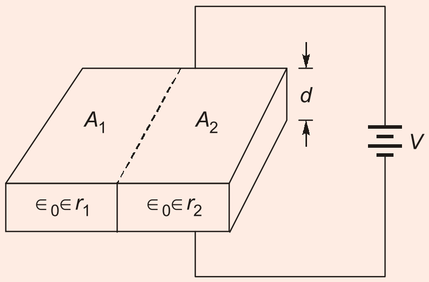


Figure (a)

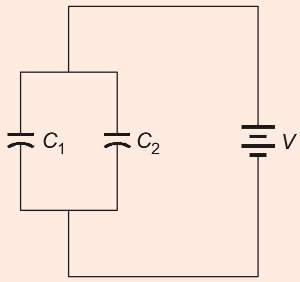
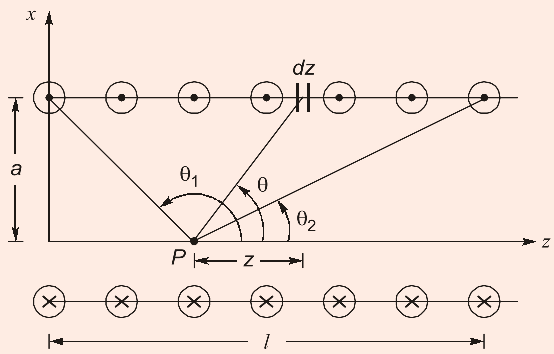


Figure (b)

[10 + 10 marks : 2022]

Solution:

(i) Consider the cross-section of the solenoid as shown in figure,



Since the solenoid consists of circular loops, therefore the contribution to the magnetic field H at point P by an element of the solenoid of length dz is

$$dH_z = \frac{I\,dl a^2}{2[a^2 + z^2]^{3/2}} = \frac{I a^2 n dz}{2[a^2 + z^2]^{3/2}}$$

Where, $$dl = ndz = \left(\frac{N}{l}\right) dz$$

From the figure, $$\tan\theta = \frac{a}{z}$$

$\therefore$ $$dZ = -a\,\text{cosec}^2\theta\, d\theta = \frac{-[z^2 + a^2]^{3/2}}{d^2} \sin\theta \cdot d\theta$$

Therefore,
$$dH_z = \frac{-nI}{2}\sin\theta\, d\theta$$

$$H_z = \frac{-nI}{2}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta$$

$$\vec{H} = \frac{nI}{2}(\cos\theta_2 - \cos\theta_1)\hat{a}_z$$

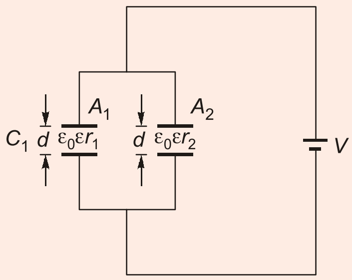
Substituting,
$$n = \frac{N}{l}, \text{ gives}$$

$$\vec{H} = \frac{NI}{2l}(\cos\theta_2 - \cos\theta_1)\hat{a}_z$$

If $l >> a$ or
$$\theta_2 \approx 0°$$
and
$$\theta_1 \approx 180°$$

$$\vec{H} = \frac{NI}{l}\hat{a}_z$$

(ii) Since flux density is continuous and $\vec{E}$ is discontinuous therefore given configuration is two parallel connected capacitors



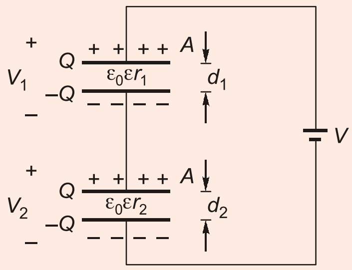
$$C_1 = \frac{\varepsilon_0 \cdot \varepsilon_{r1} A_1}{d}$$

and
$$C_2 = \frac{\varepsilon_0 \cdot \varepsilon_{r2} A_2}{d}$$

When two capacitor are connected in parallel then equivalent capacitance is given as

$$C_{eq} = C_1 + C_2 = \frac{\varepsilon_0 \cdot \varepsilon_{r1} A_1}{d} + \frac{\varepsilon_0 \cdot \varepsilon_{r2} A_2}{d}$$

From figure (b) as the result needed, given capacitors must be connected in series, so we are providing solution accordingly,



Where, $C_1 = \dfrac{\varepsilon_0 \cdot \varepsilon_{r1} A}{d_1}$

and $C_2 = \dfrac{\varepsilon_0 \cdot \varepsilon_{r2} A}{d_2}$

$$V = V_1 + V_2$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\frac{\varepsilon_0 \varepsilon_{r_1} A}{d_1}} + \frac{1}{\frac{\varepsilon_0 \varepsilon_{r_2} A}{d_2}}$$

End of Solution

Q.8 (c) (i) Find the general solution of the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 9(e^{3x} + 10xy^2).$$

(ii) Find the root of $xe^x - 1 = 0$ correct to four decimal places by the Newton-Raphson method starting with $x_0 = 0.6$.

[10 + 10 marks : 2022]

Solution:

(i) Given, $\dfrac{\partial^3 z}{\partial x^3} - \dfrac{2\partial^3 z}{\partial x^2 \partial y} = 9(e^{3x} + 10xy^2)$

$$D^3 - 2D^2D' = 9(e^{3x} + 10xy^2)$$

$\Rightarrow$ Auxillary equation,

$$m^3 - 2m^2 = 0$$

$\Rightarrow$ $m^2(m-2) = 0$

$\Rightarrow$ $m = 0, 0, 2$

So, $\text{CF} = F_1(y) + xF_2(y) + F_3(y + 2x)$

$$PI_1 = \frac{9e^{3x}}{D^3 - 2D^2D'}$$

Here, $D = 3$ and $D' = 0$

$$PI_1 = \frac{1}{3}e^{3x}$$

$\Rightarrow$ For $PI_2 = \dfrac{90xy^2}{D^3 - 2D^2D'}$

$$PI_2 = \frac{1}{D^3\left(1 - \frac{2D'}{D}\right)}(90xy^2)$$

$$= \frac{90}{D^3}\left(1-\frac{2D'}{D}\right)^{-1}(xy^2)$$

$$= \frac{90}{D^3}\left(1+\frac{2D'}{D}+\frac{4D'^2}{D^2}+\frac{8D'^3}{D^3}+....\right)xy^2$$

$$= \frac{90}{D^3}\left(xy^2+\frac{2}{D}\cdot D'(xy)^2+\frac{4}{D^2}D'^2(xy^2)+\frac{8}{D^3}D'^3(xy^2)+....\right)$$

$$= \frac{90}{D^3}\left(xy^2+\frac{2}{D}(2xy)+\frac{4}{D^2}(2x)+0\right) = 90\left(\frac{y^2}{D^3}(x)+\frac{4y}{D^4}(x)+\frac{8}{D^5}(x)\right)$$

$$= 90\left(y^2\cdot\frac{x^4}{2.3.4}+4y\frac{x^5}{2.3.4.5}+8\frac{x^6}{2.3.4.5.6}\right)$$

$$= 90\left(\frac{x^4y^2}{24}+\frac{x^5y}{30}+\frac{x^6}{90}\right)$$

Hence, complete solution is

$$Z = F_1(y)+xF_2(y)+F_3(y+2x)+\frac{1}{3}e^{3x}+90\left(\frac{x^4y^2}{24}+\frac{x^5y}{30}+\frac{x^6}{90}\right)$$

(ii) Given, Equation, $f(x) = xe^x - 1 = 0$

and $x_0 = 0.6$

For Newton-Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

So, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$\Rightarrow$ $f'(x_0) = e^x + xe^x = (x+1)e^x$

$\Rightarrow$ $x_1 = x_0 - \frac{(x_0e^{x_0}1)}{(x_0+1)e^{x_0}}$

$\Rightarrow$ $x_1 = 0.5680$

Similarly, $x_2 = x_1 - \frac{(x_1e^{x_1}-1)}{(x_1+1)e^{x_1}}$

$\Rightarrow$ $x_2 = 0.56714$

Similarly, $x_3 = x_2 - \frac{(x_2e^{x_2}-1)}{(x_2+1)e^{x_2}}$

$\Rightarrow$ $x_3 = 0.56714$

Hence, the root of the equation correct upto 4 digit is 0.567.

End of Solution

■■■■