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ESE 2022

Main Exam Detailed Solutions

Electrical Engineering

PAPER-II

EXAM DATE : 26-06-2022 | 2:00 PM to 05:00 PM

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ANALYSIS

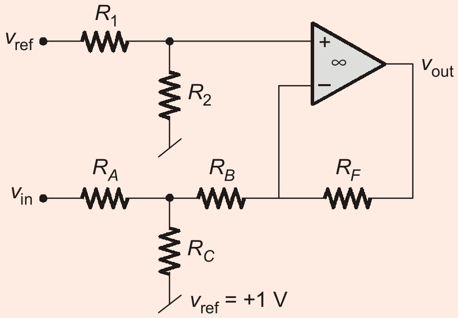
Electrical Engineering Paper-II
ESE 2022 Main Examination

Sl.	Subjects	Marks
1.	Analog and Digital Electronics	54
2.	Power Systems	72
3.	Systems & Signal Processing	72
4.	Control Systems	84
5.	Electrical Machines	104
6.	Power Electronics	84
7.	Communication Systems	10
		Total 480

Scroll down for detailed solutions

Section-A

Q.1 (a) Draw the transfer characteristic of the circuit shown below. Assume the op-amp to be ideal.



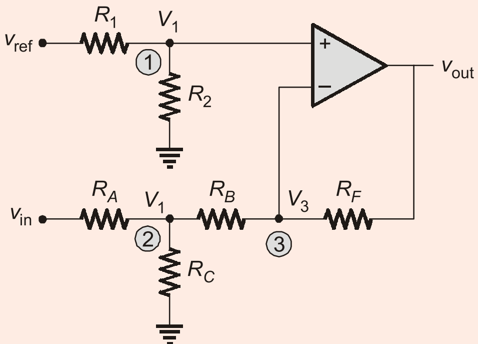
$$R_1 = R_2 = R_A = R_C = 100\text{ k}\Omega$$
$$R_B = 50\text{ k}\Omega,\ R_F = 1\text{ M}\Omega$$

[12 marks : 2022]

Solution:

Given :

$$V_{ref} = 1\text{ V}$$
$$R_1 = R_2 = R_A = R_C = 100\text{ k}\Omega$$
$$R_B = 50\text{ k}\Omega,\ R_F = 1\text{ M}\Omega$$



At node (1), apply voltage divider rule

$$V_1 = V_{ref} \times \frac{R_2}{R_2 + R_1}$$

$$= 1 \times \frac{100k}{100k + 100k} = \frac{1}{2}\text{V} \quad \text{...(1)}$$

At node (3), apply nodal analysis,

$$\frac{V_3 - V_2}{R_B} + \frac{V_3 - V_0}{R_F} = 0$$

$$V_3\left(\frac{1}{50k} + \frac{1}{1000k}\right) = \frac{V_2}{50k} + \frac{V_0}{1000k}$$

$$V_3 = \frac{20}{21}V_2 + \frac{V_0}{21} \quad \text{...(2)}$$

At node (2) apply nodal analysis,

$$\frac{V_2 - V_{in}}{R_A} + \frac{V_2}{R_C} + \frac{V_2 - V_3}{R_B} = 0$$

$$V_2\left(\frac{1}{100k} + \frac{1}{100k} + \frac{1}{50k}\right) = \frac{V_{in}}{100k} + \frac{V_3}{50k} \quad \text{...(3)}$$

$$V_2 = \frac{V_{in}}{4} + \frac{V_3}{2} \quad \text{...(3)}$$

Put eqn. (3) into eqn. (2)

$$V_3 = \frac{40}{21}\left(\frac{V_{in}}{4} + \frac{V_3}{2}\right) + 21V_0$$

$$\frac{11}{21}V_3 = \frac{5}{21}V_{in} + 21V_0$$

By using virtual ground theory, $V^+ = V^-$

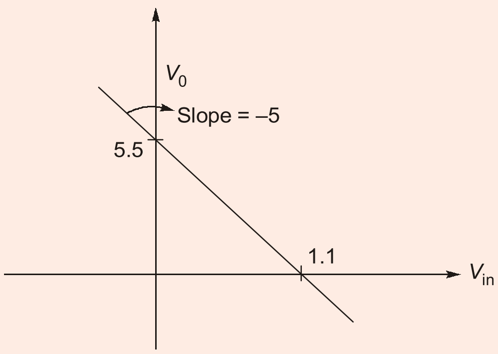
$$V_3 = V_1$$

$$\therefore \quad V_3 = \frac{1}{2}V$$

$$\therefore \quad \frac{11}{21} \times \frac{1}{2} = \frac{5}{21}V_{in} + \frac{V_0}{21}$$

$$V_o = -5V_{in} + 5.5$$

Transfer characteristic of the circuit,



End of Solution

Q.1 (b) Let an analog filter have a transfer function

$$H(s) = \frac{s+0.5}{(s+0.5)^2+16}$$

Use bilinear transformation method to convert analog filter into a digital filter.

Assume the resonant frequency of $\omega_r = \frac{\pi}{2}$ for the digital IIR filter.

[12 marks : 2022]

Solution:

Given : $\omega_r = \frac{\pi}{2}$

From the system transfer function, we note that $\Omega_c = 4$. The sampling period can be determined as

$$\Omega_c = \frac{2}{T_s}\tan\frac{\omega_r}{2}$$

$$4 = \frac{2}{T_s}\tan\frac{\pi}{4}$$

$$T_s = \frac{1}{2}\text{sec}$$

Using bilinear transformation

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{(z-1)}{(z+1)}}$$

$$H(z) = \frac{\frac{4(z-1)}{(z+1)}+0.5}{\left[\frac{4(z-1)}{(z+1)}+0.5\right]^2+16}$$

$$= \frac{[4(z-1)+0.5(z+1)](z+1)}{[4(z-1)+0.5(z+1)]^2+16(z+1)^2}$$

$$= \frac{(4z-4+0.5z+0.5)(z+1)}{16(z-1)^2+0.25(z+1)^2+4(z-1)(z+1)+16(z+1)^2}$$

$$= \frac{(4.5z-3.5)(z+1)}{16(z^2-2z+1)+16.25(z^2+2z+1)+4z^2-4}$$

$$H(z) = \frac{4.5z^2+z-3.5}{36.25z^2+0.5z+28.25}$$

$$H(z) = \frac{4.5+z^{-1}-3.5z^{-2}}{36.25+0.5z^{-1}+28.25z^{-2}} \quad \text{(Required bilinear transformation)}$$

End of Solution

Q.1 (c) An LTI system is described by the following equations :

$$X = AX + Bx$$
$$y = CX + Dx$$

where

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix};\ B = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix};$$

$$C = [1 \quad 0 \quad 1 \quad 0];\ D = 0.$$

Draw the block diagram of the system and therefrom, identify the state variable(s) that are

(i) not controllable.

(ii) not observable.

(iii) neither controllable nor observable.

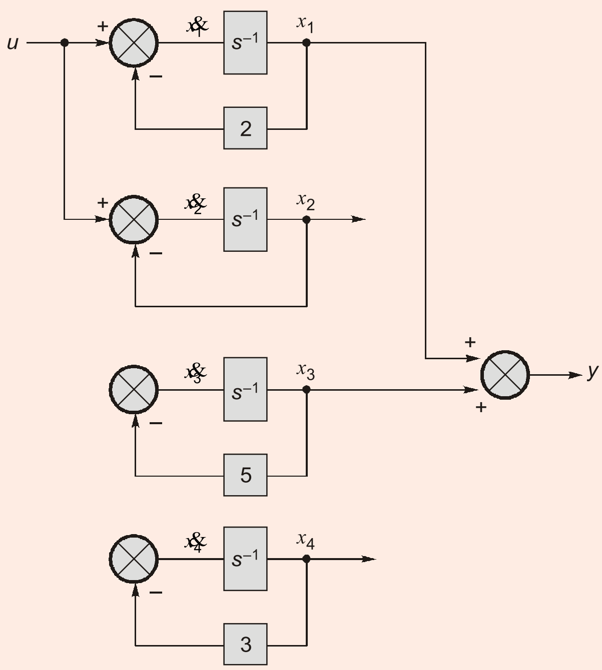
[12 marks : 2022]

Solution:

From given state model state and output equations can be written as

$$\dot{x}_1 = -2x_1 + U$$
$$\dot{x}_2 = -x_2 + U$$
$$\dot{x}_3 = -5x_3$$
$$\dot{x}_4 = -3x_4$$
$$y = x_1 + x_3$$

State diagram :



(i) State variable x_3 and x_4 are not controllable.
(ii) State variable x_2 and x_4 are unobservable.
(iii) State variable x_4 is uncontrollabel and unobservable.

End of Solution

Q.1 (d) The efficiency of a 10 kVA, 110/220 V, 50 Hz, single phase transformer is 96% at three-fourth of full load at 0.8 pf leading and 96.5% at full load upf.
Determine the :
(i) constant loss "P_i".
(ii) maximum efficiency at upf.

[12 marks : 2022]

Solution:

(i) $S = 10 \text{ kVA}, f = 50 \text{ Hz}$

Let constant loss = P_i, Copper loss = P_{Cu}

Case I : Efficiency, $\eta = 96\%, x = \frac{3}{4}$ of full-load

Power factor = 0.8

$$\text{Efficiency, } \eta = \frac{xS \times Pf}{Pf \times xs + P_i + x^2 P_{Cu}}$$

$$0.96 = \frac{\frac{3}{4} \times 10 \times 0.8}{0.8 \times \frac{3}{4} \times 10 + P_i + \left(\frac{3}{4}\right)^2 \times P_{Cu}}$$

$$P_i + \frac{9}{16} P_{Cu} = 0.25 \quad \text{...(i)}$$

Case II : Efficiency, $\eta = 96.5\%, x = 1$ (full-load)

Power factor = 1

$$0.965 = \frac{1 \times 10 \times 1}{1 \times 1 \times 10 + P_i + 1^2 \times P_{Cu}}$$

$$P_i + P_{Cu} = 0.3626 \quad \text{...(ii)}$$

Now, solve eqn. (i) and eqn. (ii)

$$P_i = 0.10522 \text{ kW} = 105.22 \text{ W}$$

$$P_{Cu} = 0.25737 \text{ kW} = 257.37 \text{ W}$$

Hence, constant loss, $P_i = 105.22$ W

(ii) For maximum efficiency

Constant loss = Copper loss

$$\text{Pi} = x^2 P_{Cu}$$

($x \rightarrow$ % load at which maximum efficiency occur)

$$105.22 = x^2 \times 257.37$$

$$x = \sqrt{\frac{105.22}{257.37}}$$

$$= 0.6393$$

So, maximum efficiency occur at 63.93% of full-load

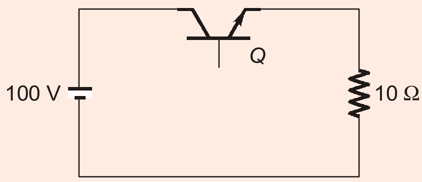
$$\text{Power factor} = 1$$

$$\eta_{max} = \frac{10 \times 0.6393 \times 1}{10 \times 0.6393 \times 1 + 2 \times 0.10522}$$

$$= 96.813\%$$

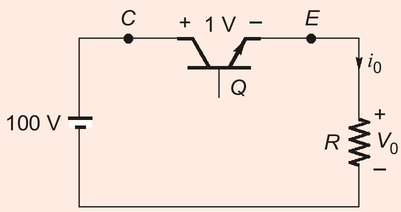
End of Solution

Q.1 (e) In the figure below, the transisotr Q is operating at 10 kHz and 40% duty ratio. If the transistor has on-state voltage drop of 1 V and $t_{on} = t_{off} = 5$ μs, find its total losses during the operation.
(Assume linear rise and fall characteristics of the voltage and currents in the device during switching).



[12 marks : 2022]

Solution:



$$t_{ON} = t_{OFF} = 5\ \mu\text{sec}$$

BJT → ON (Ideal),

$$i_o = \frac{V_o}{R} = \frac{V_s}{R} = \frac{100}{10} = 10\ \text{A}$$

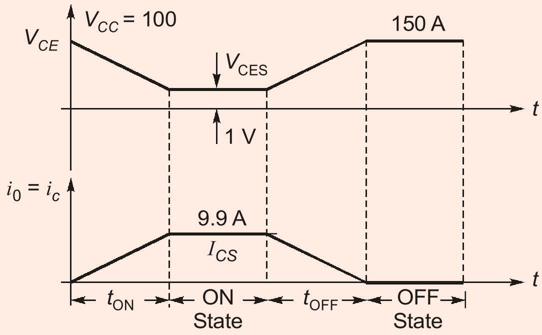
Considering V-drop :

$$i_o = \frac{V_s - V_{drop}}{R}$$

$$i_o = \frac{100 - 1}{10}$$

$$= \frac{99}{10}$$

$$i_o = 9.9\ \text{A}$$



t_{ON} process :

$$i_C = \frac{I_{CS}}{t_{ON}} \cdot t$$

$$V_{CE} = -\left(\frac{V_{CC} - V_{CES}}{t_{ON}}\right)t + V_{CC}$$

Average power loss during t_{ON} process

$$= \frac{1}{T}\int_0^{t_{ON}} V_{CE} \cdot i_C \cdot dt$$

$$= \frac{1}{T}\int_0^{t_{ON}} \left[-\left(\frac{V_{CC} - V_{CES}}{t_{ON}}\right)t + V_{CC}\right] \cdot \left(\frac{I_{CS}}{t_{ON}} \cdot t\right) dt$$

$$= f \cdot I_{CS} \cdot t_{ON}\left[\frac{V_{CC}}{2} - \frac{V_{CC} - V_{CES}}{3}\right]$$

$$= 10 \times 10^3 \times 9.9 \times 5 \times 10^{-6}\left[\frac{100}{2} - \frac{100-1}{3}\right]$$

$$= 8.415 \text{ W}$$

t_{OFF} process :

Average power loss during t_{OFF} process

$$= \frac{1}{T}\int_0^{t_{OFF}} V_{CE} \cdot i_C \cdot dt$$

$$= \frac{1}{T}\int_0^{t_{OFF}} \left[\left(\frac{V_{CC} - V_{CES}}{t_{OFF}}\right) \cdot t + V_{CES}\right] \cdot \left[\frac{-I_{CS}}{t_{OFF}} \cdot t + I_{CS}\right] dt$$

$$\simeq f \cdot t_{OFF} \cdot \frac{I_{CS}}{6}(V_{CC} - V_{CES})$$

$$= 10 \times 10^3 \times 5 \times 10^{-6} \times \frac{9.9}{6}(100 - 1)$$

$$= 8.1675 \text{ W}$$

On State :

$$V_{CE} = 1\text{ V},\ i_C = 9.9\text{ A}$$

Duty ratio : $$\alpha = \frac{T_{ON}}{T}$$

$$\alpha = T_{ON}.f$$

$$\therefore \quad T_{ON} = \frac{\alpha}{f} = \frac{0.4}{10.10^3}$$

$$T_{ON} = 40\ \mu s$$

∴ Average power loss during ON state

P_{Avg} (On state) :

$$P_{Avg}\text{ (On State)} = \frac{1}{T}\int_0^{T_{ON}} V_{CE}\cdot i_C \cdot dt$$

$$= f\int_0^{T_{ON}} 1\times 9.9\times dt$$

$$= f.T_{ON}.9.9$$

$$= 10\times 10^3\times 40\times 10^{-6}\times 9.9$$

$$= 3.96\text{ W}$$

$$P_{Avg}\text{ (OFF State)} = \frac{1}{T}\int_0^{T_{OFF}} V_{CE}\cdot i_C dt$$

$$= 0$$

∴ Total average power loss

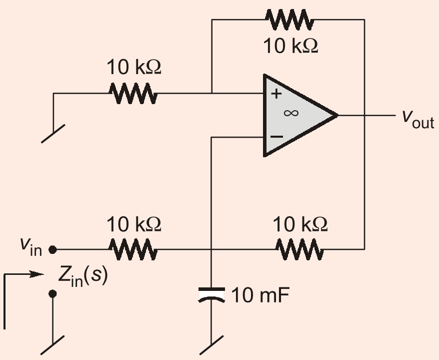
$$= P_{Avg}\,(t_{ON\ process}) + P_{Avg}\text{ (ON state)} + P_{Avg}\,(t_{OFF\ process}) + P_{Avg}\text{ (OFF state)}$$

$$= 8.415\text{ W} + 3.96\text{ W} + 8.1675 + 0$$

$$= 20.54\text{ W}$$

End of Solution

Q.2 (a) (i) Determine the transfer function of the circuit shown below and determine therefrom, the function performed by the circuit. Also determine the input impedance $Z_{in}(s)$, as indicated in the figure.

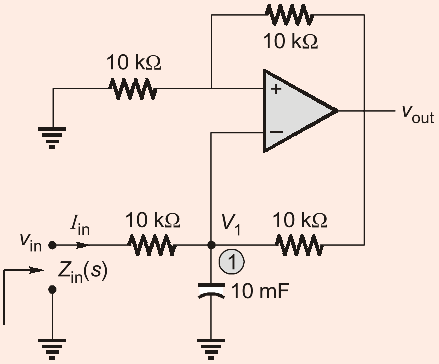


(ii) The transfer function of a 2nd order filter has a zero at the origin of the s-plane while its poles are located at $s = -3 \pm j4$ rad/sec. Identify the filter, if the magnitude of the voltage gain at 5 rad/sec = 1. Also plot the magnitude of the voltage gain as the input signal frequency is varied between $\omega = 0$ to $\omega = \infty$.

[12 + 8 marks : 2022]

Solution:

(i) At node (1), apply KCL,



$$\frac{V_1 - V_{in}}{10k} + \frac{V_1}{\frac{1}{s} \times (0.01)} + \frac{V_1 - V_{out}}{10k} = 0$$

$$V_1\left(\frac{1}{10k} + 0.01s + \frac{1}{10k}\right) = \frac{V_{in}}{10k} + \frac{V_{out}}{10k}$$

$$V_1 = \frac{V_{in} + V_{out}}{2(1+50s)} \quad \text{...(1)}$$

We know that, for non-inverting amplifier,

$$V_{out} = \left(1 + \frac{R_2}{R_1}\right)V_1 = \left(1 + \frac{10k}{10k}\right)V_1 = 2V_1$$

$$\therefore \quad V_{out} = 2\left(\frac{V_{in}}{2(1+50s)} + \frac{V_{out}}{2(1+50s)}\right)$$

$$V_{out}\left(1 - \frac{1}{1+50s}\right) = \frac{V_{in}}{1+50s}$$

$$V_{out} \times \frac{50s}{1+50s} = \frac{V_{in}}{1+50}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{50s}$$

$$I_{in} = \frac{V_{in} - V_1}{10k} = \frac{V_{in} - \frac{(V_{in} + V_{out})}{2(1+50s)}}{10k}$$

$$= \frac{V_{in}(2(1+50s)) - V_{in} - \frac{V_{in}}{50s}}{2 \times 10k(1+50s)}$$

$$= V_{in}\left(\frac{2+100s-1-\frac{1}{50s}}{20k(1+50s)}\right)$$

$$= V_{in}\left(\frac{(1+100s)(50s)-1}{(50s)(20k)(1+50s)}\right)$$

Input impedance, $Z_i = \frac{V_{in}}{I_{in}}$

$\therefore$ $Z_i = \frac{1000s(1+50s)}{5000s^2+50s-1}$ kΩ

(ii) Poles : $s = -3 \pm j4$

Zeros : $s = 0$

$\therefore$ Transfer function of 2nd order is given as

$$H(s) = \frac{K.s}{(s+3)^2+16} = \frac{Ks}{s^2+6s+25}$$

$$|H(j\omega)||_{\omega=5} = \left|\frac{K(5)}{-25+30j+2s}\right| = 1$$

$$\frac{5K}{30} = 1$$

$$K = 6$$

$\therefore$ $$H(j\omega) = \frac{6(j\omega)}{2s-\omega^2+6j\omega}$$

$$|H(j\omega)| = \frac{6\omega}{\sqrt{(2s-\omega^2)^2+(6\omega)^2}}$$

At $\omega = 0$, $|H(j\omega)| = 0$

At $\omega = 1$, $|H(j\omega)| = 0.242$

At $\omega = 10$, $|H(j\omega)| = 0.624$

At $\omega = 15$, $|H(j\omega)| = 0.410$

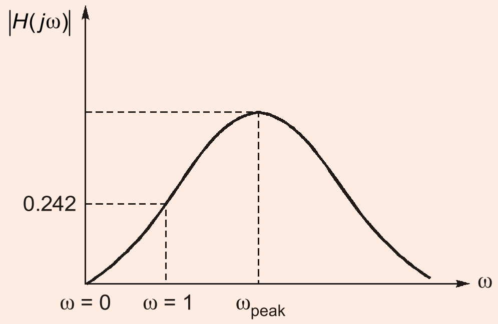
At $\omega = 20$, $|H(j\omega)| = 0.304$

At $\omega = 30$, $|H(j\omega)| = 0.2014$

At $\omega = 50$, $|H(j\omega)| = 0.12$

At $\omega = 100$, $|H(j\omega)| = 0.059$

Plot $|H(j\omega)|$ (voltage gain) $V_s \omega$



From the plot, we can see that, it is a band pass filter.

End of Solution

Q.2 (b) Consider a mechanical vibratory system as shown in Figure (a) below. Two kilograms of force is applied to the system in the form of step input resulting in the oscillation of mass. Determine the system parameters *m*, *b* and *k* of the mechanical vibratory system if the step response curve obtained is as shown in Figure (b). Displacement '*x*' may be measured from the equilibrium position.

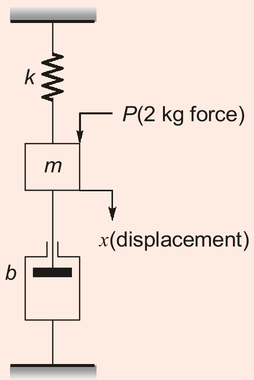


Figure (a)

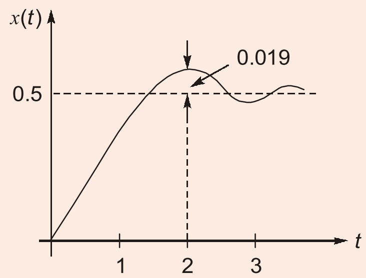


Figure (b)

[20 marks : 2022]

Solution:

We know,

$$F = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + Kx$$

Take Laplace transform of above

$$F(s) = ms^2X(s) + bsX(s) + KX(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + K}$$

$$= \frac{1/m}{s^2 + \frac{b}{m}s + \frac{K}{m}} \quad \text{...(i)}$$

From the graph,

$$\% M_P = \frac{0.019}{0.05} \times 100$$

$$= 38\%$$

$$e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = 0.38$$

$$\pi\xi = \ln\left(\frac{1}{0.38}\right)\left(\sqrt{1-\xi^2}\right)$$

Squaring both sides,

$$(\pi\xi)^2 = \left(\ln\frac{1}{0.38}\right)^2 (1-\xi^2)$$

$$(\pi\xi)^2 = 0.9362(1-\xi^2)$$

$$\xi^2 = \frac{0.9362}{\pi^2 + 0.9362}$$

$$\xi = 0.2943$$

From graph peak time (t_P) = 2 sec

$$t_P = \frac{\pi}{\omega_d} = 2$$

$$\omega_d = \frac{\pi}{2} = 1.57 \text{ rad/sec}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_n = \frac{1.57}{\sqrt{1-(0.2943)^2}} = 1.642 \text{ rad/sec}$$

From eqn. (i) characteristic equation is

$$s^2 + \frac{b}{m}s + \frac{K}{m} = 0 \quad \text{...(ii)}$$

Standard second-order characteistic equation is given by

$$s^2 + 2\xi\omega_n + \omega_n^2 = 0 \quad \text{...(iii)}$$

$$\text{Steady state value} = 0.05$$

From eqn. (i)

$$\frac{X(s)}{F(s)} = \frac{1/m}{s^2 + \frac{b}{m}s + \frac{K}{m}}$$

$$X(s) = \frac{1/m}{s^2 + \frac{b}{m}s + \frac{K}{m}} F(s)$$

Here, $F(t) = 2u(t)$

$$F(s) = \frac{2}{s}$$

$$X(s) = \frac{1/m}{s^2 + \frac{b}{m}s + \frac{K}{m}} \cdot \frac{2}{s}$$

Steady state value, $$X(s) = \lim_{s\to 0} sX(s)$$

$$= \lim_{s\to 0} s \frac{1/m}{s^2 + \frac{b}{m}s + \frac{K}{m}} \cdot \frac{2}{s}$$

$$= \frac{1/m}{K/m} = \frac{2}{K} = 0.05$$

$$K = \frac{1}{0.05} = 40$$

Now, compare eqn. (ii) and eqn. (iii)

$$\frac{b}{m} = 2\xi\omega_n$$

$$\frac{K}{m} = \omega_n^2$$

$$\Rightarrow \quad \frac{40}{m} = (1.642)^2$$

$$\Rightarrow \quad m = 14.83$$

$$\frac{b}{14.83} = 2 \times 0.2943 \times 1.642$$

$$b = 14.32$$

Hence, $m = 14.83$ kg

$b = 14.32$

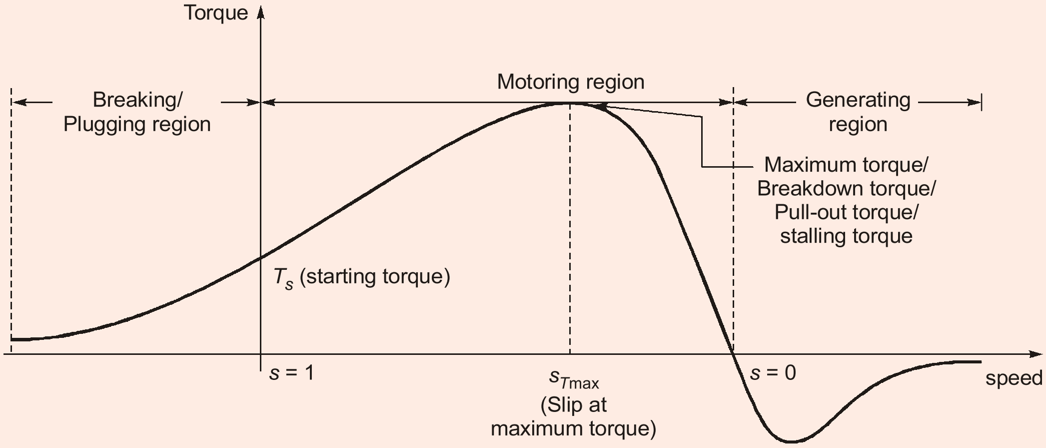
$K = 40$

End of Solution

Q.2 (c) Draw the Speed-Torque characteristics of a three-phase induction motor. A 3-phase, 50 Hz, 400 V, 8-pole, star-connected induction motor has the following circuit parameters referred to stator side in an exact equivalent circuit model :
$R_1 = 0.21\ \Omega$, $R_2' = 0.32\ \Omega$, $X_1 + X_2' = 4.8\ \Omega$, $R_i = 138\ \Omega$, $X_m = 18.1\ \Omega$
Assuming $X_1 = X_2'$ and slip $S = 4\%$, find the
(i) total impedance seen from stator side (per-phase).
(ii) input line current.
(iii) input power drawn.
(iv) gross mechanical power developed.
(v) gross torque developed.

[20 marks : 2022]

Solution:



Given : Parameter of 3-ϕ induction motor

$$R_1 = 0.21\ \Omega$$
$$X_1 + X_2' = 4.8\ \Omega$$
$$X_m = 18.1\ \Omega$$
$$R_2' = 0.32\ \Omega$$
$$R_i = 138\ \Omega$$
Slips, $s = 4\% = 0.04$

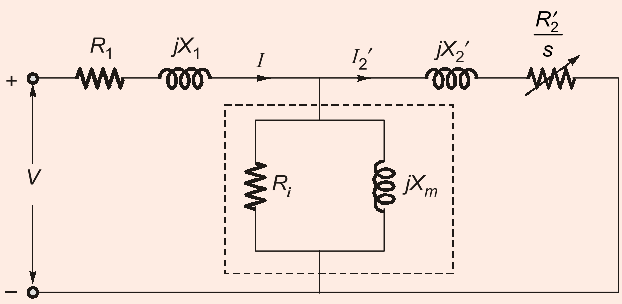
and $X_1 = X_2'$

frequency, $f = 50$ Hz

Number of poles, $P = 8$

Supply voltage, $V = 400$ V

As $X_1 = X_2'$ ⇒ $X_1 = X_2' = 2.4\ \Omega$



(i) Total impedance,
$$Z = R_1 + jX_1 + \left[R_i \,||\, jX_m \,||\left(\frac{R_2'}{s} + jX_2'\right)\right]$$

⇒
$$Z = 0.21 + 2.4j + \left[138 \,||\, 18.1j \,||\left(\frac{0.32}{0.04} + j2.4\right)\right]$$

$$Z = (5.5335 + 6.3144j)\ \Omega$$

⇒
$$Z = 8.396\angle 48.771^\circ\ \Omega$$

(ii) Input line current,
$$I = \frac{V}{\sqrt{3}Z} \text{ A}$$
$$I = 27.506\angle -48.771 \text{ Å}$$

(iii) Input power drawn,
$$P = \text{Re}\left\{\sqrt{3}VI^*\right\}$$
$$P = \text{Re}\{12559.887 + j14332.38\}$$
$$P = 12.56 \text{ kW}$$

(iv) Net impedance of shunt branch,
$$Z_i = (138 \parallel 18.1j)\ \Omega$$
$$\Rightarrow \quad Z_i = (2.334 + 17.794j)\ \Omega$$
$$\Rightarrow \quad Z_i = 17.9463\angle 82.528°\ \Omega$$

So, by current division,
$$I'_2 = I \times \frac{Z_i}{Z_i + 8 + 2.4j}$$
$$I'_2 = 21.761\angle -29.143 \text{ A}$$

So, Air gap power,
$$P_g = 3|I'_2|^2\left(\frac{R'_2}{s}\right)$$
$$P_g = 11.365 \text{ kW}$$

Gross mechanical power developed,
$$P_d = P_g(1 - s)$$
$$P_d = 10.910 \text{ kW}$$

(v) Gross torque developed,
$$T_d = \frac{P_d}{\frac{2\pi N_s}{60}(1-s)}$$
$$\Rightarrow \quad T_d = \frac{P_g \times 60}{2\pi N_s}$$

Synchronous speed,
$$N_s = \frac{120f}{P} \Rightarrow N_s = \frac{120 \times 50}{8}$$
$$\Rightarrow \quad N_s = 750 \text{ rpm}$$
$$T_d = 144.704 \text{ Nm}$$

End of Solution

Q.3 (a) Consider the unilateral Laplace transform pair cos $(2t)u(t) \xleftrightarrow{Lu} X(s)$. Using the Laplace transform properties, determine the time signals corresponding to the following :

(i) $X(2s)$

(ii) $(s + 4)X(s)$

(iii) $\frac{d}{ds}(e^{-5s}X(s))$

(iv) $s^{-2}X(s)$

[20 marks : 2022]

Solution:

$$x(t) = \cos(2t)u(t) \rightleftharpoons X(s) = \frac{s}{s^2+4}$$

(i) Time-scaling property : $x(at),\ a \neq 0 \rightleftharpoons \frac{1}{|a|}X\left(\frac{s}{a}\right)$

Put $a = \frac{1}{2}$

$$x\left(\frac{t}{2}\right) \rightleftharpoons 2X(2s)$$

$$\frac{1}{2}x\left(\frac{t}{2}\right) \rightleftharpoons X(2s)$$

Let $F(s) = X(2s)$

Therefore, $f(t) = \frac{1}{2}x\left(\frac{t}{2}\right) = \frac{1}{2}\cos 2\left(\frac{t}{2}\right) = \frac{1}{2}\cos t$

(ii) Differentiation in time property :

$$\frac{dx(t)}{dt} \rightleftharpoons sX(s) - x(0^-) \quad ...(1)$$

$\because$ $x(t) = \cos 3tu(t)$

$\therefore$ $x(0^-) = 0$

From (1), $\frac{dx(t)}{dt} \rightleftharpoons sX(s)$

Let $F(s) = (s + 4)X(s) = sX(s) + 4X(s)$

Therefore, $f(t) = \frac{dx(t)}{dt} + 4x(t)$

$$= \frac{d}{dt}[\cos 2t.u(t)] + 4\cos 2tu(t)$$

$\Rightarrow$ $f(t) = [-2\sin 2tu(t) + \cos 2t\delta(t)] + 4\cos 2tu(t)$

$= -2\sin 2t.u(t) + \delta(t) + 4\cos 2t.u(t)$

$\Rightarrow$ $f(t) = [4\cos 2t - 2\sin 2t]u(t) + \delta(t)$

$[\because \cos 2t.\delta(t) = \cos 0\,\delta(t) = \delta(t)]$

(iii) Time-shifting property :

$$x(t-t_o) \rightleftharpoons X(s).e^{-st_o}$$

Put $t_o = 5$

$$x(t-5) \rightleftharpoons X(s).e^{-5s}$$

Let $$F(s) \rightleftharpoons e^{-5s}X(s)$$

Therefore, $$f(t) = x(t-5)$$

By applying differentiation in frequency property,

$$tf(t) \rightleftharpoons \frac{-dF(s)}{ds}$$

$\Rightarrow$ $$-tf(t) \rightleftharpoons \frac{dF(s)}{ds}$$

$\Rightarrow$ $$-tx(t-5) \rightleftharpoons \frac{d}{ds}[e^{-5s}X(s)]$$

(iv) $$s^{-2}X(s) = s^{-2}\cdot\frac{s}{s^2+4}$$

$$= \frac{1}{s(s^2+4)}$$

$$= \frac{A}{s}+\frac{Bs+C}{s^2+4}$$

$$= \frac{As^2+4A+Bs^2+Cs}{s(s^2+4)}$$

$$A + B = 0,\ 4A = 1,\ C = 0$$

$\Rightarrow$ $$B = -\frac{1}{4},\ A = \frac{1}{4}$$

$\Rightarrow$ $$s^{-2}X(s) = \frac{\frac{1}{4}}{s}-\frac{\frac{1}{4}s}{s^2+4}$$

Take laplace inverse of above expression

$$x(t) = \frac{1}{4}u(t)-\frac{1}{4}\cos(2t)u(t)$$

$$= \frac{1}{4}[1-\cos(2t)]u(t)$$

End of Solution

Q.3 (b) Let the steady state velocity error be defined as the difference in velocity between input and output of the unity feedback system as time approaches infinity. Find the expression for the error in the velocity, $\dot{e}(\infty) = \dot{r}(\infty) - \dot{i}(\infty)$ and fill the following table for the error in velocity for different inputs in Type 0, 1, 2 system. Notations carry their usual meaning.

		Type		
		0	1	2
Input	Step			
	Ramp			
	Parabola			

[20 marks : 2022]

Solution:

$$\dot{e}(\infty) = \dot{r}(\infty) - \dot{i}(\infty) \text{ (given)}$$

$$e_{ss} = \text{Input} - \text{Output}$$

$e_{ss} \rightarrow$ Steady state error

$$e_{ss} = R(s) - R(s)\frac{G(s)}{1+G(s)} = \frac{R(s)}{1+G(s)}$$

(i) Position error constant (K_p). It is defined for unit step input.

$$\therefore \quad R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s\to 0}\frac{sR(s)}{1+G(s)} = \lim_{s\to 0}\frac{s\times\frac{1}{s}}{1+G(s)}$$

$$\therefore \quad e_{ss} = \frac{1}{1+\lim_{s\to 0}G(s)} = \frac{1}{1+G(0)} = \frac{1}{1+K_p}$$

$$K_p = G(0) \rightarrow \text{Position error constant}$$

$\therefore$ For type 0 system, $e_{ss} = \dfrac{1}{1+K_p}$

Type 1 system, $K_p = \infty,\ e_{ss} = 0$

Type 2 system, $K_p = \infty,\ e_{ss} = 0$

(ii) Velocity error constant : It is defined for unit ramp input, i.e.,

$$R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s\to 0}\frac{sR(s)}{1+G(s)} = \lim_{s\to 0}\frac{s\times\frac{1}{s^2}}{1+G(s)} = \lim_{s\to 0}\frac{1}{s+sG(s)}$$

$$= \frac{1}{\lim_{s\to 0}sG(s)} = \frac{1}{K_V}$$

$$\therefore \quad K_V = \lim_{s\to 0}sG(s) \rightarrow \text{Velocity error constant}$$

For type 0, $K_V = 0,\ e_{ss} = \infty$

For type 1, $K_V = \text{finite},\ e_{ss} = \dfrac{1}{K_V}$

For type 2, $K_V = \infty,\ e_{ss} = 0$

(iii) Acceleration error constant : It is defined for unit parabolic input, i.e.,

$$R(s) = \frac{1}{s^3}$$

$$e_{ss} = \lim_{s\to 0}\frac{sR(s)}{1+G(s)} = \lim_{s\to 0}\frac{s\times\frac{1}{s^3}}{1+G(s)}$$

$$= \lim_{s\to 0}\frac{1}{s^2+s^2G(s)} = \frac{1}{\lim\limits_{s\to 0}s^2G(s)} = \frac{1}{K_a}$$

$\therefore$ $K_a = \lim\limits_{s\to 0}s^2G(s) \rightarrow$ Acceleration error constant

For type 0, $K_a = 0,\ e_{ss} = \dfrac{1}{K_a} = \infty$

For type 1, $K_a = 0,\ e_{ss} = \dfrac{1}{K_a} = \infty$

For type 2, $K_a = \text{finite},\ e_{ss} = \dfrac{1}{K_a}$

Table for the error in velocity for different inputs in Type 0, 1, 2 system

		Type 0	Type 1	Type 2
Input	Step	$\frac{1}{1+K_p}$	0	0
	Ramp	∞	$\frac{1}{K_v}$	0
	Parabola	∞	∞	$\frac{1}{K_a}$

End of Solution

Q.3 (c) (i) What do you mean by Tuned Power Lines? A 3-phase, 50 Hz, 500 kV, lossless transmission line is 600 km long. The line inductance is 1 mH/km/phase and capacitance is 10 nF/km/phase. Determine the phase constant β, surge impedance Z_c, velocity of propagation v, and line wavelength λ.

(ii) A 200 km long, 50 Hz, 3-phase transmission line has a total series impedance of $40 + j125\ \Omega$ and shunt admittance of 10^{-3} ℧. It is feeding to a load of 62.5 MVA at 220 kV with 0.8 pf lagging. Find the sending end voltage, current and power factor using nominal π-method.

[10 + 10 marks : 2022]

Solution:

(i) A tuned line is where the receiving-end voltage and current are numerically equal to the corresponding sending-end values so that there is no voltage drop on load.

$$V_{L\text{-}L} = 500 \text{ kV}$$
$$f = 50 \text{ Hz}$$
$$L = 1 \text{ mH/km/phase}$$
$$C = 10 \text{ nF/km/phase}$$

Surge impedance, $$Z_C = \sqrt{\frac{L}{C}}$$
$$= \sqrt{\frac{1\times 10^{-3}}{10\times 10^{-9}}} = 316.22\ \Omega$$

Velocity of propagation, $$V = \frac{1}{\sqrt{LC}}$$
$$= \frac{1}{\sqrt{1\times 10^{-3}\times 10\times 10^{-9}}}$$
$$= 3.162\times 10^5 \text{ km/sec}$$

Line wavelength, $$\lambda = \frac{V}{f}$$
$$= \frac{3.162\times 10^5}{50}$$
$$= 6324.55 \text{ km}$$

Phase constant, $$\beta = \frac{2\pi}{\lambda} \text{ rad/km}$$
$$= \frac{2\pi}{6324.55} \text{ rad/km}$$
$$= 9.929\times 10^{-4} \text{ rad/km}$$

(ii) Given that $$Z = 40 + j125$$
$$Y = j10^{-3}\ \mho$$
$$V_s = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$
$$V_R = \frac{220}{\sqrt{3}} = 127 \text{ kV} \quad \text{(Phase)}$$
$$I_R = \frac{62.5\times 10^3}{\sqrt{3}\times 220} = 164 \text{ A}$$
$$\phi_R = \cos^{-1}(0.8) = 36.86°$$

A, B, C, D for nominal π can be calculated as

$$A = D = 1+\frac{YZ}{2}$$

$$= 1+\frac{j10^{-3}(40+j125)}{2} = 0.9377\angle 1.22^\circ$$

$$B = Z = 40 + j125 = 131.24\angle 72.25^\circ$$

$$C = \left(1+\frac{YZ}{4}\right)Y$$

$$= \left(1+\frac{j10^{-3}(40+j125)}{4}\right)j10^{-3}$$

$$= 96.88\times 10^{-5}\angle 90.59^\circ$$

$$V_s = 0.9377\angle 1.22 \times 127\times 10^3\angle 0^\circ + 131.24\angle 72.25^\circ \times 164\angle -36.86^\circ$$

$$= 137.42\angle 6.2^\circ$$

$$I_s = 96.88\times 10^{-5}\angle 90.59^\circ \times 127\times 10^3\angle 0^\circ + 0.9377\angle 1.22 \times 164\angle -36.86^\circ$$

$$= 128.13\angle 15.1^\circ$$

$$\phi_s = 15.1^\circ - 6.2^\circ = 8.9^\circ$$

$$\text{P.F.} = \cos\phi_s = \cos 8.9^\circ = 0.987 \text{ lead}$$

End of Solution

Q4 (a) A 240 V shunt motor takes a current of 3.5 A on no load. The armature resistance is of 0.4 Ω, and shunt field resistance of 160 Ω when motor operates at full load. It takes 24 A and runs at 2400 rpm when the motor is converted to long shunt compound type by adding a field winding of 0.1 Ω. There is a change of 10% in the total flux, when the compound motor develops the same torque. Determine the power developed and speed of the motor when it is connected as

(i) a cumulative compound motor.

(ii) a differential compound motor.

[20 marks : 2022]

Solution:

At full load, $I_L = 24\text{ A}, N = 2400$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{240}{160} = 1.5\text{ A}$$

$$I_a = I_L - I_{sh} = 22.5\text{ A}$$

$$E_b = 240 - 22.5(0.4) = 231\text{ V}$$

(i) It is mentioned that shunt became long shunt cumulative with change in flux 10%. Considering cumulative flux increased by 10%.

So, $\phi_2 = 1.1\phi_1$

Mentioned same torque, $T \propto \phi I_a$

$\therefore$ $\phi_1 I_{a1} = \phi_2 I_{a2}$

$$I_{a2} = \frac{\phi_1}{\phi_2} I_{a1} = \frac{\phi_1}{1.1\phi_1} 22.5$$

$$I_{a2} = 20.454 \text{ A}$$

$\therefore$
$$E_{b2} = V - I_{a2}(R_a + R_{se})$$
$$= 240 - 20.454(0.4 + 0.1) = 229.77 \text{ V}$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$$\frac{N_2}{2400} = \frac{229.77}{231} \times \frac{\phi_1}{1.1\phi_1}$$

$\therefore$ $N_2 = 2170.2$ rpm

Power developed : $E_{b2} I_{a2} = 229.77 \times 20.454$
$= 4699.7$ W

(ii) Considering differentially compounded flux change given 10%.

So $\phi_2 = 0.9\phi_1$

Torque is same given

$\therefore$ $\phi_1 I_{a1} = \phi_2 I_{a2}$

$$I_{a2} = \frac{\phi_1}{0.9\phi_1} \times 22.5 = 25 \text{ A}$$

$$E_{b2} = V - I_{a2}(R_a + R_{se}) = 240 - 25(0.4 + 0.1)$$
$$= 227.5 \text{ V}$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$$\frac{N_2}{2400} = \frac{227.5}{231} \times \frac{\phi_1}{0.9\phi_1}$$

$N_2 = 2626.26$ rpm

Power developed : $E_{b2} \times I_{a2} = 227.5 \times 25$
$= 5687.5$ W

End of Solution

Q.4 (b) What do you mean by the steady state stability limit of a power system? Find the steady state power limit of a power system consisting of a generator with equivalent reactance of 0.5 pu, connected to an infinite bus through a series reactance of 0.1 pu. The terminal voltage of the generaor is held at 1.2 pu and the infinite bus voltage at 1.0 pu.

[20 marks : 2022]

Solution:

Steady state stability is defined as the ability of an electric power system to maintain synchronism between machines within the system and an external tie line following a small and slow disturbance.

So, steady state stability limit refers to the maximum power which can be transferred without loss of stability.

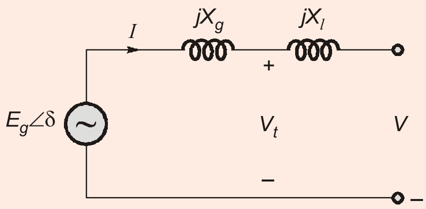
Given : Generation reactance, $\chi_g = 0.5$ pu

Service reactance of line, $\chi_2 = 0.1$ pu

Terminal voltage of generator, $V_b = 1.2$ pu

Infinite bus voltage, $V = 1.0$ pu

Let the generator voltage be '$E_g\angle\delta°$, then



Current flowing, $$I = \frac{E_g\angle 90° - V}{jX_g + jX_l} = \frac{V_t\angle\theta - V}{jX_l}$$

Let $V_l = 1.2\angle\theta$

So, $$I = \frac{1.2\angle\theta - 1}{0.1j}$$

$\Rightarrow$ $$E_g = 1.2\angle\theta + j0.5\left[\frac{1.2\angle\theta - 1}{0.1j}\right]$$

$\Rightarrow$ $$E_g = 1.2\angle\theta + 6\angle\theta - 5$$
$$= 7.2\angle\theta - 5$$
$$= 7.2\cos\theta - 5 + j7.2\sin\theta$$

As for steady state limit, $\delta = 90°$

So, $\text{Re}\{E_g\} = 0 \Rightarrow 7.2\cos\theta - 5 = 0$

$\Rightarrow$ $$\cos\theta = \frac{5}{7.2} \Rightarrow \theta = 46.017°$$

So, $E_g = 7.2\angle 46.017 - 5$

$\Rightarrow$ $|E_g| = 5.1807$ pu

So, steady state limit, $\text{SSSL} = \dfrac{|E_g||V|}{|(X_g)+jX_l|}$

$\text{SSSL} = 8.6346 \text{ pu}$

End of Solution

Q.4 (c) A single phase full bridge fully controlled converter is supplying a battery at 100 V for charging application. The converter is connected to 240 V, 50 Hz AC supply. The interconnecting inductor between converter and battery is very large to make the load current flat, continuous and ripple free at 10 A. The inductor has a resistance of 1.5 Ω. Find the

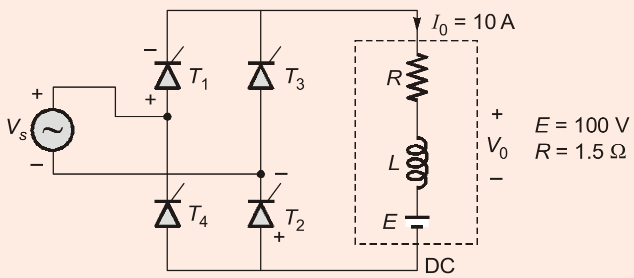
(i) operating triggering angle.
(ii) average and rms values of converter output voltage.
(iii) power supplied to the battery.
(iv) converter input power.
(v) input power factor and displacement factor.
(vi) thyristor rms current.

Draw source current, thyristor current and output voltage waveforms of the converter.

Assume a conduction voltage drop of 1.5 V for each thyristor.

[20 marks : 2022]

Solution:



(i) $T_1T_2 \rightarrow$ ON : KVL : Consider drop of T_1 and T_2

$$-v_s + 1.5 + v_o + 1.5 = 0$$

$$v_o = v_s - 3\text{V}$$

$$v_o = V_m.\sin \omega t - 3$$

$$V_o = \frac{1}{T}\int_{\alpha}^{\pi+\alpha} (V_M \sin\omega t - 3)d(\omega t)$$

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} (V_M \sin\omega t - 3)d(\omega t)$$

$$= \frac{2V_M}{\pi}\cos\alpha - \frac{3}{\pi}\int_{\alpha}^{\pi+\alpha} d(\omega t)$$

$$= \frac{2V_M}{\pi}\cos\alpha - \frac{3}{\pi}(\omega t)_a^{\pi+\alpha}$$

$$V_o = \frac{2V_m}{\pi}\cos\alpha - 3$$

$$V_o = E + I_o R$$

$$= 100 + (10 \times 1.5)$$

$$V_o = 115\text{ V}$$

$$\frac{2V_m}{\pi}\cos\alpha - 3 = 115$$

$$\frac{2\times 240\sqrt{2}}{\pi}\cos\alpha = 118$$

$$\cos\alpha = 0.546$$

$$\alpha = 56.89^\circ$$

Operating triggering angle, $\alpha = 56.9^\circ$

(ii) Average value of converter output voltage

$$V_o = 115\text{ V}$$

$T_1T_2 \rightarrow$ ON : Total voltage drop of Thyristor

$$= V_T = 1.5 + 1.5 = 3\text{ V}$$

$$V_{or} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}(V_m\sin\omega t - V_T)^2 d(\omega t)\right]^{1/2}$$

$$V_{or} = \left[\frac{1}{\pi}\left\{\int_{\alpha}^{\pi+\alpha} V_m^2\sin^2\omega t d(\omega t) + \int_{\alpha}^{\pi+\alpha} -2V_m\sin\omega t\cdot V_T d(\omega t) + \int_{\alpha}^{\pi+\alpha} V_T^2 d(\omega t)\right\}\right]^{1/2}$$

$$V_{or} = \left[\frac{1}{\pi}\left\{\left(\frac{V_m^2}{2} + V_T^2\right)\pi - 2V_m V_T\cos\alpha\right\}\right]^{1/2}$$

$$V_{or} = \left[\frac{1}{\pi}\left\{(240^2 + 3^2)\pi - 2.240\sqrt{2}\cdot 3\cos 56.89\right\}\right]^{1/2}$$

rms value of converter output voltage

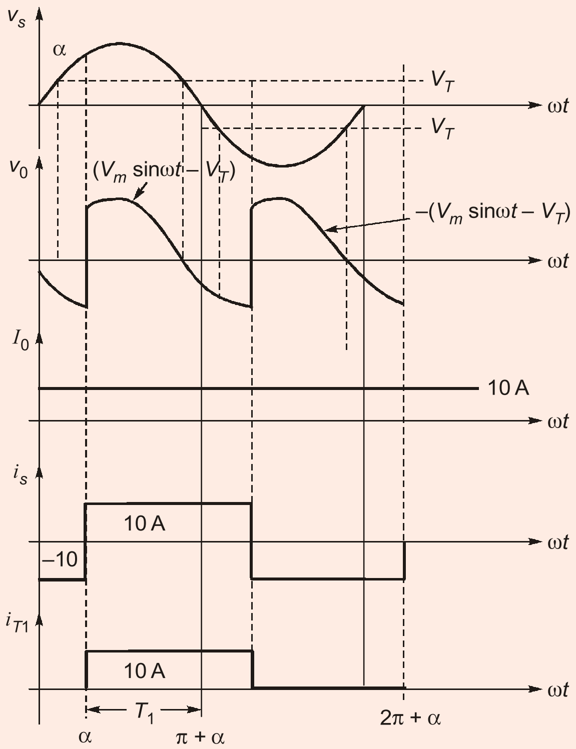
$$V_{or} = 238.54\text{ V}$$

(iii) Power supplied to battery

$$= EI$$

$$= 100 \times 10$$

$$= 1000\text{ W}$$



(iv) Converter input power

$$P_{in} = P_1 = V_{sr} \cdot I_{s1} \cos\phi_1$$

$$= 240 \cdot \frac{2\sqrt{2}}{\pi} I_o \cos\alpha$$

$$= 240\left(\frac{2\sqrt{2}}{\pi} \cdot 10\right)\cos(56.89)$$

$$= 1{,}179.89 \text{ W}$$

(v) Input power factor, $PF = g.FDF = 0.9 \cos\alpha$

$$= 0.9 \times \cos(56.89)$$

$$= 0.4916$$

Displacement factor,

$$FDF = \cos\alpha = \cos(56.89)$$

$$FDF = 0.546$$

(vi) Thyristor rms current,

$$(I_T)_{rms} = I\left(\frac{\pi}{2\pi}\right)^{1/2} = \frac{I_o}{\sqrt{2}}$$

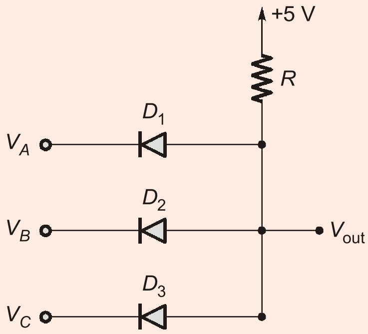
$$= \frac{10}{\sqrt{2}}$$

$$= 7.07 \text{ A}$$

End of Solution

Section-B

Q.5 (a) (i) Draw the truth table of the three input logic gate shown below :



+5 V may be taken as logic '1' while 0 V may be taken as logic '0'. The diodes D_1, D_2 and D_3 are ideal diodes.

(ii) Given the logical function of three variables

$$f(A, B, C) = A + \bar{B}C$$

express f in the standard product-of-sum form.

[6 + 6 marks : 2022]

Solution:

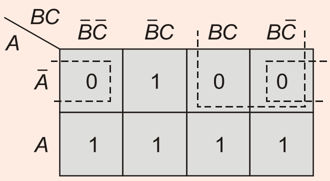
(i) If $V_A = 0$, then diode D_1 will be on, then $V_{out} = 0$ V, irrespective of V_B and V_C.
Similarly, $V_B = 0$, then diode D_2 will be on, then $V_{out} = 0$ V. Same applies for V_C also.
But if $V_A = V_B = V_C = 1$, then D_1, D_2 and D_3 will be off, then $V_{out} = 1$.

Truth Table :

A	B	C	V_{out}
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(ii)

$$f(A, B, C) = A + \bar{B}C$$



$$\text{POS from } f = (A + C)(A + \bar{B})$$

Standard POS form.

End of Solution

Q.5 (b) The characteristic equation of a unity feedback (negative) system is given by $s^3 + 3ks^2 + (k + 2)s + 4 = 0$.

(i) Determine the forward path transfer function $G(s)$.

(ii) Using Routh-Hurwitz criteria, find the range of k for which the system is stable.

[12 marks : 2022]

Solution:

(i) Characteristic equation

$$s^3 + 3Ks^2 + (K+2)s + 4 = 0$$

$$s^3 + 3Ks^2 + Ks + 2s + 4 = 0$$

$$1 + \frac{K(3s^2 + s)}{s^3 + 2s + 4} = 0$$

Standard characteristic equation of a unity feedback (negative) system is given by

$$1 + G(s)H(s) = 0 \qquad \{H(s) = 1\}$$

On comparison,

$$G(s) = \frac{K(3s^2 + s)}{s^3 + 2s + 4}$$

(ii)

$$CE = s^3 + 3Ks^2 + (K+2)s + 4$$

Routh-hurwitz Array is given by

s^3	1	$K+2$
s^2	$3K$	4
s	$\frac{3K(K+2)-4}{3K}$	0
s^0	4	

For system to be stable

$$\frac{3K(K+2) - 4\times 1}{3K} > 0$$

$$3K^2 + 6K - 4 > 0$$

$$(K - 0.5)(K + 2.5) > 0$$

$$\therefore \quad K > 0.5$$

End of Solution

Q.5 (c) A star-delta transformer has delta side disconnected between A and D while supply is connected to the star side as shown in the Figure (a) below. An rms voltmeter between A and B reads 460 V under this condition. When A and D are connected, the same voltmeter between A and B reads 415 V. Find the voltage between A and D when they are disconnected.

Now the delta side is reconnected as star as shown in the Figure (b) with supply remaining the same on the other side. Find the voltages between P and N and between P and Q.

(Assume odd harmonics upto 7^{th} order).

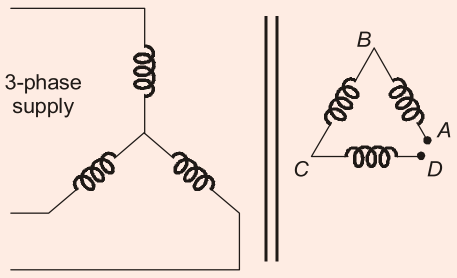


Figure (a)

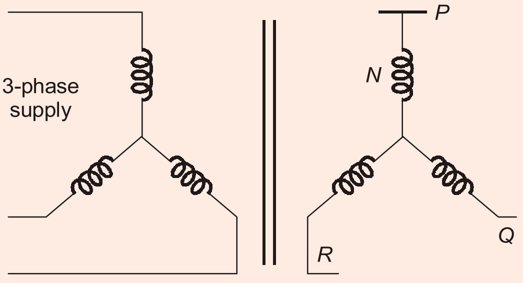
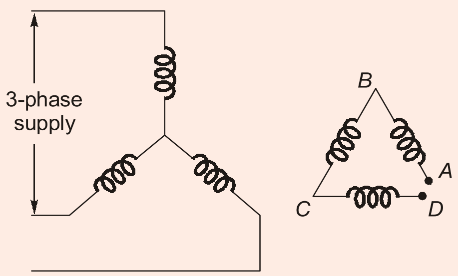


Figure (b)

[12 marks : 2022]

Solution:



First case : *A* and *D* disconnected.

RMS voltmeter between *A* and *B* = 460 V when *A* and *D* are disconnected.

Δ is opened, so third harmonic voltage exist, along with fundamental fifth and seventh.

$$\therefore \quad V_{AB} = \sqrt{E_1^2 + E_3^2 + E_5^2 + E_7^2} = 460 \text{ V}$$

$$\text{or,} \quad E_1^2 + E_3^2 + E_5^2 + E_7^2 = 460^2 \quad ...(1)$$

Second case : When *A* and *D* are connected.

Δ is closed. So, third harmonic voltages will be eliminated due to closed delta.

Same voltmeter now records 415 V.

$$\therefore \quad V_{AB} = \sqrt{E_1^2 + E_5^2 + E_7^2} = 415 \text{ V}$$

$$\text{or} \quad E_1^2 + E_5^2 + E_7^2 = 415^2 \quad ...(2)$$

By solving eqn. (1) and (2), we get E_3

$$\therefore \quad E_3 = 198.431 \text{ Volts}$$

Find voltage between *A* and *D* when they are disconnected which means imagine a voltmeter across *A* and *D*.

Delta is still opened.

Third harmonics do exist.

$$\therefore \quad V_{AD} = 3E_3$$

$$= 3(198.431) = 595.293 \text{ V}$$

If secondary is in Y. The connection is Y-Y ungrounded.

Third harmonics exist in phases but cancelled in lines.

$\therefore$ Voltage between P & $N \Rightarrow \sqrt{E_1^2 + E_3^2 + E_5^2 + E_7^2}$

Voltage between P & Q (lines) = $\sqrt{3} \times \sqrt{E_1^2 + E_5^2 + E_7^2}$

From (1) and (2) equations

$$V_{PN} = 460 \text{ V}$$

and

$$V_{PQ} = \sqrt{3} \times 415 = 718.8 \text{ V}$$

End of Solution

Q.5 (d) A three phase overhead transmission line in delta configuration is operating at 400 kV between phases at 50 Hz. The overall conductor diameter is 5.0 cm each. Find the allowable minimum spacing between the conductors to avoid corona loss under fair weather condition.

Also find the corona loss under stormy weather condition where the disruptive critical voltage reduces by 20%.

(Assume air density factor of 0.95, irregularity factor of 0.85 and disruptive critical voltage of 450 kV (L-L) under fair weather condition).

[12 marks : 2022]

Solution:

Given : Supply voltage, V = 400 kV (Δ-connected)

Critical disruptive voltage, V_C = 450 kV

Air density factor, δ = 0.95

Irregularity factor, M = 0.85

As Corona loss $\propto (V - V_C)^2$ ...(1)

Diameter of conductor, D = 5.0 cm

Radius, r = 2.5 cm

Frequency, f = 50 Hz

And breakdown strength of air, g = 21.1 kV/cm (rms)

So,

$$V_C = Mg\delta_r \ln\left(\frac{d}{r}\right)$$

where d is the distance between the conductor.

So, minimum distance for no corona loss is

when

$$V_C = \frac{V}{\sqrt{3}} \quad \text{(from (1))}$$

$$\Rightarrow \quad \frac{400}{\sqrt{3}} = 0.85 \times 21.1 \times 0.95 \times 2.5 \times \ln\left(\frac{d}{2.5}\right)$$

$$\Rightarrow \quad d = 5.6565 \text{ m}$$

If critical disruptive voltage reduces by 20%,

i.e.,

$$V_{C1} = 450 \times (1 - 0.2)$$

$$V_{C1} = 360 \text{ kV}$$

Corona Power loss,

$$P_L = 242.2 \times 10^{-5} \times \frac{f+25}{\delta} \times \sqrt{\frac{r}{d}} \times (V_{Ph} - V_C)^2 \text{ kW/km/Phase}$$

$$= 242.2 \times 10^{-5} \times \frac{50+25}{0.95} \times \sqrt{\frac{2.5}{565.65}} \times (230.94 - 207.84)^2 \text{ kW/km/Phase}$$

$= 6.783$ kW/km/Phase

End of Solution

Q.5 (e) The configuration of a single phase bridge inverter is shown in the figure below. The DC source voltage is 100 V and the connected load is a pure inductor of 1 H. The switching frequency of the inverter is 50 Hz.

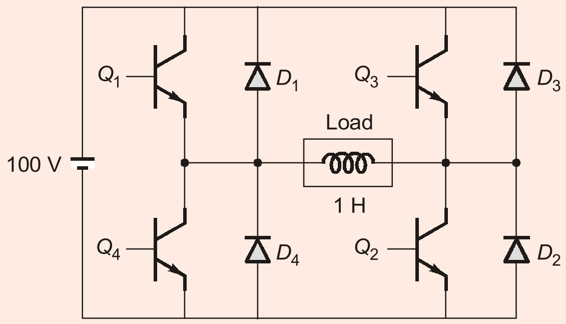
Determine :

(i) the maximum current through the transistors and diodes.

(ii) the rms currents of the diodes and transistors.

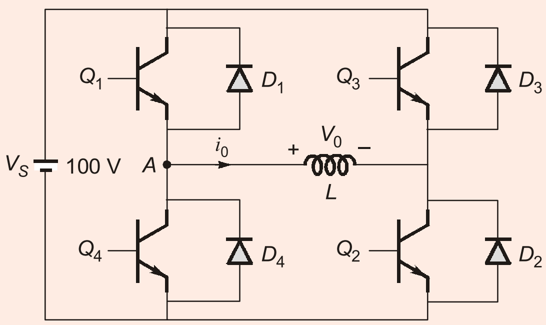
(iii) Sketch the load voltage and current waveforms indicating conducting devices.

(Assume ideal devices with no losses and square wave switching for the inverter).

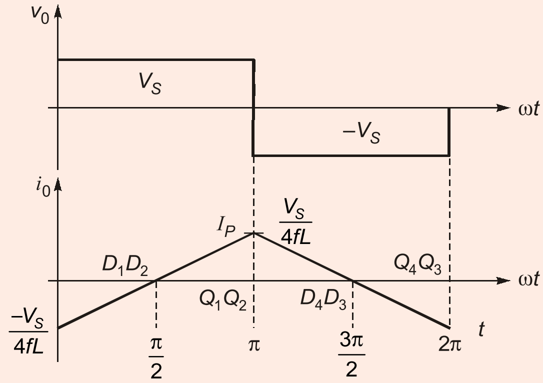


[12 marks : 2022]

Solution:



$$V_S = 100 \text{ V}, L = 1 \text{ H}, f = 50 \text{ Hz}$$



Let peak current passing through load current is I_P (after reaching steady state).

$\frac{\pi}{2}$ to π : $Q_1Q_2 \rightarrow$ ON

$$V_o = V_s$$

$$L \cdot \frac{di}{dt} = V_s$$

$$di = \frac{V_s}{L} \cdot dt$$

$$\int_0^{I_P} di = \frac{V_s}{\omega L} \int_{\pi/2}^{\pi} d(\omega t)$$

$$I_P = \frac{V_s}{\omega L} \cdot \frac{\pi}{2} = \frac{V_s}{4fL}$$

$$= \frac{100}{4 \times 50 \times 1}$$

$$= 0.5 \text{ A}$$

(i) Maximum current through transistors and diodes

$$= I_P = \frac{V_s}{4fL} = 0.5 \text{ A}$$

(ii)

$$I_{o,rms} = \frac{I_P}{\sqrt{3}} = \frac{0.5}{\sqrt{3}} = 0.2886 \text{ A}$$

$$(I_D)_{rms} = (I_Q)_{rms} = I_{o,rms}\left(\frac{\pi/2}{2\pi}\right)^{1/2} = \frac{I_{o,rms}}{\sqrt{4}}$$

$$= \frac{0.2886}{2} = 0.1443 \text{ A}$$

End of Solution

Q.6 (a) (i) A modulated signal is represented below as :

$$e(t) = 1500\sin(2\pi \times 10^9 t + 2\sin\pi \times 10^4 t)$$

Determine :

(1) type of modulation.

(2) carrier frequency.

(3) transmitted power, if the signal is applied to a 75 Ω antenna.

(4) modulation index.

(5) frequency deviation.

(ii) Explain the mathematical function that is performed by the following instructions of 8085 microprocessor :

```
MVI A, 07H
RLC
MOV B, A
RLC
RLC
ADD B
```

[10 + 10 marks : 2022]

Solution:

(i)
$$e(t) = 1500\sin(2\pi \times 10^5 t + 2\sin(\pi \times 10^4 t)) \quad ...(1)$$

For angle modulated signal, expression is given as :

$$s(t) = A_c \sin(2\pi f_c t + \beta\sin 2\pi f_m t) \quad ...(2)$$

$$A_c = 1500,\ f_m = \frac{10^4}{2}\text{Hz}$$

(1) Type of modulation is angle modulation because angle of the carrier signal is changing with respect to the signal $m(t)$.

(2) Comparing (1) and (2) equn.

$$f_c = 10^9 \text{ Hz}$$

(3) Transmitted power $= \dfrac{\left(\dfrac{A_c}{\sqrt{2}}\right)^2}{R} = \dfrac{A_c^2}{2R}$

$$= \frac{(1500)^2}{2 \times 75} = 15000 \text{ W}$$

(4) Modulation index = β, comparing eqn. (1) and (2), we get

$$\beta = 2$$

(5) Frequency deviation $= \Delta f = \beta \times f_m$

$$\therefore \quad \Delta f = 2 \times \frac{10^4}{2} = 10^4 \text{ Hz}$$

(ii) → MVIA, 07H, move the data immediately to accumulator

[A] ← 07H

→ RLC

Rotate the elements of accumulator to left without carry,

A → 0000 0111 ⇒ 0000 1110

[A] 0EH (Data is multiplied by 2)

→ MOV B, A

[B] ← [A], More contents of A to B

[B] ← 0EH

→ RLC

Rotate contents of accumulator to left without carry

$$00001110 \Rightarrow \underbrace{0001}_{1}\ \underbrace{1100}_{C}$$

[A] ← 1CH (Multiplication by 2)

→ RLC

$$00011100 \Rightarrow \underbrace{0011}_{3}\ \underbrace{1000}_{8}$$

[A] ← 38H (Multiplication by 2)

→ ADD B

[A] ← [A] + [B],

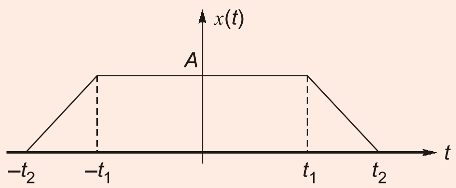
$$\begin{array}{r} 38 \\ 06 \\ \hline 47 \end{array}$$

[A] ← 47H

At last we can see that the decimal value of [A] is almost 70 which is 10 times the initial value. Hence, we can say that multiplication by 10 is performed in above instructions.

End of Solution

Q.6 (b) Consider the trapezoidal function $x(t)$ as shown below. Express $x(t)$ in terms of ramp function and find its Fourier transform. Specify the Fourier transform property used (if any).



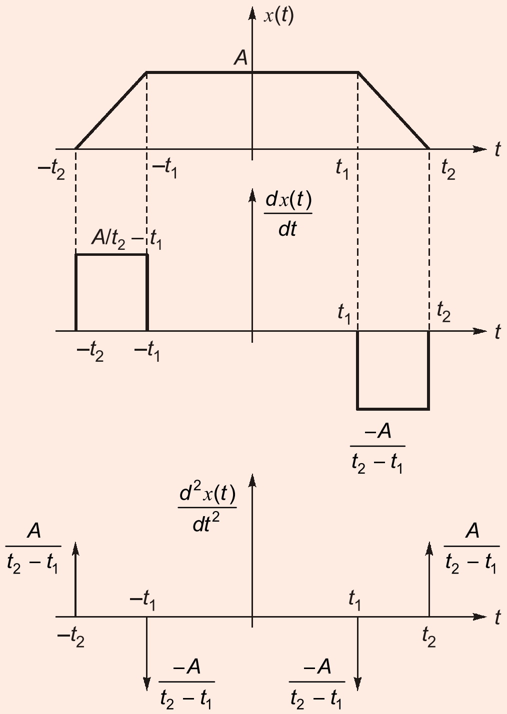
[20 marks : 2022]

Solution:

Given signal $x(t)$ be represented in terms of ramp signal as

$$x(t) = \frac{A}{t_2 - t_1} r(t + t_2) - \frac{A}{t_2 - t_1} r(t + t_1) - \frac{A}{t_2 - t_1} r(t - t_1) + \frac{A}{t_2 - t_1} r(t - t_2)$$

$$x(t) = \frac{A}{(t_2 - t_1)}[r(t+t_2) - r(t+t_1) - r(t-t_1) + r(t-t_2)]$$



Now, $$\frac{d^2x(t)}{dt^2} = \frac{A}{(t_2 - t_1)}[\delta(t+t_2) - \delta(t+t_1) - \delta(t-t_1) + \delta(t-t_2)] \quad ...(1)$$

As we know,

$$\delta(t) \xleftrightarrow{F.T.} 1$$

$$\delta(t+t_2) \xleftrightarrow{F.T.} e^{j\omega t_2} \quad \text{(Time shifting property)}$$

$$\delta(t-t_2) \xleftrightarrow{F.T.} e^{-j\omega t_2}$$

$$\delta(t+t_1) \xleftrightarrow{F.T.} e^{j\omega t_1}$$

$$\delta(t-t_1) \xleftrightarrow{F.T.} e^{-j\omega t_1}$$

Also using the differentiation property

$$x(t) \xleftrightarrow{F.T.} X(\omega)$$

$$\frac{dx(t)}{dt} \xleftrightarrow{F.T.} j\omega X(\omega)$$

$$\frac{d^2x(t)}{dt^2} \xleftrightarrow{F.T.} (j\omega)^2 X(\omega)$$

Substituting all the above results in eqn. (1)

$$(j\omega)^2 X(\omega) = \frac{A}{(t_2 - t_1)}[e^{j\omega t_2} - e^{+j\omega t_1} - e^{-j\omega t_1} + e^{-j\omega t_2}]$$

$$X(\omega) = \frac{A}{(j\omega)^2 (t_2 - t_1)}[e^{j\omega t_2} + e^{-j\omega t_2} - (e^{j\omega t_1} + e^{-j\omega t_1})]$$

$$= \frac{2A}{\omega^2 (t_2 - t_1)}\left[\frac{(e^{j\omega t_1} + e^{-j\omega t_1})}{2} - \frac{(e^{j\omega t_2} + e^{-j\omega t_2})}{2}\right]$$

$$X(\omega) = \frac{2A}{\omega^2 (t_2 - t_1)}[\cos\omega t_1 - \cos\omega t_2]$$

End of Solution

Q.6 (c) A 25 kVA, 2500 V/250 V two winding transformer is to be used as autotransformer with a source voltage of 2500 V. Find the maximum power that can be supplied by the autotransformer to load at unity power factor. Find the conducted and transformed power under this condition.

Also find the efficiency of the autotransformer at unity power factor and maximum output power condition if the efficiency of the single phase transformer at half load, 0.8 pf lag and rated voltage is 90%. Assume the no load losses for the single phase transformer as 500 W.

[20 marks : 2022]

Solution:

Given : Rated kVA, $S = 25$ kVA

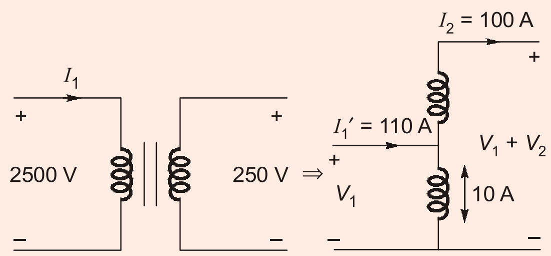
Voltage rating, $\frac{V_1}{V_2} = \frac{2500 \text{ V}}{250 \text{ V}}$

Power factor, $\cos\phi = 1$

Current on 250 V side, $I_2 = \frac{S}{V_2 \cos\phi}$ A

$\Rightarrow$ $I_2 = \frac{25000}{250 \times 1}$ A

$\Rightarrow$ $I_2 = 100$ A



2 winding Auto transformer

Current on primary side, $I_1 = \frac{S}{V_1 \cos\phi} \Rightarrow I_1 = \frac{25000}{2500} \Rightarrow I_1 = 10 \text{ A}$

Total power supplied at unity power factor,

$$P_T = (2500 + 250) \times 100$$
$$P_T = 2750 \times 100$$
$$P_T = 275 \text{ kW}$$
$$I_1' = \frac{P_T}{V_1} \Rightarrow I_1' = \frac{275000}{2500} \text{A} \Rightarrow I_1' = 110 \text{ A}$$

Induction power transfer, $P_I = 2500 \times 10$

$$P_I = 25 \text{ kW at u.p.f.}$$

Conductive power transfer, $P_C = P_T - P_I$

$\Rightarrow$ $P_C = 250 \text{ kW at u.p.f.}$

Efficiency of 2 winding transformer at power factor 0.8 lag at half load,

$$\eta = 90\% \Rightarrow 0.9 = \frac{2.5 \times 0.8 \times 1000 \times 0.5}{0.5 \times 25 \times 1000 \times 0.8 + P_L}$$

where, P_L is losses.

$$P_L = 1111.111 \text{ W}$$
$$P_L = \frac{1}{4} P_{Cu} + P_i$$

where P_{Cu} is full load copper loss and P_i is no load loss.

$$P_i = 500 \text{ W (given)}$$

$\Rightarrow$ $P_{Cu} = 2444.44 \text{ W}$

The losses in 2 winding transformer and auto-transformer are same.

So, efficiency of auto-transformer,

$$\eta_a = \frac{275000}{275000 + P_{Cu} + P_i} \times 100\%$$

$\Rightarrow$ $\eta_a = 98.94\%$

End of Solution

Q.7 (a) The loop transfer function G(s)H(s) of a single-feedback-loop control system is given by

$$G(s)H(s) = \frac{K}{s^2(s+2)(s+10)}$$

Apply the Nyquist criterion and determine the value of K for the system to be stable. Also determine the gain margin if $K = 10$.

[20 marks : 2022]

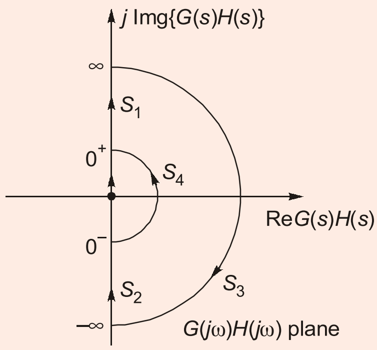
Solution:

Given : $G(s)H(s) = \frac{K}{s^2(s+2)(s+10)}$

For s_1 :

$$G(j\omega)H(j\omega) = \frac{-K}{\omega^2(j\omega+2)(j\omega+10)}$$

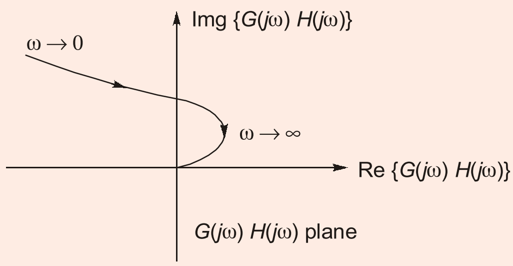
$$= \frac{-K}{\omega^2}\left[\frac{1}{-\omega^2+12j\omega+20}\right]$$



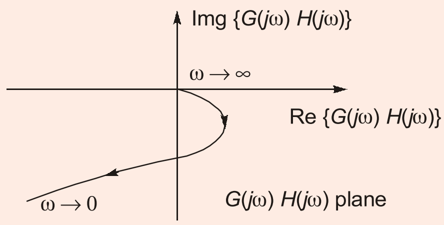
$$G(j\omega)H(j\omega) = \frac{-K[(20-\omega^2)-12j\omega]}{\omega^2[(20\omega^2)^2+(12\omega)^2]}$$

$$= \frac{-K[20-\omega^2]}{\omega^2[(20-\omega^2)^2+(12\omega)^2]+\dfrac{j12K}{\omega[(20-\omega^2)^2+(12\omega)^2]}}$$

ω	0	1	2	5	∞
Re{$G(j\omega)H(j\omega)$}	∞	−0.037	-4.807×10^{-3}	5.513×10^{-5}	0
Img{$G(j\omega)H(j\omega)$}	∞	0.023	7.211×10^{-3}	6.62×10^{-4}	0



For s_2 : s_2 is mirror image of s_1.

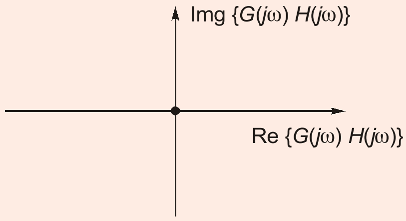


For s_3 :

$$s_3 = \lim_{\substack{R\to\infty \\ \theta\to\frac{\pi}{2}\text{to}-\frac{\pi}{2}}} Re^{j\theta}$$

$$G(s_3)H(s_3) = \frac{K}{s_3^2(s+2)(s+10)}$$

$$= 0.e^{-2\pi \text{ to } 2\pi}$$

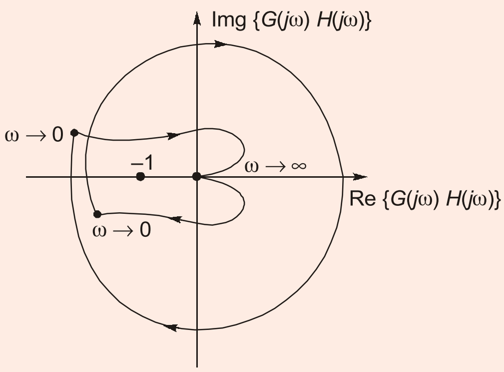


For s_4 :

$$s_4 = \lim_{\substack{R\to 0 \\ \theta\to-\frac{\pi}{2}\text{to}\frac{\pi}{2}}} Re^{j\theta}$$

$$\Rightarrow \quad G(s_4)H(s_4) = \frac{K}{s_4^2(s_4+2)(s_4+10)}$$

$$\Rightarrow \quad G(s_4)H(s_4) = \infty.e^{+\pi \text{ to } -\pi}$$



From Nyquist criteria,

Number of encirclements, $N = P - Z$

P is the open loop poles on R.H.S. of s-plane.

Z is the closed loop poles on R.H.S. of s-plane.

$\Rightarrow \quad -2 = 0 - Z$

As N is in clockwise direction,

$\Rightarrow \quad Z = 2$

Hence, the system is unstable.

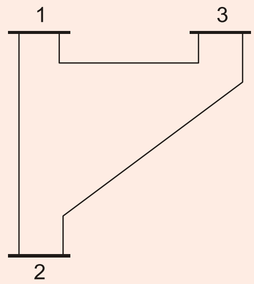
For all values of K system is unstable.

$\text{Img}\{G(j\omega)H(j\omega)\}$ is not possible, hence gain margin is undefined.

End of Solution

Q.7 (b) Why are the load flow studies necessary in power system?
For a 3-bus system as shown in figure (data are given in table), construct the Y-bus.

Bus	Impedance Z_{in} (pu)	Line charging $y'_{ik}/2$ (pu)
1 - 2	0.02 + j0.06	j0.03
1 - 3	0.08 + j0.24	j0.025
2 - 3	0.06 + j0.18	j0.02



[20 marks : 2022]

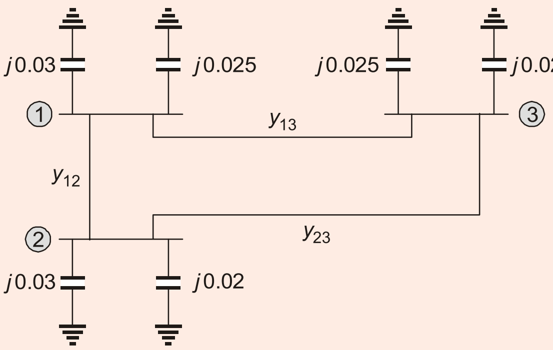
Solution:

Load flow studies are one of the most important aspects of power system planning and operation. The load flow gives us the sinusoidal steady state of the entire system – voltages, real and reactive power generated and absorbed and line losses.

Significance of load flow analysis in power system are as follows :

- We can obtain the voltage magnitudes and angles at each bus in the steady state. This is rather important as the magnitudes of the bus voltages are required to be held within a specified limit. Once the bus voltage magnitudes and their angles are computed using the load flow, the real and reactive power flow through each line can be computed.
- Also based on the difference between power flow in the sending and receiving ends, the losses in a particular line can also be computed.
- From the line flow we can also determine the over and under load conditions.
- The load flow study of a power system is essential to decide the best operation of existing system and for planning the future expansion of the system.
- It helps in designing a new power system network.
- It helps in system loss minimization and transformer tap setting for economic operation.

As per the data, the equivalent network is



$$y_{12} = \frac{1}{z_{12}} = \frac{1}{0.02 + j0.06} = \frac{0.02 - j0.06}{(0.02)^2 + (0.06)^2} = 5 - j15$$

$$y_{13} = \frac{1}{z_{13}} = \frac{1}{0.08 + j0.24} = \frac{0.08 - j0.24}{(0.08)^2 + (0.24)^2} = 1.25 - j3.75$$

$$y_{23} = \frac{1}{z_{23}} = \frac{1}{0.06 + j0.18} = \frac{0.06 - j0.18}{(0.06)^2 + (0.18)^2} = 1.67 - j5.0$$

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$= \begin{bmatrix} y_{12} + y_{13} + j0.03 + j0.025 & -y_{12} & -y_{13} \\ -y_{12} & y_{12} + y_{23} + j0.03 + j0.02 & -y_{23} \\ -y_{13} & -y_{23} & y_{13} + y_{23} + j0.025 + j0.02 \end{bmatrix}$$

$$= \begin{bmatrix} 6.25 - j18.695 & -5.0 + j15 & -1.25 + j3.75 \\ -5.0 + j15 & 6.67 - j19.95 & -1.67 + j5.0 \\ -1.25 + j3.75 & -1.67 + j5.0 & 2.92 - j8.705 \end{bmatrix}$$

End of Solution

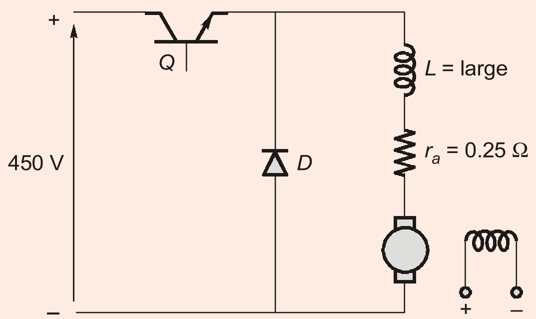
Q.7 (c) A separately excited DC motor of 10 hp, 200 V, 1750 rpm is fed from a DC-DC chopper for speed control purpose. The motor has armature resistance r_a = 0.25 Ω and the series inductance is such that the motor current is flat and continuous. The chopper is fed from a 450 V DC source. The schematic diagram is shown in the figure below. Find :

(i) the duty ratio, transistor rms current rating under rated condition.

(ii) the motor speed under no load condition at rated intput voltage.

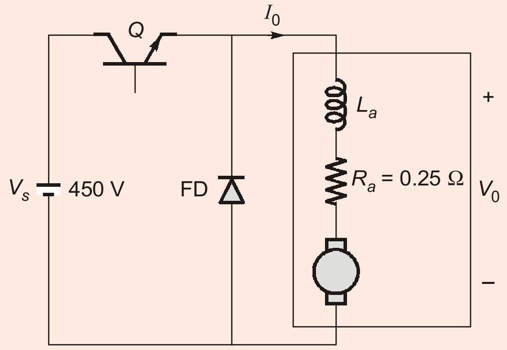
(iii) Now the motor is relocated at a distance with a connecting cable of 1.5 Ω resistance and the speed is to be reduced to 1000 rpm at half-rated torque. Find the new duty ratio and converter output voltage.

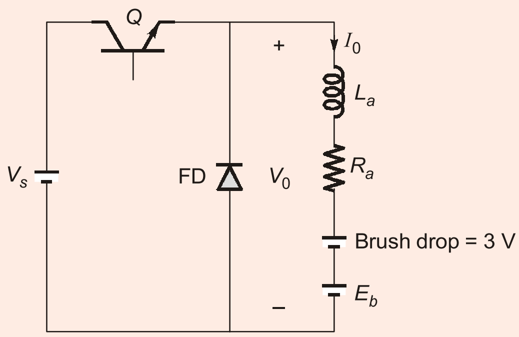
Assume negligible device losses, a total brush drop of 3.0 V for the motor under operation and a 10 kHz switching frequency for the chopper.

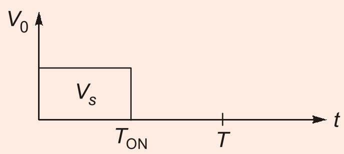


[20 marks : 2022]

Solution:







$$V_o = \alpha . V_s$$

$$V_{o(\text{Rated})} = \alpha . 450$$

$$\alpha = \frac{200}{450} = 0.444$$

$$P_{o(\text{Rated})} = V_{o(\text{Rated})} . I_{o(\text{Rated})}$$

$$10 \text{ HP} = 200 I_{o(\text{Rated})}$$

$$(10 \times 745.699) \text{ W} = 200 I_{o(\text{Rated})}$$

$$I_{o(\text{Rated})} = 37.28 \text{ A}$$

$$(I_{\text{Transistor}})_{\text{rms}} = I_{o(\text{Rated})} \left(\frac{T_{ON}}{T} \right)^{1/2}$$

$$= \sqrt{\alpha} I_{o(\text{Rated})}$$

$$= \sqrt{0.44} \times (37.28)$$

$$= 24.85 \text{ A}$$

(i)

$$V_{o(\text{Rated})} = E_b + \text{Brush drop voltage} + I_{o(\text{Rated})} R_a$$

$$200 = K_m \cdot \frac{2\pi}{60} N_{\text{Rated}} + 3 + (37.28 \times 0.25)$$

$$200 = K_m \cdot \frac{2\pi}{60} \cdot 1750 + 3 + (37.28 \times 0.25)$$

$$K_m = 1.02 \text{ V-S/rad}$$

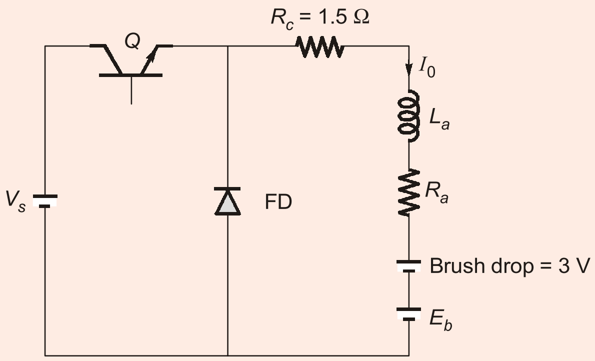
(ii) At No Load ($I_o = 0$)

$$\therefore \quad V_{o(\text{Rated})} = K_m \cdot \frac{2\pi}{60} N + \text{Brush drop voltage} + I_o R_a$$

$$200 = 1.02 \times \frac{2\pi}{60} N + 3 + (0 \times 0.25)$$

$$N = 1844.32 \text{ rpm}$$

(iii)



$R_c \rightarrow$ Connecting cable resistance

$$T_a \propto I_o \qquad [\because \phi \rightarrow \text{Const}]$$

At half rated torque,

$$I_o = \frac{I_{o(\text{Rated})}}{2} = \frac{37.28}{2}$$

$$I_o = 18.64 \text{ A}$$

$$V_o = E_b + \text{Brush drop voltage} + I_o(R_a + R_c)$$

$$V_o = K_m \frac{2\pi}{60} N + 3 + I_o(0.25 + 1.5)$$

$$= 1.02 \times \frac{2\pi}{60} \times 1000 + 3 + (18.64 \times 1.75)$$

$$V_o = 106.814 + 3 + 32.62$$

$$V_o = 142.434 \text{ V}$$

$$V_o = \alpha V_s$$

$$142.434 = \alpha \times 450$$

$$\alpha = 0.316$$

End of Solution

Q.8 (a) Consider the sequence $x(n) = \left(\frac{1}{2}\right)^n u(n)$ and $y(n) = \left(\frac{1}{3}\right)^{n-2} u(n-2)$. Use the convolution property of Z-transform to find the convolution of the two sequences, $q(n) = x(n)*y(n)$. Validate your result using time-domain method of convolution.

[20 marks : 2022]

Solution:

Given :
$$x(n) = \left(\frac{1}{2}\right)^n u(n)$$
$$y(n) = \left(\frac{1}{3}\right)^{n-2} u(n-2)$$

Given a new signal,
$$q(n) = x(n) * y(n)$$
$$\Big\Downarrow\Big\Uparrow \text{Z.T.}$$
$$Q(z) = X(z).Y(z) \quad ...(1)$$

As we know,
$$x(n) = \left(\frac{1}{2}\right)^n u(n) \xrightleftharpoons{Z.T.} X(z) = \frac{z}{z-0.5};\ |z| > 0.5$$
$$y_1(n) = \left(\frac{1}{3}\right)^n u(n) \xrightleftharpoons{Z.T.} Y_1(z) = \frac{z}{z-\frac{1}{3}};\ |z| > \frac{1}{3} \quad ...(2)$$

Now,
$$y(n) = y_1(n-2)$$
$$\Big\Downarrow\Big\Uparrow \text{Z.T.}$$
$$y(z) = z^{-2}Y_1(z)$$

Substituting above result in eqn. (2)
$$Y(z) = \frac{z^{-1}}{z-\frac{1}{3}}$$

On putting values of $X(z)$ and $Y(z)$ in eqn. (1)
$$Q(z) = \frac{1}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$$
$$Q(z) = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{3}}$$
$$A = Q(z)\cdot\left(z-\frac{1}{2}\right)\Bigg|_{z=\frac{1}{2}} = \frac{1}{\frac{1}{2}-\frac{1}{3}} = 6$$

$$B = Q(z)\cdot\left(z-\frac{1}{3}\right)\Bigg|_{z=\frac{1}{3}} = \frac{1}{\frac{1}{3}-\frac{1}{2}} = -6$$

Therefore,

$$Q(z) = \frac{6}{z-\frac{1}{2}} - \frac{6}{z-\frac{1}{3}} = z^{-1}\left(\frac{6z}{z-\frac{1}{2}} - \frac{6z}{z-\frac{1}{3}}\right)$$

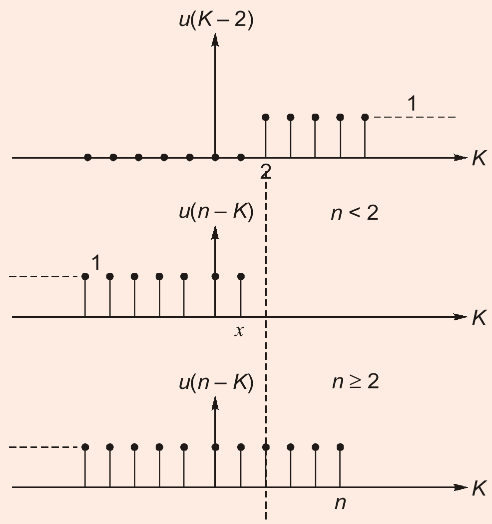
$$\Updownarrow \text{ I.Z.T.}$$

$$q(n) = 6\left(\frac{1}{2}\right)^{n-2} u(n-1) - 6\left(\frac{1}{3}\right)^{n-1} u(n-1) \qquad \ldots(1)$$

By using time domain approach :

$$q(n) = \sum_{K=-\infty}^{\infty} y(k)\cdot x(n-k)$$

$$= \sum_{K=-\infty}^{\infty} \left(\frac{1}{3}\right)^{K-2} u(K-2)\cdot\left(\frac{1}{2}\right)^{n-K} u(n-K)$$



Thus,

$$q(n) = \begin{cases} 0, & \text{for } n < 2 \\ \sum_{K=2}^{n} \left(\frac{1}{3}\right)^{K-2}\cdot\left(\frac{1}{2}\right)^{n-K} & \text{for } n \geq 2 \end{cases}$$

$$= \begin{cases} 0, & \text{for } n < 2 \\ 9\cdot\left(\frac{1}{2}\right)^{n} \sum_{K=2}^{n}\left(\frac{2}{3}\right)^{K} & \text{for } n \geq 2 \end{cases}$$

$$= \begin{cases} 0, & \text{for } n < 2 \\ 9\cdot\left(\frac{1}{2}\right)^n \cdot \dfrac{\left(\frac{2}{3}\right)^2\left[1-\left(\frac{2}{3}\right)^{n-1}\right]}{1-\frac{2}{3}} & \text{for } n \geq 2 \end{cases}$$

$$\Rightarrow \qquad q(n) = \begin{cases} 0, & \text{for } n < 2 \\ \dfrac{9\cdot\left(\frac{1}{2}\right)^n \cdot \frac{4}{9}\left[1-\left(\frac{2}{3}\right)^{n-1}\right]}{\frac{1}{3}} & \text{for } n \geq 2 \end{cases}$$

$$= \begin{cases} 0, & \text{for } n < 2 \\ 3\cdot\left(\frac{1}{2}\right)^{n-2}\left[1-\left(\frac{2}{3}\right)^{n-1}\right] & \text{for } n \geq 2 \end{cases}$$

$$= \begin{cases} 0, & \text{for } n < 2 \\ 3\left[\left(\frac{1}{2}\right)^{n-2} - 2\left(\frac{1}{3}\right)^{n-1}\right] & \text{for } n \geq 2 \end{cases}$$

$$= \begin{cases} 0, & \text{for } n < 2 \\ 6\left[\left(\frac{1}{2}\right)^{n-1} - \left(\frac{1}{3}\right)^{n-1}\right] & \text{for } n \geq 2 \end{cases}$$

$$= 6\left[\left(\frac{1}{2}\right)^{n-1} - \left(\frac{1}{3}\right)^{n-1}\right]u(n-2) \qquad \text{...(1)}$$

Since, $q(n)|_{n=1} = 0$

So, we can write eqn. (1) in the following form also,

$$q(n) = 6\left[\left(\frac{1}{2}\right)^{n-1} - \left(\frac{1}{3}\right)^{n-1}\right]u(n-1)$$

Hence, result obtained from convolution property of z-transform is validated using time domain approach of convolution.

End of Solution

Q.8 (b) A 6.6 kV star-connected synchronous motor has synchronous impedance $Z_s = (2 + j10)\ \Omega$ per phase. Its excitation is so adjusted that for an input power of 500 kW, the motor pf is 0.7 leading. Now the motor is connected in parallel with a load drawing 1000 kW power at 0.8 pf lag while its input power is changed to 600 kW with same excitation as above.

Find :

(i) the new load angle of the motor.

(ii) the total input power factor for the load and synchronous motor combination.

[20 marks : 2022]

Solution:

Given : $V_t = 6.6$ kV

$Z_s = (2 + j10)\ \Omega$/Ph

Y-connected synchronous motor.

(i) Given : $P_{in} = 500$ kW, $\cos\phi = 0.7$ leading

$\phi = -45.57°$

As we know,

$$P_{in} = \sqrt{3}V_L I_L \cos\phi$$

Input line current of motor,

$$\vec{I}_a = \frac{P_{in}}{\sqrt{3} \times V_L \cos\phi}$$

$$= \frac{500}{\sqrt{3} \times 6.6 \times 0.7} = 62.48\angle 45.57° \text{ A}$$

Excitation emf of motor is given as,

$$\vec{E}_f = \vec{V}_t - \vec{Z}_s \vec{I}_a$$

$$= \frac{6600}{\sqrt{3}}\angle 0° - (2 + j10)(62.48\angle 45.57°)$$

$$= 4202.34\angle -7.19° \text{ Volt (per phase)}$$

$$\vec{E}_f = \sqrt{3} \times 4202.34\angle -7.19°$$

$$= 7278.65\angle -7.19° \text{ V (line to line)}$$

Now motor input power changed to 600 kW

$P_{in} = 600$ kW

$|E_f| = 7278.65$ Volts

Now,
$$P_{in} = \frac{V_t^2}{Z_s}\cos\theta_s - \frac{E_f \cdot V_t}{Z_s}\cos(Q_s + \delta)$$

where, δ = new load angle

$$600 = \frac{6.6^2 \times 10^3}{10.198}\cos(78.70°) - \frac{7278.65 \times 6.6}{10.198}\cos(78.70 + \delta)$$

$$\frac{7278.65 \times 6.6}{10.198}\cos(78.70 + \delta) = 836.96 - 600 = 236.96$$

$$\cos(78.70 + \delta) = 0.05$$

New power angle, $\delta = \cos^{-1}(0.05) - 78.70° = 8.42°$

Therefore, $E_f\angle -\delta = 7278.65\angle -8.42°$ Volts (line to line)

(ii) From part (i)

$$\vec{E}_f = 7278.65\angle{-8.42°} \text{ Volts}$$

Motor input current,
$$\vec{I}_a = \frac{\vec{V}_t - \vec{E}_f}{\vec{Z}_s}$$

$$\vec{I}_a = \frac{6600\angle 0° - 7278.65\angle -8.42°}{(2 + j10)\sqrt{3}}$$

$$= 119.94\angle 40.70° \text{ A}$$

New power factor of motor,
$$\cos\phi = \cos(40.69°) = 0.7582 \text{ leading}$$

Total input power of system
$$\vec{s}_{in} = \vec{s}_L + \vec{s}_M$$

$$= 1250\angle 36.86° + 791.34\angle{-40.69°}$$

$$\vec{s}_{in} = 1617.16\angle 8.316° \text{ kVA}$$

Therefore, power factor of overall system is
$$\cos\phi = \cos(8.136°) = 0.9894 \text{ lagging}$$

End of Solution

Q.8 (c) A 3-phase, star-connected, 4-pole, 415 V, 50 Hz induction motor has equivalent circuit parameters of R_s = 1.1 Ω, R_r = 0.9 Ω, $X_{ls} = X_{lr}$ = 2.7 Ω and X_m = 67 Ω, all referred from stator side. The motor is driving a load whose torque is proportional to square of speed. For a load torque of 35 N-m the speed is 1440 rpm in the load characteristics. The motor is supplied from a three phase PWM inverter and the speed control is achieved through voltage control method. Now, if the motor speed is to be brought down to 1250 rpm, find the

(i) load torque.
(ii) inverter applied voltage.
(iii) motor input current and power factor.
(iv) motor input power.
(v) motor developed output power.

(Assume negligibel no load losses and no input harmonics to the motor).

[20 marks : 2022]

Solution:

$$R_s = 1.1\ \Omega,\ R_r = 0.9\ \Omega,\ X_{ls} = X_{lr} = 2.7\ \Omega$$

$$X_m = 67\ \Omega$$

$$V_{LL} = 415 \text{ V},\ f = 50 \text{ Hz},\ P = 4$$

$$N_S = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

For load-torque = 35 N-m,
$$N = 1440 \text{ rpm}$$

$$\text{Torque} \propto N^2$$

(i)
$$\frac{T_1}{T_2} = \frac{N_1^2}{N_2^2}$$

$$\frac{35}{T_2} = \frac{(1440)^2}{(1250)^2}$$

$$T_2 = 26.373 \text{ N-m}$$

$$N_2 = 1250$$

$$s = \frac{1500-1250}{1500} = 0.166$$

(ii)
$$\text{Load torque} = \frac{3V_{Ph}^2}{\omega_s\left[\left(r_1+\frac{r_2'}{5}\right)^2+(X_1+X_2')^2\right]} \times \frac{r_2'}{s}$$

$$26.373 = \frac{3 \times V_{ph}^2}{\frac{2\times 3.14 \times 1500}{60}\left[\left(1.1+\frac{0.9}{0.1666}\right)^2+(2.7+2.7)^2\right]} \times \frac{0.9}{0.1666}$$

Fundamental rms voltage of inverter

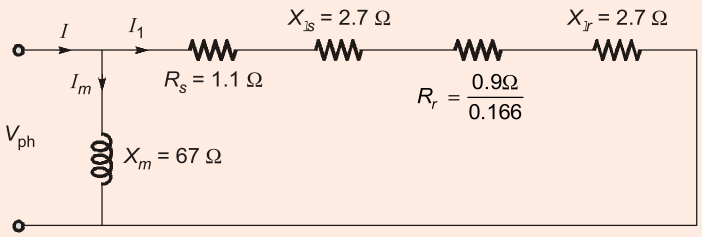
$$V_{Ph} = 135.098 \text{ V (per phase)}$$

$$V_{L-L} = \sqrt{3} \times 135.098 = 233.997 \text{ V}$$

(iii)
$$I = I_m + I_1$$

$$= \frac{135.098}{j67} + \frac{135.098}{\left(1.1+\frac{0.9}{0.166}\right)+j(2\times 2.7)}$$

$$= 17.311\angle -44.77^\circ$$



Input current, $I = 17.311$ A

$$\text{Power factor} = \cos 44.77^\circ$$

$$= 0.709 \text{ (lagging)}$$

(iv)
$$\text{Motor input power} = \sqrt{3} \times V_{LL} \times I \times Pf$$

$$= \sqrt{3} \times 233.997 \times 17.311 \times 0.709$$

$$= 4974.384 \text{ W}$$

(v)
$$I_1 = \frac{135.098}{\left(1.1+\frac{0.9}{0.166}\right)+j(2\times 2.7)}$$
$$= 15.955\angle{-39.624°}$$

Motor developed output power

$$= 3I_1^2 R_r\left(\frac{1}{s}-1\right)$$
$$= 3\times 15.955^2\times 0.9\left(\frac{1}{0.166}-1\right)$$
$$= 3453.149 \text{ W}$$

End of Solution