GATE 2023

INSTRUMENTATION ENGINEERING

Memory based Questions & Solutions

Exam held on 11th Feb, 2023 Forenoon Session

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SECTION - A GENERAL APTITUDE

Q.1 How many 5-digit number can be formed by using 3, 4, 5 and 6 if exactly One digit appearing twice.

Ans. (240)

We have 5 digits out of which 2 are identical.

$$\text{Number of ways} = \frac{5!}{2!} = \frac{120}{2} = 60$$

Out of four digit exactly one digit appearing twice.

$$\text{Number of ways} = {}^4C_1 = 4$$

$$\text{Total 5-digit number} = 4 \times 60 = 240$$

End of Solution

Q.2 Disagree : Protest :: Agree : _______

(a) Refuse
(b) Recommend
(c) Refute
(d) Pretext

Ans. (b)

End of Solution

Q.3 The village was nested in a green spot ____ the ocean and the field

(a) in
(b) at
(c) through
(d) between

Ans. (d)

End of Solution

Q.4 Residency is a famous housing complex with many well-established individuals among its residents. A recent survey conducted among the residents of the complex revealed that all of those residents who are well established in their respective fields happen to the academicians. The survey also revealed that most of these academicians are authors of some best-selling books. Based only on the information provided above, which one of the following statements can be logically inferred with certainty?

(a) Some authors of best-selling books are residents of the complex who are well-established in their fields
(b) All academicians residing in the complex are well establishing in their fields.
(c) Some residents of the complex who are well established in their fields are also authors of some best-selling books.
(d) Some academicians residing in the complex are well established in their fields.

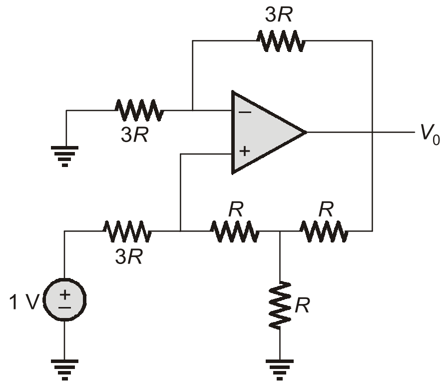
Ans. (d)

End of Solution

■■■■

SECTION - B TECHNICAL

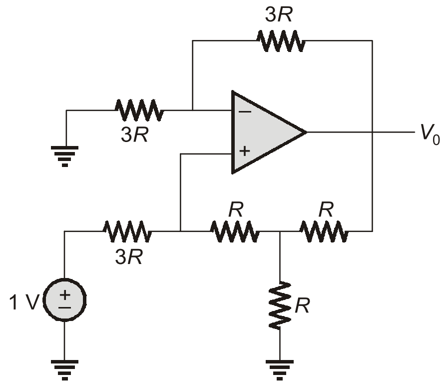
Q.5 For Ideal op-amp shown below:



The value of V_0 is ________ V.

Ans. (2)

Assuming op-amp is not saturated.



$$\frac{V_A - 1}{4R} + \frac{V_A}{R} + \frac{V_A - V_o}{R} = 0$$

$$\frac{V_A - 1}{4} + V_A + V_A - V_o = 0$$

$$9V_A - 1 - 4V_o = 0$$

$$V_A = \frac{1}{9} + \frac{4}{9}V_o$$

∴ Current flowing through $3R\ \Omega$.

$$i = \frac{1 - V_A}{4R} = \frac{1}{4R}\left[1 - \left(\frac{1}{9} + \frac{4V_o}{9}\right)\right]$$

∴ Voltage at non-inverting terminal

$$V_+ = 1 - \frac{1}{4R} \times 3R\left[1 - \left(\frac{1}{9} + \frac{4V_o}{9}\right)\right]$$

$$V_+ = 1 - \frac{3}{4}\left[\frac{8}{9} - \frac{4V_o}{9}\right]$$

$$V_+ = 1 - \frac{2}{3} + \frac{V_o}{3}$$

$$V_+ = \frac{1}{3} + \frac{V_o}{3}$$

Using virtual short concept

$$V_+ = V_- = \frac{1}{3} + \frac{V_o}{3}$$

Applying KCL at inverting node

$$\frac{0 - V_-}{3R} = \frac{V_- - V_o}{3R}$$

$$-V_- = V_- - V_o$$

$$V_o = 2V_-$$

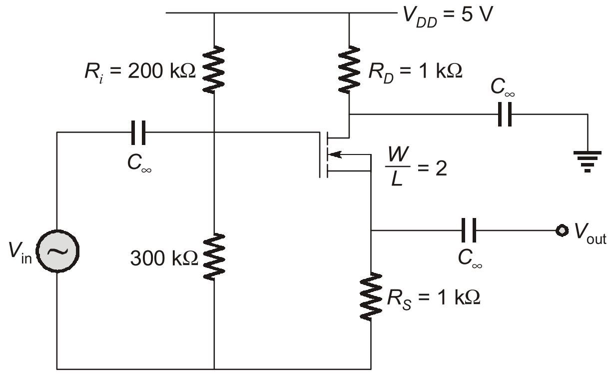
$$V_o = 2\left[\frac{1}{3} + \frac{V_o}{3}\right]$$

$$3V_o = 2 + 2V_o$$

$$V_o = 2 \text{ Volt}$$

End of Solution

Q.6 Figure below shows a feedback amplifier constructed using an nMOS transistor. Assume that $\mu_n C_{ox}$ = 1 mA/V^2, threshold voltage V_T = 1 V and W/L = 2. The bias voltage at the drain terminal is 4 V. The capacitor C_∞ offer zero impedance once at signal frequency. The ratio V_{out}/V_{in} is _____.



Ans. (0.66)

Given : $V_D = 4$ V, $\frac{W}{L} = 2$, $V_T = 1$ V

$\mu_n C_{ox} = 1$ mA/V^2

Given amplifier is common drain amplifier.

Now,
$$g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)$$
$$V_{GS} = V_G - V_S$$
$$V_{GS} = \frac{5\times 300}{500} - I_D R_S$$
$$V_{GS} = 3 - I_D R_S$$

Also,
$$I_D = \frac{5 - V_D}{R_D} = \frac{5-4}{1} = 1 \text{ mA}$$

∴
$$V_{GS} = 3 - 1$$
$$V_{GS} = 2 \text{ Volt}$$

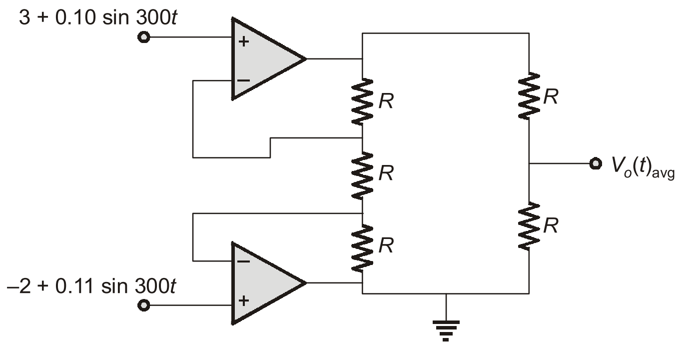
Hence,
$$g_m = 1\times 10^{-3}\times 2[2-1]$$
$$g_m = 2 \text{ mA/V}$$

∴ For common drain amplifier,

$$\frac{V_{out}}{V_{in}} = \frac{g_m R_s}{1 + g_m R_s} \qquad (\text{Taking } r_{ds} = \infty)$$
$$= \frac{2\times 1}{1 + 2\times 1}$$
$$\frac{V_{out}}{V_{in}} = \frac{2}{3} \text{ V/V}$$

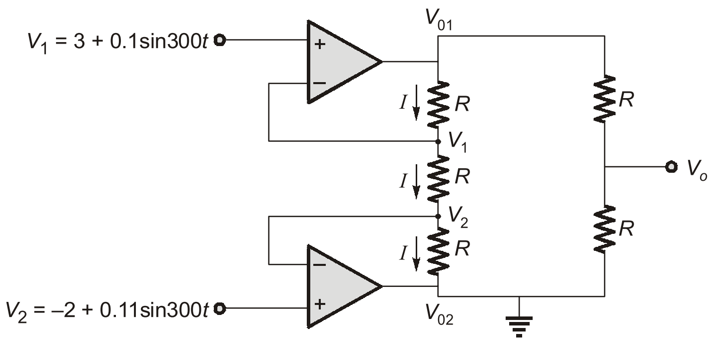
End of Solution

Q.7 For the circuit shown below :



The value of $V_o(t)_{avg}$ =

Ans. (0.5)



$$V_{o_1} = IR + V_1$$

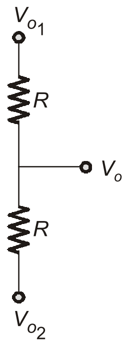
$$= \frac{(V_1 - V_2)}{R} R + V_1 = 2V_1 - V_2 \quad \text{...(i)}$$

$$V_{o_2} = V_2 - IR = V_2 - \frac{(V_1 - V_2)}{R} \times R$$

$$= 2V_2 - V_1 \quad \text{...(ii)}$$

$$V_o = \frac{V_{o_1} \times R}{R+R} + \frac{V_{o_2} \times R}{R+R} = \frac{V_{o_1} + V_{o_2}}{2}$$

$$= \frac{2V_1 - V_2 + 2V_2 - V_1}{2} = \frac{V_1 + V_2}{2}$$



$$V_o = \frac{V_1 + V_2}{2} = \frac{3 + 0.10\sin 300t - 2 + 0.11\sin 300t}{2}$$

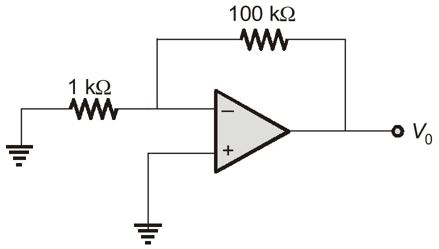
$$= \frac{1 + 0.21\sin 300t}{2} = \frac{1}{2} + \frac{0.21}{2}\sin 300t$$

$$= 0.5 + 0.105\sin 300t$$

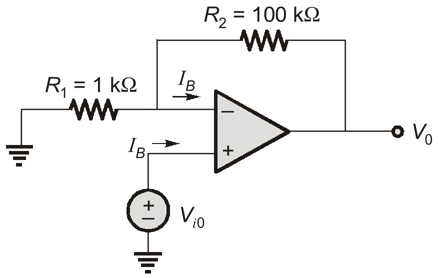
$$V_{o(\text{avg})} = 0.5 \text{ V}$$

End of Solution

Q.8 The opamp in the circuit shown is ideal, except that it has an input current of 1 nA and an input offset voltage of 10 μV. The resulting worst - case output voltage will be ± _______ μV.



Ans. (1110)



KCL at inverting terminal:

$$\frac{0-V_{i0}}{R_1}+\frac{V_0-V_{i0}}{R_2} = I_B$$

$$\frac{V_0}{R_2} = I_B+\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_{i0}$$

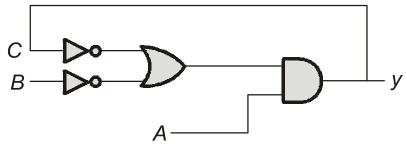
$$V_0 = I_B R_2+\left(1+\frac{R_2}{R_1}\right)V_{i0}$$

$$V_0 = 10^{-9}\times 100\times 10^{3}+\left(1+\frac{100}{1}\right)\times 10\times 10^{-6}$$

$$V_0 = 1110\ \mu\text{V}$$

End of Solution

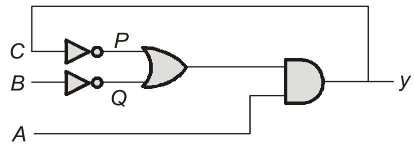
Q.9 Given $A = B = 1$, initially $C = 1$, when circuit is turned on. If delays of AND, OR, NOT gates are 10 ns, 10 ns, 5 ns respectively then frequency of steady state oscillations of output y is _____ MHz.



Ans. (20)

Given $A = B = 1$

$C = 1$ (Initially)



From circuit $P = \bar{C} = \bar{y}$

$Q = \bar{B}$

Next state of the output

$$y^+ = A \cdot (\bar{B} + \bar{C})$$

$A = 1$ and $B = 1$

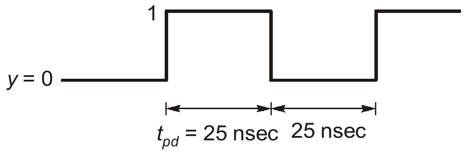
$$y^+ = 1 \cdot (0 + \bar{C}) = 0 + \bar{y} = \bar{y}$$

$$y^+ = \bar{y}$$

Overall propagation delay of circuit,

$$t_{Pd} = 5 + 10 + 10 = 25 \text{ nsec}$$

So, after every 25 nsec the output will toggle.



Frequency of steady state oscillations,

$$f = \frac{1}{50 \times 10^{-9}} = 20 \times 10^6 \text{ Hz}$$

$$f = 20 \text{ MHz}$$

End of Solution

Q.10 Simplify $F(w, x, y, z) = \Sigma m(4, 5, 10, 11, 12, 13, 14, 15)$

(a) $x\bar{y} + wy$

(b) $wx + \bar{w}x\bar{y} + w\bar{x}y$

(c) $wx + wy + \bar{x}y$

(d) $\bar{x}y + \bar{w}\bar{y}$

Ans. **(a)**

Given: $F(w, x, y, z) = \Sigma m(4, 5, 10, 11, 12, 13, 14, 15)$

K-Map:

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$				
$\bar{w}x$	1	1		
wx	1	1	1	1
$w\bar{x}$			1	1

$F(w, x, y, z) = x\bar{y} + wy$

End of Solution

Q.11 Match the following **List-1** to **List-2**.

(1) $x \oplus x$

(P) 1

(2) $x \oplus \bar{x}$

(Q) 0

(3) $x \oplus 0$

(R) $\bar{x}$

(4) $x \oplus 1$

(S) x

Ans. **(##)**

Truth table for EXOR gate

Inputs		Output	
A	B	Y	
0	0	0	→ Same inputs
1	1	1	→ Different inputs
1	0	1	→ Different inputs
1	1	0	→ Same inputs

Inputs		Output
A	B	Y
0	0 →	0
0	1 →	1
1	0	1
1	1	0

If one input of EXOR gate is 0, then the output is same as second input.

Inputs		Output
A	B	Y
0	0	0
0	1	1
1	0 →	1
1	1 →	0

If one output of EXOR gate is 1, then the output is the complement of second input.

(1) → (Q)
(2) → (P)
(3) → (S)
(4) → (R)

End of Solution

Q.12 The following excitation table is corresponding to which of the following Flip-flop?

$Q(t)$	$Q(t+1)$	Input
0	0	0
0	1	1
1	0	1
1	1	0

(a) D-Flip-flop
(b) JK-Flip-flop
(c) SR-Flip-flop
(d) T-Flip-flop

Ans. (d)

When, Input = 0

$Q(t+1) = Q(t)$

When, Input = 1

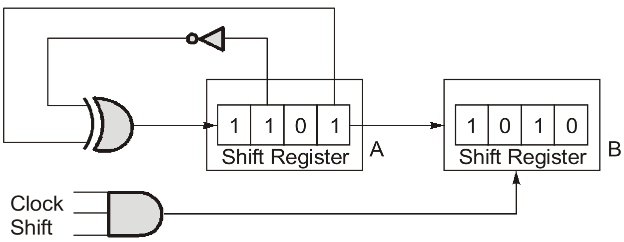
$Q(t+1) = \bar{Q}(t)$

Here, the output toggle for input = 1

So, it is a T-flip-flop.

End of Solution

Q.13 In the circuit shown, the initial binary content of shift register A is 1101 and that of shift register B is 1010. The shift register are +ve edge triggered, and gates have no delay. When the shift control is high, what will be the binary content of the shift register A and B after 4 clock pulse.



(a) A = 1101, B = 1101
(b) A = 0101, B = 1101
(c) A = 1010, B = 1111
(d) A = 1110, B = 1001

Ans. (b)

	Register A	Register B
	1 1 0 1	1 0 1 0
Clock 1:	1 1 1 0	1 1 0 1
Clock 2:	0 1 1 1	0 1 1 0
Clock 3:	1 0 1 1	1 0 1 1
Clock 4:	0 1 0 1	1 1 0 1

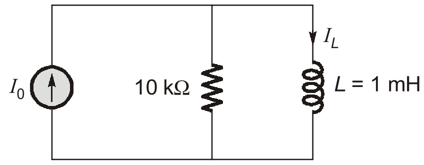
After 4th clock pulse,

$A = 0\ 1\ 0\ 1$

$B = 1\ 1\ 0\ 1$

End of Solution

Q.14 The circuit is excited by step current $I_0u(t)$ at $t = 0^-$, I_L flows $I_L = \frac{I_0}{5}$. The minimum time taken for the current of inductor to reach 99% of its final value is ______ μs.



Ans. (0.438)

The current equation for inductor

$$i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$i_L(0^+) = \frac{I_o}{5}$$

$$i_L(\infty) = I_o$$

$$i_L(t) = I_o + \left[\frac{I_o}{5} - I_o\right]e^{-t/\tau} = I_o - \frac{4I_o}{5}e^{-t/\tau}$$

The inductor current reaches to 99% of its final value.

i.e. $$i_L(t) = 0.99\, I_o$$

$$0.99\, I_o = I_o\left[1 - \frac{4}{5}e^{-t/\tau}\right]$$

$$\frac{4}{5}e^{-t/\tau} = 0.01$$

$$t = -\tau \ln\left(\frac{0.05}{4}\right) = 4.382\tau$$

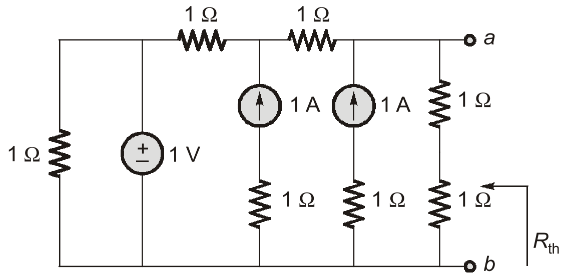
$$\tau = \frac{L}{R} = \frac{1\times10^{-3}}{10\times10^{3}} = 10^{-7} \text{ sec}$$

$$t = 4.382 \times 10^{-7} \text{ sec}$$

$$t = 0.4382\ \mu\text{s}$$

End of Solution

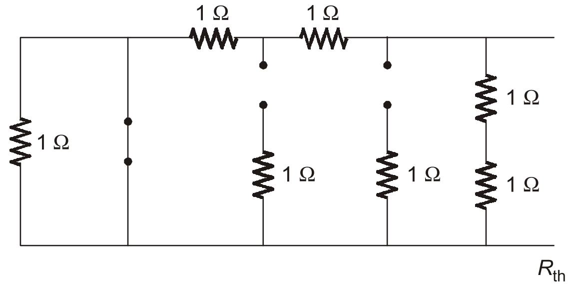
Q.15 For the circuit shown below:



The equivalent resistance R_{th} is ________.

Ans. (1)

For Thevenin resistance, we need to deactivate all the independent source.



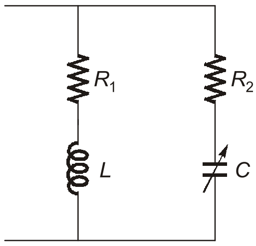
$$R_{th} = (1+1)\,||\,(1+1) = 2\,||\,2$$

$$= \frac{2\times 2}{2+2} = \frac{4}{4} = 1\,\Omega$$

$$R_{th} = 1\ \Omega$$

End of Solution

Q.16 For the circuit shown below:



$R_1 = R_2 = 2.2\ \Omega$

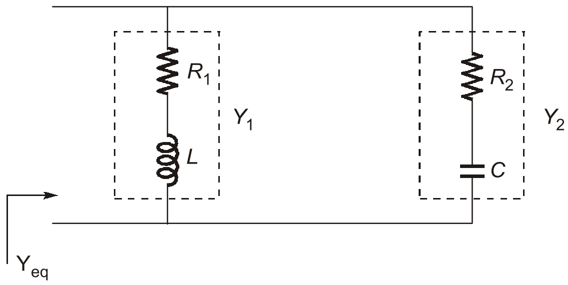
$L = 7$ mH

and $\omega = 100\,\pi$ rad/sec then the value of C (in mF) is _______.

Ans. (1.45)

Given : $R_1 = R_2 = 2.2\ \Omega$

$L = 7$ mH, $\omega = 100\pi$ rad/sec



$$Y_{eq} = Y_1 + Y_2$$

$$= \frac{1}{R_1 + j\omega L} + \frac{1}{R_2 - \dfrac{j}{\omega C}}$$

$$= \frac{(R_1 - j\omega L)}{\sqrt{R_1^2 + (\omega L)^2}} + \frac{R_2 + \dfrac{j}{\omega C}}{\sqrt{R_2^2 + \left(\dfrac{1}{\omega C}\right)^2}}$$

For resonant frequency,

$$I_m\{Y_{eq}\} = 0$$

$$\frac{-j\omega L}{\sqrt{R_1^2 + (\omega L)^2}} + \frac{\dfrac{j}{\omega C}}{\sqrt{R_2^2 + \left(\dfrac{1}{\omega C}\right)^2}} = 0$$

$$\omega L\sqrt{R_2^2 + \left(\frac{1}{\omega C}\right)^2} = \frac{1}{\omega C}\sqrt{R_1^2 + (\omega L)^2}$$

$$\omega^2 L^2\left(R_2^2 + \frac{1}{\omega^2 C^2}\right) = \frac{1}{\omega^2 C^2}(R_1^2 + \omega^2 L^2)$$

$$\omega^2 L^2 R_2^2 + \frac{L^2}{C^2} = \frac{R_1^2}{\omega^2 C^2} + \frac{L^2}{C^2}$$

$$\omega^2 L^2 R_2^2 = \frac{R_1^2}{\omega^2 C^2}$$

$$C^2 = \frac{R_1^2}{R_2^2} \cdot \frac{1}{\omega^4 L^2}$$

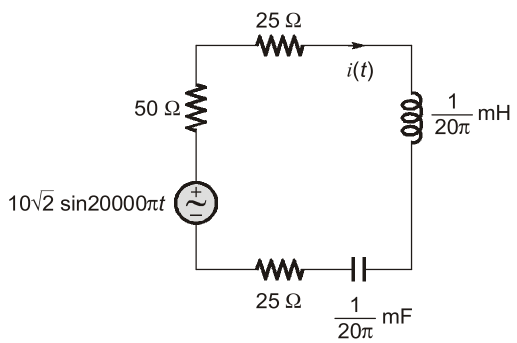
$$C = \frac{R_1}{R_2} \cdot \frac{1}{\omega^2 L}$$

$$= \frac{2.2}{2.2} \times \frac{1}{(100\pi)^2 \times 7 \times 10^{-3}}$$

$$C = 1.45 \times 10^{-3} \text{ F}$$

End of Solution

Q.17 For the circuit shown below:



rms value of $i(t)$ is ________.

Ans. (0.1)

The resonant frequency,

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{20\pi}\times 10^{-3}\times\frac{1}{20\pi}\times 10^{-3}}}$$

$$= 20\pi \times 10^3 \text{ rad/sec}$$

$$\omega_o = 20000\pi \text{ rad/sec}$$

Hence, the circuit is under resonance.

So, the total current,

$$i(t) = \frac{10\sqrt{2}\angle 0^\circ}{50+25+25}$$

$$= 0.1\sqrt{2}$$

$$\text{RMS value of } i(t) = \frac{0.1\sqrt{2}}{\sqrt{2}} = 0.1 \text{ A}$$

$$i(t)|_{rms} = 0.1 \text{ Amp}$$

End of Solution

Q.18 Force per unit length between two infinitely long parallel conductors, with a gap of 2 cm between them is 10 μN/m, when the gap is doubled, then force per unit length will be _____ μN/m.

Ans. (5)

We know that,

Force is inversely proportional to 'd'

$$F \propto \frac{1}{d}$$

$$\frac{F_1}{F_2} = \frac{d_2}{d_1}$$

$$\frac{10}{F_2} = \frac{4\text{ cm}}{2\text{ cm}}$$

$$F_2 = \frac{10}{2} = 5\ \mu\text{N/m}$$

End of Solution

Q.19 If $f(x) = x\sin\left(\frac{1}{x}\right)$; then $\lim_{x\to 0} f(x) = ?$

Ans. (0)

$$\lim_{x\to 0} x\sin\frac{1}{x} = \lim_{x\to 0}\frac{\sin\frac{1}{x}}{\frac{1}{x}}$$

$$\lim_{y\to\infty}\frac{\sin y}{y} = \frac{\text{finite}}{\infty} = 0$$

End of Solution

Q.20 If $f(z) = i\frac{1-z}{1+z}$ then $f^{-1}(z)$ maps real axis to

(a) unit circle with center not at origin
(b) real axis
(c) imaginary axis
(d) unit circle with center at origin

Ans. (d)

Given : $w = u + iv = \frac{i(1-z)}{(1+z)}$

$$= \frac{i - ix - i^2 y}{1 + x + iy}$$

$$= \frac{i(1-x) + y}{(1+x) + iy} \times \frac{(1+x) - iy}{(1+x) - iy}$$

$$u + iv = \frac{y(1-x) + y(1+x)}{(1+x)^2 + y^2} + i\frac{(1-x^2) - y^2}{(1+x)^2 + y^2}$$

To find image of real axis is w-plane.

We take $V = 0$

$$\Rightarrow \quad \frac{(1-x^2) - y^2}{(1+x)^2 + y^2} = 0$$

$$\Rightarrow \quad 1 - x^2 - y^2 = 0$$

$$\Rightarrow \quad x^2 + y^2 = 1$$

$\therefore$ Under the mapping of $f^{-1}(z)$ real axis in w-plane maps to unit circle in z-plane.

End of Solution

Q.21 $A = \begin{bmatrix} 1 & 4 \\ -3 & \alpha \\ \beta & 6 \end{bmatrix}$ If Rank of matrix A is 1 then the ratio of $\frac{\alpha}{\beta}$ is ______.

Ans. **(–8)**

Given : $\rho(A_{3\times2}) = 1$

So, $C_2 = KC_1$

$\Rightarrow$ $4 = K(1)$

$\Rightarrow$ $K = 4$

Also, $\alpha = -3K$ and $6 = K\beta$

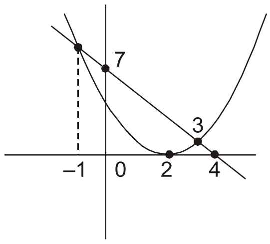
$\therefore$ $\frac{\alpha}{\beta} = -\frac{1}{2}K^2 = -\frac{1}{2}\times 4^2 = -8$

End of Solution

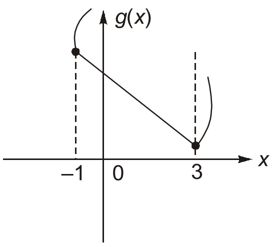
Q.22 $g(x) = \max\{(x-2)^2, -2x+7\}$; $x \in (-\infty, \infty)$. Minimum value of $g(x)$ = _____.

Ans. **(1)**

$$g(x) = \max\{(x-2)^2, -2x+7\}$$



Then $g(x)$



From the figure:

Minimum value of $g(x)$ is at

$x = 3$

$\therefore$ $g_{min}(x) = g(3)$

$= (3-2)^2$ or $(-2x+7)_{x=3}$

$= 1$

End of Solution

Q.23 If X is a discrete random variable

X	0	1	2
P	0.25	0.5	?

then value of $[E(X) - E(X^2)]$ = ______.

Ans. **(–0.5)**

x is discrete R.V

W.K.T: $\sum_{i=1}^{n} P(x = x_i) = 1$

$\Rightarrow \quad 0.25 + 0.5 + k = 1$

$\Rightarrow \quad k = 1 - 0.75$

$k = 0.25$

$\therefore \quad E(x) = \sum_i x_i P_i$

$= 0 \times 0.25 + 1 \times 0.5 + 2 \times 0.25$

$0.5 + 0.5$

$E(x) = 1$

$E(x^2) = \sum_i x_i^2 P_i$

$= 0^2 \times 0.25 + 1^2 \times 0.5 + 2^2 \times 0.25$

$= 0 + 0.5 + 1$

$E(x^2) = 1.5$

$\therefore \quad E(x) - E(x^2) = 1 - 1.5 = -0.5$

End of Solution

Q.24 Choose solution 'S' for set of equations

$x - 2y + z = 0$

$x - z = 0$

(a) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha, \beta \in R \right\}$

(b) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \middle| \alpha \in R \right\}$

(c) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \middle| \alpha, \beta \in R \right\}$

(d) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha \in R \right\}$

Ans. (b)

Given : $x - 2y + z = 0$

$x - z = 0$

Solution is given by solving also equations.

	x	y	z	
–2	1	1	–2	
0	–1	1	0	

$$\frac{x}{2} = \frac{y}{2} = \frac{z}{2} = K$$

$\Rightarrow$ $x = K,\ y = K,\ z = K$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ K \in R$$

End of Solution

Q.25 In a P-i-n photodiode, a pulse of light containing 8×10^{12} incident photons at wavelength $\lambda_0 = 1.55\ \mu m$ gives rise to an average 4×10^{12} electrons collected at the terminals of the device. The quantum efficiency of the photodiode at its wavelength is _______%.

(a) 54.2
(b) 62.5
(c) 80
(d) 50

Ans. (d)

$$\text{quantum efficiency, } \eta = \frac{\text{Electrons out}}{\text{Photons incident}} = \frac{4 \times 10^{12}}{8 \times 10^{12}} = 0.5$$

$\therefore$ $\eta = 50\%$

End of Solution

Q.26 LED is operated under ______ and photodiode is operated under _______.

(a) Forward biased and Reverse biased
(b) Forward biased and Forward biased
(c) Reverse biased and Reverse biased
(d) Reverse biased and Forward biased

Ans. (a)

End of Solution

Q.27 A silica glass fibre has a core refractive index of 1.47 and a cladding refractive index of 1.44. If the cladding is completely stripped out and core is dipped in water having a refractive index of 1.33, the numerical aperture of modified fiber is _____.

Ans. (0.62)

Numerical Aperture, $N.A = \sqrt{n_{core}^2 - n_{water}^2} = \sqrt{1.47^2 - 1.33^2}$

$N.A = 0.62$

End of Solution

Q.28 Number of times Nyquist plot of $G(s)H(s) = \dfrac{(s-1)(s-2)}{2(s+1)(s+2)}$ encircles the origin is _____.

Ans. (2)

$$G(s)H(s) = \frac{(s-1)(s-2)}{2(s+1)(s+2)}$$

∴ Number of poles which are in RHS, $P = 0$

Number of zeros which are in RHS, $Z = 2$

(Here it is asking for number of encirclement around the origin).

∴ Encirclement, $N = P - Z$

$= 0 - 2$

$N = -2$

∴ 2 clockwise encirclement.

End of Solution

Q.29 Number of zeros of $s^3 + 2s^2 + 5s + 80$ in right half plane is _____.

Ans. (2)

$s^3 + 2s^2 + 5s + 80$

Forming RH array

s^3	1	5
s^2	2	80
s^1	$\dfrac{10-80}{2} = -35$	
s^0	80	

∴ There are two sign change.

Hence, 2 open loop zeros in RHS.

End of Solution

Q.30 If $G(s) = \dfrac{2(1-s)}{(1+s)^2}$ then phase margin of $G(s)$ = ________ degree.

Ans. (0)

$$G(s) = \frac{2(1-s)}{(1+s)^2}$$

Phase margin is calculated at gain crossover frequency ω_{gc}.
ω_{gc} is the frequency at which the magnitude of $G(j\omega)H(j\omega)$ is 1.

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = \frac{2\sqrt{1+\omega_{gc}^2}}{(1+\omega_{gc}^2)} = \frac{2}{\sqrt{1+\omega_{gc}^2}} = 1$$

$$2 = \sqrt{1+\omega_{gc}^2}$$

$$4 = 1+\omega_{gc}^2$$

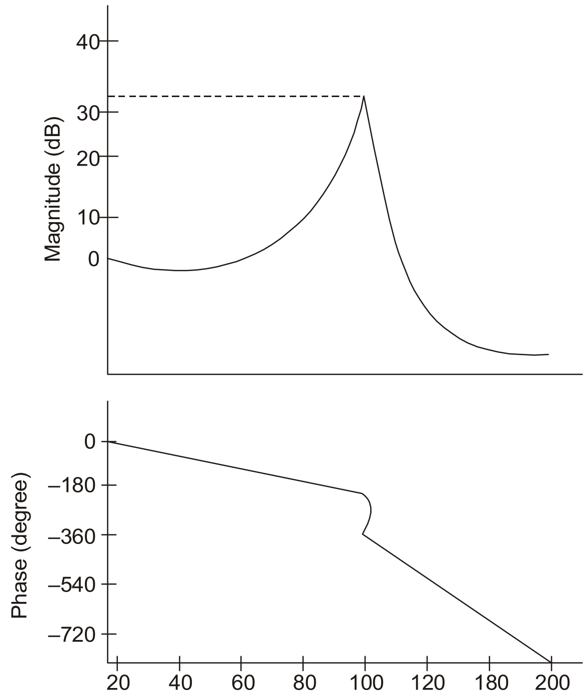
$$\omega_{gc} = \sqrt{3} \text{ rad/sec}$$

Now
$$\phi \text{ at } \omega_{gc} = \tan^{-1}[-\omega_{gc}] - 2\tan^{-1}[\omega_{gc}]$$
$$= -3\tan^{-1}[\omega_{gc}]$$
$$= -3\tan^{-1}(\sqrt{3})$$
$$= -180°$$

∴
$$\text{Phase margin} = 180 + \phi \text{ at } \omega_{gc}$$
$$= 180° - 180° = 0°$$

End of Solution

Q.31 Magnitude and phase plot shown in figure. Match with the transfer function.



(a) $\dfrac{10000}{s^2+2s+10000}e^{-0.5\times10^{-12}s}$

(b) $\dfrac{10000}{s^2+2s+10000}$

(c) $\dfrac{10000}{s^2+2s+10000}e^{-0.05s}$

(d) $\dfrac{100}{s^2+2s+100}$

Ans. (c)

From the figure it is clear that there should be phase lag.

Hence, option (a) and (c) could be answer.

From the figure, at ω = 100 rad/sec, Phase is –360°.

Now considering option (c) only delay element

$$e^{-0.05s} = e^{-0.05 \times 100j} = e^{-j5}$$

Hence, phase is = –5 radians = –286.5°

and from the rest of transfer function $\dfrac{1000}{s^2 + 2s + 10000}$

at $\omega = \omega_n$ = 100 rad/sec phase is –90°.

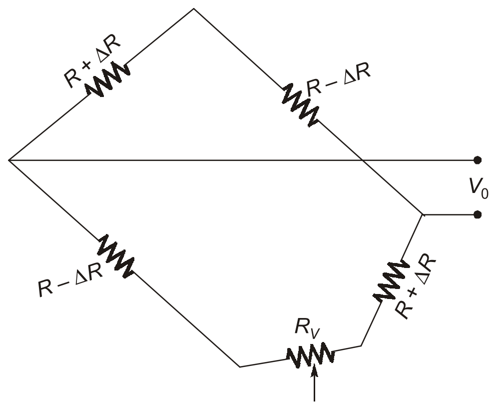
$$= -286.5 - 90°$$

$$= -376.5°$$

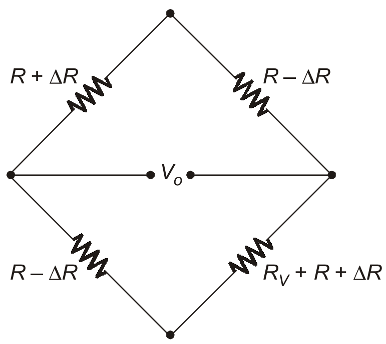
Hence, option (c) satisfy the graph.

End of Solution

Q.32 Initially bridge was balanced and after applying force resistances are changed then find minimum value of R_V to balance the bridge.



Ans. (##)



$$V_o = E\left[\frac{R + \Delta R}{R + \Delta R + R - \Delta R} - \frac{R - \Delta R}{R - \Delta R + (R_V + R + \Delta R)}\right]$$

$$V_o = E\left[\frac{R + \Delta R}{2R} - \frac{R - \Delta R}{2R + R_V}\right]$$

For a balanced bridge $\Rightarrow V_b = 0$

$$0 = E\left[\frac{R+\Delta R}{2R} - \frac{R-\Delta R}{2R+R_V}\right]$$

$$\frac{R+\Delta R}{2R} = \frac{R-\Delta R}{2R+R_V} \Rightarrow 2R + R_V = 2R\left[\frac{R-\Delta R}{R+\Delta R}\right]$$

$$R_V = 2R\left[\frac{R-\Delta R}{R+\Delta R} - 1\right] = 2R\left[\frac{R-\Delta R-R-\Delta R}{R+\Delta R}\right]$$

$$= 2R\left[-\frac{2\Delta R}{R+\Delta R}\right] = \frac{-4R\Delta R}{R\left[1+\frac{\Delta R}{R}\right]} = -\frac{4\Delta R}{1+\frac{\Delta R}{R}} \qquad \left(\frac{\Delta R}{R}\text{ neglected}\right)$$

$$= -4\Delta R \Rightarrow \text{minimum value}$$

End of Solution

Q.33 The Laplace transform of the continuous time signal $x(t) = e^{-3t}u(t-5)$ is ______. $u(t)$ denotes the continuous time unit step signal.

(a) $\frac{e^{-5(s-3)}}{s-3}$, Real{s} > –3
(b) $\frac{e^{-5(s+3)}}{s+3}$, Real{s} > –3
(c) $\frac{e^{-5(s-3)}}{s+3}$, Real{s} > –3
(d) $\frac{e^{-5s}}{s+3}$, Real{s} > –3

Ans. **(b)**

We have, $x(t) = e^{-3t}u(t-5)$

We know, $u(t) \rightarrow \frac{1}{s}$

$$u(t-5) \rightarrow \frac{1}{s}\cdot e^{-5s}$$

$$e^{-3t}u(t-5) \rightarrow \frac{1}{(s+3)}\cdot e^{-5(s+3)}$$

$\therefore$ $$X(s) = \frac{e^{-5(s+3)}}{s+3}; \qquad \text{Re}(s) > -3$$

End of Solution

Q.34 If $y(t) = x(4t)$ where $x(t)$ is a continuous time periodic signal with fundamental period of 100 sec, then the fundamental period of $y(t)$ is ________ sec.

Ans. (25)

Let $\quad x(t) = \sin(\omega_o t)$

We have, $\quad T = 100$ sec

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{100} \text{ rad/sec}$$

Now, $\quad y(t) = x(4t) = \sin(4\omega_o t)$

$$\therefore \quad \omega' = 4\omega_o = 4 \times \frac{2\pi}{100}$$

$$\frac{2\pi}{T'} = 4 \times \frac{2\pi}{100}$$

$$T' = 25 \text{ sec}$$

End of Solution

Q.35 For LTI system with input $x(t)$ and output $y(t)$ have transfer function $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{s-\pi}{s+\pi}$ if $y(t) = \sin \pi t$ then $x(t)$ is

(a) $\cos\left[\pi t + \frac{\pi}{4}\right]$

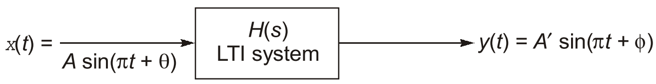
(b) $\sin\left[\pi t - \frac{\pi}{2}\right]$

(c) $\sin\left[\pi t + \frac{\pi}{2}\right]$

(d) $\sin \pi t$

Ans. (b)

We know,



where, $\quad A' = A \cdot |H(\omega)|_{\omega=\omega_o}$

$$\phi = \theta + \angle H(\omega)|_{\omega=\omega_o}$$

We have, $\quad \omega_o = \pi, \; A' = 1, \; \phi = 0$

Now, $$|H(\omega)|_{\omega=\pi} = \frac{\sqrt{\omega^2+\pi^2}}{\sqrt{\omega^2+\pi^2}} = 1$$

$$\angle H(\omega)|_{\omega=\pi} = 180° - \tan^{-1}\left(\frac{\omega}{\pi}\right) - \tan^{-1}\left(\frac{\omega}{\pi}\right)$$

$$= 180° - 45° - 45°$$

$$= 90°$$

$$\therefore \quad A = 1$$

$$\theta = -90°$$

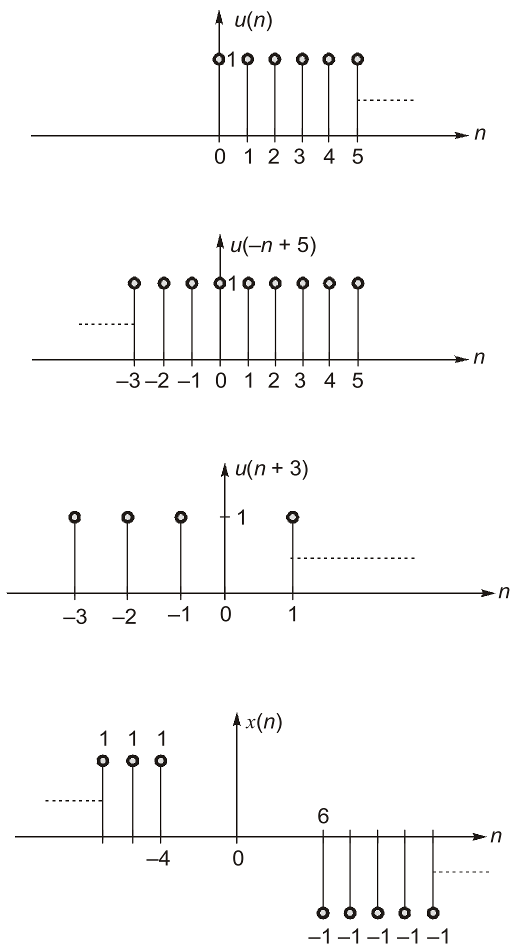
Now, $$x(t) = \sin(\pi t - 90°) = \sin\left(\pi t - \frac{\pi}{2}\right)$$

End of Solution

Q.36 Consider the discrete time signal $x(n) = u(-n+5) - u(n+3)$ where $u(n) = \begin{cases} 1; & n \geq 0 \\ 0; & n < 0 \end{cases}$.

The smallest 'n' for which $x(n) = 0$ is ________.

Ans. (–3)



Smallest 'n' for which $x[n] = 0$

$\therefore \quad n = -3$

End of Solution

Q.37 The impulse response of an LTI system is $h(t) = \delta(t) + 0.5\delta(t-4)$, where $\delta(t)$ is the continuous time unit impulse signal. If the input signal $x(t) = \cos\left(\frac{7\pi}{4}t\right)$ then output is

(a) $0.5\sin\left(\frac{7\pi}{4}t\right)$
(b) $1.5\sin\left(\frac{7\pi}{4}t\right)$
(c) $0.5\cos\left(\frac{7\pi}{4}t\right)$
(d) $1.5\cos\left(\frac{7\pi}{4}t\right)$

Ans. **(c)**

We have; $h(t) = \delta(t) + 0.5\delta(t-4)$

Taking FT;

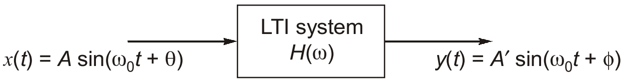
$$H(\omega) = 1 + 0.5e^{-j4\omega}$$

at $\omega = \frac{7\pi}{4}$

$$H(\omega) = 1 + 0.5e^{-j4\left(\frac{7\pi}{4}\right)}$$
$$= 1 + 0.5e^{-j7\pi}$$
$$= 0.5$$

$\therefore$ $|H(\omega)|_{\omega = \frac{7\pi}{4}} = 0.5$

$$\angle H(\omega)|_{\omega = \frac{7\pi}{4}} = 0$$



$$A' = A_0 |H(\omega)|_{\omega = \omega_0}$$
$$\phi = \theta + \angle H(\omega)|_{\omega = \omega_0}$$

$\therefore$ $y(t) = 0.5\cos\left(\frac{7\pi}{4}t\right)$

End of Solution

Q.38 A continuous real valued signal $x(t)$ has finite positive energy and $x(t) = 0$, $t < 0$. From the list given below select all the signal whose continuous time Fourier transform is purely imaginary.

(a) $x(t) + x(-t)$
(b) $j(x(t) + x(-t))$
(c) $x(t) - x(-t)$
(d) $j(x(t) - x(-t))$

Ans. **(b, c)**

If $x(t)$ is real, then its Fourier transform will be complex conjugate.

i.e., $x(t) \longrightarrow X(\omega)$

$x(-t) \longrightarrow X(-\omega) = X^*(\omega)$

Option (a) $\rightarrow$ $x(t) + x(-t) \xleftrightarrow{FT} X(\omega) + X(-\omega) = X(\omega) + X^*(\omega) \Rightarrow$ Real

Option (b) $\rightarrow$ $j[(x(t) + x(-t)]$

$= j[X(\omega) + X(-\omega)]$ ... Taking Fourier Transform

$= j(X(\omega) + X^*(\omega))$

$=$ Purely Imaginary

Option (c) $\rightarrow$ $x(t) - x(-t) \xleftrightarrow{F.T} X(\omega) - X(-\omega) = X(\omega) - X^*(\omega) \Rightarrow$ Purely imaginary

Option (d) $\rightarrow$ $j[x(t) - x(-t)] \xleftrightarrow{F.T} j[X(\omega) - X^*(\omega)] \Rightarrow$ Purely Real

End of Solution

Q.39 If $F(z) = \dfrac{1}{1-z}$ expanded as power series around $z = 2$ would result in $f(z) = \sum_{k=0}^{\infty} a_k (z-2)^k$ with region of convergence $|z - 2| < 1$ then value of a_k is

(a) $\left(\dfrac{1}{2}\right)^k$

(b) $(-1)^k$

(c) $(-1)^{k+1}$

(d) $\left(\dfrac{1}{z}\right)^{k+1}$

Ans. **(c)**

Power series expansion for

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \text{........} \quad \text{when } |x| < 1$$

Now, given that, $F(z) = \dfrac{1}{1-z} = -\left[\dfrac{1}{1+(z-2)}\right]$

$= -[1 - (z - 2) + (z - 2)^2 - (z - 2)^3 + \text{......}],$ when $|z - 2| < 1$

$$= -\sum_{K=0}^{\infty} (-1)^K (z-2)^K, \quad |z - 2| < 1$$

$$= \sum_{K=0}^{\infty} (-1)^{K+1} (z-2)^K, \quad |z - 2| < 1$$

$$= \sum_{K=0}^{\infty} a_K (z-2)^K, \quad |z - 2| < 1$$

$\therefore$ $a_K = (-1)^{K+1}$

End of Solution

■■■■