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Detailed Solutions

ESE-2024 Mains Test Series

E & T Engineering Test No : 6

Section A : Electromagnetics + Basic Electrical Engineering

Q.1 (a) Solution:

(i) We have,

from open circuit test,

$$P_i = 396 \text{ W}, I_0 = 9.65 \text{ A}, V_{OC} = 120 \text{ V}$$

We know that,

$$P_i = V_1 I_0 \cos \phi_0$$

$$396 = (120) I_0 \cos \phi_0$$

$$I_0 \cos \phi_0 = I_w = \frac{396}{120} = 3.3 \text{ A}$$

Since,
$$I_0^2 = I_w^2 + I_\mu^2$$

where I_w and I_μ represent the core-loss and magnetizing components of no load current I_0.

$$I_\mu^2 = I_0^2 - I_w^2$$

$$I_\mu = \sqrt{(9.65)^2 - (3.3)^2}$$

$$I_\mu = 9.07 \text{ A}$$

and,

$$I_w R_0 = V_{OC} \qquad I_\mu X_0 = V_{OC}$$

$$R_0 = \frac{V_{OC}}{I_w} = \frac{120}{3.3} \qquad X_0 = \frac{V_{OC}}{I_\mu} = \frac{120}{9.07}$$

$$R_0 = 36.36\ \Omega \qquad X_0 = 13.23\ \Omega$$

We have,

from short circuit test,

$$P_{cfl} = 810\text{ W}, I_{1fl} = 20.8\text{ A}, V_{SC} = 92\text{ V}$$

We know,

$$I_{1fl}^2 R_{e1} = P_{cfl} \qquad V_{sc} = I_{1fl} Z_{e1}$$

$$R_{e1} = \frac{810}{(20.8)^2} = 1.87\ \Omega \qquad 92 = (20.8) Z_{e1}$$

$$Z_{e1} = 4.42\ \Omega$$

Since, $$R_{e1}^2 + X_{e1}^2 = Z_{e1}^2$$

$$X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2} = \sqrt{(4.42)^2 - (1.87)^2}$$

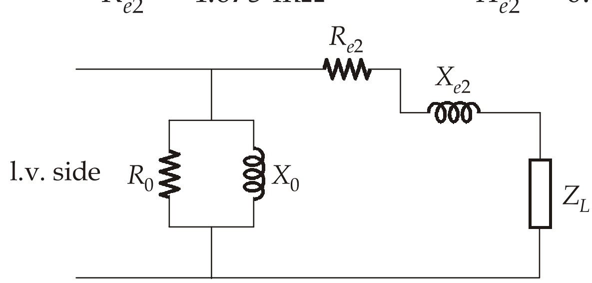
$$X_{e1} = 4\ \Omega$$

Thus, referring R_{e1} and X_{e1} to lv side we get,

$$R_{e2} = R_{e1}\left(\frac{T_2}{T_1}\right)^2 \qquad X_{e2} = X_{e1}\left(\frac{T_2}{T_1}\right)^2$$

$$= 1.87\left(\frac{120}{2400}\right)^2 \qquad = 4\left(\frac{120}{2400}\right)^2$$

$$R_{e2} = 4.675\text{ m}\Omega \qquad X_{e2} = 0.01\ \Omega$$



(ii) We know that,

$$\text{efficiency at full load, } \eta_{fl} = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + P_{cfl}}$$

$$= \frac{50 \times 1000 \times 0.8}{50 \times 1000 \times 0.8 + 396 + 810}$$

$$= 0.9707 = 97.07\%$$

(iii) Voltage regulation,

$$R = \frac{I_1 R_{e1}}{V_1}\cos\phi_2 + \frac{I_1 X_{e1}}{V_1}\sin\phi_2$$

$$= \left(\frac{20.8 \times 1.87}{2400} \times 0.8\right) + \left(\frac{20.8 \times 4}{2400} \times 0.6\right)$$

$$= 0.0338 = 3.38\%$$

Q.1 (b) Solution:

We have, Number of poles = 4

Number of slots = 60

$$\text{Slots per pole per phase, } m = \frac{\text{slots}}{\text{poles} \times \text{phases}} = \frac{60}{4 \times 3} = 5$$

$$\text{Slot angle } \beta = \frac{180° \times \text{poles}}{\text{slots}} = \frac{180° \times 4}{60} = 12°$$

$$\text{Slots per pole} = \frac{\text{slots}}{\text{poles}} = \frac{60}{4} = 15$$

For full pitch coil, the coil span is 15 slots. For the given coil, the coil span is 13 – 1 = 12 slots.

$$\therefore \quad \alpha = (15 - 12) \text{ slot angles} = 3\beta \quad \Rightarrow \quad 3 \times 12° = 36°$$

(i) For fundamental frequency waveform,

$$\Rightarrow \quad \text{Coil span factor } K_{c1} = \cos\frac{\alpha}{2} = \cos\frac{36°}{2} = 0.951$$

$$\text{Distribution factor, } K_{d1} = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)} = \frac{\sin\left(\frac{5 \times 12°}{2}\right)}{5\sin\left(\frac{12°}{2}\right)} = 0.957$$

Winding factor

$$K_{w1} = K_{c1} K_{d1} = 0.951 \times 0.957 = 0.91$$

(ii) For third-harmonic frequency waveform,

$$K_{c3} = \cos\left(\frac{3\alpha}{2}\right) = \cos\left(\frac{3\times 36°}{2}\right)$$

$$K_{c3} = 0.588$$

$$K_{d3} = \frac{\sin\left(\frac{3m\beta}{2}\right)}{m\sin\frac{3\beta}{2}} = \frac{\sin\left(\frac{5\times 3\times 12°}{2}\right)}{5\sin\left(\frac{3\times 12°}{2}\right)}$$

$$K_{d3} = 0.647$$

Winding factor, $K_{w3} = K_{c3} \times K_{d3} = 0.588 \times 0.647 = 0.38$

(iii) For fifth-harmonic frequency waveform,

$$\Rightarrow \quad K_{c5} = \cos\left(\frac{5\alpha}{2}\right) = \cos\left(\frac{5\times 36°}{2}\right)$$

$$K_{c5} = 0$$

Winding factor, $K_{w5} = K_{c5} \times K_{d5}$

$$K_{w5} = 0 \times k_{d5}$$

$$= 0$$

Q.1 (c) Solution:

We have,

A shunt generator delivering 50 kW at 250 V with

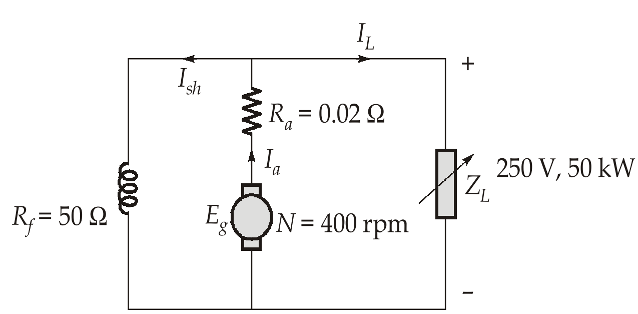
Rotation per minute, $N = 400$ rpm

Armature field resistance, $R_a = 0.02\ \Omega$

Field resistance, $R_f = 50\ \Omega$

Brush contact drop, $V_L = 1$ V per brush

The, shunt generator circuit is drawn as below:



Now, Load current, $I_L = \dfrac{50\times 10^3}{250} = 200$ A

Shunt field current, $I_{sh} = \frac{V}{R_{sh}} = \frac{250}{50} = 5\text{ A}$

Armature current, $I_a = I_L + I_{sh}$

$= 200 + 5 = 205\text{ A}$

Generated emf, $E_{g1} = V + I_a R_a + \text{Voltage drop in the brushs}$

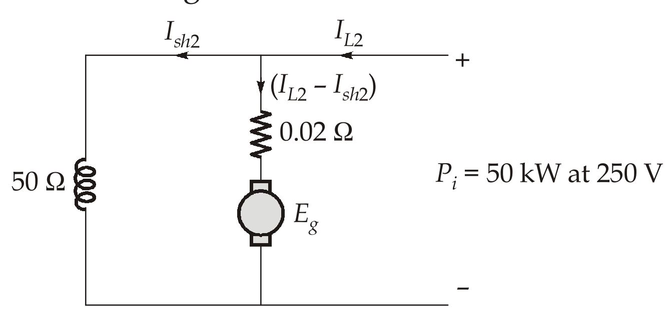
$= 250 + (205 \times 0.02) + (2 \times 1)$

$= 256.1\text{ V}$

Speed of the generator, $N_1 = 400\text{ rpm}$

Now,

Assume the same circuit working as motor,



The input current, $I_{L2} = \frac{P_i}{V} = \frac{50 \times 10^3}{250} = 200\text{ A}$

$I_{sh2} = \frac{V}{R_{sh}} = \frac{250}{50} = 5\text{ A}$

Armature current, $I_{a2} = I_{L2} - I_{sh2}$

$= 200 - 5 = 195\text{A}$

$E_{g2} = V - I_{a2}R_a - \text{brush drop}$

$= 250 - 195 \times 0.02 - 2 \times 1$

$= 244.1\text{ V}$

We know that the speed of the DC machine, $E \propto N\phi$, where E is the generated emf/ back emf, N is the speed of the machine and ϕ is the flux per pole. Therefore,

$$\frac{N_2}{N_1} = \frac{E_{g2}}{E_{g1}} \times \frac{\phi_1}{\phi_2}$$

Since, the field current is constant, $\phi_2 = \phi_1$

$$\therefore \quad N_2 = \frac{E_{g2}}{E_{g1}} \times N_1 = \frac{244.1}{256.1} \times 400$$

$$N_2 = 381.3\text{ rpm}$$

Q.1 (d) Solution:

(i) Maxwell's equation in differential form are:

1. $$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} \quad ...(i)$$

where, $\vec{E}$ = Electric field intensity and $\frac{\partial \vec{B}}{\partial t}$ = time derivative of magnetic field density.

2. $$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad ...(ii)$$

$\vec{H}$ = Magnetic field intensity

$\vec{J}$ = Conduction current density

$\frac{\partial \vec{D}}{\partial t}$ = Time derivative of electric field density

$\vec{D}$ = Electric flux density

3. $$\nabla \cdot \vec{D} = \rho_V \quad ...(iii)$$

$\vec{D}$ = Electric flux density

ρ_V = Volume charge density

4. $$\nabla \cdot \vec{B} = 0 \quad ...(iv)$$

$\vec{B}$ = Magnetic flux density

Maxwell equation in integral form

1. $$\oint \vec{E} \cdot d\vec{l} = \iint \left(\frac{-\partial \vec{B}}{\partial t}\right) \cdot d\vec{s} \quad ...(v)$$

2. $$\oint \vec{H} \cdot d\vec{l} = \iint \vec{J} \cdot d\vec{s} + \iint \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s} \quad ...(vi)$$

3. $$\oiint \vec{D} \cdot d\vec{s} = \iiint_V \rho_V \cdot dV \quad ...(vii)$$

4. $$\oiint \vec{B} \cdot d\vec{s} = 0$$

Statements:

1. EMF around a closed line (path) is equal to the time derivative of magnetic flux density crossing the open surface which is inside the closed line.

2. MMF (Magnetomotive force) around a closed line is equal to the conduction current density plus time derivative of electric flux density crossing open surface.
3. Total electric flux crossing any closed surface is equal to the total charge inside the volume enclosed by that closed surface.
4. Total magnetic flux crossing any closed surface is zero.

(ii) Given,

Characteristic impedance, $Z_0 = 80\ \Omega$

[1 neper = 2.71; 1 neper (dB) = 8.686 dB = 20 log 2.71)]

$$\alpha = 0.05\ \text{dB/m}$$

Converting the attenutation constant in Np/m,

$$1\ \text{neper} \equiv 8.686\ \text{dB}$$

$$\Rightarrow \quad 8.686\ \text{dB} \equiv 1\ \text{neper}$$

$$0.05\ \text{dB} \equiv \frac{1}{8.686} \times 0.05 = 5.756 \times 10^{-3}\ \text{neper/m} = \alpha$$

$$C = 0.3 \times 10^{-9}\ \text{F/m}$$

1. In distortionless transmission line

$$Z_0 = \sqrt{\frac{L}{C}} = 80 \text{ and } \alpha = \sqrt{RG}$$

$$\frac{L}{0.3 \times 10^{-9}} = 6400$$

$$L = 1.92\ \mu\text{H/meter}$$

$$\alpha = \sqrt{RG} \qquad \text{...(i)}$$

In distortionless transmission line

$$\frac{R}{L} = \frac{G}{C}$$

$$R = \frac{G}{C} L \qquad \text{...(ii)}$$

From equation (i) and (ii), we get

$$\alpha = \sqrt{\frac{G^2 L}{C}} = GZ_0$$

$$G = \frac{\alpha}{Z_0} = \frac{5.75 \times 10^{-3}}{80}$$

$$= 7.1875 \times 10^{-5}$$

$$= 71.875\ \mu\mho/\text{m}$$

From equation (ii),

$$R = \frac{G}{C}L = \frac{71.875\times10^{-6}}{0.3\times10^{-9}}\times1.92\times10^{-6} = 0.46\ \Omega/\text{m}$$

2. Phase velocity, $V_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.92\times10^{-6}\times0.3\times10^{-9}}}$

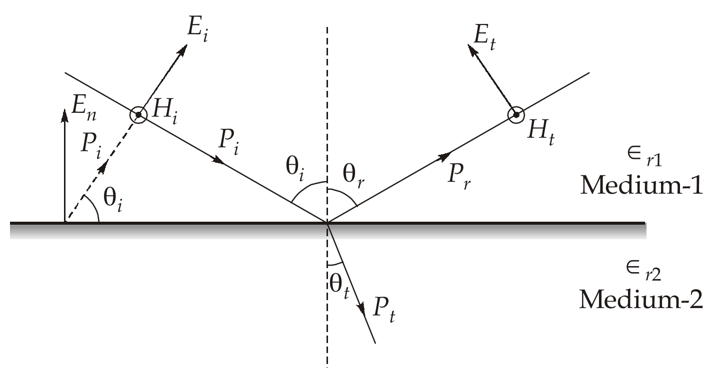
$$V_p = 4.16\times10^7\ \text{m/s}$$

Q.1 (e) Solution:

Consider the two nonmagnetic dielectric media separated by the interface with

$$\mu_0 = \mu_1 = \mu_2,\ \eta_1 = \frac{\eta_0}{\sqrt{\epsilon_{r_1}}},\ \eta_2 = \frac{\eta_0}{\sqrt{\epsilon_{r_2}}}$$

Considering parallel polarization or p-polarization i.e. the electric field is parallel to the plane of incidence as shown below:



$$E_{t1} = E_{t2}$$

$$E_i\cos\theta_i + E_r\cos\theta_r = E_t\cos\theta_t$$

$$1+\Gamma_{\parallel} = \tau_{\parallel}\frac{\cos\theta_t}{\cos\theta_i} \quad \text{...(i)}$$

The angle between power flow and normal to the surface is the same as the angle between the field and the surface.

Similarly applying H field boundary condition for the second equation.

$$\Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\Gamma_{\parallel} = \frac{\frac{1}{\sqrt{\epsilon_{r_2}}}\cos\theta_t - \frac{1}{\sqrt{\epsilon_{r_1}}}\cos\theta_i}{\frac{1}{\sqrt{\epsilon_{r_2}}}\cos\theta_t + \frac{1}{\sqrt{\epsilon_{r_1}}}\cos\theta_i}$$

We have, $$\sqrt{\epsilon_{r_1}}\sin\theta_i = \sqrt{\epsilon_{r_2}}\sin\theta_t \rightarrow \text{By Snell's law}$$

$$\frac{\sqrt{\epsilon_{r_2}}}{\sqrt{\epsilon_{r_1}}} = \frac{\sin\theta_i}{\sin\theta_t}$$

$$\Gamma_{||} = \frac{\cos\theta_t - \dfrac{\sin\theta_i}{\sin\theta_t}\cos\theta_i}{\cos\theta_t + \dfrac{\sin\theta_i}{\sin\theta_t}\cos\theta_i} = \frac{\sin\theta_t\cos\theta_t - \sin\theta_i\cos\theta_i}{\sin\theta_t\cos\theta_t + \sin\theta_i\cos\theta_i}$$

$$= \frac{\sin 2\theta_t - \sin 2\theta_i}{\sin 2\theta_t + \sin 2\theta_i} = \frac{\sin(\theta_t - \theta_i)\cos(\theta_t + \theta_i)}{\cos(\theta_t - \theta_i)\sin(\theta_t + \theta_i)}$$

$$\Gamma_{||} = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)}$$

We know that,

$$\tau_{||} = \frac{2\eta_2\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t} = \frac{\dfrac{2}{\sqrt{\epsilon_{r_2}}}\cos\theta_i}{\dfrac{1}{\sqrt{\epsilon_{r_2}}}\cos\theta_t + \dfrac{1}{\sqrt{\epsilon_{r_1}}}\cos\theta_i}$$

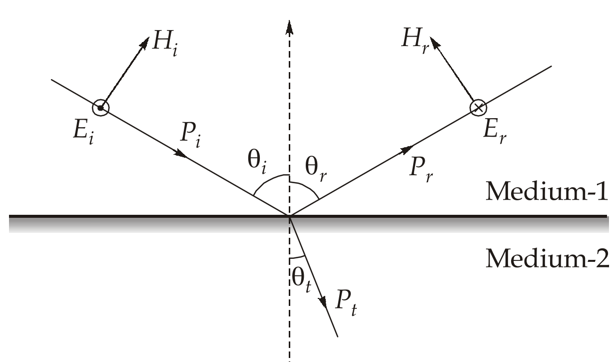
$$= \frac{2\cos\theta_i}{\cos\theta_t + \dfrac{\sin\theta_i}{\sin\theta_t}\cos\theta_i}$$

$$= \frac{2\cos\theta_i\sin\theta_t}{\sin\theta_t\cos\theta_t(\sin^2\theta_i + \cos^2\theta_i) + \sin\theta_i\cos\theta_i(\sin^2\theta_t + \cos^2\theta_t)}$$

$$= \frac{2\cos\theta_i\sin\theta_t}{(\sin\theta_i\cos\theta_t + \sin\theta_t\cos\theta_i)(\cos\theta_i\cos\theta_t + \sin\theta_i\sin\theta_t)}$$

$$\tau_{||} = \frac{2\cos\theta_i\sin\theta_t}{\sin(\theta_i + \theta_t)\cos(\theta_t - \theta_i)}$$

For ⊥ polarization or surface polarization or S-polarization,



$$\Gamma_{\perp} = \frac{\eta_2 \sec\theta_t - \eta_1 \sec\theta_i}{\eta_2 \sec\theta_t + \eta_1 \sec\theta_i}$$

$$= \frac{\frac{1}{\sqrt{\epsilon_{r_2}}}\cos\theta_i - \frac{1}{\sqrt{\epsilon_{r_1}}}\cos\theta_t}{\frac{1}{\sqrt{\epsilon_{r_2}}}\cos\theta_i + \frac{1}{\sqrt{\epsilon_{r_1}}}\cos\theta_t}$$

$$= \frac{\cos\theta_i - \frac{\sin\theta_i}{\sin\theta_t}\cos\theta_t}{\cos\theta_i + \frac{\sin\theta_i}{\sin\theta_t}\cos\theta_t} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}$$

$$\Gamma_{\perp} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}$$

Also,

$$\tau_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = \frac{\frac{2}{\sqrt{\epsilon_{r_2}}}\cos\theta_i}{\frac{1}{\sqrt{\epsilon_{r_2}}}\cos\theta_i + \frac{1}{\sqrt{\epsilon_{r_1}}}\cos\theta_t}$$

$$= \frac{2\cos\theta_i}{\cos\theta_i + \frac{\sin\theta_i}{\sin\theta_t}\cos\theta_t} = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_t + \theta_i)}$$

$$\tau_{\perp} = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_t + \theta_i)}$$

Q.2 (a) Solution:

According to question,

maximum efficiency of transformer is 98% at 15 kVA at unity power factor (upf).

We know that

$$\text{maximum efficiency, } \eta_{max} = \frac{\text{rating of transformer}}{\text{rating of transformer + 2(iron loss)}}$$

$$0.98 = \frac{15 \times 10^3}{15 \times 10^3 + 2P_i}$$

$$14700 + 1.96P_i = 15000$$

$$P_i = 153.06 \text{ Watt}$$

Since, Copper loss (P_{cu}) = iron loss (P_i) ..... at maximum efficiency

$$P_{cu} = 153.06 \text{ Watt .... at 15 kVA}$$

Thus, copper loss at 20 kVA,

$$P_{\text{cu (at 20 kVA)}} = \left(\frac{20}{15}\right)^2 \times 153.06$$

$$= 272.11 \text{ Watt}$$

Calculation of all day efficiency,

(i) full load of 20 kVA 12 hours/day and no load rest of the day (at unity power factor)

$$\Rightarrow \quad \text{Output per hour (kWh)} = 20 \times 10^3 \times 1 \times 12$$

$$= 240 \text{ kWh}$$

full load copper losses for 12 hours = 272.11 × 12

$$= 3265.32 \text{ Watt}$$

Iron losses = 153.06 × 24 = 3673.44 kWh

$$\text{all day efficiency, } \eta = \frac{\text{Output kWh}}{\text{Output kWh + losses}}$$

$$= \frac{240 \times 1000}{(240 \times 1000) + 3265.32 + (153.06 \times 24)} = 97.19\%$$

(ii) full load of 20 kVA 4 hours/day and 0.4 full load rest of the day.

$$\Rightarrow \quad \text{output per hour (kWh)} = (20 \times 10^3 \times 4) + (0.4 \times 20 \times 10^3 \times 20)$$

$$= 240000 \text{ Watt hour}$$

$$= 240 \text{ kWh}$$

$$\text{Copper losses} = [(0.4)^2 \times 272.11 \times 20] + (272.11 \times 4)$$

$$= 1959.192 \text{ Watt hour}$$

$$\text{Iron losses} = 153.06 \times 24$$

$$= 3673.44 \text{ Watt hour}$$

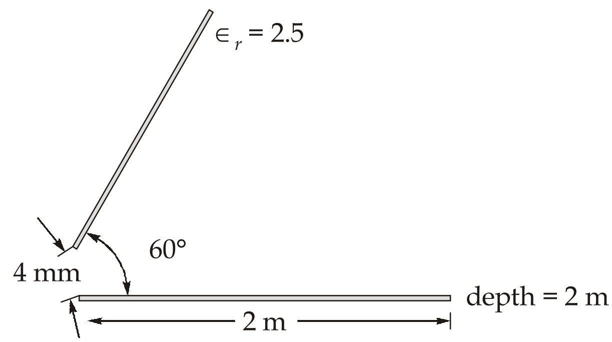
$$\eta_{\text{all day}} = \frac{\text{Output kWh}}{\text{Output kWh + losses}}$$

$$= \frac{240 \times 1000}{(240 \times 1000) + 1959.192 + 3673.44}$$

$$= 97.71\%$$

Q.2 (b) Solution:

(i) Given diagram,



Apply Poisson's equation,

$$\nabla^2 V = \frac{-\rho_V}{\in}$$

Here, $\rho_V = 0$

$$\nabla^2 V = 0$$

Solving the Laplace's equation in cylindrical basis,

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Between plates $V = f(\phi)$ and independent of ρ or z.

$$\therefore \qquad \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} = 0$$

Integrating both sides,

$$\frac{\partial V}{\partial \phi} = C_1$$

$$V(\phi) = C_1\phi + C_2$$

$$V\left(\phi = \frac{\pi}{3}\right) = C_1\frac{\pi}{3} + C_2$$

$$V(\phi = 0) = C_2$$

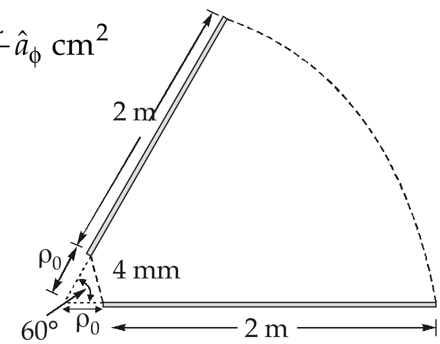
$$\nabla V = V(\phi = \pi/3) - V(\phi = 0) = C_1\frac{\pi}{3} = 60 \text{ V}$$

$$C_1 = \frac{180}{\pi} \text{ V/rad}$$

Electric field between plates,

$$\vec{E} = -\nabla V = -\frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi = \frac{-C_1}{\rho}\hat{a}_\phi = \frac{-180}{\pi\rho}\hat{a}_\phi \text{ V/m}$$

$$\vec{D} = \epsilon_0\epsilon_r \vec{E} = \frac{-180\,\epsilon_0\epsilon_r}{\pi\rho}\hat{a}_\phi \text{ cm}^2$$



Gauss's law $\oint \vec{D}\cdot d\vec{s} = Q_{\text{enclosed}} = \iint \rho_s \cdot ds$

$$-\frac{180\,\epsilon_0\epsilon_r}{\pi}\int_{\rho_0}^{\rho_0+2}\frac{1}{\rho}d\rho = Q$$

$$Q = \frac{-180\,\epsilon_0\epsilon_r}{\pi}[\ln\rho]_{\rho_0}^{\rho_0+2}$$

$$= \frac{-180\,\epsilon_0\epsilon_r}{\pi}\ln\left(\frac{\rho_0+2}{\rho_0}\right)$$

$$= \frac{180\,\epsilon_0\epsilon_r}{\pi}\ln\left(\frac{\rho_0}{2+\rho_0}\right)$$

Using cosine rule

$$(4\text{ mm})^2 = \rho_0^2 + \rho_0^2 - 2\rho_0^2\cos 60°$$

$$\rho_0^2 = \frac{16\times10^{-6}}{2-2\times\frac{1}{2}} = \frac{16\times10^{-6}}{1}$$

$$\rho_0 = 4\text{ mm}$$

$$Q = \frac{180\times2.5}{\pi\times36\pi\times10^9}\ln\left(\frac{4\times10^{-3}}{2+4\times10^{-3}}\right) = -7.87\text{ nC}$$

$$Q \cong -8\text{ nC}$$

(ii) Here,
$$H_0 = 10,\ \beta = \frac{1}{2}$$

$$\eta = 150\angle30°$$

$$\frac{E_0}{H_0} = \eta = 150\angle30°$$

$$E_0 = 1500\angle30°$$

$\therefore$
$$\vec{E} = \text{Re}\left[1500\angle30° e^{-\gamma x}e^{j\omega t}\hat{a}_E\right]\text{V/m}$$

$$E = 1500e^{-\alpha x}\cos\left(\omega t - \frac{1}{2}x + \frac{\pi}{6}\right)(-a_z)\text{ V/m}$$

Now, we know that,

$$\alpha = \omega\sqrt{\frac{\mu\in}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}-1\right)}$$

and $$\beta = \omega\sqrt{\frac{\mu\in}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}+1\right)}$$

$\therefore$ $$\frac{\alpha}{\beta} = \frac{\sqrt{\left[\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}-1\right]}}{\sqrt{\left[\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}+1\right]}}$$

But $\frac{\sigma}{\omega\in} = \tan 2\theta_n = \tan 60° = \sqrt{3}$

$$\frac{\alpha}{\beta} = \frac{\sqrt{\left(\sqrt{1+3}-1\right)}}{\sqrt{\left(\sqrt{1+3}\right)+1}} = \frac{1}{\sqrt{3}}$$

$$\alpha = \beta\times\frac{1}{\sqrt{3}} = \frac{1}{2\sqrt{3}} = 0.2807 \text{ Np/m}$$

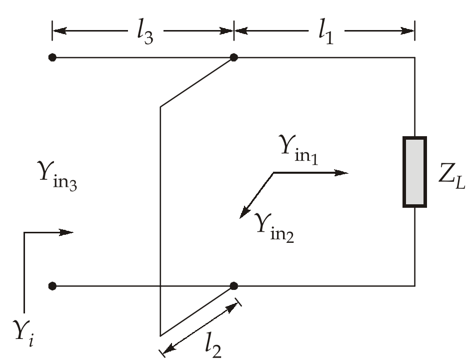
The skin depth is given as,

$$\delta = \frac{1}{\alpha} = 2\sqrt{3} = 3.464 \text{ m}$$

Since the wave has only z-component of the electric field it is polarized along the z-direction.

Q.2 (c) Solution:

(i) Given transmission line is



The input impedance of a transmission line is given by

$$Z_{\text{in}} = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$$

1. For line with $l_1 = \frac{\lambda}{4}, \beta l_1 = \frac{\pi}{2} \Rightarrow Z_{in1} = \frac{Z_0^2}{Z_L}$ *or* $Y_{in1} = \frac{Z_L}{Z_0^2}$

$$Y_{\text{in1}} = \frac{150 + 200j}{(120)^2} = (10.416 + 13.88j) \text{ mS}$$

For line with $l_2 = \frac{\lambda}{8}, \beta l_2 = \frac{\pi}{4} \quad \Rightarrow \quad Z_{\text{in2}} = Z_0 \times \left(\frac{Z_L + jZ_0}{Z_0 + jZ_L}\right)$

Due to short circuit, $Z_L \to 0$

$$Z_{\text{in2}} = \lim_{Z_L \to 0} Z_0 \times \left(\frac{Z_L + jZ_0}{Z_0 + jZ_L}\right) = jZ_0$$

$$Y_{\text{in2}} = \frac{-j}{Z_0} = \frac{1}{120j} = -8.34j \text{ mS}$$

For line with $l_3 = \frac{7\lambda}{8}, \beta l_3 = \frac{7\pi}{4} \quad \Rightarrow \quad Z_{in3} = Z_0\left[\frac{Z_i + jZ_0 \tan\frac{7\pi}{4}}{Z_0 + jZ_i \tan\frac{7\pi}{4}}\right]$

$$= \frac{Z_0[Z_i - jZ_0]}{[Z_0 - jZ_i]}$$

But,
$$Y_i = Y_{\text{in1}} + Y_{\text{in2}} = [(10.416 + 13.88j) - 8.34j] \text{ mS}$$
$$= (10.416 + 5.54j) \text{ mS}$$

$$Z_i = \frac{1}{Y_i} = \frac{1000}{10.416 + 5.54j} = (74.835 - 39.8j)\,\Omega$$

$$Y_{\text{in3}} = \frac{Z_0 - jZ_i}{Z_0(Z_i - jZ_0)} = \frac{120 - 74.835j - 39.8}{120(74.835 - 39.8j - 120j)}$$

$$Y_{\text{in3}} = (4.806 + 1.93j)\text{mS}$$

2. If the short circuit section were open,

$$Y_{\text{in1}} = (10.416 + 13.88j)\text{mS}$$

For open circuited line with $l_2 = \lambda/8, \beta l_2 = \pi/4$

$$Z_{\text{in2}} = \frac{-jZ_0}{\tan\frac{\pi}{4}} = -jZ_0$$

$$Y_{in2} = \frac{1}{-jZ_0} = \frac{j}{120} = 8.34j \text{ mS}$$

$$\text{For line with } l_3 = \frac{7\lambda}{8}, \beta l_3 = \frac{7\pi}{4}. \text{ Hence,}$$

$$Z_{in3} = Z_0\left[\frac{Z_i + jZ_0 \tan\frac{7\pi}{4}}{Z_0 + jZ_i \tan\frac{7\pi}{4}}\right]$$

$$= \frac{Z_0\left[Z_i - jZ_0\right]}{(Z_0 - jZ_i)}$$

$$Y_i = Y_{in1} + Y_{in2} = [(10.416 + 13.88j) + 8.34j] \text{ mS}$$

$$= (10.416 + 22.22j) \text{ mS}$$

$$Z_i = \frac{1}{Y_i} = \frac{1000}{10.416 + 22.22j} = 17.295 - 36.896j$$

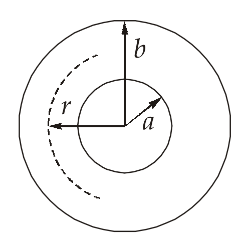
$$Y_{in3} = \frac{Z_0 - jZ_i}{Z_0(Z_i - jZ_0)} = \frac{120 - j(17.295 - 36.896j)}{120(17.295 - 36.896j - j120)}$$

$$= \frac{120 - j17.295 - 36.896}{120(17.295 - 36.896j - j120)}$$

$$Y_{in3} = (1.388 + 4.26j) \text{ mS}$$

(ii) Both inner and outer conductors are ideal.

The electric field intensity at a distance 'r' from the centre of the table is given by:



$$\vec{E} = \frac{Q}{2\pi \in_r}\hat{a}_r \text{ V/m} \quad \text{...(i)}$$

The work done in moving a unit positive charge from conductor to sheath, which is the potential difference between conductor and sheath is given by

$$V = \int_{r=a}^{b} \vec{E}\cdot d\vec{r} = \int_{a}^{b} \frac{Q}{2\pi \in_r} dr$$

$$V = \frac{Q}{2\pi \in} \cdot \ln\left(\frac{b}{a}\right) \quad \text{...(ii)}$$

Substituting the value of V from equation (ii) in equation (i), we get,

$$\vec{E} = \frac{V}{r\ln\left(\frac{b}{a}\right)}\hat{a}_r$$

As the wave propagates along the conductor which is along $\hat{z}$, then associated $\vec{H}$ is given by

$$\vec{H} = (\text{Unit propagation vector}) \times \frac{\vec{E_r}}{\eta}$$

We get,

$$\hat{a}_H = \hat{a}_\rho \times \hat{a}_E = \hat{a}_z \times \hat{a}_r = \hat{a}_\phi$$

$\therefore$

$$\vec{H} = \frac{1}{\eta}\frac{V}{r\ln\left(\frac{b}{a}\right)}\hat{a}_\phi$$

Average power flow,

$$P = \frac{1}{2}\oint\left(\vec{E}\times\vec{H}\right)ds$$

$$= \frac{1}{2}\int_{\phi=0}^{2\pi}\int_{r=a}^{b}\left[\frac{1}{\eta}\cdot\frac{V}{r\ln\left(\frac{b}{a}\right)}\right]^2 rd\,rd\phi$$

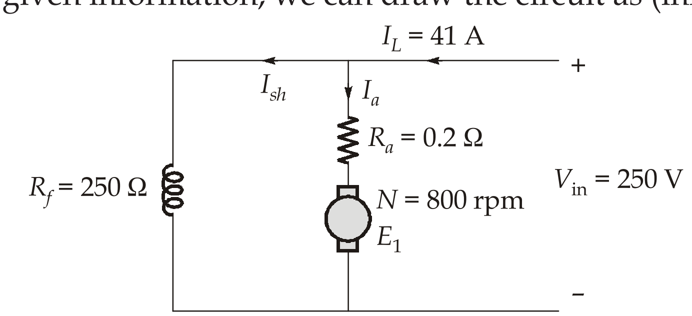
$$= \frac{1}{2\eta}\frac{V^2}{\left[\ln\left(\frac{b}{a}\right)\right]^2}\ln\left(\frac{b}{a}\right)\times 2\pi$$

$\therefore$

$$P = \frac{\pi V^2}{\eta\ln\left(\frac{b}{a}\right)}\ \text{W/m}^2$$

Q.3 (a) Solution:

(i) According to given information, we can draw the circuit as (initially)



Armature current, $I_a = I_L - I_{sh}$

$$= 41 - \frac{250}{250}$$

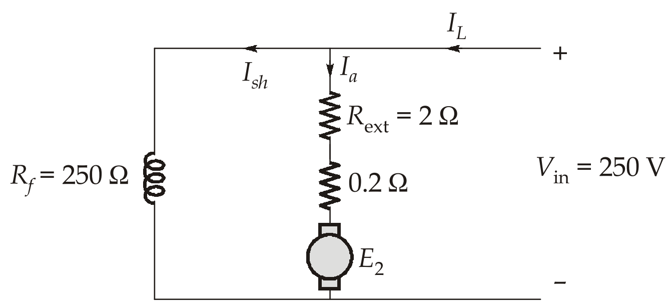
$$I_a = 40\ \text{A}$$

Back emf on full load,

$$E_1 = V - I_a R_a$$

$$E_1 = 250 - (40 \times 0.2) = 242 \text{ V}$$

Now, when a resistance of 2 Ω is placed in series with the armature, the back emf is



$$E_2 = V - I_a(0.2 + 2)$$

$$= 250 - 40(2.2)$$

$$= 250 - 88$$

$$= 162 \text{ V}$$

(ii) Double-full load torque condition:

For a dc shunt motor, torque $T \propto I_a$. Hence, at double full-load torque, the armature current

$$I_{a2} = 40 \times 2 = 80 \text{ A}$$

With 2 Ω resistance in the armature circuit, the back emf at double full load torque,

$$E_3 = V - I_{a2}\,(0.2 + 2)$$

$$E_3 = 250 - 80(2.2)$$

$$= 74 \text{ V}$$

For a dc motor, back emf $E_b \propto N\phi$, where N is the speed of the DC motor and ϕ is the flux per pole which is given to be constant throughout. Given $N_1 = 800$ rpm at full load, we get

$$N_3 = \frac{E_3}{E_1} N_1$$

$$= \frac{74}{242} \times 800 = 244.6 \text{ rpm}$$

(iii) Stalling torque,

⇒ Under stalling condition, the speed is zero and therefore the back emf is zero.

Let I_{a0} be the armature current taken by the motor under stalling condition. Hence,

$$E_{b0} = V - I_{a0}(0.2 + 2)$$

$$0 = 250 - 2.2I_{a0}$$

$$I_{a0} = 113.64 \text{ A}$$

For dc shunt motor, $T \propto I_a$ $\quad (\because \phi \text{ is constant})$

Therefore, stalling torque ∝ stalling current.

and full load torque ∝ full load current

$$\frac{\text{Stalling torque}}{\text{full load torque}} = \frac{\text{Stalling current}}{\text{full load armature current}}$$

$$= \frac{113.64}{40} = 2.84$$

Thus, $\quad$ stalling torque = 2.84 × full-load torque

Q.3 (b) Solution:

(i) Given that

$$E_z = 5\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{3\pi y}{b}\right)\cos(10^{12}t - \beta z) \text{ V/m}$$

$$a = 6 \text{ cm and } b = 3 \text{ cm}$$

Intrinsic impedance, η = ?

Average power flow, P_{avg} = ?

$E_z \neq 0$, This must be TM_{23} mode ($m = 2$, $n = 3$).

The cut-off frequency for TM_{mn} mode is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Since $a = 2b$,

$$f_c = \frac{c}{4b}\sqrt{m^2 + 4n^2}$$

$$= \frac{3\times 10^8}{4\times 3}\times 100\sqrt{4+36} = \frac{3\times 10^8}{12}\times 100\sqrt{40}$$

$$= 15.811 \text{ GHz}$$

From the given expression, operating frequency

$$f = \frac{\omega}{2\pi} = \frac{10^{12}}{2\pi} = 159.154 \text{ GHz}$$

$$\text{Intrinsic impedance, } \eta_{TM} = \eta_0\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 377\sqrt{1-\left(\frac{15.811}{159.154}\right)^2}$$

$$= 375.135\ \Omega$$

The Poynting vector is given by

$$\vec{P} = \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\eta_{TM}}\hat{a}_z$$

We have,

$$E_{xs} = -\frac{j\beta}{h^2}\left(\frac{m\pi}{a}\right)E_0 \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$E_{ys} = -\frac{j\beta}{h^2}\left(\frac{n\pi}{b}\right)E_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

Thus,
$$\vec{P} = \frac{\beta^2 E_0^2}{2h^4\eta_{TM}}\left[\left(\frac{2\pi}{a}\right)^2\cos^2\left(\frac{2\pi x}{a}\right)\sin^2\left(\frac{3\pi y}{b}\right) + \left(\frac{3\pi}{b}\right)^2\sin^2\left(\frac{2\pi x}{a}\right)\cos^2\left(\frac{3\pi y}{b}\right)\right]\hat{a}_z$$

The average power flow,

$$P_{\text{avg}} = \int \vec{P}\,\vec{ds} = \int_{x=0}^{a}\int_{y=0}^{b} P_{avg}\cdot dx\,dy$$

$$= \frac{\beta^2 E_0^2}{2h^4\eta_{TM}}\frac{ab}{4}\left[\frac{4\pi^2}{a^2} + \frac{9\pi^2}{b^2}\right] = \frac{\beta^2 E_0^2 ab}{8h^2\eta_{TM}}$$

where,
$$\beta = \frac{\omega}{c}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{10^{12}}{3\times 10^8}\sqrt{1-\left(\frac{15.811}{159.154}\right)^2} = 3.317\times 10^3 \text{ rad/m}$$

$$h^2 = \frac{4\pi^2}{a^2} + \frac{9\pi^2}{b^2} = \frac{10\pi^2}{b^2} = 1.096\times 10^5$$

$$P_{\text{avg}} = \frac{(3.317\times 10^3)^2 \times 5^2 \times 18\times 10^{-4}}{8\times(1.096\times 10^5)\times 375.135} = 1.5 \text{ mW}$$

(ii) We obtain the fields for TM_{11} mode ($m = 1$, $n = 1$) as

$$E_x = \frac{\beta}{h^2}\left(\frac{\pi}{a}\right)E_0\cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)\sin(\omega t - \beta z)$$

$$E_y = \frac{\beta}{h^2}\left(\frac{\pi}{b}\right)E_0\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\sin(\omega t - \beta z)$$

$$E_z = E_0\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)\cos(\omega t - \beta z)$$

$$H_x = -\frac{\omega\in}{h^2}\left(\frac{\pi}{b}\right)E_0\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\sin(\omega t - \beta z)$$

$$H_y = \frac{\omega \in}{h^2}\left(\frac{\pi}{a}\right)E_0 \cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)\sin(\omega t - \beta z)$$

$$H_z = 0$$

For the electric field lines,

$$\frac{dy}{dx} = \frac{E_y}{E_x} = \frac{a}{b}\tan\left(\frac{\pi x}{a}\right)\cot\left(\frac{\pi y}{b}\right)$$

For the magnetic field lines,

$$\frac{dy}{dx} = \frac{H_y}{H_x} = \frac{-b}{a}\cot\left(\frac{\pi x}{a}\right)\tan\left(\frac{\pi y}{b}\right)$$

Notice that $\left(\frac{E_y}{E_x}\right)\left(\frac{H_y}{H_x}\right) = -1,$ showing that the electric and magnetic field lines are mutually orthogonal. This is observed in figure, where the field lines are sketched.

The surface current density on the walls of the waveguide is given by

$$K = \hat{a}_n \times \vec{H} = \hat{a}_n \times (H_x, H_y, 0)$$

At $x = 0,\ \hat{a}_n = \hat{a}_x, \vec{H} = H_y(0, y, z, t)\hat{a}_y,$ that is

$$K = \frac{\omega \in}{h^2}\left(\frac{\pi}{a}\right)E_0 \sin\left(\frac{\pi y}{b}\right)\sin(\omega t - \beta z)\hat{a}_z$$

At $x = a,\ \hat{a}_n = -\hat{a}_x, \vec{H} = -H_y(a, y, z, t)\hat{a}_y,$ or

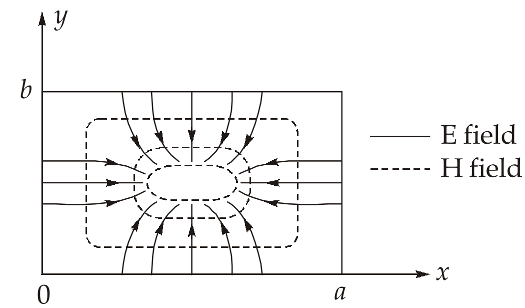
$$K = \frac{\omega \in}{h^2}\left(\frac{\pi}{a}\right)E_0 \sin\left(\frac{\pi y}{b}\right)\sin(\omega t - \beta z)\hat{a}_z$$

At $y = 0,\ \hat{a}_n = \hat{a}_y, \vec{H} = -H_x(x, 0, z, t)\hat{a}_x,$ or

$$K = \frac{\omega \in}{h^2}\left(\frac{\pi}{b}\right)E_0 \sin\left(\frac{\pi x}{a}\right)\sin(\omega t - \beta z)\hat{a}_z$$

At $y = b,\ \hat{a}_n = -\hat{a}_y, \vec{H} = H_x(x, b, z, t)\hat{a}_x,$ or

$$K = \frac{\omega \in}{h^2}\left(\frac{\pi}{b}\right)\sin\left(\frac{\pi x}{a}\right)\sin(\omega t - \beta z)\hat{a}_z$$



Field lines for TM_{11} mode

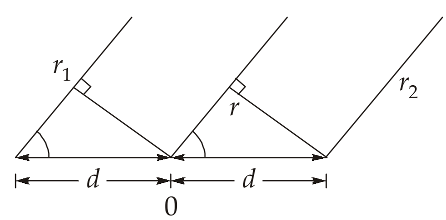
Q.3 (c) Solution:

(i) For the given array, phase difference, $\psi = \beta d \cos\theta + \alpha$

In this case, $d = \frac{\lambda}{2}$, $\alpha = 0$, therefore,

$$\psi = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\cos\theta + 0$$

$$\psi = \pi \cos\theta$$



As the currents are in the ratio 1:2:1, the total far field at distance point P is given as

$$E_t = E_0\left[e^{-j\psi} + 2 + e^{j\psi}\right]$$

$$= E_0[2 + 2\cos\psi]$$

$$= 2E_0[1 + \cos(\pi\cos\theta)]$$

Maximum value $|E_t| = 4E_0$

So, the normalized value $E_{\text{normalized}} = \frac{E_t}{|E_t|}$

$$= \frac{[1 + \cos(\pi\cos\theta)]}{2}$$

Maxima Direction:

For the maximum field value, $\frac{1 + \cos(\pi\cos\theta)}{2}$ should be maximum. Therefore,

$$1 + \cos(\pi\cos\theta) = 2$$

$$\cos(\pi\cos\theta) = 1$$

$$\pi \cos \theta_{max} = 0$$

$$\cos \theta_{max} = 0$$

$$\theta_{max} = \frac{\pi}{2}, \frac{3\pi}{2}$$

Minima Direction:

For the minimum field value, (1 + cos (π cos θ)) should be minimum. Therefore,

$$1 + \cos(\pi \cos \theta) = 0$$

$$\cos(\pi \cos \theta) = -1$$

$$\pi \cos \theta = \pm\pi$$

$$\cos \theta = \pm 1$$

$$\theta_{min} = 0, \pi, 2\pi.....$$

Half-power points:

At half power point, (1 + cos (π cos θ)) should be $\frac{1}{\sqrt{2}}$.

Therefore,

$$\frac{1}{2}(1 + \cos \pi \cos \theta_{HPP}) = \frac{1}{\sqrt{2}}$$

$$\cos(\pi \cos(\theta_{HPP})) = 1.41 - 1$$

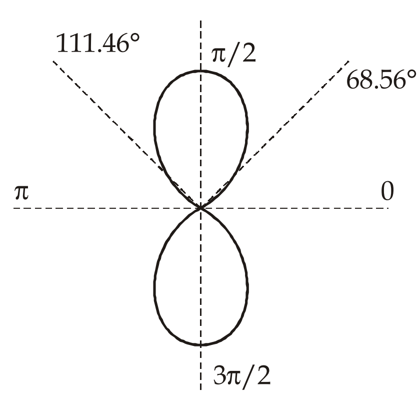
$$\cos(\pi \cos(\theta_{HPP})) = 0.41$$

$$\pi \cos(\theta_{HPP}) = +65.79°$$

$$\cos \theta_{HPP} = \pm 0.366$$

$$\theta_{HPP} = 68.56, 111.46°$$

From above, the radiation pattern of antenna array is drawn as below:



(ii) **1.** Given radiation intensity

$$U(\theta, \phi) = 5(1 + \sin^2\theta \sin^2\phi),\ 0 < \theta < \pi,\ 0 < \phi < 2\pi$$

We know that, Directive Gain is given by

$$G_d(\theta, \phi) = \frac{U(\theta,\phi)}{U_{avg}}$$

$$U_{\text{avg}} = \frac{W_r}{4\pi} = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} 5(1+\sin^2\theta\sin^2\phi)\sin\theta d\theta d\phi$$

$$= \frac{5}{4\pi}\left[\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi} d\phi + \int_0^{\pi}\sin^3\theta d\theta\int_0^{2\pi}\sin^2\phi d\phi\right]$$

$$= \frac{5}{4\pi}\left[2\pi[-\cos\theta]_0^{\pi} + \int_0^{\pi}\frac{(3\sin\theta - \sin 3\theta)}{4}d\theta\int_0^{2\pi}\frac{(1-\cos 2\phi)}{2}d\phi\right]$$

$$= \frac{5}{4\pi}\left[4\pi + \left\{\frac{3}{4}[-\cos\theta]_0^{\pi} - \frac{1}{4}\left[\frac{-\cos 3\theta}{3}\right]_0^{\pi}\right\}\left[\frac{1}{2}\phi - \frac{\sin 2\phi}{4}\right]_0^{2\pi}\right]$$

$$= \frac{5}{4\pi}\left[4\pi + \left(\frac{3}{2} - \frac{2}{12}\right)\times\pi\right]$$

$$= \frac{5}{4\pi}\left[4\pi + \frac{4}{3}\pi\right]$$

$$= 5 + \frac{5}{3} = \frac{20}{3}$$

$$G_d(\theta, \phi) = \frac{U(\theta,\phi)}{U_{avg}} = \frac{15}{20}(1+\sin^2\theta\sin^2\phi)$$

$$G_d(\theta, \phi) = 0.75\,(1 + \sin^2\theta \sin^2\phi)$$

The directivity of the antenna is given by

$$D = G_{d(\max)} = 1.5$$

2. Given that,

Effective height of antenna, $l_e = 100$ m

Current, $I_{\text{rms}} = 450$ A

$f = 4 \times 10^4$ Hz

Total Resistance of antenna circuit, $R_t = 1.12\ \Omega$

We have, $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{4 \times 10^4} = 7500 \text{ m}$

Thus, $l_e = \frac{\lambda}{75}$ (i.e. Hertezian dipole)

Radiation Resistance of Hertzian dipole,

$$R_r = 80\pi^2 \left(\frac{l_e}{\lambda}\right)^2$$

$$= 80\pi^2 \left(\frac{100}{7500}\right)^2 = 80\pi^2 \frac{1}{(75)^2} = 0.14\ \Omega$$

Power radiated,

$$P_r = R_r \cdot I_{rms}^2$$

$$= 0.14 \times (450)^2$$

$$P_r = 28.424 \text{ kW}$$

$$R_t = 1.12\ \Omega,\ \eta = ?$$

Efficiency of the antenna,

$$\eta = \frac{R_r}{R_r + R_l} = \frac{\text{Radiation resistance}}{\text{Total antenna resistance}}$$

$$\%\eta = \frac{0.14}{1.12} \times 100 = 12.5\%$$

$$\%\eta = 12.5\%$$

Q.4 (a) Solution:

(i) **1. Pulverizer :** The basic function of a pulveriser in a thermal power plant is to break down and crush the coal lumps into fine particle form before it is taken to boiler for burning. Pulverisation of coal basically increases surface area which is important to facilitate efficient burning of coal.

2. Superheaters : Superheater is a heat exchanger which is used to remove the last traces of moisture (1 to 2%) from the saturated steam coming out of the boiler and to increase its temperature sufficiently above saturation temperature. Superheating increases overall efficiency. In addition, it reduces the moisture content in last stages of the turbine therefore, there is lesser erosion of turbine blades.

3. **Economisers :** An economiser is a heat exchanger which raises the temperature of the feed water leaving the highest pressure feed water heater to about the saturation temperature corresponding to the boiler pressure. This is done by means of hot flue gasses leaving the superheater at a temperature varying from 370°C to 620°C.
4. **Condenser :** The purpose a surface condenser is to condense the exhaust steam from the steam turbine to obtain maximum efficiency, and also to convert exhaust steam to pure water, so that it may be reused in steam generator or boiler as boiler feed water.
5. **Cooling tower :** Cooling tower serves to dissipate heat into the atmosphere through the cooling of a water stream to a lower temperature.

(ii) Merits of nuclear power plant :

1. It consumes very small quantity of fuels. Thus, fuel transportation cost is very small.
2. A nuclear power plant is completely free from air pollution.
3. It reduces the demand of coal, oil and gas. Thus it conserves these fossil fuels.
4. A nuclear power plant requires less space as compared to any other type of the same size.
5. The nuclear power plant is very economical for producing bulk electric power.
6. It can be located near the load centres because it does not require large quantities of water and need not be near coal mines. Therefore, the cost of primary distribution is reduced.

Limitations :

1. The nuclear power plants are not suitable for ranging load conditions as the reactors can not be easily controlled.
2. Initial cost of nuclear power plant is higher as compared to a hydro or steam power plant of equal capacity.
3. Nuclear waste disposal is a serious problem. It may produce serious environmental hazards, if not disposed properly.

Q.4 (b) Solution:

Poynting Theorem: This theorem states that the vector product, $\vec{S} = \vec{E} \times \vec{H}$ at any point is a measure of rate of energy flow per unit area at that point and is perpendicular to both $\vec{E}$ and $\vec{H}$. This theorem states about the conservation of energy in an electromagnetic wave.

Derivation: By modified Ampere's circuital law in differential form,

$$\vec{\nabla}\times\vec{H} = \vec{J}+\frac{\partial\vec{D}}{\partial t} = \vec{J}+\in\frac{\partial\vec{E}}{\partial t}$$

$$\vec{J} = \vec{\nabla}\times\vec{H}-\in\frac{\partial\vec{E}}{\partial t}$$

Multiplying both sides by $\vec{E}$, we get

$$\vec{E}\cdot\vec{J} = \vec{E}\cdot(\vec{\nabla}\times\vec{H})-\vec{E}\cdot\in\frac{\partial\vec{E}}{\partial t} \qquad ...(i)$$

We use the vector identity

$$\vec{\nabla}(\vec{E}\times\vec{H}) = \vec{H}(\vec{\nabla}\times\vec{E})-\vec{E}(\vec{\nabla}\times\vec{H})$$

Thus from equation (i)

$$\vec{E}\cdot\vec{J} = \vec{H}(\vec{\nabla}\times\vec{E})-\vec{\nabla}(\vec{E}\times\vec{H})-\vec{E}\in\frac{\partial\vec{E}}{\partial t}$$

$$= -\mu\vec{H}\frac{\partial\vec{H}}{\partial t}-\in\vec{E}\frac{\partial\vec{E}}{\partial t}-\nabla(\vec{E}\times\vec{H}) \qquad ...(ii)$$

$$\left[\because \vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t} = -\mu\frac{\partial\vec{H}}{\partial t}\right]$$

Now, $$\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}H^2 \text{ and } \vec{E}\cdot\frac{\partial E}{\partial t} = \frac{1}{2}\frac{\partial(E^2)}{\partial t}$$

So, from equation (ii),

$$\vec{E}\cdot\vec{J} = -\frac{1}{2}\mu\frac{\partial}{\partial t}(H^2)-\frac{1}{2}\in\frac{\partial(E^2)}{\partial t}-\nabla(\vec{E}\times\vec{H})$$

On integrating over a volume,

$$\int_V \vec{E}\cdot\vec{J}\,dV = \int_V -\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2+\frac{1}{2}\in E^2\right)dV-\int_V \vec{\nabla}(\vec{E}\times\vec{H})dV$$

$$\int_V \vec{E}\cdot\vec{J}\,dV = -\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\mu H^2+\frac{1}{2}\in E^2\right)dV-\int_V \vec{\nabla}(\vec{E}\times\vec{H})\,dV$$

Using Divergence Theorem,

$$\int_V \vec{E}\cdot\vec{J}\,dV = -\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\mu H^2+\frac{1}{2}\in E^2\right)dV-\oint_s(\vec{E}\times\vec{H})d\vec{s}$$

Where s is the closed surface bounding the volume (V). On rearrangement, we get,

$$\int_V \vec{E}\cdot\vec{J}dV = -\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\mu H^2 + \frac{1}{2}\in E^2\right)dV - \oint(\vec{E}\times\vec{H})d\vec{s}$$

In other words,

(Ohmic power Dissipated) = (Rate of decrease in energy stored in electric and magnetic field) – (Total power leaving the volume)

This is mathematical form of poynting theorem.

Significance of Terms:

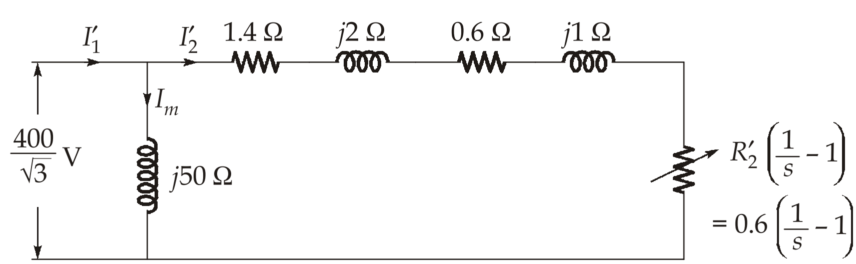
1. The term $\int \vec{E}\cdot\vec{J}\,dV$ represents the rate at which the energy is being dissipated within the volume V.

2. The first term in RHS $-\frac{\partial}{\partial t}\int\left(\frac{1}{2}\mu H^2 + \frac{1}{2}\in E^2\right)dV$ represents the time rate at which stored electric field energy and magnetic field energy is decreasing within the volume V.

3. The second term in RHS, $\oint(\vec{E}\times\vec{H})d\vec{s}$ represents the rate at which energy is escaping or leaving the volume V through closed surface S. This forms the law of conservation of energy.

Q.4 (c) Solution:

According to given parameters, we can draw the approximate equivalent circuit of induction motor as below:



Rotational loss = 275 ω.

Now,

(i) For slip, s = 0.03

$$R_2'\left(\frac{1}{s}-1\right) = 0.6\left(\frac{1}{0.03}-1\right) = 19.4\ \Omega$$

We have,
$$I_2' = \frac{(400/\sqrt{3})}{(1.4+0.6+19.4)+j(2+1)}$$
$$I_2' = 10.69\angle -7.98°\text{A}$$

and
$$I_m = \frac{(400/\sqrt{3})}{(j50)}$$
$$I_m = 4.62\angle{-90°}\text{ A}$$
$$\text{Rotor power output} = 3I_2'^2\left(R_2'\left(\frac{1}{s}-1\right)\right)$$
$$= 3(10.69)^2 \times 19.4$$
$$= 6.65\text{ kW}$$
$$I_1' = I_m + I_2' = 4.62\angle{-90°} + 10.69\angle{-7.98°}$$
$$I_1' = 12.216\angle{-29.964°}\text{A}$$
$$\text{Power factor} = \cos(29.964°) = 0.866\text{ lagging}$$
$$\text{Power input} = \sqrt{3}V_1 I_1' \cos\phi$$
$$= \sqrt{3}\times 400\times 12.216\times 0.866$$
$$= 7332.257\text{ W}$$
$$= 7.33\text{ kW}$$

(ii)
$$\text{Rotor power output} = 6.65\text{ kW}$$
$$\text{Mechanical output, } P_{sh} = 6.65\text{ kW} - 0.275\text{ kW} = 6.375\text{ kW}$$
$$\text{Rotor speed, } N_r = N_s(1-s)$$
$$= 1000(1-0.03)$$
$$N_r = 970\text{ rpm}$$
$$P_{sh} = T_{sh}\cdot\frac{2\pi N_r}{60}$$
$$\text{Shaft Torque, } T_{sh} = \frac{P_{sh}\times 60}{2\pi N_r}$$
$$T_{sh} = 62.759\text{ N-m}$$

(iii)
$$\%\eta = \frac{P_{\text{output}}}{P_{\text{i/p}}} = \frac{6.375\times 1000}{7332.257}$$
$$\%\eta \approx 87\%$$

Section B : Computer Organization and Architecture-1 + Materials Science-1 + Electronic Devices & Circuits-2 + Advanced Communications Topics-2

Q.5 (a) Solution:

(i) CISC (Complex Instruction Set Computers)

- Large instruction set
- Instruction formats are of different lengths
- Instructions perform both elementary and complex operations
- Control unit is micro-programmed
- Not pipelined or less pipelined
- Single register set
- Numerous memory addressing options for operands
- Emphasis on hardware
- Includes multi-clock complex instructions
- Memory-to-memory: "LOAD" and "STORE" incorporated in instructions
- Small code sizes, high cycles per instruction
- Transistors used for storing complex instructions

Examples of CISC processors:

- VAX
- PDP-11
- Motorola 68000 family
- Intel x86 architecture based processors.

RISC (Reduced Instruction Set Computers)

- Compact instruction set
- Instruction formats are all of the same length
- Instructions perform elementary operations
- Control unit is simple and hardwired
- RISC is highly pipelined.
- Multiple register set
- RISC processors provide very few addressing modes
- Emphasis on software
- Single-clock, reduced instruction only
- Register to register: "LOAD" and "STORE" are independent instructions

- Low cycles per instruction, large code sizes
- Spends more transistors on memory registers

  Examples of RISC processors
  - Apple iPods (custom ARM7TDMI SOC)
  - Apple iPhone (Samsung ARM1176JZF)
  - Nintendo Game Boy Advance (ARM7)
  - Sony Network Walkman (Sony in house ARM based chip)

(ii) There are two types of microinstructions:

1. Horizontal micro-programmed instructions
2. Vertical micro-programmed instructions

The basic difference between them are listed below:

Horizontal Microinstructions

- In the horizontal microprogammed microprocessor, control signal is expressed in a decoded binary format.
- It supports longer control field.
- No need of external decoder to generate the control signals (CS), so it is faster than vertical microinstructions.
- It allows high degree of parallelisms.
- It require one bit for each control signal.

Vertical Microinstructions

- In the vertical microprogammed microprocessor, control signals are expressed in a encoded binary format.
- It supports shorter control field.
- Need of a external decoder to generate the control signal, so it is slower than horizontal microinstructions.
- It allows easy implementation of new instruction, so it is more flexible.
- It allows a low degree of parallelism i.e., the degree of parallelism is either 0 or 1.
- From 'n' bit, we get $\log_2 n$ control signals.

Q.5 (b) Solution:

The key properties of insulating laminates play a crucial role in their effectiveness in electronic applications. Here are some important properties and their contributions;

1. **Dielectric Strength:** This property measures the ability of the insulating laminates to withstand high electric fields without breaking down. High dielectric strength ensures that the laminate can be used in applications with high voltage without risk of electrical breakdown, thus providing safety and reliability in electronic circuits.
2. **Mechanical Strength:** Insulating laminates must posses adequate mechanical strength to withstand handling, assembly, and environmental stresses. A strong laminate ensures the structural integrity of electronic devices, preventing deformation or damage during operation and handing.
3. **Dimensional Stability:** Insulating laminates should maintain their dimensions and shape under varying temperatures and humidity levels. Dimensional stability ensures the accuracy and reliability of electronic components and circuits, especially in precision applications where tight tolerances are required.
4. **Thermal conductivity:** Effective heat dissipation is critical for preventing overheating and maintaining the performance and reliability of electronic devices. Insulating laminates with good thermal conductivity help to efficiently transfer heat away from heat-generating components, ensuring optimal operating temperatures and extending the life span of electronics.
5. **Chemical Resistance:** Insulating laminates should be resistant to chemical substances such as solvents, fluxes and cleaning agents commonly encountered during assembly and operation. Chemical resistance prevents degradation or corrosion of the laminate, ensuring long term reliability and performance in harsh environments.
6. **Flame Retardancy:** In applications where fire safety is a concern, insulating laminates should exhibit flame retardant properties to inhibit the spread of fire and minimize the release of toxic gases. Flame retardant laminates enhance the safety of electronic devices, particularly in critical applications such as aerospace, automotive, and consumer electronics.
7. **Environmental Sustainability:** With increasing emphasis on sustainability, insulating laminates are being developed with environmentally friendly materials and manufacturing processes. Sustainable laminates reduce the environmental impact of electronic devices throughout their life cycle, from production to disposal, while maintaining performance and reliability.

   By possessing these key properties, insulating laminates contribute to the effectiveness, safety, reliability, and longevity of electronic devices and circuits in various applications, from consumer electronics to industrial automation and beyond.

Q.5 (c) Solution:

(i) In the base region,

Minority carrier diffusion length,

$$L_p = \sqrt{D_P \tau_P} = \sqrt{10 \times 10^{-7}} = 10^{-3}\text{ cm} >> W$$

$$P_{n0} = \frac{n_i^2}{N_B} = \frac{(9.65 \times 10^9)^2}{10^{17}}$$

$$P_{n0} = 9.31 \times 10^2\text{ cm}^{-3}$$

Similarly, in the emitter region,

$$L_E = \sqrt{D_E \tau_E} = \sqrt{1 \times 10^{-8}} = 10^{-4}\text{ cm and}$$

$$n_{E0} = \frac{n_i^2}{N_E} = \frac{(9.65 \times 10^9)^2}{10^{19}} = 9.31\text{ cm}^{-3}$$

The current components are given by $I_{Ep} = \frac{qAD_P P_{n0}}{W} e^{\frac{qV_{EB}}{KT}}$

$$I_{Ep} = \frac{1.6 \times 10^{-19} \times 5 \times 10^{-4} \times 10 \times 9.31 \times 10^2}{0.6 \times 10^{-4}} \times e^{\frac{0.6}{0.0259}}$$

$$I_{Ep} = 1.428 \times 10^{-4}\text{ A}$$

$$I_{En} = \frac{qAD_E n_{E0}}{L_E}(e^{qV_{EB}/kT} - 1)$$

$$= \frac{1.6 \times 10^{-19} \times 5 \times 10^{-4} \times 1 \times 9.31}{10^{-4}} \times \left[e^{\frac{0.6}{0.0259}} - 1\right]$$

$$= 8.5687 \times 10^{-8}\text{ A}$$

We have, $I_{Cp} = \alpha_T I_{Ep}$, where α_T is the base transport factor given by

$$\alpha_T = \frac{1}{1 + 0.5(W/L_P)^2} = \frac{1}{1 + 0.5(0.06)^2} = 0.9982$$

Therefore, $I_{Cp} = 0.9982 \times 1.428 \times 10^{-4} = 1.425 \times 10^{-4}$ A

Therefore, the common base current gain α_0 is

$$\alpha_0 = \frac{I_{Cp}}{I_{Ep} + I_{En}} = \frac{1.425 \times 10^{-4}}{1.428 \times 10^{-4} + 8.5687 \times 10^{-8}}$$

$$\alpha_0 = 0.9973$$

(ii) The early effect, also known as the base width modulation effect, is a phenomenon that occurs in bipolar junction transistor (BJTs) where the width of the base region varies due to a variation in the applied base-to-collector voltage, thereby affecting the collector current.

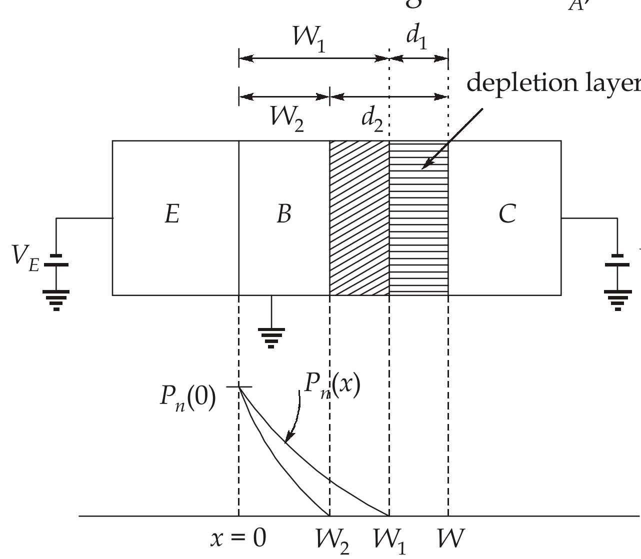
As the collector base voltage (V_{CB}) increases, the width of the base region decreases. This narrowing of the base region effectively reduces the distance for the charge carriers (electrons or holes) to travel from the base to the collector.

The reduction in base width due to the early effect increases the collector current (I_C). This effect becomes significant at higher collector base voltages.

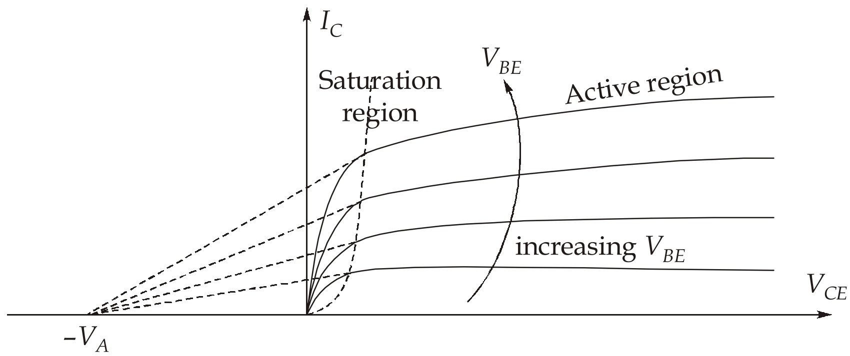
The early effect is typically modeled by including an early voltage (V_A) in the BJT's small signal equivalent circuit. This voltage represents the slope of the collector current with respect to the collector base voltage.

Figure shows the position of depletion layer for two voltages V_1 and V_2. As voltage is increased from V_1 to V_2, the depletion layer width increases from d_1 to d_2 and correspondingly the effective base width shrinks from W_1 to W_2. Its effect on hole concentration profile is also shown. This effect of change in base width due to change in collector voltage is known as "Base width modulation" or "Early Effect".

As a result of reduction in base width, the collector current increases. This results in upward sloping output characteristics as shown below. The slope introduced by the Early effect is almost linear with I_c, and the common-emitter characteristics extrapolate to an intersection with the voltage axis at V_A, called the Early voltage.



Characteristics



Q.5 (d) Solution:

The emitter base junction is forward biased.

The built in voltage,
$$V_{bi} = KT\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$V_{bi} = 0.0259\ln\left(\frac{5\times10^{18}\times2\times10^{17}}{(9.65\times10^9)^2}\right)$$

$$V_{bi} = 0.956 \text{ V}$$

W_1 is the depletion layer width of the emitter base junction given as

$$W_1 = \sqrt{\frac{2\varepsilon}{q}\left(\frac{N_A}{N_D}\right)\left(\frac{1}{N_A+N_D}\right)(V_{bi}-V)}$$

$$W_1 = \sqrt{\frac{2\times1.05\times10^{-12}}{1.6\times10^{-19}}\left(\frac{5\times10^{18}}{2\times10^{17}}\right)\left(\frac{1}{5\times10^{18}+2\times10^{17}}\right)(0.956-0.5)}$$

$$W_1 = 5.364\times10^{-6} \text{ cm}$$

$$W_1 = 5.364\times10^{-2}\ \mu\text{m}$$

Similarly, we obtain for the reverse-biased base collector junction

$$V_{bi} = 0.0259\ln\left[\frac{2\times10^{17}\times10^{16}}{(9.65\times10^9)^2}\right]$$

$$= 0.795 \text{ V}$$

and
$$W_2 = \sqrt{\frac{2\times1.05\times10^{-12}}{1.6\times10^{-19}}\left(\frac{10^{16}}{2\times10^{17}}\right)\left(\frac{1}{10^{16}+2\times10^{17}}\right)\times(0.795+5)}$$

$$W_2 = 4.254\times10^{-6} \text{ cm} = 4.254\times10^{-2}\ \mu\text{m}$$

Therefore, the neutral base width is

$$W = W_B - W_1 - W_2$$
$$W = 1 - 5.364 \times 10^{-2} - 4.254 \times 10^{-2}$$
$$W = 0.904\ \mu\text{m}$$

The Minority carrier concentration at the emitterbase junction is given by,

$$P_n(0) = P_{n0} e^{\frac{qV_{EB}}{KT}} = \frac{n_i^2}{N_D} e^{\frac{qV_{EB}}{KT}}$$
$$P_n(0) = \frac{(9.65 \times 10^9)^2}{2 \times 10^{17}} e^{\frac{0.5}{0.0259}}$$
$$P_n(0) = 1.127 \times 10^{11}\ \text{cm}^{-3}$$

Q.5 (e) Solution:

(i)

$$\text{Overall loss} = 10 \log_{10} \frac{\text{input power}}{\text{output power}}$$
$$= 10 \log_{10} \frac{12 \times 10^{-6}}{6 \times 10^{-6}}\ \text{dB}$$
$$= 10 \log_{10} 2 = 3.0103\ \text{dB}$$
$$\text{Overall loss/km} = \frac{3.0103}{12} = 0.25086\ \frac{\text{dB}}{\text{km}}$$

For 25 km, loss occurred

$$= 0.25086 \times 25\ \text{dB}$$
$$= 6.2715\ \text{dB}$$

For $\left(\frac{25}{1.25} - 1\right)$ splices at 1.25 km intervals, attenuation = $\left(\frac{25}{1.25} - 1\right) \times 1.2$ dB, since for each splice, attenuation is 1.2 dB. Hence, loss due to splices = 22.8 dB.

Therefore, the overall signal attenuation

$$= 6.2715 + 22.8 = 29.0715\ \text{dB} \cong 29\ \text{dB}$$

The numerical power input/output ratio can be calculated as follows:

$$10 \log_{10} \left(\frac{P_{\text{input}}}{P_{\text{output}}}\right) = 29$$
$$\log_{10} \frac{P_{\text{input}}}{P_{\text{output}}} = \frac{29}{10}$$
$$\frac{P_{\text{input}}}{P_{\text{output}}} = 10^{2.9} = 794.32$$

(ii) 1. Dispersion per unit length

$$= \frac{\text{Pulse broadening}}{\text{Distance}}$$

$$= \frac{0.3 \times 10^{-6}}{18} = 0.01667 \times 10^{-6} \text{ s/km}$$

$$= 0.01667 \times 10^{3} \times 10^{-9} \text{ s/km}$$

$$= 16.67 \text{ ns/km}$$

2. Optimum bandwidth of fiber

$$= \frac{1}{2 \times \text{Pulse broadening}}$$

$$= \frac{1}{2 \times 0.3 \times 10^{-6}} = \frac{1}{0.6} \times 10^{6} \text{Hz}$$

$$= 1.67 \text{ MHz}$$

3. Bandwidth length product

$$= 1.67 \times 18$$

$$= 30.06 \text{ MHz km}$$

Q.6 (a) Solution:

(i) We know, the pinch off voltage of a JFET is given by

$$V_p = \frac{a^2 q N_A}{2\varepsilon} \quad \text{...(1)}$$

But, $$qN_A = \frac{qN_A \mu_p}{\mu_p} = \frac{\sigma}{\mu_p}$$

Put $\sigma = \frac{1}{\rho} = \frac{1}{10}$ S/cm and $\mu_p = 500 \ \frac{\text{cm}^2}{\text{V-sec}}$

$\therefore$ $$qN_A = 2 \times 10^{-4} \text{ C/cm}^3$$

Dielectric constant $$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$= 8.85 \times 10^{-12} \times 12$$

$$= 106.2 \times 10^{-12} \text{ F/m}$$

$$= 1.062 \times 10^{-12} \text{ F/cm}$$

and, half channel height, $a = 2 \times 10^{-6} \text{ m} = 2 \times 10^{-4} \text{ cm}$

Putting these values in (1), we get,

$$V_p = \frac{(2 \times 10^{-4})^2 \times 2 \times 10^{-4}}{2 \times 1.062 \times 10^{-12}} = 3.766 \text{ V}$$

(ii) The pn junction built in potential barrier is given by

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$V_{bi} = (0.0259)\ln\left(\frac{10^{16} \times 10^{19}}{(1.5 \times 10^{10})^2}\right)$$

$$V_{bi} = 0.874 \text{ V}$$

The zero biased source-substrate p-n junction width is

$$x_{d0} = \sqrt{\frac{2\varepsilon_s}{q}\frac{V_{bi}}{N_A}}$$

$$x_{d0} = \sqrt{\frac{2 \times 11.7 \times 8.85 \times 10^{-14} \times 0.874}{1.6 \times 10^{-19} \times 10^{16}}}$$

$$x_{d0} = 3.36 \times 10^{-5} \text{ cm} = 0.336 \text{ µm}$$

The reverse biased drain-substrate *p-n* junction width is given by

$$x_d = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_{DS})}{qN_A}} \quad \text{...(i)}$$

At punch through, we will have

$$x_{d0} + x_d = L, \text{ where } L \text{ is the channel length}$$

or

$$0.336 + x_d = 1.2$$

$$x_d = 1.2 - 0.336$$

$$x_d = 0.864 \text{ µm}$$

From equation (i),

$$V_{bi} + V_{DS} = \frac{x_d^2 q N_A}{2\varepsilon_s} = \frac{(0.864 \times 10^{-4})^2 \times (1.6 \times 10^{-19})(10^{16})}{2 \times 11.7 \times (8.85 \times 10^{-14})}$$

$$= 5.764 \text{ V}$$

The punch through voltage is then found as

$$V_{DS} = 5.764 - 0.874$$

$$V_{DS} = 4.89 \text{ V}$$

Q.6 (b) Solution:

(i) Soft magnetic materials:

1. These materials are easy to magnetize and demagnetize.
2. Their retentivity is low
3. Their coercivity is low.
4. These materials favour rapid switching of magnetization to the applied AC field.
5. They have high permeability and susceptibility.
6. They have high magnetic saturation.
7. They have high curie temperature.
8. They have low hysteresis loss (because of small area of hysteresis loop).
9. Nature of hysteresis loop is tall and narrow.

Examples:

1. Fe-Si alloy or soft-iron or Si-steel
2. Metallic glass
3. Fe-Ni alloy like permalloy, Mu-metal and super alloy.
4. Ferrites

Application: These are used in high frequency applications. These are used for construction of transformer and inductor cores.

Hard magnetic materials:

1. These materials are difficult to demagnetize.
2. Their retentivity is high.
3. Their coercivity is high.
4. They have low permeability and susceptibility.
5. They have high hysteresis loss (because of large area of hysteresis is loop).
6. They require very large value of magnetizing force (H) for magnetization.
7. They have a tall and wide hysteresis curve.

Examples:

1. Carbon Steel
2. Tungsten Steel
3. Alnico [Al + Ni + Co]
4. Cunife (Cu + Ni + Fe]
5. Neodymium: Neodymium - iron - boron (Nd Fe B)

Applications:

1. These are used in permanent magnets.
2. These are used in Magnetic Sensors:

   These materials are used in Hall effect sensors, magnetoresistive sensors, and other devices that detect and measures magnetic fields.
3. It is also used in data storage:

   Hard magnetic materials are employed in magnetic storage devices, such as hard disk drives, to store data.

(ii) 1. Given, number of Si atoms

$$N = 8 \times 10^{28}\ \text{m}^{-3}$$

$$\in_r = 11.7$$

From the Clausius-Mossotti equation, the electronic polarizability

$$\alpha_e = \frac{3 \in_0}{N}\left(\frac{\in_r - 1}{\in_r + 2}\right)$$

$$\alpha_e = \frac{3(8.85 \times 10^{-12})}{8 \times 10^{28}} \times \frac{11.7 - 1}{11.7 + 2}$$

$$\alpha_e = 2.592 \times 10^{-40}\ \text{Fm}^2$$

2. We know that,

Local electric field,

$$E_{LOC} = E + \frac{1}{3 \in_0} P$$

where, polarization,

$$P = \chi_e \in_0 E$$

$$P = (\in_r - 1) \in_0 E$$

$$E_{LOC} = E + \frac{1}{3 \in_0}(\in_r - 1) \in_0 E$$

$$= E\left[1 + \frac{\in_r - 1}{3}\right] = E\left[1 + \frac{11.7 - 1}{3}\right]$$

$$\frac{E_{LOC}}{E} = 4.56$$

3. Since polarization is due to valence electrons and there are four valence electrons per Si atom, hence $Z = 4$.

$$\text{Resonant frequency, } \omega_0 = \left(\frac{Ze^2}{m_e \alpha_e}\right)^{1/2}$$

$$\omega_0 = \left[\frac{4 \times (1.6 \times 10^{-19})^2}{(9.1 \times 10^{-31})(2.592 \times 10^{-40})}\right]^{1/2}$$

$$\omega_0 = 2.0835 \times 10^{16} \text{ rad/sec}$$

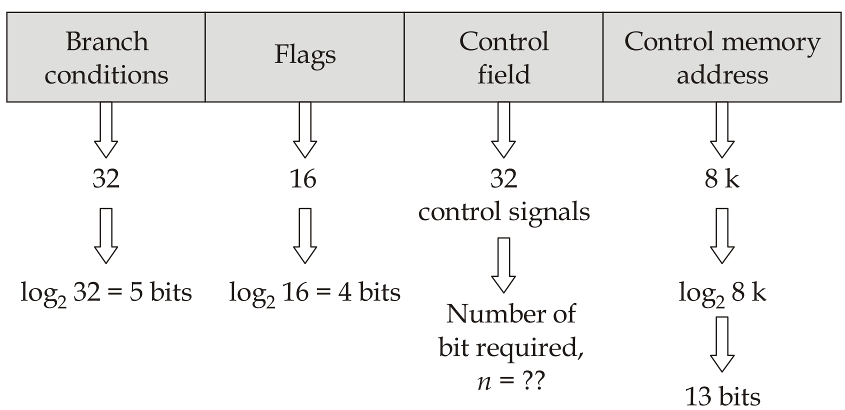
Q.6 (c) Solution:

We have,

- Hypothetical control unit supports, Control signal = 32

  Control word memory = 8 k
- CPU uses 32 branch condition to control the branch logic processor containing 16 flags.

On the basis of given data, the format of the control word is given as below:



Now,

(i) Considering horizontal microprogrammed instructions:

For horizontal microprogrammed instruction, one control signal requires one bit. Hence, for 32 control signals, number of bits required for control field is 32.

Now,

Horizontal control word size = $(5 + 4 + n + 13)$ bits

$= (5 + 4 + 32 + 13)$ bits = 54 bits.

Horizontal Control memory size = 8 k × Horizontal control word size

$= 8 \text{ k} \times 54$ bits

$= \left(\frac{8\text{k} \times 54}{8}\right)$ Bytes

$= 54$ k Bytes

(ii) Considering vertical microprogrammed instructions:

For vertically microprogrammed instructions, 'x' control signals can be represented by '$\log_2 x$' bits.

Hence, for 32 control signals, number of bits required is $\log_2 32 = 5$.

Thus,

$$\text{Vertical control word size} = (5 + 4 + 5 + 13) \text{ bits}$$
$$= (5 + 4 + 5 + 13) \text{ bits} = 27 \text{ bits}$$

$$\text{Vertical control memory size} = 8 \text{ k} \times \text{vertical control word size}$$
$$= 8 \text{ k} \times 27 \text{ bits} = \left(\frac{8 \text{ k} \times 27}{8}\right) \text{bytes}$$
$$= 27 \text{ k bytes}$$

Q.7 (a) Solution:

Gain of a dish antenna

$$G = \eta\left(\frac{4\pi A_e}{\lambda^2}\right) = \frac{4\pi^2 \dfrac{D^2}{4} \times \eta}{\lambda^2}$$

$$G = \frac{\pi^2 D^2}{\lambda^2} \times \eta$$

where, D is diameter of antenna and η is aperture efficiency.

given that, $\eta = 65\% = 0.65$

Now, $$G = \frac{\pi^2 D^2}{\lambda^2} \times 0.65 = \frac{6.42 D^2}{\lambda^2}$$

$\therefore$ Gain G_T of antenna at satellite end,

$$G_T = \frac{6.42 D^2}{\lambda^2}$$

where, $D_T = 0.5$ m

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{12 \times 10^9} \qquad (\because f = 12 \text{ GHz})$$

$$\lambda = 0.025 \text{ m} = 2.5 \text{ cm}$$

$\therefore$ $$G_T = 6.42\left(\frac{0.5}{0.025}\right)^2$$

$$G_T = 2568$$

Gain of the antenna at earth station,

$$G_r = 6.42\left(\frac{D_r}{\lambda}\right)^2$$

where, $D_r = 10$ m

$$G_r = 6.42\left(\frac{10}{0.025}\right)^2 = 1027200$$

We have, $$\text{path loss} = \left(\frac{4\pi d}{\lambda}\right)^2$$

where, $d = 36000 \text{ km} = 36 \times 10^6$ m

$$\text{Path loss} = \left(\frac{4\pi \times 36 \times 10^6}{0.025}\right)^2$$

$$= 3.2744 \times 10^{20}$$

$$\text{Path loss in dB} = 10 \log_{10}(3.2744 \times 10^{20})$$

$$= 205.151 \text{ dB}$$

Transmitted power

$$P'_T = 40 \text{ dBm}$$

$$P'_{T(\text{dBm})} = 10\log_{10}\left(\frac{P_T}{10^{-3}}\right)$$

$$P'_{T(\text{dBm})} = 10 \log_{10} P_T - 30$$

$\Rightarrow$ $$P_{T(\text{dB})} = 40 - 30 = 10 \text{ dB}$$

$$10 \log_{10} P_T = P_{T(\text{dB})} = 10 \text{ dB}$$

$$P_T = 10 \text{ W}$$

Now received power at earth station is given by

$$P_r = \frac{P_t G_t G_r}{\left(\frac{4\pi d}{\lambda}\right)^2}$$

$$\frac{C}{N} = \frac{P_r}{KTB}$$

where,

$K \rightarrow$ Boltzmann's constant = 1.38×10^{-23} J/K

$T \rightarrow$ Equivalent noise temperature = 270 K

$B \rightarrow$ Bandwidth = 10 MHz = 10^7 Hz

$$\therefore \quad \frac{C}{N} = \frac{P_t G_t G_r}{\left(\frac{4\pi d}{\lambda}\right)^2 \times KTB}$$

$$\left(\frac{C}{N}\right)_{dB} = 10\log_{10} P_t + 10\log_{10} G_t + 10\log G_r - 10\log_{10}\left(\frac{4\pi d}{\lambda}\right)^2$$
$$- 10 \log_{10} K - 10 \log_{10} T - 10\log_{10} B$$

$$\left(\frac{C}{N}\right)_{dB} = 10 + 10 \log 2568 + 10 \log 1027200 - 205.151$$
$$- 10 \log(1.38 \times 10^{-23}) - 10 \log 270 - 10 \log 10^7$$

$$\left(\frac{C}{N}\right)_{dB} = 10 + 34.0959 + 60.1165 - 205.151 + 228.6012$$
$$- 24.3136 - 70$$

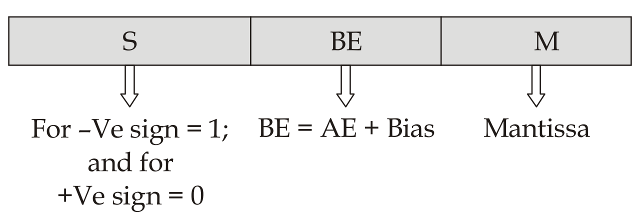
$$\left(\frac{C}{N}\right)_{dB} = 33.349$$

Q.7 (b) Solution:

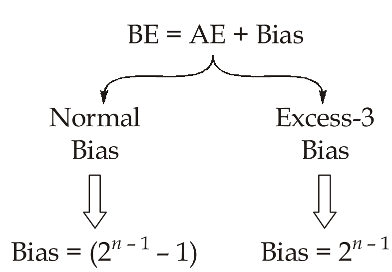
We have,

$$\text{Data} = -14.75 \times 2^{12}$$

Now, converting the number 14.75 into its binary equivalent, we get $-1110.11 * 2^{+12}$



- sign:- sign bit is 1 for negative number and 0 for positive number
- Bias Exponent (BE):- It is defined as sum of Actual Exponent (AE) + Bias



(n is the number of bits used for exponent)

If nothing is mentioned, then by default go for normal bias.

$$\text{Bias} = (2^{n-1} - 1) = (2^{7-1} - 1) = 63$$

In binary, Bias = 0111111

- Mantissa (M):– bits after the decimal point is called as mantissa or it is defined as the significant digits of a floating point number.

(i) Without Normalisation:

⇒ The binary equivalent of $(-14.75) * 2^{12} = -1110.11 * 2^{+12}$

- Sign $(S) = 1$
- Bias Exponent (BE) = Actual Exponent (AE) + Bias

$$= (0001100) + (2^{n-1} - 1)_2 \qquad \text{(Here, } n = 7\text{)}$$

$$= (0001100)_2 + (0111111)_2 = 100\ 1011$$

- Mantissa : $1110.\underbrace{11}_{M}$

$M = 11$

S	BE	M
1	1001011	1100 0000 0000

1 1 0 0 | 1 0 1 1 | 1 1 0 0 | 0 0 0 0 | 0 0 0 0

C B C 0 0

Hence, without normalisation $(-14.75) * 2^{12}$ is represented as $(CBC00)_H$ in the above mentioned format.

(ii) with normalisation:

The binary equivalent of $(-14.75) * 2^{12} = -1110.11 * 2^{12}$.

After normalisation, we can represent it as $-1.11011 * 2^{15}$.

- Sign Bit, $S = 1$
- Bias Exponent (BE) = Actual Exponent (AE) + Bias

$$= 0001111 + 0111111 = (1001110)$$

- Mantissa (M) = $1.\underbrace{11011}_{M}$

$M = 11011$

Sign	BE	M
1	1001110	11011 0000 000

1 1 0 0 | 1 1 1 0 | 1 1 0 1 | 1 0 0 0 | 0 0 0 0

C E D 8 0

Hence, with normalisation (–14.75) $*$ 2^{12} is represent as $(CED80)_H$, in the above mentioned format.

Now, write the $(CED80)_H$, in binary form as 1100 1110 11011000 0000.

On Comparing the data with given format, we have

Sign	BE	M
1	1001110	11011 0000 000
1 bit	7 bits	12 bits

- Sign bit is 1 $\Rightarrow$ number is negative
- BE = 1001110

We know that

$$BE = AE + Bias$$
$$AE = BE - Bias$$
$$= (1001110 - 0111111) = (0001111)$$

The binary equivalent of AE is 15.

- Mantissa, M = 11011

Combining all the information, we have

Number as $(-1.11011) * 2^{15} = (-1110.11) * 2^{12}$

Now, converting the binary number in decimal form:

$$(1110.11)_2 = 2^0(0) + 2^1(1) + 2^2(1) + 2^3(1) + 2^{-1}(1) + 2^{-2}(1)$$
$$= 0 + 2 + 4 + 8 + \frac{1}{2} + \frac{1}{4}$$
$$= 14.75$$

Hence, we get data as (–14.75) $*$ 2^{12} from normalised form $(CED80)_H$.

Q.7 (c) Solution:

(i) Ceramic materials exhibit a variety of structures, each influencing their properties and applications. Here are some common ceramic material structures:

1. **Crystalline Structure:** Many ceramic materials have a crystalline structure, where atoms are arranged in a regular, repeating pattern known as a crystal lattice. This structure provides ceramics with their characteristic strength and stability. e.g:- Al_2O_3 (Alumina), silicon carbide (SiC), and silicon nitride (Si_3 N_4).
2. **Amorphous Structure:** Some ceramic materials have an amorphous or non-crystalline structure, lacking long-range order in atomic arrangement.

Examples include glass and certain ceramic composites.

Amorphous ceramics typically exhibit isotropic properties and can be formed into complex shapes through processes like melting and solidification.

3. **Layered Structures:** Certain ceramic materials, such as clay minerals, have layers held together by weak forces. This structure allows for properties like plasticity and easy shaping, making clay-based ceramics suitable for pottery and construction materials.
4. **Composite Structure:** Ceramic composites consist of a ceramic matrix reinforced with other materials like fibers or particles. These composites combine the properties of ceramics with the strength and toughness of the reinforcing phase.

   Examples include, ceramic matrix composites (CMCs) used in aero space applications for their high strength-to-weight ratio.
5. **Perovskite Structure:** Some ceramic materials, such as barium titanate ($BaTiO_3$), exhibit a perovskite crystal structure characterized by a cubic unit cell with a central titanium atom surrounded by oxygen atom in a distorted octahedral arrangement. Perovskite ceramics are known for their piezoelectric and ferroelectric properties making them useful in electronic and sensor applications.
6. **Spinel Structure:** Spinel ceramics, like magnesium aluminate spinel ($Mg\,Al_2\,O_4$), have a cubic crystal structure where cations occupy both tetrahedral and octahedral sites within the lattice. Spinel ceramics are valued for their high thermal shock resistance and chemical stability, making them suitable for refractory applications and used as transparent ceramic materials.

Applications of ceramic materials:

1. It is used in electronic components such as insulators, capacitors, resistors and substrates for integrated circuits.
2. It is used in automotive components such as catalytic, converters, spark plugs, brake pads and engine components.
3. Ceramics play a crucial role in aerospace applications due to their ability to withstand high temperatures, corrosion resistance and light weight properties.
4. Ceramics are employed in various medical applications including dental implants, bone substitutes, surgical instruments, and prosthetic devices.
5. Ceramics are used in chemical processing equipments such as reactors, pumps, valves and lining materials due to their corrosion resistance, chemical inertness and high temperature stability.

(ii) 90% Cu 10% Ni alloy is given to have a resistivity of $1.9 \times 10^{-7}\ \Omega$m

For additions of a single impurity that forms a solid solution, the impurity resistivity ρ_i is related to the impurity concentration C_i in terms of the atom fraction as follows:

$\rho_i = AC_i(1 - C_i)$ ...(i)

$\rho_i \rightarrow$ resistivity

$C_i \rightarrow$ impurity concentration

$A \rightarrow$ composition independent constant

According to Matthiessen's rule, the total resistivity is given by

$\rho_{(total)} = \rho_i + \rho_t = 1.9 \times 10^{-7}\ \Omega$-m , where ρ_i represent the impurity resistivity and ρ_t represent thermal resistivity.

For the 90 Cu-10 Ni alloy, the Nickel impurity contribution to the total conductivity is computed as

$$\rho_{i(1)} = \rho_{total_{(1)}} - \rho_t = 1.9 \times 10^{-7} - 1.67 \times 10^{-8}\ (\Omega\text{-m})$$
$$= 1.73 \times 10^{-7}\ (\Omega\text{-m})$$

In the problem statement, the impurity (i.e., nickel) concentration is expressed in weight percent. However equation (i) calls for concentration in atom fraction (i.e., atom % divided by 100). Consequently, conversion from weight percent to atom fraction is necessary.

Assume the atom fraction of nickel as C'_{Ni} and the weight percent of Ni and Cu by C_{Ni} and C_{Cu} respectively. Using these notations, the conversion of 90 Wt% Cu – 10 Wt% Ni may be accomplished by using

$$C'_{Ni} = \frac{C'_{Ni}}{100} = \frac{C_{Ni}A_{Cu}}{C_{Ni}A_{Cu} + C_{Cu}A_{Ni}} \quad \text{...(ii)}$$

$$A_{Ni} = \text{Atomic weight of Nickel} = 58.69\ \text{g/mol}$$

$$A_{Cu} = \text{Atomic weight of copper} = 63.55\ \text{g/mol.}$$

$$C'_{Ni(1)} = \frac{(10\ \text{Wt\%})63.55\ \text{g/mol}}{(10\ \text{Wt\%})(63.55\ \text{g/mol}) + (90\ \text{Wt\%}) \times 58.69}$$
$$= 0.107$$

Now rearranging the equation (i), we get

$$A = \frac{\rho_{i(1)}}{C'_{Ni(1)}(1 - C'_{Ni(1)})}$$

$$A = \frac{1.73 \times 10^{-7}(\Omega\text{-m})}{(0.107)(1 - 0.107)} = 1.81 \times 10^{-6}(\Omega\text{-m}) \quad \text{...(iii)}$$

To achieve a total resistivity of 2.5 × 10^{-7} Ω-m, the impurity concentration must be

$$\rho_{i(2)} = \rho_{total(1)} - \rho_t$$
$$= 2.5 \times 10^{-7}(\Omega\text{-m}) - 1.67 \times 10^{-8}\ (\Omega\text{-m})$$
$$\rho_{i(2)} = 2.33 \times 10^{-7}(\Omega\text{-m}) \quad ...(iv)$$

From equation (i),

$$\rho_{i(2)} = AC'_{Ni} - AC'^2_{Ni}$$

$$AC'^2_{Ni} - AC'_{Ni} + \rho_{i(2)} = 0$$

$$C'_{Ni} = \frac{A \pm \sqrt{A^2 - 4A\rho_{i(2)}}}{2A}$$

Substituting $A = 1.81 \times 10^{-6}\ (\Omega\text{-m})$

$$\rho_{i(2)} = 2.33 \times 10^{-7}\ (\Omega\text{-m})$$

$$C'_{Ni(2)} = \frac{1.81 \times 10^{-6} \pm \sqrt{(1.81 \times 10^{-6})^2 - 4 \times (1.81 \times 10^{-6}) \times (2.33 \times 10^{-7})}}{(2 \times 1.81 \times 10^{-6})}$$

$$= \frac{1.81 \times 10^{-6} \pm 1.26 \times 10^{-6}}{3.62 \times 10^{-6}}$$

$$C'_{Ni(2)} = 0.848,\ 0.152$$

Taking 0.152 as the concentration of Ni (in at % 100), then 0.848 gives the concentration of Cu in terms of atom %.

$$\left.\begin{aligned} C'_{Ni(2)} &= 0.152 \times 100 = 15.2 \text{ at } \% \\ C'_{Cu(2)} &= 0.848 \times 100 = 84.8 \text{ at } \% \end{aligned}\right\}$$

Concerting the atomic % composition to weight %,

$$C_{Ni(2)} = \frac{C'_{Ni(2)}A_{Ni}}{C'_{Ni(2)}A_{Ni} + C'_{Cu(2)}A_{Cu}} \times 100$$

$$= \frac{(15.2 \text{ at } \%)\,(58.69 \text{ g/mol})}{(15.2 \text{ at } \%)(58.69 \text{ g/mol}) + (84.8 \text{ at } \%)(63.55 \text{ g/mol})} \times 100$$

$$= 14.2\%$$

Thus, the composition of Alloy must be 85.8 Wt% Cu - 14.2 wt% Ni to achieve a total resistivity of 2.5 × 10^{-7} Ω-m at room temperature.

Q.8 (a) Solution:

The miss penalty to main memory is $\dfrac{100 \text{ ns}}{0.25 \dfrac{\text{ns}}{\text{clockcycle}}}$ = 400 clock cycle

The effective CPI with one level of caching is given by

Total CPI = Base CPI + memory-stall cycles per instruction

For the processor with one level of caching i.e. only primary cache,

Total CPI = 1 + memory stall cycles per instruction

$$= 1 + 400 \times \frac{2}{100} = 9$$

With two levels of caching, a miss in the primary (or first-level) cache can be satisfied either by the secondary cache or by main memory. The miss penalty for an access to the second-level cache is

$$\frac{5 \text{ ns}}{0.25 \dfrac{\text{ns}}{\text{clockcycle}}} = 20 \text{ clock cycles}$$

If the miss is satisfied in the secondary cache, then this is the entire miss penalty. If the miss needs to go to main memory, then the total miss penalty is the sum of the secondary cache access time and main memory access time.

Thus, for a two-level cache, total CPI is the sum of the stall cycles from both levels of cache and the base CPI.

Total CPI = 1 + Primary stalls per instruction
+ secondary stalls per instruction

$$= 1 + (2\% \text{ of } 20) + \left(\frac{0.5 \times 400}{100}\right)$$

$$= 1 + 0.4 + 2 = 3.4$$

Thus, the processor with the secondary cache is faster by

$$\frac{9.0}{3.4} = 2.6 \text{ times}$$

Q.8 (b) Solution:

(i) 1. Shot noise current

$$= \sqrt{2 \times \text{Charge of electron} \times \text{Dark current} \times \text{Bandwidth}}$$

$$= \sqrt{2 \times 1.6 \times 10^{-19} \times 0.15 \times 520}$$

$= 4.99 \times 10^{-9} = 4.99$ nA

2. Thermal noise current

$$= \sqrt{\frac{4 \times \text{Boltzmann constant} \times \text{absolute temperature} \times \text{bandwidth}}{\text{Total resistance}}}$$

$$= \sqrt{\frac{4 \times 1.38 \times 10^{-23} \times (273 + 28) \times 520}{(12 \times 10^6 + 120)}}$$

$$= \sqrt{\frac{4 \times 1.38 \times 10^{-23} \times 301 \times 520}{12{,}000{,}120}}$$

$= 8.485 \times 10^{-13}$ A

3. Signal to noise ratio

$$= \frac{I_{\text{signal}}}{I_{\text{noise}}}$$

$$I_{\text{noise}} = \sqrt{(\text{Shot noise current})^2 + (\text{Thermal noise current})^2}$$

$$= \sqrt{(4.996 \times 10^{-9})^2 + (8.485 \times 10^{-13})^2}$$

$= 4.99 \times 10^{-9}$ A

I_{signal} = Responsitivity × Minimum radiant input

$I_{\text{signal}} = 0.6 \times 3 \times 10^{-6}$

$= 1.8 \times 10^{-6}$ A

$$\therefore \quad \text{Signal to noise ratio} = \frac{1.8 \times 10^{-6}}{4.99 \times 10^{-9}} = 360.72$$

Signal to noise ratio in dB

$= 20 \log_{10} 360.72$

$= 51.143$ dB

4. Noise equivalent power: It is defined as the incident optical power required to produce a signal to noise ratio of unity in a 1 Hz bandwidth. It is defined as

$$\text{NEP} = \frac{\text{Noise current}}{\text{Peak radiant sensitivity}}$$

$$= \frac{4.99 \times 10^{-9}}{0.6} = 8.31 \times 10^{-9} \text{ W}$$

(ii) 1. The maximum line-of-sight distance between the two antennas having heights h_t (in m) and h_r (in m) above the earth is given by

$$\text{LOS} = 4.12\left[\sqrt{h_t} + \sqrt{h_r}\right] \text{Km}$$

$$= 4.12\left[\sqrt{120} + \sqrt{16}\right]$$

$$= 61.61 \text{ km}$$

2. The electric strength at the receiving antenna is given by

$$E_R = \frac{88\sqrt{P}\,h_t h_r}{\lambda d^2}, \text{ where } \lambda = \frac{c}{f} = \frac{3 \times 10^8}{50 \times 10^6} = 6 \text{ m}$$

$$E_R = \frac{88\sqrt{15000} \times 120 \times 16}{6 \times (12 \times 10^3)^2}$$

$$E_R = 0.02395 \text{ V/m} = 23.95 \text{ mV/m}$$

3. Let us assume at distance 'd', the electric field strength is 1 mV/m. Thus,

$$d^2 = \frac{88\sqrt{P}\,h_t h_r}{\lambda E_R}$$

$$d^2 = \frac{88\sqrt{15 \times 10^3} \times 120 \times 16}{6 \times 1 \times 10^{-3}}$$

$$d = \sqrt{3448881558}$$

$$d = 58727.17 \text{ km}$$

Q.8 (c) Solution:

We know, in a BJT, $\dfrac{T_j - T_A}{P_D} = \theta$

where T_j = junction temperature

T_A = ambient temperature

θ = thermal resistance

P_D = collector junction power dissipation

$$P_D = \frac{T_j - T_A}{\theta} = \frac{\Delta T}{\theta}$$

$$\therefore \quad \frac{dP_D}{dT} = \frac{1}{\theta}$$

Also, power dissipated (P_D) when the transistor is cut-off is given by [$I_C = I_{CO}$ at cut-off]

$$P_D = (V_{CC} - I_{CO}R_C)I_{CO}$$

$$P_D = V_{CC}I_{CO} - I_{CO}^2R_C \quad \text{...(i)}$$

As
$$\frac{dP_D}{dT} = \frac{dP_D}{dI_{CO}} \times \frac{dI_{CO}}{dT}$$

Differentiating equation (i) w.r.t I_{CO}, we get

$$\frac{dP_D}{dI_{CO}} = [V_{CC} - 2I_{CO}R_C]$$

$$\therefore \quad \frac{dP_D}{dT} = [V_{CC} - 2I_{CO}R_C] \times (0.07I_{CO})$$

Because $\frac{dI_{CO}}{dT} = 0.07\, I_{CO}$ (Given)

To avoid thermal run away, we need that

$$\frac{dP_D}{dT} < \frac{1}{\theta}$$

or
$$(V_{CC} - 2I_{CO}R_C)(0.07I_{CO}) < \frac{1}{\theta}$$

i.e., $-0.14R_CI_{CO}^2 + 0.07V_{CC}I_{CO} - \frac{1}{\theta} < 0$

The Thermal run-away will occur for the roots of the below quadratic equation for which the following relation is satisfied:

$$\therefore \quad 0.14R_CI_{CO}^2 - 0.07V_{CC}I_{CO} + \frac{1}{\theta} < 0$$

The roots of the equation are given by

$$I_{CO} = \frac{0.07V_{CC} \pm \left[(0.07V_{CC})^2 - 4(0.14R_C)\frac{1}{\theta}\right]^{1/2}}{2(0.14R_C)}$$

$$\therefore \quad \frac{V_{CC} - \left[V_{CC}^2 - \frac{8R_C}{0.07\theta}\right]^{1/2}}{4R_C} \le I_{CO} \le \frac{V_{CC} + \left[V_{CC}^2 - \frac{8R_C}{0.07\theta}\right]^{1/2}}{4R_C}$$

Thermal runaway occurs for

$$V_{CE} > \frac{V_{CC}}{2}$$

or $$V_{CC} - I_{CO}R_C > \frac{V_{CC}}{2}$$

$$2I_{CO}R_C < V_{CC}$$

$\therefore$ $$I_{CO} < \frac{V_{CC}}{2R_C}$$

Hence, if thermal runaway is not destructive, the collector current I_{CO} after runaway can never exceed $V_{CC}/2R_C$.

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