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Detailed Solutions

ESE-2024 Mains Test Series

Electrical Engineering Test No : 8

Section A : Electromagnetic Theory + Digital Electronics + Communication Systems

Q.1 (a) Solution:

Let, at a distance x from the plate of dielectric constant ε_1 , the permittivity be

$$\varepsilon = \varepsilon_1 + Kx$$

Now, at $x = D$, the permittivity is ε_2

$$\therefore \quad K = \frac{\varepsilon_2 - \varepsilon_1}{D}$$

$$\therefore \quad \varepsilon = \varepsilon_1 + \left(\frac{\varepsilon_2 - \varepsilon_1}{D}\right)x$$

$\therefore$ Field intensity at a distance x from the plate of permittivity ε_1

$$E(x) = \frac{D}{\varepsilon} = \frac{\sigma}{\varepsilon}, \qquad \sigma = \text{surface charge density on plate-1}$$

$$= \frac{\sigma}{\varepsilon_1 + \left(\frac{\varepsilon_2 - \varepsilon_1}{D}\right)x}$$

$\therefore$ Potential difference between the plates is

$$V = \int_0^D E(x)\,dx = \sigma\int_0^D \frac{1}{\varepsilon_1 + \left(\frac{\varepsilon_2 - \varepsilon_1}{D}\right)x}dx$$

$$= \frac{\sigma D}{(\varepsilon_2 - \varepsilon_1)}\left[\ln\left(\frac{\varepsilon_2 - \varepsilon_1}{D}x + \varepsilon_1\right)\right]_{x=0}^{x=D}$$

$$= \frac{\sigma AD}{(\varepsilon_2 - \varepsilon_1)A} \ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right)$$

$$= \frac{QD}{A(\varepsilon_2 - \varepsilon_1)} \ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right) \qquad \{\because Q = \sigma A\}$$

So, the capacitance of the parallel plate capacitor is

$$C = \frac{Q}{V} = \frac{A}{D} \frac{\varepsilon_2 - \varepsilon_1}{\ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right)}$$

For $\varepsilon_2 = \varepsilon_1$;

$$C = \frac{A}{D} \frac{\left(\frac{\varepsilon_2}{\varepsilon_1} - 1\right)\varepsilon_1}{\ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right)}$$

Let, $\frac{\varepsilon_2}{\varepsilon_1} = K \qquad (\varepsilon_1 = \varepsilon_2 \to \varepsilon)$

$$C = \lim_{K\to1} \frac{A}{D} \frac{(K-1)\varepsilon}{\ln(K)} = \lim_{K\to1} \frac{A}{D} \frac{\varepsilon}{1/K}$$

$$C = \lim_{K\to1} \frac{A}{D} \times \varepsilon \times \frac{\varepsilon_2}{\varepsilon_1} = \frac{\varepsilon A}{D}$$

Q.1 (b) Solution:

From the above circuit shown,

$$S = A \cdot B$$

$$R = \bar{A} \cdot B$$

Drawing characteristics table for AB flip-flop,

A	B	Q_n	$S = (A \cdot B)$	$R = (\bar{A} \cdot B)$	Q_{n+1}
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	0	0	1
1	1	0	1	0	1
1	1	1	1	0	1

Finding Q_{n+1} in terms of A , B , Q_n

$$Q_{n+1} = A \cdot B + \overline{B} \cdot Q_n$$

This is required characteristics equation,

K-map:

A \ BQ_n	00	01	11	10
0	0	1	0	0
1	0	1	1	1

$$Q_{n+1} = \overline{B}Q_n + AB$$

For $B = 0$, from characteristics equation,

$$Q_{n+1} = Q_n$$

For $B = 1$, from characteristic equation,

$$Q_{n+1} = A$$

Hence, we see that

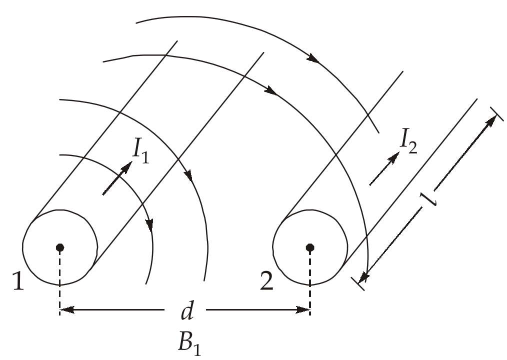
$$Q_{n+1} = Q_n \text{, when } B = 0,$$

$$Q_{n+1} = A \text{, when } B = 1$$

For $B = 1$; A is latched as the output and for $B = 0$, the output does not change.

Q.1 (c) Solution:

Consider two straight, long parallel lines situated in space as shown below,



Conductor 1 produces a field around and its value is B_1. At the location of conductor 2 its value is given by

$$B_1 = \mu_0 \frac{I_1}{2\pi d}$$

While conductor 2 is carrying a current of I_2 and is situated in the field with flux density B_1.

$$\vec{dF} = I\, d\vec{l} \times \vec{B}$$

We can write for length l,

$$F_2 = I_2\, l \times B_1$$

$$F_2 = \mu_0 \frac{I_1 I_2 l}{2\pi d}$$

$$F_2 = \frac{4\pi \times 10^{-7} \times 6 \times 6 \times 4}{2\pi \times 5 \times 10^{-3}} = 5.76 \times 10^{-3} \text{ N}$$

According to right hand thumb rule flux between the conductors is nullified when same direction of current is flowing in conductors, so there will be force of attraction between conductors.

If direction of I_2 is reversed then the flux between the conductor is aiding so force of repulsion is between the conductors of same magnitude.

$$F = 5.76 \times 10^{-3} \text{ N}$$

Q.1 (d) Solution:

(i) $(A + B)(B + C)(C + A) = AB + BC + CA$

$$\begin{aligned}
\text{L.H.S.} &= (A + B)(B + C)(C + A) \\
&= (AB + AC + B + BC)(C + A) \\
&= [B + AC + BC](C + A) \\
&= BC + ACC + BCC + AB + AAC + ABC \\
&= BC + AC + BC + AB + AC + ABC \\
&= BC + AB(1 + C) + AC \\
&= BC + AB + AC
\end{aligned}$$

Hence Proved.

(ii) $(A+B)(\bar{A}+C) = AC + \bar{A}B + BC$

$$\begin{aligned}
\text{L.H.S.} &= (A+B)(\bar{A}+C) \\
&= A\bar{A} + AC + \bar{A}B + BC \\
&= 0 + AC + \bar{A}B + BC
\end{aligned}$$

Hence Proved.

Q.1 (e) Solution:

For sinusoidal modulation, current relation between carrier and modulated signal is given by,

$$\frac{I_t}{I_c} = \sqrt{1 + \frac{m^2}{2}} \quad \text{...(i)}$$

Where, I_t = modulated current

I_c = carrier current

m = modulation index

According to question, $I_t = 11$ A, when $m = 0.4$

∴ From equation (i),

$$I_c = \frac{I_t}{\sqrt{1+\frac{m^2}{2}}} = \frac{11}{\sqrt{1+\frac{0.4^2}{2}}} = 10.585 \text{ A}$$

Now, simultaneous modulation by another audio sine wave:

∴ Again from equation (i),

$$m_t = \sqrt{2\left(\left(\frac{I_t}{I_c}\right)^2 - 1\right)}$$

$$m_t = \sqrt{2\left(\left(\frac{12}{10.585}\right)^2 - 1\right)} = 0.7553$$

Modulation index due to second wave,

$$m_2 = \sqrt{m_t^2 - m^2}$$

$$= \sqrt{(0.7569)^2 - (0.4)^2} = 0.641$$

Modulation index due to second wave is 0.641.

Q.2 (a) Solution:

(i) Since $\hat{a}_z$ is the normal to the boundary plane,

the normal component is

$$E_{1n} = \vec{E}_1 \cdot \hat{a}_n = \vec{E}_1 \cdot \hat{a}_z = 3$$

∴ $$\vec{E}_{1t} = \vec{E}_1 - \vec{E}_{1n} = 5\hat{a}_x - 2\hat{a}_y$$

By boundary conditions

$$\vec{E}_{2t} = \vec{E}_{1t} = 5\hat{a}_x - 2\hat{a}_y$$

and $$\varepsilon_{r2}\vec{E}_{2n} = \varepsilon_{r1}\vec{E}_{1n}$$

$$\vec{E}_{2n} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}\vec{E}_{1n} = \frac{4}{3}\vec{E}_{1n} = 4\hat{a}_z$$

So, the field in second medium is given as

$$\vec{E}_2 = (\vec{E}_{2t} + \vec{E}_{2n}) = \left(5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z\right) \text{ kV/m}$$

(ii) Let, α_1 and α_2 be the angles $\vec{E}_1$ and $\vec{E}_2$ make with the interface while θ_1 and θ_2 are the angles they make with the normal to the interface,

$$\alpha_1 = (90° - \theta_1) \text{ and } \alpha_2 = (90° - \theta_2)$$

$$\therefore \quad \tan\theta_1 = \frac{E_1 t}{E_1 n} = \frac{\sqrt{5^2 + 2^2}}{3} = \frac{\sqrt{29}}{3} = 1.795$$

$$\therefore \quad \theta_1 = 60.9°$$

Similarly,

$$\therefore \quad \tan\theta_2 = \frac{E_2 t}{E_2 n} = \frac{\sqrt{5^2 + 2^2}}{4} = \frac{\sqrt{29}}{4} = 1.346$$

$$\theta_2 = 53.4°$$

Hence, the angles between electric field intensity and the normal to the boundary surface in both media are given as

$$\theta_1 = 60.9° \text{ and } \theta_2 = 53.4°$$

(iii) The energy densities are given as

$$W_{E1} = \frac{1}{2}\varepsilon_1 \left|\vec{E}_1\right|^2 = \frac{1}{2} \times 4 \times \frac{10^{-9}}{36\pi}(25 + 4 + 9) \times 10^6$$

$$= 672 \ \mu\text{J/m}^3$$

$$W_{E2} = \frac{1}{2}\varepsilon_2 \left|\vec{E}_2\right|^2 = \frac{1}{2} \times 3 \times \frac{10^{-9}}{36\pi}(25 + 4 + 16) \times 10^6$$

$$= 597 \ \mu\text{J/m}^3$$

(iv) At the centre (3, 4, –5) of the cube of side 2 m, $z = -5 < 0$, ie., the cube is in region-2 with $2 \leq x \leq 4$, $3 \leq y \leq 5$, $-6 \leq z \leq -4$

Hence, the energy with in the cube is

$$W_E = \int W_{E2}\, dV = \int_{x=2}^{4} \int_{y=3}^{5} \int_{z=-6}^{-4} W_{E2}\, dz\, dy\, dx$$

$$= W_{E2} \times 2 \times 2 \times 2$$

$$= 597 \times 8 \ \mu\text{J} = 4.776 \text{ mJ}$$

Q.2 (b) Solution:

(i) Given, $c(t) = 5\cos(2\pi \times 10^5 t)$

$m(t) = 3\cos(2\pi \times 10^3 t) + 6\cos(3\pi \times 10^3 t)$

Frequency sensitivity,

$$K_f = 10^3 \text{ Hz/Volt}$$

The time domain expression for FM wave is

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int m(t)dt\right]$$

$$= 5\cos\left[2\pi \times 10^5 t + 2\pi \times 10^3 \int (3\cos(2\times 10^3 \pi t) + 6\cos(3\times 10^3 \pi t))dt\right]$$

$$= 5\cos[2\pi \times 10^5 t + 2\pi \times 10^3]\ [3\cos(2 \times 10^3\ \pi t) + 6\cos(3 \times 10^3\ \pi t)]dt$$

$$= 5\cos\left[2\pi \times 10^5 t + 2\pi \times 10^3 \left(\frac{3\sin(2\times 10^3 \pi t)}{2000\pi} + \frac{6\sin(3\times 10^3 \pi t)}{3000\pi}\right)\right]$$

$$S_{FM}(t) = 5\cos[2\pi \times 10^5 t + 3\sin(2 \times 10^3\ \pi t) + 4\sin(3 \times 10^3\ \pi t)]$$

(ii) The instantaneous frequency of FM signal is,

$$f_i = f_c + K_f m(t)$$

Maximum frequency deviation,

$$\Delta f = K_f m(t)\big|_{max},$$

$$= 10^3[3\cos(2 \times 10^3 \pi t) + 6\cos(3 \times 10^3 \pi t)]_{max},$$

$$= 10^3 \times (3 + 6)$$

$$\Delta f = 9 \text{ kHz}$$

Maximum carrier swing, $2\Delta f = 18$ kHz

Modulation index, $$\beta = \frac{\Delta f}{f_{m\,max}}$$

where, $$f_{m,\,max} = \max\left\{\frac{2000\pi}{2\pi}, \frac{3000\pi}{2\pi}\right\} = 1.5\,\text{kHz}$$

$$\therefore \quad \beta = \frac{9 \text{ kHz}}{1.5 \text{ kHz}} \simeq 6$$

(iii) Transmission bandwidth,

According to Carson's rule,

$$\text{Bandwidth} = 2(\Delta f + f_{m,\,max},)$$

$$= 2(9 + 1.5) \text{ kHz}$$
$$= 21 \text{ kHz}$$

(iv) Average power in the FM wave:

Since, modulation index, $\beta > 1$, hence the given is bandwidth FM signal.

Hence, the power in the FM wave is equal to carrier power i.e,

$$P_t = \frac{A_c^2}{2} = \frac{(5)^2}{2} = 12.5 \text{ W}$$

Q.2 (c) (i) Solution:

Given, modulating signal,

$$x(t) = 5 \sin(4\pi\, 10^3 t - 10\pi \cos 2\pi\, 10^3\, t) \text{ V}$$

the standard phase modulated signal can be

$$s(t) = A_c \cos(\omega_c t + K_p\, x(t))$$

Instantaneous angle of the modulated signal is,

$$\theta(t) = \omega_c t + K_p x(t)$$

Instantaneous frequency, $\omega_i(t) = \dfrac{d\theta(t)}{dt}$

$$f_i(t) = \frac{1}{2\pi}\left(\omega_c + K_p \frac{dx(t)}{dt}\right) = f_c + \frac{K_p}{2\pi}\frac{dx(t)}{dt}$$

$$f_i(t) = f_c + \frac{25}{2\pi}\begin{bmatrix}\cos(4000\pi t - 10\pi \cos 2000\pi t)\\(4000\pi + 20000\pi^2 \sin 2000\pi t)\end{bmatrix}$$

At $t = 0.5$ ms,

$$f_i(0.5 \text{ ms}) = f_c + \frac{25}{2\pi}[\cos(2\pi - 10\pi\cos(\pi))]\left[4000\pi + 2000\pi^2 \sin(\pi)\right]$$

$$= f_c + \frac{25}{2\pi}[\cos(12\pi)](4000\pi) = f_c + 50 \text{ kHz}$$

$$= 20 \text{ kHz} + 50 \text{ kHz} \qquad (\text{given, } f_c = 20 \text{ kHz})$$

$$\therefore \quad f_i(0.5 \text{ ms}) = 70 \text{ kHz}$$

Q.2 (c) (ii) Solution:

1. From the given waveform,

$$S_{max} = A_C[1 + \mu] = 9$$
$$S_{min} = A_C[1 - \mu] = -9$$

Modulation index, $\mu = \dfrac{9-3}{9+3} = \dfrac{6}{12} = \dfrac{1}{2}$

$$\therefore \qquad A_C\left[1+\frac{1}{2}\right] = 9$$

$$A_C = \frac{9}{\left(\frac{3}{2}\right)} = 6\text{ V}$$

2. Transmission efficiency of AM,

$$\eta = \frac{\mu^2}{2+\mu^2} = \frac{\left(\frac{1}{2}\right)^2}{2+\left(\frac{1}{2}\right)^2}$$

$$\therefore \qquad \eta = \frac{1}{9}\times 100\% = 11.11\%$$

3. Given modulation index, $\mu = 0.3$

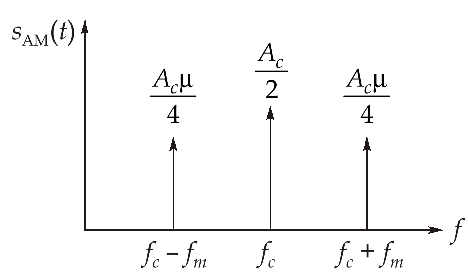
But, $\mu = \frac{A_M}{A_C}$

where, $A_M = 3$ from the given waveform,

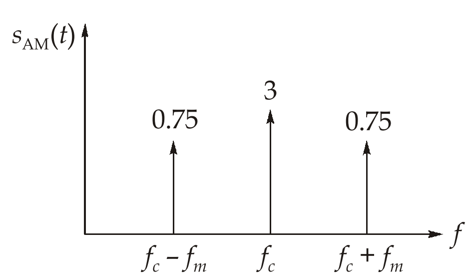
$$\therefore \qquad A_C' = \frac{A_M}{\mu} = \frac{3}{0.3} = 10\text{ V}$$

So, the amplitude of the carrier should be added is 4V to achieve 0.3 modulation index.

4. General frequency spectrum of AM waveform is



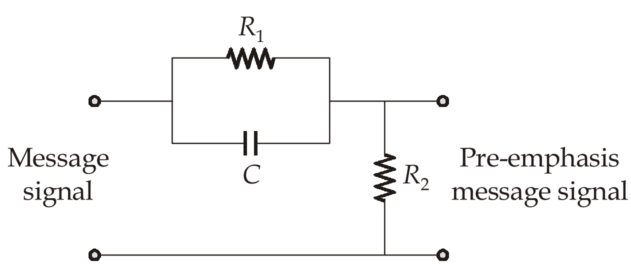
For $\mu = \frac{1}{2}$, $A_c = 6$ V

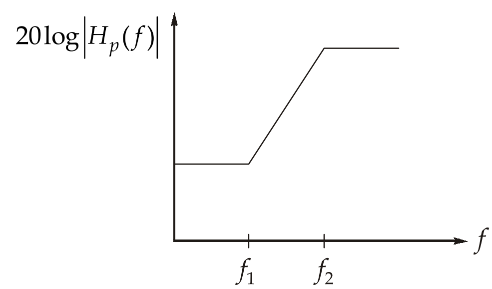


Q.3 (a) Solution:

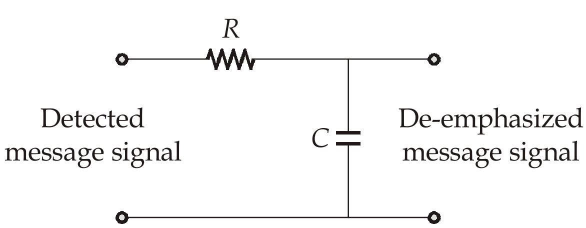
In FM, the noise increases linearly with frequency. By this, the higher frequency components of message signal are badly affected by the noise. To solve this problem we can use a pre-emphasis filter of transfer function $H_p(f)$ at the transmitter to boost the higher frequency components before modulating. Similarly, at the receiver, the de-emphasis filter of transfer function $H_d(f)$ can be used after demodulator to attenuate the higher frequency components thereby restoring the original message signal.

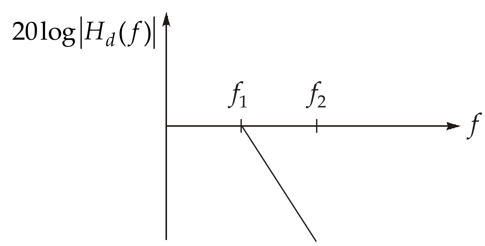
The pre-emphasis network and its frequency response are as shown,





The de-emphasis network and its frequency response are as shown,





In FM broadcasting f_1 and f_2 are normally chosen to be 2.1 kHz and 30 kHz respectively. The frequency response of pre-emphasis network is

$$H_p(f) = \left(\frac{\omega_2}{\omega_1}\right)\frac{j\omega + \omega_1}{j\omega + \omega_2}$$

Where, $\omega_1 = 2\pi f_1$, $\omega = 2\pi f$

For $\omega << \omega_1$,

$$H_p(f) \approx 1$$

For $\omega_1 << \omega << \omega_2$,

$$H_p(f) \approx \frac{j2\pi f}{\omega_1}$$

So, the amplitude of frequency components less than 2.1 kHz are left unchanged and greater than that are increased proportional to f.

The frequency response of de-emphasis network is,

$$H_d(f) = \frac{\omega_1}{j2\pi f + \omega_1}$$

For $\omega << \omega_2$,

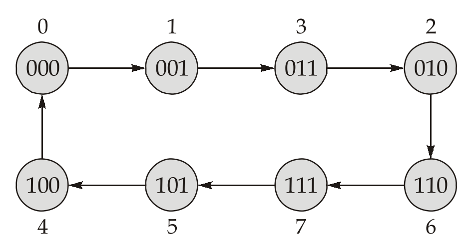
$$H_p(f) \approx \frac{j2\pi f + \omega_1}{\omega_1}$$

Such that, $$H_p(f)\, H_d(f) = 1$$

Q.3 (b) Solution

As it is required to design a 3-bit counter, the number of flip-flops required = 3

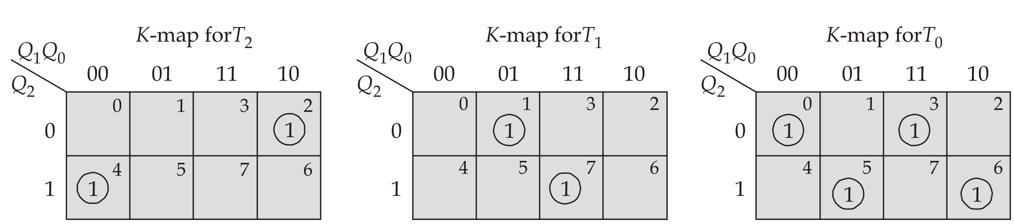
Sequence diagram:



Excitation table:

Present state			Next state			Required excitations		
Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	T_2	T_1	T_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	1	0	1	0
0	1	0	1	1	0	1	0	0
0	1	1	0	1	0	0	0	1
1	0	0	0	0	0	1	0	0
1	0	1	1	0	0	0	0	1
1	1	0	1	1	1	0	0	1
1	1	1	1	0	1	0	1	0

Minimization:



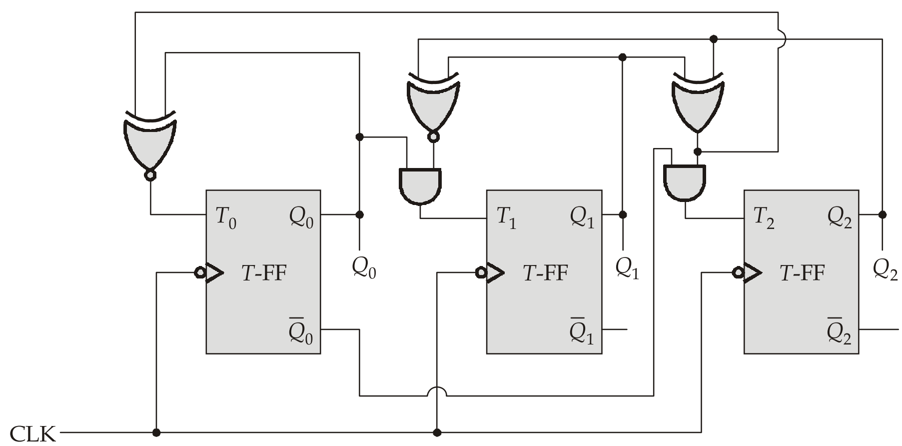
$$T_2 = Q_2\bar{Q}_1\bar{Q}_0 + \bar{Q}_2Q_1\bar{Q}_0 = (Q_2 \oplus Q_1)\bar{Q}_0$$

$$T_1 = \bar{Q}_2\bar{Q}_1Q_0 + Q_2Q_1Q_0 = (Q_2 \odot Q_1)Q_0$$

$$T_0 = \bar{Q}_2(Q_1 \odot Q_0) + Q_2(Q_1 \oplus Q_0)$$

$$= Q_2 \oplus (Q_1 \odot Q_0) = (Q_2 \oplus Q_1) \odot Q_0$$

Logic circuit:



Q.3 (c) Solution:

For a single-tone AM signal,

Transmission efficiency, $\eta = \dfrac{\mu^2}{2+\mu^2} \times 100\%$

Antenna current, $I_t = I_c\sqrt{1+\dfrac{\mu^2}{2}}$

When $m(t) = m_1(t)$, $\dfrac{\mu_1^2}{2+\mu_1^2} = 0.20 \Rightarrow \mu_1^2 = 0.40 + (0.20)\mu_1^2$

$$(0.80)\mu_1^2 = 0.40 \Rightarrow \mu_1^2 = 0.50$$

When $m(t) = m_1(t) + m_2(t)$,

$$\frac{I_t}{I_{t_1}} = 1.05 \Rightarrow \frac{\sqrt{2+\mu^2}}{\sqrt{2+\mu_1^2}} = 1.05$$

$\mu^2 = \mu_1^2 + \mu_2^2$; μ_2 = Modulation index due to $m_2(t)$ alone

$$\frac{2+\mu_1^2+\mu_2^2}{2+\mu_1^2} = (1.05)^2$$

$$\frac{2.5+\mu_2^2}{2.5} = (1.05)^2 \quad \because \mu_1^2 = 0.50$$

$$\mu_2^2 = \left[(1.05)^2 \times 2.5\right] - 2.5 = 0.25625$$

When $m(t) = m_2(t)$,

$$\eta = \frac{\mu_2^2}{2+\mu_2^2} \times 100\% = \frac{0.25625}{2.25625} \times 100\% \simeq 11.36\%$$

Q.4 (a) (i) Solution:

Here, $l = 50$ cm, $\lambda_1 = 2\ \mu C/m$

$\lambda_2 = 1\ \mu C/m$, $\lambda_3 = 1\ \mu C/m$

In this case total field intensity at the centre is given as

$$E = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

Where, $\vec{E}_1$ = Field at the centre due to side-1

$$= \frac{2\times 10^{-6} \times 50 \times 10^{-2}}{2\pi\varepsilon_0 r\sqrt{l^2 + 4r^2}}\hat{a}_y \qquad \left[\text{Here, } r = \frac{50}{2}\tan 30^\circ = \frac{25}{\sqrt{3}}\text{cm}\right]$$

$$= \frac{2\times 10^{-6} \times 50 \times 10^{-2}}{2\pi\varepsilon_0 \frac{25}{\sqrt{3}} \times 10^{-2} \sqrt{\left[50^2 + 4\left(\frac{25}{\sqrt{3}}\right)^2\right] \times 10^{-4}}}\hat{a}_y$$

$$= \frac{6\times 10^{-6}}{\pi\varepsilon_0}\hat{a}_y \text{ V/m}$$

E_2 = Field at the centre due to side 2

$$= \frac{1\times 10^{-6} \times 50 \times 10^{-2}}{2\pi\varepsilon_0 r\sqrt{l^2 + 4r^2}}(-\hat{a}_x \sin 60^\circ - \hat{a}_y \cos 60^\circ)$$

$$\left[\text{Here, } r = \frac{50}{2}\tan 30^\circ = \frac{25}{\sqrt{3}}\text{cm}\right]$$

$$= \frac{2\times 10^{-6} \times 50 \times 10^{-2}}{2\pi\varepsilon_0 \frac{25}{\sqrt{3}} \times 10^{-2} \sqrt{\left[(50)^2 + 4\left(\frac{25}{\sqrt{3}}\right)^2\right] \times 10^{-4}}}(-\hat{a}_x \sin 60^\circ - \hat{a}_y \cos 60^\circ)$$

$$= \frac{3\times 10^{-6}}{\pi\varepsilon_0}(-\hat{a}_x \sin 60^\circ - \hat{a}_y \cos 60^\circ) \text{ V/m}$$

$$\vec{E}_3 = \text{Field at the centre due to side-3}$$

$$= \frac{1\times10^{-6}\times50\times10^{-2}}{2\pi\varepsilon_0\frac{25}{\sqrt{3}}\times10^{-2}\sqrt{\left[50^2+4\left(\frac{25}{\sqrt{3}}\right)^2\right]\times10^{-4}}}(\hat{a}_x\sin60^\circ-\hat{a}_y\cos60^\circ)$$

$$= \frac{3\times10^{-6}}{\pi\varepsilon_0}(\hat{a}_x\sin60^\circ-\hat{a}_y\cos60^\circ)\text{V/m}$$

Hence, total field at the centroid is given as

$$\vec{E}_1 = \vec{E}_1+\vec{E}_2+\vec{E}_3$$

$$= \frac{6\times10^{-6}}{\pi\varepsilon_0}\hat{a}_y+\frac{3\times10^{-6}}{\pi\varepsilon_0}(-\hat{a}_x\sin60^\circ-\hat{a}_y\cos60^\circ)$$

$$+\frac{3\times10^{-6}}{\pi\varepsilon_0}(-\hat{a}_x\sin60^\circ-\hat{a}_y\cos60^\circ)$$

$$= \frac{10^{-6}}{\pi\varepsilon_0}(6-6\cos60^\circ)\hat{a}_y = \frac{10^{-6}\times36\pi}{\pi\times10^{-9}}(6-3)\hat{a}_y$$

$$= 10^3\times36\times3\hat{a}_y = 108\text{ kV/m}$$

Q.4 (a) (ii) Solution

An odd parity bit generator gives the output 1 when the number of 1's in the data bits is even, so that the total number of 1's in the data bits and the parity bit together is odd. Thus the truth table for this condition using four bit input is

A	B	C	D	$\bar{F}$
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

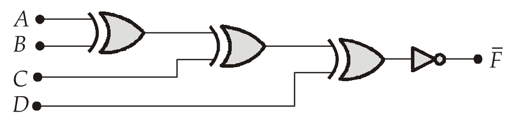
K-map

AB \ *CD*	00	01	11	10
00	1		1	
01		1		1
11	1		1	
10		1		1

From the K-map, the output is

$$\begin{aligned} F &= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + AB\bar{C}\bar{D} + ABCD + \bar{A}B\bar{C}D + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} \\ &= \bar{A}\bar{B}(\overline{C \oplus D}) + AB(\overline{C \oplus D}) + \bar{A}B(C \oplus D) + A\bar{B}(C \oplus D) \\ &= (\overline{C \oplus D})\,(\overline{A \oplus B}) + (C \oplus D)(A \oplus B) \\ &= \overline{(A \oplus B) \oplus (C \oplus D)} \end{aligned}$$

Logic diagram :



Q.4 (b) Solution:

Given two cones θ = 20° and θ = 160° and sphere of 0.2 m radius.

Applying Ampere's circuital law encircling the cone, we get

$$2\pi\rho\, H_\phi = 50$$

$$H_\phi = \frac{25}{\pi\rho} \text{ in cylindrical coordinate system}$$

$$2\pi r \sin\theta\, H_\phi = 50$$

$$H_\phi = \frac{25}{\pi r \sin\theta} \text{ in spherical coordinate system}$$

(i) The magnetic field intensity

$$H = \frac{25}{\pi r \sin\theta} a_\phi \text{ A/m}$$

(ii) The stored energy density,

$$W_H = \frac{1}{2}\mu_0 \left(\frac{25}{\pi r \sin\theta}\right)^2 = \frac{625\mu_0}{2\pi^2 r^2 \sin^2\theta} \text{ J/m}^3$$

The stored energy,

$$W_H = \frac{625\mu_0}{2\pi^2} \int_0^{2\pi} \int_{20^\circ}^{160^\circ} \int_0^{0.2} \frac{1}{r^2 \sin^2\theta} \cdot r^2 \sin\theta \cdot dr\, d\theta\, d\phi$$

$$= \frac{625\mu_0}{2\pi^2} \times 2\pi \times 0.2 \int_{20}^{160} \frac{1}{\sin\theta} \cdot d\theta$$

$$= \frac{125\mu_0}{\pi} \ln\left|\tan\frac{\theta}{2}\right|_{20^\circ}^{160^\circ} = 50 \times 10^{-6} \ln\left|\frac{\tan 80^\circ}{\tan 10^\circ}\right| = 173.5\ \mu\text{J}$$

Q.4 (c) Solution:

(i) For the given 4-bit R-$2R$ ladder converter output voltage,

$$V_0 = V_R \cdot \frac{R_f}{R}\left[\frac{b_1}{2^1}+\frac{b_2}{2^2}+\frac{b_3}{2^3}+\frac{b_4}{2^4}\right]$$

Given that,

$$V_R = 10 \text{ V}$$

$$R = 5 \text{ k}\Omega$$

$$\text{value of 1 LSB} = 1 \text{ volt}$$

$$1 = 10 \times \frac{R_f}{5\times 10^3}\cdot\left[\frac{1}{2^4}\right]$$

$$R_f = \frac{5\times 10^3 \times 2^4}{10} = 500 \times 2^4 = 8000\ \Omega$$

$$R_f = 8 \text{ k}\Omega$$

(ii) For binary value of 1000

$$b_1 = 1;\ b_2 = b_3 = b_4 = 0$$

$$\therefore \quad 8 = 10\times \frac{R_f}{5\times 10^3}\left[\frac{1}{2}\right]$$

$$R_f = \frac{8\times 2\times 5\times 10^3}{10} = 8 \text{ k}\Omega$$

(iii) Thus for getting a full scale voltage of 10 V,

$$b_1 = b_2 = b_3 = b_4 = 1$$

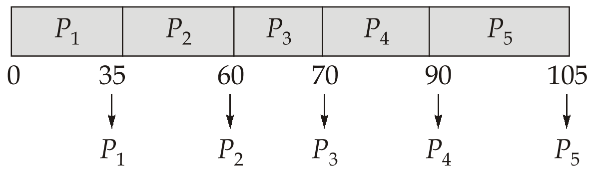
$$\frac{R_f \times 10}{5\times 10^3}\left[\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}\right] = 10$$

$$\frac{R_f}{5\times 10^3}[0.9375] = 1$$

$$R_f = \frac{5\times 10^3}{0.9375} = 5.333 \text{ k}\Omega$$

Section B : Computer Fundamentals - 1 + Electrical and Electronic Measurements - 1 Power Electronics & Drives - 2 + Engineering Mathematics - 2

Q.5 (a) Solution:



Calculating TAT:

$P_1 : 35 - 0 = 35$

$P_2 : 60 - 5 = 55$

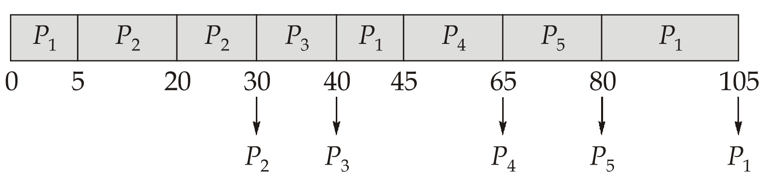
$P_3 : 70 - 20 = 50$

$P_4 : 90 - 45 = 45$

$P_5 : 105 - 65 = 40$

$$\text{Avg TAT} = \frac{35+55+50+45+40}{5} = 45$$

SRT:



Calculating TAT:

$P_1 : 105 - 0 = 105$

$P_2 : 30 - 5 = 25$

$P_3 : 40 - 20 = 20$

$P_4 : 65 - 45 = 20$

$P_5 : 80 - 65 = 15$

$$\text{Avg TAT} = \frac{105+25+20+20+15}{5} = 37$$

$$\text{Difference} = 45 - 37 = 8$$

Q.5 (b) Solution:

For moving instrument, we have

$$k\theta = \frac{I^2}{2}\frac{dL}{d\theta} \quad \text{or} \quad \theta = \frac{I^2}{2k}\frac{dL}{d\theta}$$

Given, $L = (0.01 + C\theta)^2$ mH

$$\therefore \quad \frac{dL}{d\theta} = 2(0.01 + C\theta) \cdot C \text{ mH/radian}$$

Also, $$\theta_1 = 90° = \frac{\pi}{2} \text{ radian},$$

$$\theta_2 = 120° = \frac{2\pi}{3} \text{ radian}, I_1 = 1.5 \text{ A}, I_2 = 2 \text{ A}$$

$$\therefore \quad \frac{\theta_1}{\theta_2} = \frac{I_1^2}{I_2^2} \times \left[\frac{2C(0.01 + C\theta_1)}{2C(0.01 + C\theta_2)}\right]$$

Putting the values, we have

$$\left(\frac{\pi/2}{2\pi/3}\right) = \left(\frac{1.5}{2}\right)^2 \cdot \left[\frac{0.01 + \frac{\pi}{2}C}{0.01 + \frac{2\pi}{3}C}\right]$$

or $$\frac{3}{4} = \left(\frac{3}{4}\right)^2 \times \left[\frac{0.01 + \frac{\pi}{2}C}{0.01 + \frac{2\pi}{3}C}\right]$$

or, $$4\left(0.01 + \frac{2\pi}{3}C\right) = 3\left(0.01 + \frac{\pi}{2}C\right)$$

or, $$C = \frac{-0.06}{7\pi} = -2.729 \times 10^{-3} \text{ or } -2.72837 \times 10^{-3}$$

$$\therefore \quad C = -2.73 \times 10^{-3}$$

Q.5 (c) Solution:

Hence the output is a square wave, value of fundamental voltage is

$$V_{01} = \frac{4V_s}{\pi\sqrt{2}} = \frac{4 \times 230}{\pi\sqrt{2}} = 207.0727 \text{ V}$$

Magnitude of load impedance at fundamental frequency is,

$$|Z_1| = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + (2\pi \times 50 \times 0.03)^2}$$

$$|Z_1| = 13.7414 \ \Omega$$

$$I_{01} = \frac{V_{01}}{Z_{01}} = \frac{207.0727}{13.7414} = 15.0692 \text{ V}$$

$$V_{03} = \frac{4V_s}{n\pi\sqrt{2}} = \frac{4\times230}{3\pi\sqrt{2}} = 69.0242 \text{ V}$$

$$|Z_3| = \sqrt{10^2 + (3\times2\pi\times50\times0.03)^2} = 29.9906\ \Omega$$

$$I_{03} = \frac{69.0242}{29.9906} = 2.3015 \text{ A}$$

Similarly,

$$I_{05} = \frac{920}{5\times\pi\times\sqrt{2}}\times\frac{1}{\sqrt{10^2+(5\times2\pi\times50\times0.03)^2}} = 0.8597 \text{ A}$$

$$I_{07} = \frac{920}{7\times\pi\times\sqrt{2}}\times\frac{1}{\sqrt{10^2+(7\times2\pi\times50\times0.03)^2}} = 0.4433 \text{ A}$$

Rms value of resultant load current,

$$I_{0,\text{ rms}} = \sqrt{I_{01}^2 + I_{03}^2 + I_{05}^2 + I_{07}^2}$$

$$I_{0,\text{ rms}} = \sqrt{(15.0692)^2 + (2.3015)^2 + (0.8597)^2 + (0.4433)^2}$$

$$= 15.27 \text{ A}$$

$$\text{Power delivered to load} = I_{0,\text{rms}}^2 R = (15.27)^2\times10$$

$$= 2331.729 \text{ W}$$

Q.5 (d) Solution:

$$\text{Page size} = 1 \text{ KB} = 2^{10} \text{ bytes}$$

$$\text{Page table entry} = 4 \text{ B} = 2^2 \text{ bytes}$$

1^{st} level page table = # page table entries × Page table entry size

$$= \frac{2^{34}}{2^{10}}\times2^2 \text{ bytes} = 2^{24}\times2^2 = 2^{26} \text{ bytes}$$

2^{nd} level page table = # page table entries × Page table entry size

$$= \frac{2^{26}}{2^{10}}\times2^2 \text{ bytes} = 2^{16}\times2^2 = 2^{18} \text{ bytes}$$

3^{rd} level page table = # page table entries × Page table entry size

$$= \frac{2^{18}}{2^{10}}\times2^2 \text{ bytes} = 2^{8}\times2^2 = 2^{10} \text{ bytes}$$

Now page size = 3^{rd} level size.

So no need of further level.

Q.5 (e) Solution:

(i) Since $f(x)$ is a probability density function, we have

$$\int_x f(x)dx = 1$$

That is
$$\int_0^4 f(x)\,dx = 1$$

$f(x)$ is a continuous function, we have

$$\int_0^2 ax\,dx + \int_2^4 a(4-x)dx = 1$$

or
$$2a + 2a = 1$$

$$a = \frac{1}{4}$$

(ii)
$$F(x) = P(-\infty < x \le x) = \int_{-\infty}^{x} f(x)\,dx$$

We have,

$x < 0$; $\quad F(x) = 0$

$0 \le x \le 2$; $\quad F(x) = \int_0^x \frac{x}{4}dx = \frac{x^2}{8}$

$2 \le x \le 4$; $\quad F(x) = \int_0^2 \frac{x}{4}dx + \int_2^x \frac{1}{4}(4-x)dx$

$$= \frac{1}{2} + \frac{1}{8}(8x - x^2 - 12)$$

$$= \frac{1}{8}(8x - x^2 - 8)$$

$x > 4$; $\quad F(x) = 1$

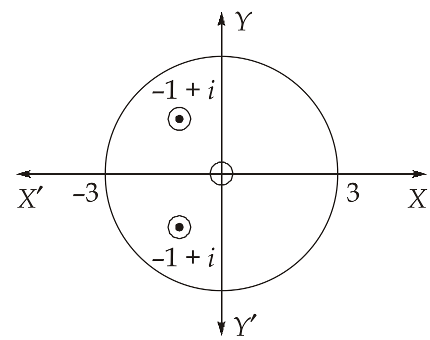
(iii)
$$P(x > 2.5) = \int_{2.5}^{4} \frac{1}{4}(4-x)\,dx$$

$$= \frac{1}{4}\left[4x - \frac{x^2}{2}\right]_{2.5}^{4} = \frac{9}{32}$$

Q.6 (a) Solution:

Here, we have

$$\frac{1}{2\pi i}\int_C \frac{e^{zt}dz}{z^2(z^2+2z+2)}$$



Poles are given by, $z = 0$ (double pole)

$z = -1 \pm i$ (simple poles)

All the four poles are inside the given circle,

$$|z| = 3$$

Residue at $z = 0$ is

$$= \lim_{z\to 0}\frac{d}{dz}z^2\frac{e^{zt}}{z^2(z^2+2z+2)} = \lim_{z\to 0}\frac{d}{dz}\frac{e^{zt}}{z^2+2z+2}$$

$$= \lim_{z\to 0}\frac{(z^2+2z+2)t\,e^{zt}-(2z+2)e^{zt}}{(z^2+2z+2)^2}$$

$$= \frac{2t\,e^0-2e^0}{4} = \frac{(t-1)}{2}$$

Residue at $z = -1 + i$

$$= \lim_{z\to -1+i}\frac{(z+1-i)e^{zt}}{z^2(z+1-i)(z+1+i)} = \lim_{z\to -1+i}\frac{e^{zt}}{z^2(z+1+i)}$$

$$= \frac{e^{-(-1+i)t}}{(-1+i)^2(-1+i+1+i)} = \frac{e^{-(-1+i)t}}{(1-2i-1)(2i)} = \frac{6^{(-1+i)t}}{4}$$

$$\int\frac{e^{zt}}{z^2(z^2+2z+2)}dz = 2\pi\, i \;\; \text{(sum of the residues)}$$

$$\frac{1}{2\pi i}\int\frac{e^{zt}}{z^2(z^2+2z+2)}dz = \frac{t-1}{2}+\frac{e^{(-1+i)t}}{4}+\frac{e^{(-1-i)t}}{4}$$

$$= \frac{t-1}{2}+\frac{e^{-t}}{4}(e^{it}+e^{it}) = \frac{t-1}{2}+\frac{e^{-t}}{4}(2\cos t)$$

$$= \frac{t-1}{2}+\frac{e^{-t}}{2}\cos t$$

Q.6 (b) (i) Solution:

The moving coil (PMMC) reads average value of current.

The Hot wire Ammeter reads rms value of current.

The electrostatic voltmeters reads rms value of voltage.

The current flowing through voltmeters will be negligible as they have high internal resistance.

Given : $i(t) = 0.5 + 0.3 \sin \omega t - 0.2 \sin 2\omega t$

$\therefore$ Average value of current, $i_{avg} = 0.5$ A

Rms value of current,
$$i_{rms} = \sqrt{(0.5)^2 + \left(\frac{0.3}{\sqrt{2}}\right)^2 + \left(\frac{0.2}{\sqrt{2}}\right)^2} = 0.561 \text{ A}$$

$\therefore$ Reading of moving coil $= i_{avg} = 0.5$ A

Reading of hot wire ammeter $= i_{rms} = 0.561$ A

Also, voltage across resistor is

$$V_R = i_{rms} \times R = 0.561 \times 1000 = 561 \text{ volt}$$

= reading of electrostatic voltmeter across 1000 Ω resistance

Also, instantaneous voltage across inductor is

$$V_L = L\frac{di}{dt} = L\,\omega(0.3\cos\omega t - 0.4\cos 2\omega t)$$
$$= 1 \times 10^{-3} \times 10^6\,(0.3\cos\omega t - 0.4\cos 2\omega t)$$
$$= 300\cos\omega t - 400\cos 2\omega t$$

$\therefore$ Reading of electrostatic voltmeter across 1 mH inductor is

$$V_L = \sqrt{\left(\frac{300}{\sqrt{2}}\right)^2 + \left(\frac{400}{\sqrt{2}}\right)^2} = 353.55 \text{ volts}$$

Q.6 (b) (ii) Solution:

Total power in the load circuit,

$$P = W_1 + W_2 = 6000 - 1000 = 5000 \text{ W}$$

$$\phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{3}\,[(6000 - (-1000))]}{6000 - 1000}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{3}(7000)}{5000}\right] = 67.58°$$

$$\cos\phi = 0.381$$

Load current per phase, $$I_P = \frac{5000}{\sqrt{3}\times 440\times 0.381} = 17.22 \text{ A}$$

Load impedance per phase, $$Z_P = \frac{V_P}{I_P} = \frac{440}{\left(\frac{17.22}{\sqrt{3}}\right)} = 44.25\ \Omega$$

Load resistance per phase, $$R_P = Z_P\cos\phi = 44.25\cos(67.58)$$

$$= 44.25 \times 0.381$$

$$R_P = 16.87\ \Omega$$

Load reactance per phase, $$X_P = Z_P\sin\phi = 44.25\sin(67.58)$$

$$X_P = 40.9\ \Omega$$

Reading of wattmeter B will be zero when power factor,

$$\cos\phi' = 0.5$$

$$\phi' = 60°$$

Since there is no change in resistance, reactance in circuit per phase,

$$X'_P = R_P\tan\phi'$$

$$X'_P = 16.87 \times \tan 60° = 29.22\ \Omega$$

Values of capacitive reactance introduced in each phase,

$$X_C = X_P - X'_P$$

$$= 40.9 - 29.22 = 11.68\ \Omega$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi\times 50\times 11.68} = 272.52\ \mu\text{F}$$

Q.6 (c) (i) Solution:

$$y = 0.516x + 33.73 \quad \text{...(i)}$$

$$x = 0.512x + 32.52 \quad \text{...(ii)}$$

$$r\frac{\partial y}{\partial x} = 0.516 \quad \text{...(iii)}$$

$$r\frac{\partial x}{\partial y} = 0.512 \quad \text{...(iv)}$$

$$r = \sqrt{\left(r\frac{\partial y}{\partial x}\right)\left(r\frac{\partial x}{\partial y}\right)} = \sqrt{0.516 \times 0.512} = 0.514$$

Coefficient of correlation = 0.514

(i) and (ii) pass through the point $(\bar{x}, \bar{y})$.

$$\therefore \quad \bar{y} = 0.516\bar{x} + 33.73 \quad ...(v)$$

$$\bar{x} = 0.512\bar{y} + 32.52 \quad ...(vi)$$

On solving (v) and (vi), we get

$$\bar{x} = 67.6,$$
$$\bar{y} = 68.61$$

Q.6 (c) (ii) Solution:

The integrand has singular points where

$$z^4 + 5z^2 + 4 = 0$$

We get, $\quad z^2 = -1, -4$

or $\quad z = \pm i, \pm 2i$

These singular points are simple poles lie inside c.

Let $z = a$ be a pole,

then $$\underset{z=a}{\text{Res}}\, f(z) \underset{z=a}{\text{Res}}\left[\frac{\phi(z)}{\psi(z)}\right] = \frac{\phi(a)}{\psi'(a)} = \frac{a\cosh \pi a}{4a^3 + 10a} = \frac{\cosh \pi a}{4a^2 + 10}$$

Hence, $$I = 2\pi i\left[\underset{z=i}{\text{Res}}\, f(z) + \underset{z=-i}{\text{Res}}\, f(z) + \underset{z=2i}{\text{Res}}\, f(z) + \underset{z=-2i}{\text{Res}}\, f(z)\right]$$

$$= 2\pi i\left[\frac{\cosh \pi i}{6} + \frac{\cosh \pi i}{6} - \frac{\cosh 2\pi i}{6} - \frac{\cosh 2\pi i}{6}\right]$$

$$= \frac{2\pi i}{3}[\cosh \pi i - \cosh 2\pi i]$$

Since, $\quad \cosh z = \cos iz$, we obtain

$$I = \frac{2\pi i}{3}[\cos(-\pi) - \cos(-2\pi)] = \frac{-4\pi i}{3}$$

Q.7 (a) Solution:

(i) Here in this case line commutated inverter means the battery is supplying power to source, since battery is supplying total power which is desired to be transferred i.e. 5000 W and $I_{0,rms}^2 R$ loss. So, according to energy balance equation.

$$E_b I_0 = 5000 + I_{0,rms}^2 R$$

$$I_{0'rms} \simeq I_0 \text{ because of large inductor}$$

$$500\, I_0 = 5000 + I_0^2\, 12.4$$

$12.4 I_0^2 - 500 I_0 + 5000 = 0$ is a quadratic equation, the roots of this equation are I_0 = 21.96 A or 18.35 A. Considering the lowest value of load current,

$$I_0 = 18.35 \text{ A}$$

$$V_0 = I_0 R - E_b$$

$$\frac{3V_{mL}}{\pi} \cos\alpha = I_0 R - E_b$$

$$\frac{3 \times \sqrt{2} \times 440}{\pi} \cos\alpha = -500 + (18.35)\,(12.4)$$

$$\alpha = \cos^{-1}\left(\frac{-272.46 \times \pi}{3 \times \sqrt{2} \times 440}\right) = 117.2920°$$

(a) Input power factor $= \dfrac{I_{s_1}}{I_s} \times \cos\alpha = \dfrac{3}{\pi}\cos(117.2920°)$

$$= 0.4378 \text{ lagging}$$

(b) Rms value of fundamental ac current

$$I_{s_1} = \frac{3}{\pi} I_s = \frac{3}{\pi}\left(I_0 \sqrt{\frac{2}{3}}\right) = \frac{\sqrt{6}}{\pi} I_0 = \frac{\sqrt{6}}{\pi} \times 18.35 = 14.307 \text{ A}$$

(c) Efficiency of energy transferred

$$= \frac{5000}{E_b I_0} = \frac{5000}{500 \times 18.35} \times 100$$

$$\eta = 54.49\%$$

(ii) The maximum usable firing angle is obtained by making load current equal to zero,

$$\frac{3V_{mL}}{\pi} \cos\alpha = -E + I_0 R$$

$$\frac{3 \times \sqrt{2} \times 440}{\pi} \cos\alpha = -500$$

$$\alpha = \cos^{-1}\left[\frac{-500}{594.208}\right]$$

$$\alpha = 147.29°$$

(iii) Rms current rating of SCR is,

$$I_{T_{rms}} = I_0\sqrt{\frac{1}{3}} = 18.35\sqrt{\frac{1}{3}} = 10.6 \text{ A}$$

SCR voltage rating = Maximum value of source voltage

$$= V_m = 440\sqrt{2} = 623 \text{ V}$$

Q.7 (b) (i) Solution:

Using Cauchy's integral formula, $\int_C \frac{\log z}{(z-1)^3} dz$

$$C : |z-1| = \frac{1}{2}$$

Poles are determined by putting denominator equal to zero,

$$(z-1)^3 = 0$$

$$z = 1, 1, 1$$

There is one pole of order three at $z = 1$ which is inside the circle C

$$\int \frac{f(z)}{(z-a)^3} dz = \frac{2\pi i f^2(a)}{2!} \qquad \left[\because \int \frac{f(z)}{(z-a)^{n+1}} dz = \frac{2\pi i}{n!} \cdot f^n(a)\right]$$

$$= \pi i\left[\frac{d^2}{dz^2}\log z\right]_{z=1} = \pi i\left[\frac{d}{dz}\left(\frac{1}{z}\right)\right]_{z=1}$$

$$= \pi i\left(\frac{-1}{z^2}\right)_{z=1} = -\pi i$$

Q.7 (b) (ii) Solution:

The given equation can be written as

$$-\frac{dr}{d\theta}\cos\theta + r\sin\theta = r^2 \qquad \text{...(i)}$$

Dividing equation (i) by $r^2 \cos\theta$, we get

$$-r^{-2}\frac{dr}{d\theta} + r^{-1}\tan\theta = \sec\theta \qquad \text{...(ii)}$$

Putting, $r^{-1} = V$

So that, $-r^{-2}\frac{dr}{d\theta} = \frac{dV}{d\theta}$ in equation (ii), we get

$$\frac{dV}{d\theta} + V\tan\theta = \sec\theta$$

$$\text{I.F.} = e^{\int \tan\theta \, d\theta} = e^{\log\sec\theta} = \sec\theta$$

Solution is

$$V\sec\theta = \int \sec\theta \sec\theta \, d\theta + C$$

$$V\sec\theta = \int \sec^2 d\theta + C$$

$$\frac{\sec\theta}{r} = \tan\theta + C$$

$$r^{-1} = (\sin\theta + C\cos\theta)$$

$$r = \frac{1}{\sin\theta + C\cos\theta}$$

Q.7 (c) Solution:

(i) Program development life cycle:

A program is needed to instruct the computer about the way a task is to be performed. The instructions in a program have three essential parts:

1. Instructions to accept the input data that needs to be processed.
2. Instructions that will act upon the input data and process it.
3. Instructions to provide the output to user.

The instructions in a program are defined in a specific sequence. Writing a computer program is not a straightforward task. A person who writes the program (computer programmer) has to follow the program development life cycle.

Following are the steps that are followed by the programmer for writing a program:

- **Problem analysis:** The programmer first understands the problem to be solved. The programmer determines the various ways in which the problem can be solved, and decides upon a single solution which will be followed to solve the problem.
- **Program design:** The selected solution is represented in a form, so that it can be coded. This requires three steps:
  - An algorithm is written, which is in English-like explanation of the solution. A flowchart is drawn, which is a diagrammatic representation of the solution.
  - The solution is represented diagrammatically, for easy understanding and clarity.

  - A pseudo code is written for the selected solution. Pseudo code uses the structured programming constructs. The pseudo code becomes an input to the next phase.
- **Program development:**
  - The computer programming languages are off different kinds low level languages, and high level languages like C, C++ and Java. The pseudo code is coded using a suitable programming language.
  - The coded pseudo code or program is compiled for any syntax errors. Syntax errors arise due to the incorrect use of programming language or due to the grammatical errors with respect to the programming language used. During compilation, the syntax errors, if any, are removed.
  - The successfully compiled program is now ready for execution.
  - The executed program generates are output result, which may be correct or incorrect. The program is tested with various inputs, to see that it generates the desired results. If incorrect results are displayed, then the program has semantic error (logical error). The semantic errors are removed from the program to get the correct results.
  - The successfully tested program is ready for use and is installed on the user's machine.
- **Program documentation:** The program is properly documented, so that later on, anyone can use it and understand its working. Any changes made to the program, after installation, forms part of the maintenance of program. The program may require updating, fixing of errors etc. during the maintenance phase.

Steps of program development cycles are:

Program analysis:

- Understand the problem
- Have multiple solutions
- Select a solution

Program design:

- Write algorithm
- Write flowchart
- Write pseudo code

Program development:

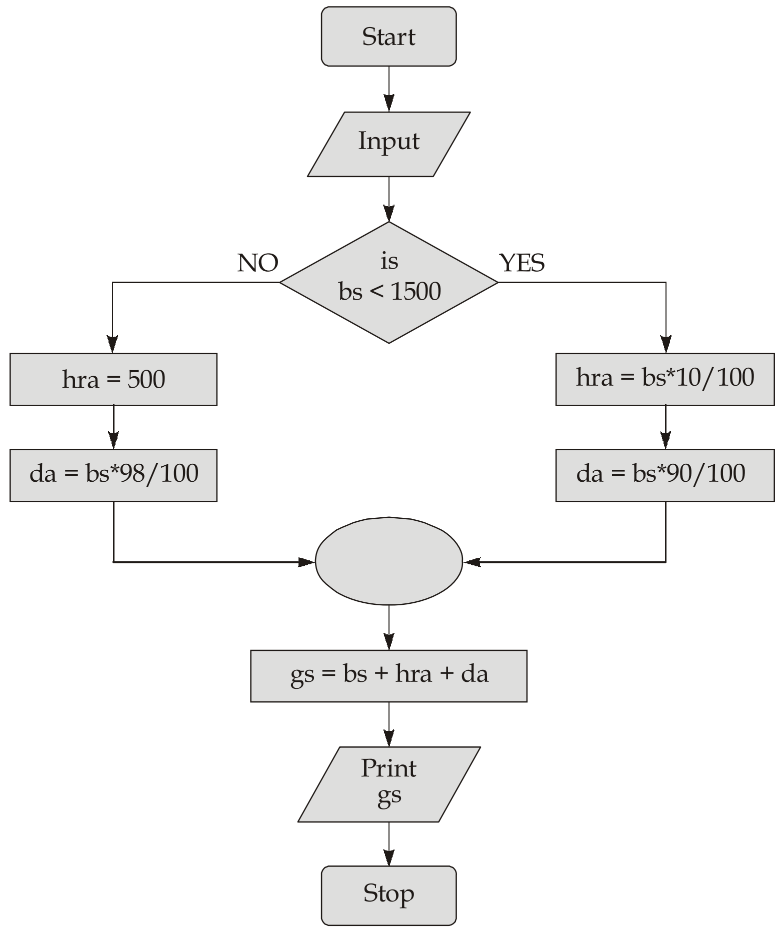
- Choose a programming language

- Write the program by converting the pseudo code, and then using the programming language.
- Compile the program and remove syntax errors, if any
- Execute the program
- Test the program. Check the output results with different inputs. If the output is incorrect, modify the program to get correct results.
- Install the tested program on the user's computer.

Program documentation and maintenance:

- Document the program for later use.
- Maintain the program for updating, removing errors, changing requirements etc.

(ii)
```
/* calculation of gross salary */
main()
{
    float bs, gs, da, hra;
    printf("Enter base salary");
    scanf("%f",&bs);
    if (bs < 1500)
{
    hra=bs*10/100;
    da=bs*90/100
}
else
{
    hra=500;
    da=bs*98/100;
}
    gs=bs+hra+da;
    printf("gross salary=Rs.%f",gs);
}
```



Q.8 (a) Solution:

Given :

$$V = 100 \text{ volt}$$

$$I = 9 \text{ A}$$

$$\cos\phi = 0.1 \text{ (lag)} \quad \Rightarrow \quad \phi = 84.26°$$

$$R_p = 3 \text{ k}\Omega, R_C = 0.1\ \Omega$$

$$L_p = 30 \text{ mH}$$

$$\text{Load Power} = \text{True Power} = VI\cos\phi = 100 \times 9 \times 0.1 = 90 \text{ Watt}$$

$$\text{p.c. reactance} = \omega L_p = 2\pi f L_p = 2\pi \times 50 \times 30 \times 10^{-3} = 9.42\ \Omega$$

Let phase angle of p.c. circuit = β, then

$$\tan\beta = \frac{\omega L_p}{R_p} = \frac{9.42}{3000} = 0.00314 \text{ radian}$$

(i) When p.c. is connected on supply side then,

$$P_{total} = \text{Total Power measured}$$
$$= P_{load} + \text{Power loss in p.c.}$$
$$P_{load} = VI\cos\phi = 90 \text{ Watt}$$

and Power loss in p.c. resistance

$$= \frac{V^2}{R_p} = \frac{(100)^2}{3000} = 3.33 \text{ Watt}$$

Also, $\text{Measured power} = [VI\cos\phi(1 + \tan\phi\tan\beta)] + I^2R_c$

$$= 90\,(1 + 9.95 \times 0.00314) + 3.33$$

$\therefore$ $W_m = 92.81 + 3.33 = 96.14 \text{ Watt}$

$\therefore$ $\%\ \text{error} = \left(\frac{W_m - W_T}{W_T}\right)\times 100$

$$= \left(\frac{96.14 - 90}{90}\right)\times 100 = 6.82\%$$

(ii) When the p.c. is connected on load side then,

Wattmeter reads, $W_m = P_{load} + P_{loss} \text{ in c.c.}$

$\therefore$ $\text{Total Power} = P_{load} + I^2 \times R_{c.c.} = 90 + (9)^2 \times 0.1 = 98.1 \text{ Watt}$

Also, $R_{load} = 11.1 \times 0.1 = 1.11\ \Omega$

$$X_{load} = 11.1 \times 0.995 = 11.05\ \Omega$$

$\therefore$ $R_{load} + R_{c.c.} = 1.11 + 0.1 = 1.21\ \Omega$

$$= R_{total} = \text{Resistance of load + Resistance of c.c.}$$
$$X_{total} + X_{cc} = 11.05\ \Omega = X_{total}$$
$$= \text{Reactance of load + Reactance of c.c.}$$

$\therefore$ Impedance of load including c.c.

$$= \sqrt{(1.21)^2 + (11.05)^2} = 11.1\ \Omega$$

$\therefore$ p.f. of load including c.c. $= \frac{1.21}{11.1} = 0.109$

$\therefore$ $\phi = 83.75°$ and $\tan\phi = 9.132$

$\therefore$ Reading of wattmeter, $W_m = VI\cos\phi(1 + \tan\phi\tan\beta)$

$$= 98.1(1 + 9.12 \times 0.00314)$$
$$= 100.9 \text{ Watt} = 100.909 \text{ W}$$
$$= \text{Measured power}$$

and True power = 90 Watt

$$\therefore \quad \% \text{ error} = \left(\frac{W_m - W_T}{W_T}\right)\times 100$$

$$= \left(\frac{100.9 - 90}{90}\right)\times 100 = 12.1\% \text{ or } 12.12\%$$

Q.8 (b) Solution:

Let $P(x, y, z)$ be any point on the sphere and $A(3, 4, 12)$ the given point so that

$$AP^2 = (x-3)^2 + (y-4)^2 + (z-12)^2 = f(x, y, z) \text{ (say)}$$

We have to find the maximum and minimum values of $f(x, y, z)$ subject to the condition

$$\phi(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$$

Let

$$F(x, y, z) = f(x, y, z) + \lambda\phi(x, y, z)$$

$$= (x-3)^2 + (y-4)^2 + (z-12)^2 + \lambda(x^2 + y^2 + z^2 - 1)$$

Then,

$$\frac{\partial F}{\partial x} = 2(x-3) + 2\lambda x$$

$$\frac{\partial F}{\partial y} = 2(y-4) + 2\lambda y$$

$$\frac{\partial F}{\partial z} = 2(z-12) + 2\lambda z$$

Also,

$$\frac{\partial F}{\partial x} = 0, \frac{\partial F}{\partial y} = 0 \text{ and } \frac{\partial F}{\partial z} = 0$$

$$x - 3 + \lambda x = 0$$

$$y - 4 + \lambda y = 0$$

$$z - 12 + \lambda z = 0$$

which gives

$$\lambda = -\frac{x-3}{x} = -\frac{y-4}{y} = -\frac{z-12}{z} \quad \text{...(iii)}$$

$$= \pm\frac{\sqrt{[(x-3)^2 + (y-4)^2 + (z-12)^2}}{\sqrt{x^2 + y^2 + z^2}}$$

$$= \pm\frac{\sqrt{f}}{1}$$

Substituting for λ in (iii), we get

$$x = \frac{3}{1+\lambda} = \frac{3}{1 \pm \sqrt{f}}$$

$$y = \frac{4}{1 \pm \sqrt{f}}$$

$$z = \frac{12}{1 \pm \sqrt{f}}$$

$$\therefore \quad x^2 + y^2 + z^2 = \frac{9+16+144}{\left(1 \pm \sqrt{f}\right)^2} = \frac{169}{\left(1 \pm \sqrt{f}\right)}$$

or $$1 \pm \sqrt{f} = \pm 13$$

$$\sqrt{f} = 12, 14$$

We have left out the negative values of $\sqrt{f}$ because $\sqrt{f}$ = AP is positive. Hence, maximum AP = 14 and minimum AP = 12.

Q.8 (c) Solution:

(i) For given expression:

$$(124)_8 - (36)_{10} + (40)_{16} = (Z\,1\,1\,X\,0\,0\,Y)_2$$

$$(124)_8 = (1\,0\,1\,0\,1\,0\,0)_2$$

$$(36)_{10} = (1\,0\,0\,1\;\,0\,0)_2$$

$$(40)_{16} = (1\,0\,0\,0\,0\,0\,0)_2$$

$$(124)_8 + (40)_{16} = (1\,0\,0\,1\,0\,1\,0\,0)_2$$

$$(124)_8 + (40)_{16} - (36)_{10} = 1\,0\,0\,1\,0\,1\,0\,0 - 1\,0\,0\,1\,0\,0 = (1\,1\,1\,0\,0\,0\,0)_2$$

On comparing with $(Z\,1\,1\,X\,0\,0\,Y)_2$

$$Z = 1, \quad X = 0, \quad Y = 0$$

(ii) Given, LHS = $(A+C)(AD+A\bar{D})+AC+C$

$= (A+C)\,A(D+\bar{D})+(A+1)\,C$ ...(Distributive property)

$= (A + C)\,A + C$ ...(Complement, Identity, Idempotent

$= AA + AC + C$ ...(Distributive)

$= A + C(A + 1)$ ...(Idempotent, Distributive)

$= A + C$ ...(Identity)

= RHS

Given, LHS = $\bar{A}(A+B)+(B+A)(A+\bar{B})$

$= \bar{A}A+\bar{A}B+BA+B\bar{B}+AA+A\bar{B}$ ...(Distributive)

$= A(\bar{A}+A)+A(B+\bar{B})+\bar{A}B+B\bar{B}$

...(Distributive, Idempotent, Identity)

$$= A+\overline{A}B+B\overline{B}$$

$$= (A+\overline{A})\,(A+B)+B\overline{B} \quad \text{...((Distributive and Absorption law)}$$

$$= A+B(1+\overline{B}) \quad \text{...(Identity, Distributive)}$$

$$= A + B = \text{RHS}$$

Given, LHS $= \overline{A+\overline{BC}+\overline{CD}}+\overline{\overline{B}C}+(\overline{A\oplus B}+C)\cdot(\overline{\overline{A}+\overline{B}})$

$$= \overline{A+\overline{BC}+\overline{CD}}+\overline{\overline{B}C}+(\overline{A}\overline{B}+AB+C)\cdot(\overline{\overline{A}+\overline{B}}) \quad \text{...(EXNOR property)}$$

$$= \overline{A}\cdot\overline{\overline{BC}}\cdot\overline{\overline{CD}}+\overline{\overline{B}}+\overline{C}+(\overline{A}\overline{B}+AB+C)\cdot(\overline{\overline{A}}\cdot\overline{\overline{B}})$$

...(By De Morgan's law)

$$= \overline{A}\cdot BC\cdot CD+B+\overline{C}+(\overline{A}\overline{B}+AB+C)\,(AB) \quad \text{...(Complement)}$$

$$= \overline{A}BCD+B+\overline{C}+\overline{A}\overline{B}AB+ABAB+CAB$$

...(Distributive, Associative)

$$= \overline{A}BCD+B+\overline{C}+0+AB+CAB$$

$$= B(1+\overline{A}CD)+\overline{C}+AB(1+C) \quad \text{...(Distributive, Identity)}$$

$$= B+\overline{C}+AB$$

$$= B(1+A)+\overline{C} \quad \text{...(Distributive,)}$$

$$= B+\overline{C} = \text{RHS}$$

Given, LHS $= A\overline{BC}[AB+\overline{C}(\overline{B\oplus C}+AC)]$

$$= A\overline{BC}[AB+\overline{C}[(\overline{B}\,\overline{C}+BC)+AC] \quad \text{...(EXNOR)}$$

$$= A\,\overline{BC}[AB+\overline{B}\,\overline{C}\,\overline{C}+BC\,\overline{C}+AC\,\overline{C}] \quad \text{...(By distributive law)}$$

$$= A\,\overline{BC}[AB+\overline{B}\,\overline{C}+0+0] \quad \text{...(Complement, Associative)}$$

$$= A\,(\overline{B}+\overline{C})\,[AB+\overline{B}\,\overline{C}] \quad \text{...(De Morgan's law)}$$

$$= A[\overline{B}\,AB+\overline{B}\,\overline{B}\,\overline{C}+\overline{C}\,AB+\overline{C}\,\overline{B}\,\overline{C}] \quad \text{...(Distributive)}$$

$$W = 0+A\,\overline{B}\,\overline{C}+A\,\overline{C}\,B+A\,\overline{C}\,\overline{B} \quad \text{...(Distributive, Complement)}$$

$$= A\,\overline{C}\,(\overline{B}+B+\overline{B}) \quad \text{...(Distributive)}$$

$$= A\,\overline{C}\,(1) = A\,\overline{C} = \text{RHS}$$

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