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Detailed Solutions

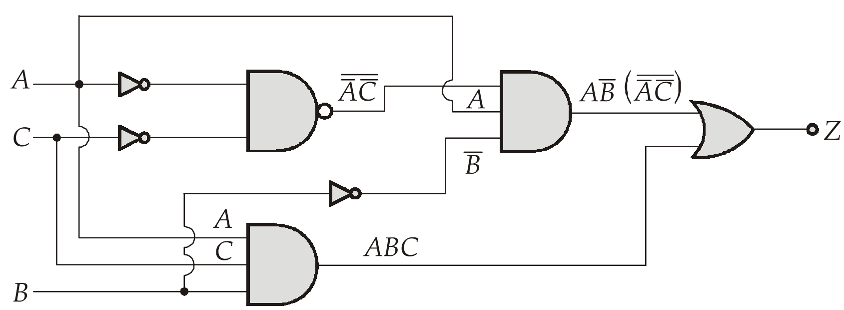
ESE-2024 Mains Test Series

E & T Engineering Test No : 8

Section A : Digital Circuits + Analog Circuits

Q.1 (a) Solution:

(i)



The output, $$Z = ABC + \left(A\bar{B}(\overline{\bar{A}\bar{C}})\right)$$

(Since Demorgan theorem says, $\overline{AB} = \bar{A} + \bar{B}$)

$$= ABC + A\bar{B}\left(\bar{\bar{A}} + \bar{\bar{C}}\right)$$

$$= ABC + A\bar{B}(A + C)$$

$$= ABC + AA\bar{B} + A\bar{B}C$$

$$= ABC + A\bar{B} + A\bar{B}C$$

$$= ABC + A\bar{B}(1 + C)$$

$$= ABC + A\bar{B}$$

$$= A\left[\bar{B} + BC\right]$$

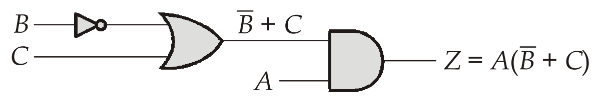
(∴ Distribution theorem says, $A + BC = (A + B)(A + C)$

$$= A\left[\left(\overline{B}+B\right)\left(\overline{B}+C\right)\right]$$

$$= A(\overline{B}+C)$$

$$= A\overline{B} + AC$$

∴ $$Z = A\overline{B} + AC = A(\overline{B}+C)$$

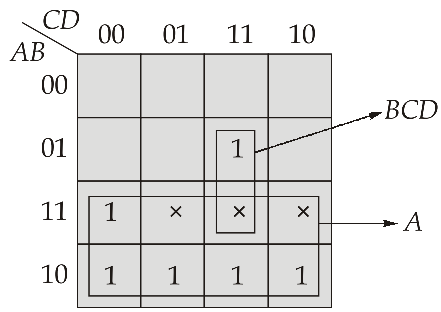


(ii) Given, $V_B = 12$ V

A/D converter's output is four bit binary number corresponding to the battery voltage in steps of 1 V with the output HIGH when battery voltage is greater than 6 V. The truth table can be, thus, written as below:

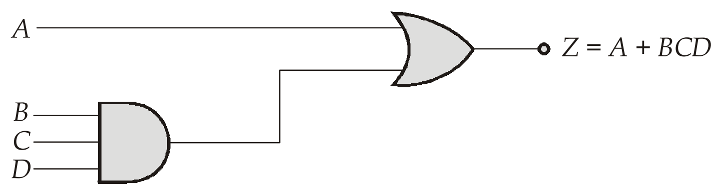
Battery Voltage (in V)	***A***	***B***	***C***	***D***	***Z***
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	×
14	1	1	1	0	×
15	1	1	1	1	×

The output expression of 'Z' can be obtained using K-Map as below:



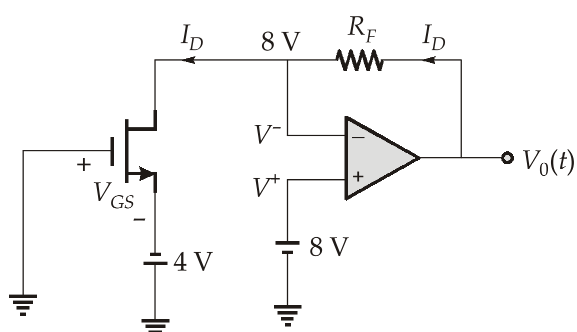
$\therefore \quad Z = A + BCD$

Logic Circuit:



Q.1 (b) Solution:

Applying DC analysis, we get,



Given data:

$$V_{th} = 2 \text{ V}$$

$$K = \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right) = \frac{140}{2} \times \left(\frac{20}{5}\right) \times 10^{-6} = 280 \times 10^{-6} \text{ A/V}^2$$

Using the concept of virtual ground, $V^- = V^+ = 8$ V. Thus,

$$V_{DS} = 8 - (-4) = 12 \text{ V}$$

$$V_{DS} \geq V_{GS} - V_{th}$$

So, MOSFET is in saturation region.

So, drain current, $\quad I_D = K(V_{GS} - V_{th})^2$

Here, $V_{GS} = 4\text{ V}$

$$I_D = 280 \times 10^{-6} (4 - 2)^2$$

$$= 280 \times 10^{-6} \times 4$$

$$I_D = 1.12\text{ mA}$$

So, output voltage V_{out} due to dc source

$$V_0 = V^- + I_D R_F$$

$$V_0 = 8 + 1.12 \times 25 = 36\text{ V}$$

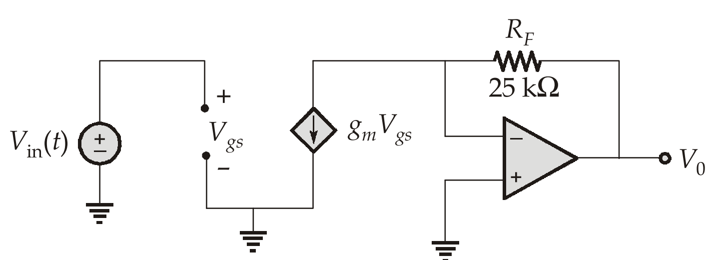
and

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = 2\,K(V_{GS} - V_{Th})$$

$$g_m = 2 \times 280 \times 10^{-6}(4 - 2)$$

$$= 1.12\text{ mA/V}$$

For AC analysis, the equivalent circuit can be drawn as below:



Here, $V_{gs} = V_{in}(t) = 230 \sin(4\pi \times 10^3 t)\text{ mV}$

Hence, $i_d = g_m V_{gs}$ or $i_d = g_m V_{in}(t)$

$$= 1.12 \times 230 \sin(4\pi \times 10^3 t) \times 10^{-6}\text{ A}$$

$$i_d = 0.2576 \sin(4\pi \times 10^3 t)\text{ mA}$$

So, output voltage due to ac source,

$$v_0 = i_d(R_F) = 0.2576 \sin(4\pi \times 10^3 t) \times 25$$

$$v_0 = 6.44 \sin(4\pi \times 10^3 t)\text{ V}$$

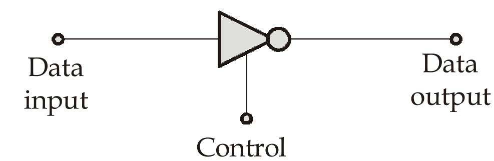
So, output voltage,

$$V_0(t) = V_0 + v_0$$

$$V_0(t) = [36 + 6.44 \sin(4\pi \times 10^3 t)]\text{ Volts}$$

Q.1 (c) Solution:

(i)

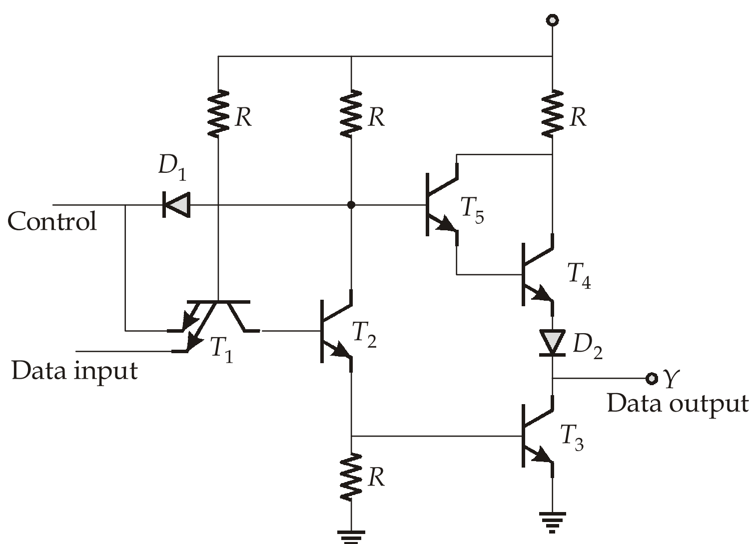


A tri-state inverter has three stable states: a high output state, a low output state, and a high-impedance state. It works as a normal NOT Gate when Control input is '1' and is in high impedance state when Control input is '0'.

Truth Table:

Data Input	Control	Data Output
0	0	High Impedance
1	0	High Impedance
0	1	1
1	1	0

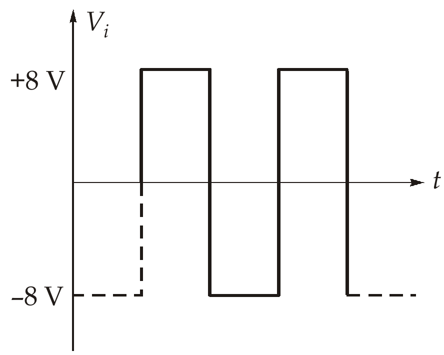
(ii)

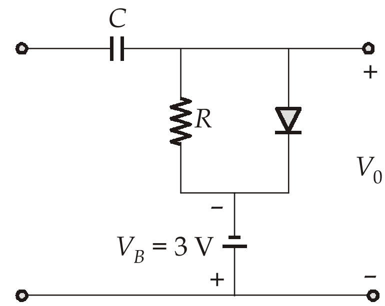


When the control input is low, the drive is removed from T_3 and T_4. The diode D_2 is forward biased and acts as a closed switch. Regardless of the state of the data input, both T_3 and T_4 are cut-off and the output is in the High impedance state. When the control input is High diode D_2 is reverse biased and acts as open circuit. The circuit works as a normal inverter and the output Y is logic 1 or 0 depending on the data input.

Q.1 (d) Solution:

(i) Given circuit is

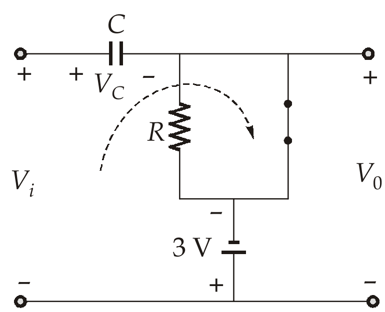




Given circuit is negative clamper circuit.

When $V_i = +8$ V

Diode is forward biased and acts as short circuit. The capacitor charges through V_i.



Applying KVL in input loop,

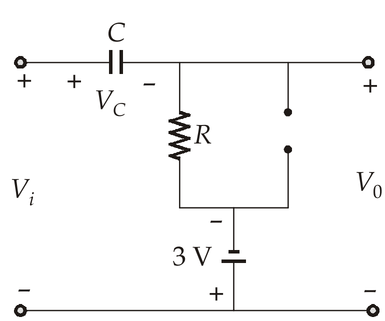
$$-V_i + V_c - 3\text{ V} = 0$$

$$V_c = V_i + 3\text{ V} \qquad \text{...(i)}$$

The capacitor is thus, charged to 11 V.

When $V_i = -8$ V

Diode is reverse biased i.e, open circuited. Further, due to very large time constant RC, the capacitor doesn't discharge and hence, the diode remains reverse biased at steady state.

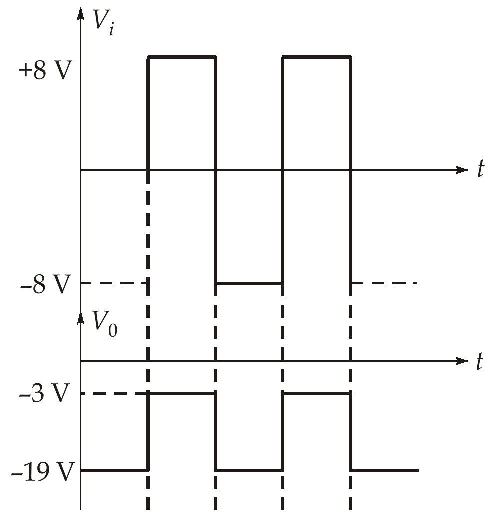


At steady state, $V_c = 11$ V and diode is reverse biased. Thus,

$$V_0 = V_i - V_C$$

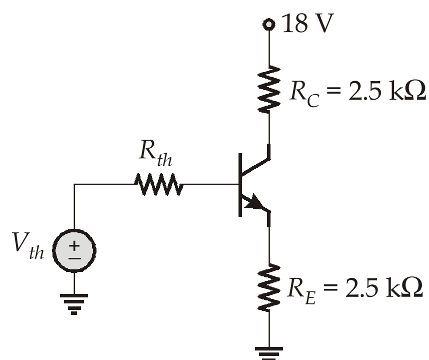
For $V_i = +8$ V $\quad V_0 = 8 - 11 = -3$ V

For $V_i = -8$ V $\quad V_0 = -8 - 11 = -19$ V



(ii) 1. DC analysis:

The Thevenin equivalent dc circuit can be drawn as below:



$$V_{th} = \frac{V_{CC} \times R_2}{R_1 + R_2} = \frac{18 \times 60}{60 + 45} = 10.285 \text{ V}$$

$$R_{th} = \frac{60 \times 45}{60 + 45} = 25.714 \text{ k}\Omega$$

Now, applying KVL in base emitter loop, we get

$$I_B = \frac{V_{th} - V_{BE}}{R_{th} + (1+\beta)R_E} = \frac{10.285 - 0.7}{25.714 + 101 \times 2.5}$$

$$= 0.0344 \text{ mA}$$

$$I_C = \beta I_B = 3.44 \text{ mA}$$

$$I_E = (1 + \beta)I_B = 101 \times 0.0344 = 3.4744 \text{ mA}$$

Now, applying KVL in collector emitter loop, we get

$$V_{CE} = V_{CC} - I_E R_E - I_C R_C$$
$$= 18 - (3.44 + 3.4744) \times 2.5 = 0.715 \text{ V}$$

So, Quiescent point,

$$(V_{ECQ}, I_{CQ}) = (0.715 \text{ V}, 3.44 \text{ mA})$$

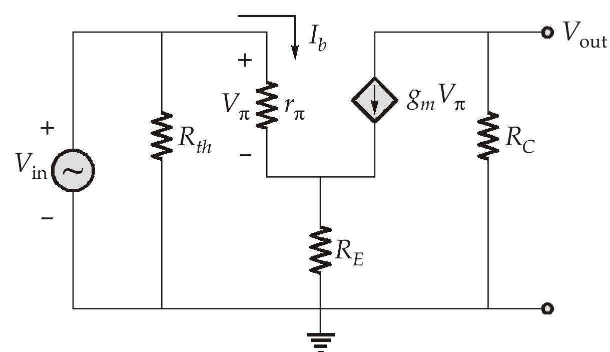
2. **AC Analysis :** The small signal parameters of the transistor can be obtained as below:

$$r_\pi = \frac{0.026}{0.0344} = 0.755 \text{ k}\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{3.44}{0.026} = 132.3 \text{ mA/V}$$

$$r_0 = \frac{V_A}{I_{CQ}} = \infty \quad \text{(given)}$$

Equivalent small signal model of the given circuit can be represented as



$$V_{out} = -g_m V_\pi R_C \quad \text{...(i)}$$

On applying KVL from input around *BE* loop, we get

$$V_{in} = V_\pi + \left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right) R_E \quad \text{...(ii)}$$

From equation (i) and (ii)

$$\frac{V_{out}}{V_{in}} = \frac{-g_m R_C}{1 + \left(\frac{1}{r_\pi} + g_m\right) R_E}$$

$$= \frac{-132.3 \times 2.5}{1 + \left(\frac{10^{-3}}{0.755} + 132.3 \times 10^{-3}\right) 2.5 \times 10^3}$$

$$A_V = -0.987 \text{ V/V}$$

Q.1 (e) Solution:

Let output of logic circuit is Y.

$$Y = \bar{C}\bar{D} + A \oplus B$$

$$\therefore \quad Y = (A \oplus B) + \overline{C + D}$$

Also, serial input is same as output Y.

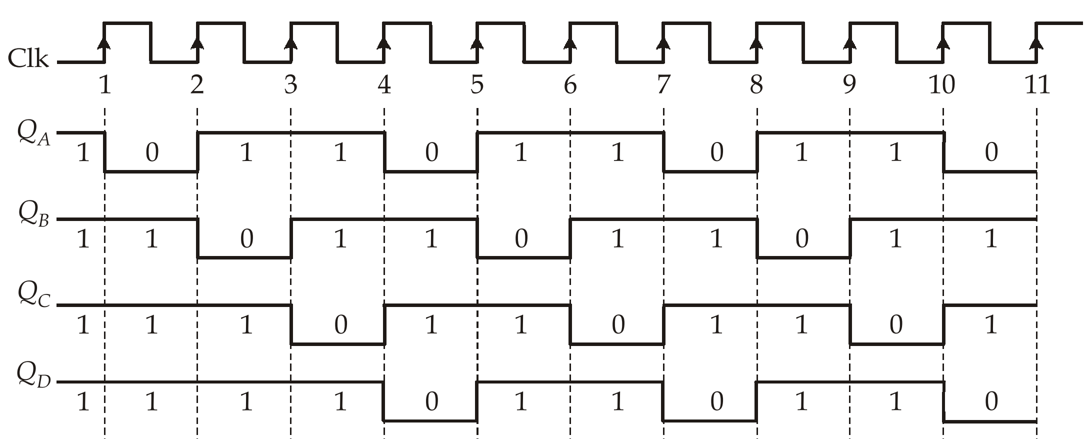
Initially, $ABCD = 1111$

$$\therefore \quad Y = (1 \oplus 1) + \overline{1+1} = 0 + 0 = 0$$

$\therefore$ Serial In = 0, in SIPO register serial input is applied to MSB flip-flop.

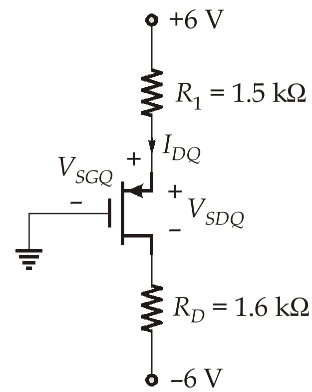
Clock	Serial In	A	B	C	D	Output, Y
–	X	1	1	1	1	0
1st	0	0	1	1	1	1
2nd	1	1	0	1	1	1
3rd	1	1	1	0	1	0
4th	0	0	1	1	0	1
5th	1	1	0	1	1	1
6th	1	1	1	0	1	0
7th	0	0	1	1	0	1
8th	1	1	0	1	1	1
9th	1	1	1	0	1	0
10th	0	0	1	1	0	1
11th	1	1	0	1	1	1

The output waveform is



Q.2 (a) Solution:

(i) For DC analysis of the given circuit, all the coupling capacitor can be open circuited and the resultant equivalent circuit will be as shown below:



By assuming that the transistor is in saturation mode and taking the numerical value of I_{DQ} in mA, we get,

$$I_{DQ} = K_p\left(V_{SGQ} - |V_{tp}|\right)^2$$

where, $V_{SGQ} = 6 - 1.5\,I_{DQ}$

So, $I_{DQ} = 2.5(6 - 1.5I_{DQ} - 1.8)^2$

$$= 2.5(4.2 - 1.5I_{DQ})^2$$

$$I_D = 2.5(4.2^2 - 2 \times 4.2 \times 1.5I_{DQ} + 2.25\,I_{DQ}^2)$$

$$I_D = 2.5\left(17.64 - 12.6I_{DQ} + 2.25I_{DQ}^2\right)$$

$$5.625I_{DQ}^2 - 32.5I_{DQ} + 44.1 = 0$$

By solving the above equation, we get

$$I_{DQ} = 3.6 \text{ mA}, 2.177 \text{ mA}$$

For $I_{DQ} = 3.6 \text{ mA}, V_{SGQ} = 6 - 1.5 \times 3.6 = 0.6 \text{ V} < |V_{tp}|$

$I_{DQ} = 2.177 \text{ mA}, V_{GSQ} = 6 - 1.5 \times 2.177 = 2.7345 \text{ V} > |V_{tp}|$

So, for the assumed case, the valid value of I_{DQ} is 2.177 mA.

$$V_{SDQ} = 12 - (1.5 + 1.6)I_{DQ} = 12 - 3.1 \times 2.177 = 5.2513 \text{ V}$$

$$V_{SGQ} - |V_{TP}| = 2.734 - 1.8 = 0.934 \text{ V}$$

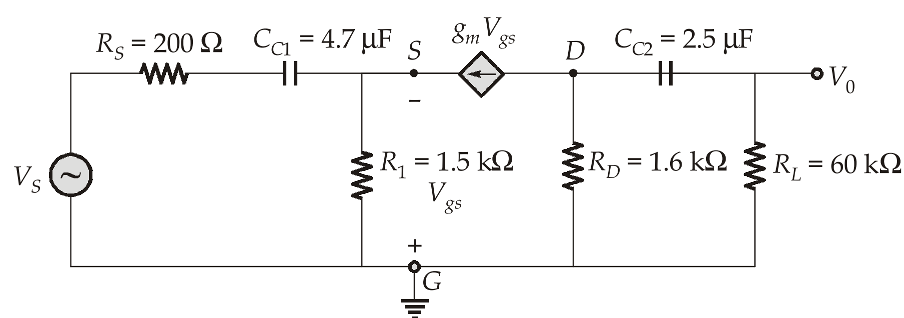
Since, $V_{SDQ} > V_{SGQ} - |V_{tp}|$, so, the initial assumption is correct about the mode of operation of transistor.

The small-signal parameters of the transistor are,

$$g_m = 2K_p(V_{SGQ} - |V_{tp}|) = 2 \times 2.5 \times 0.934$$
$$= 4.67 \text{ mA/V}$$

$$r_0 = \frac{V_A}{I_{DQ}} = \frac{1}{\lambda I_{DQ}} = \infty$$

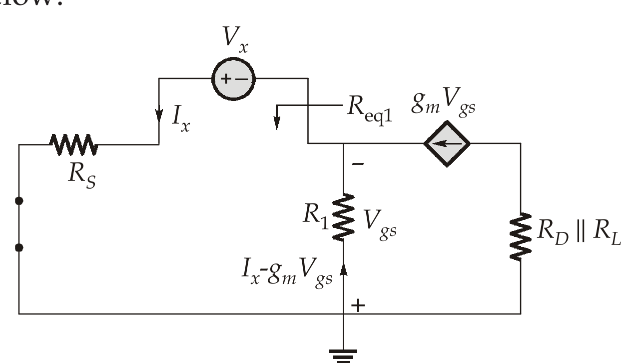
(ii) The small signal equivalent of the given amplifier will be



Calculation of time constant (τ_1) associated with C_{C1}:

$$\tau_1 = R_{eq1}\, C_{C1}$$

While calculating R_{eq1}, C_{C2} must be short circuited and voltage source V_s should be deactivated. To calculate R_{eq1}, a voltage source V_x is connected across the capacitor C_{C1} as shown below:



$$V_x = R_s I_x + V_{gs} \quad \text{...(1)}$$

and $$V_{gs} = R_1(I_x - g_m V_{gs})$$

∴ $$V_{gs} = \frac{R_1 I_x}{1 + g_m R_1}$$

Put in equation (1) we get,

$$\frac{V_x}{I_x} = \left(R_s + \frac{R_1}{1 + g_m R_1}\right)$$

$$R_{eq1} = \frac{V_x}{I_x} = R_s + \frac{R_1}{1+g_m R_1} = 200 + \frac{1500}{1+4.67\times1.5}$$

$$= 387.382\ \Omega$$

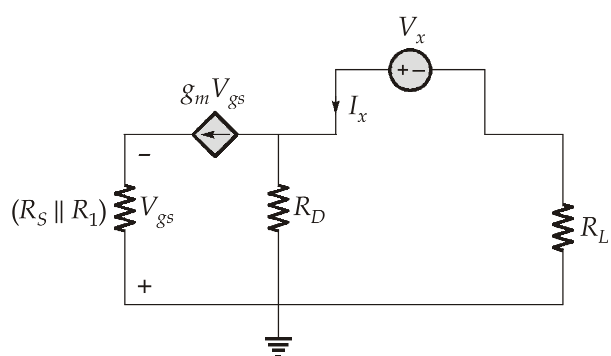
$$\tau_1 = R_{eq1}\, C_{C1} = 387.382 \times 4.7 \times 10^{-6}$$

$$= 1.82\text{ ms}$$

Calculation of time constant (τ_2) associated with C_{C2}:

$$\tau_2 = R_{eq2}C_{C2}$$

While calculating R_{eq2}, C_{C1} must be short circuited and the voltage source must be deactivated. To calculate R_{eq2}, a voltage source V_x is connected across the capacitor C_{C2} as shown below:



$$V_{gs} = -g_m\, V_{gs}(R_s \parallel R_1)$$

So, $$V_{gs} = 0$$

$$R_{eq2} = \frac{V_x}{I_x} = R_D + R_L = 60 + 1.6 = 61.6\text{ k}\Omega$$

$$\tau_2 = R_{eq2}C_{C2} = 61.6 \times 10^3 \times 2.5 \times 10^{-6}$$

$$= 0.154 = 154\text{ ms}$$

(iii) The corner frequency associated with C_{C1} is

$$f_{C1} = \frac{1}{2\pi\tau_1} = \frac{1}{2\pi\times1.82\times10^{-3}} = 87.447\text{ Hz}$$

The corner frequency associated with C_{C2} is

$$f_{C2} = \frac{1}{2\pi\tau_2} = \frac{1}{2\pi\times154\times10^{-3}} = 1.033\text{ Hz}$$

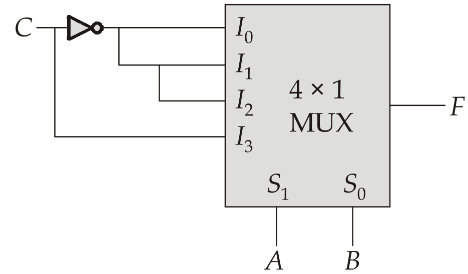
So, the corner frequency due to C_{C1} dominates that due to C_{C2}. Hence, the lower cut-off frequency of the amplifier can be given by

$$f_L = f_{C1} = 87.447\text{ Hz}$$

Q.2 (b) Solution:

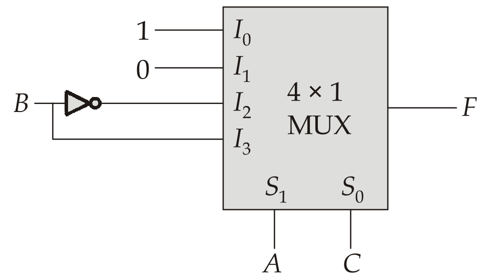
(i) If A and B are connected to the select lines, the inputs to the data lines of 4×1 MUX can be obtained as below:

Select Line	I_0	I_1	I_2	I_3
$AB \rightarrow$	00	01	10	11
$\bar{C}$	(0)	(2)	(4)	6
C	1	3	5	(7)
	$\bar{C}$	$\bar{C}$	$\bar{C}$	C



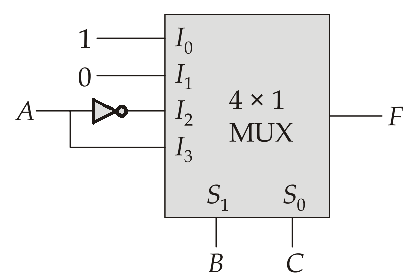
(ii) If A and C are connected to select lines, the inputs to the data lines of 4×1 MUX can be obtained as below:

Select Line	I_0	I_1	I_2	I_3
$AC \rightarrow$	00	01	10	11
$\bar{B}$	(0)	1	(4)	5
B	(2)	3	6	(7)
	1	0	$\bar{B}$	B



(iii) If B and C are select lines, the inputs to the data lines of 4×1 MUX can be obtained as below:

Select Line $\rightarrow$	I_0	I_1	I_2	I_3
BC	00	01	10	11
$\bar{A}$	(0)	1	(2)	3
A	(4)	5	6	(7)
	1	0	$\bar{A}$	A



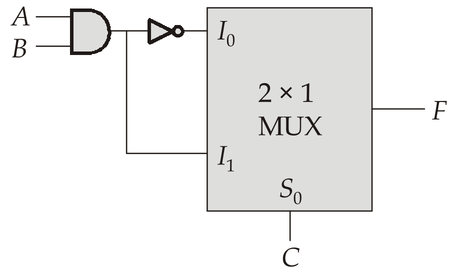
(iv) Now drawing K-map of the above function, we get,

$A \backslash BC$	00	01	11	10
0	1	0	0	1
1	1	0	1	0

$\therefore$

$$f(A, B, C) = \Sigma m(0, 2, 4, 7) = ABC + \bar{B}\bar{C} + \bar{A}\bar{C} = \bar{C}(\overline{AB}) + C(AB)$$

Thus, using 'C' as the select line, the function $f(A, B, C)$ can be implemented using 2×1 as below:



Q.2 (c) Solution:

For truth table of seven segment display when inputs $b_3\, b_2\, b_1\, b_0$ = 0000, a decimal 0 will be displayed i.e., all segments lit except X_5.

$$\therefore \quad X_0 = X_1 = X_2 = X_3 = X_4 = X_6 = 1; X_5 = 0$$

and E = 0 because 0000 is a valid input combination.

The values at the outputs for all other input combinations can be derived in similar manner as below:

Decimal digit	b_3	b_2	b_1	b_0	E	X_6	X_5	X_4	X_3	X_2	X_1	X_0
0	0	0	0	0	0	1	0	1	1	1	1	1
1	0	0	0	1	0	0	0	1	1	0	0	0
2	0	0	1	0	0	1	1	0	1	1	0	1
3	0	0	1	1	0	1	1	1	1	1	0	0
4	0	1	0	0	0	0	1	1	1	0	1	0
5	0	1	0	1	0	1	1	1	0	1	1	0
6	0	1	1	0	0	1	1	1	0	1	1	1
7	0	1	1	1	0	0	0	1	1	1	0	0
8	1	0	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	0	1	1	1	1	1	1	0
Invalid	1	0	1	0	1	1	1	0	0	1	1	1
Invalid	1	0	1	1	1	1	1	0	0	1	1	1
Invalid	1	1	0	0	1	1	1	0	0	1	1	1
Invalid	1	1	0	1	1	1	1	0	0	1	1	1
Invalid	1	1	1	0	1	1	1	0	0	1	1	1
Invalid	1	1	1	1	1	1	1	0	0	1	1	1

From the above, we can write

$E = \Sigma m(10, 11, 12, 13, 14, 15)$

$X_6 = \Sigma m(0, 2, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15)$

$X_5 = \Sigma m(2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15)$

$X_4 = \Sigma m(0, 1, 3, 4, 5, 6, 7, 8, 9)$

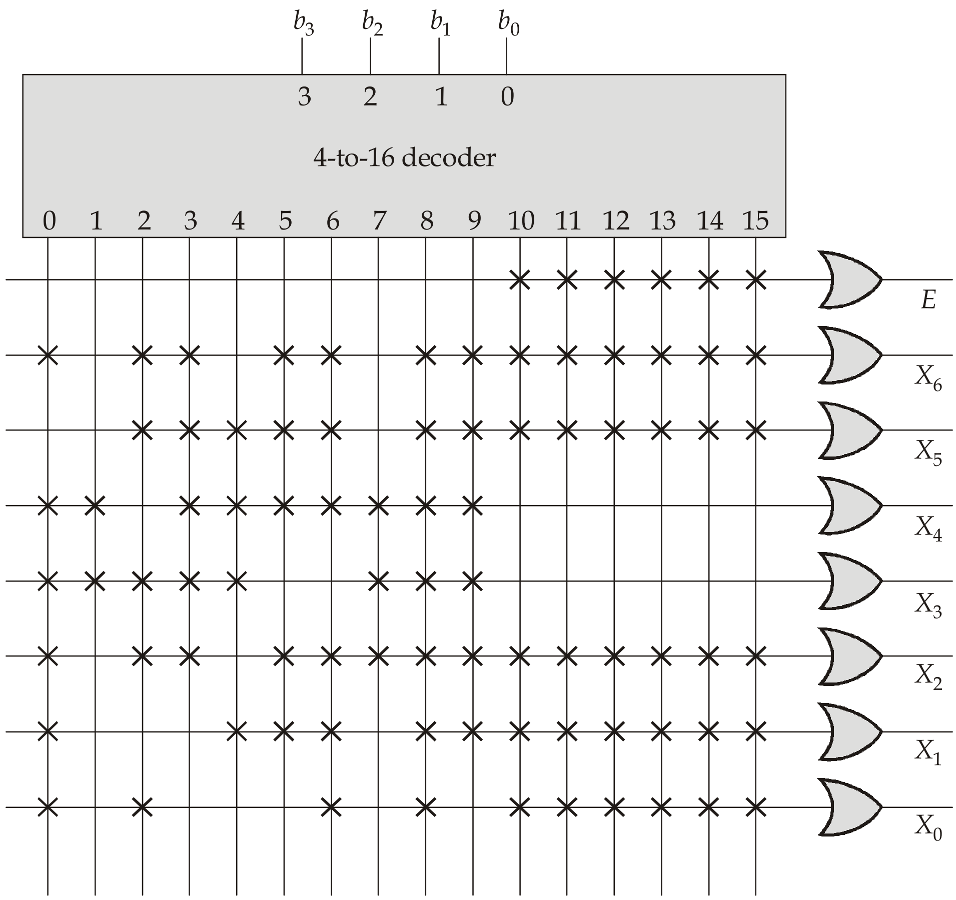
$X_3 = \Sigma m(0, 1, 2, 3, 4, 7, 8, 9)$

$X_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$

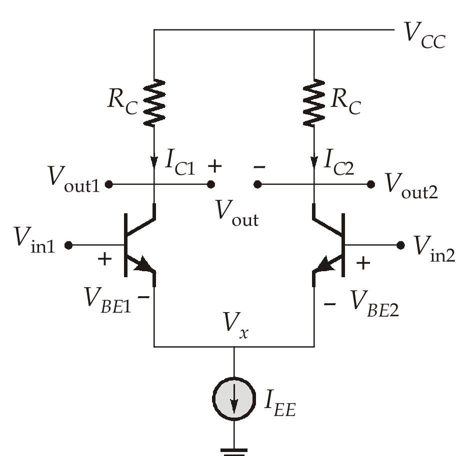
$X_1 = \Sigma m(0, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15)$

$X_0 = \Sigma m(0, 2, 6, 8, 10, 11, 12, 13, 14, 15)$

The above can be implemented using a ROM as below:



Q.3 (a) Solution:



From circuit,

$$V_{out1} = V_{CC} - I_{C1} R_C \quad ...(i)$$

$$V_{out2} = V_{CC} - I_{C2} R_C \quad ...(ii)$$

So, $$V_{out} = V_{out1} - V_{out2} = -R_C(I_{C1} - I_{C2}) \quad ...(iii)$$

Applying KVL at input node,

$$V_{in1} - V_{BE1} = V_x = V_{in2} - V_{BE2}$$

$$V_{in1} - V_{in2} = V_{BE1} - V_{BE2}$$

But the collector current, $$I_C = I_s e^{\frac{V_{BE}}{\eta V_T}} \quad [\eta = 1]$$

For transistor Q_1

$$I_{C1} = I_{s1} e^{\frac{V_{BE1}}{V_T}}$$

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_{s1}}\right)$$

For transistor Q_2,

$$I_{C2} = I_{s2} e^{\frac{V_{BE2}}{V_T}}$$

$$V_{BE2} = V_T \ln\left[\frac{I_{C2}}{I_{s2}}\right]$$

We have, $$V_{in1} - V_{in2} = V_{BE1} - V_{BE2}$$

$\therefore$ $$V_{in1} - V_{in2} = V_T \ln\left(\frac{I_{C1}}{I_{C2}}\right) \quad ...(iv)$$

$$I_{C1} = I_{C2} e^{\left(\frac{V_{in1} - V_{in2}}{V_T}\right)} \quad ...(v)$$

Also, $$I_{C1} + I_{C2} = I_{EE} \quad ...(vi)$$

So, $$I_{C2} e^{\left(\frac{V_{in1} - V_{in2}}{V_T}\right)} + I_{C2} = I_{EE}$$

$$I_{C2}\left[e^{\frac{V_{in1} - V_{in2}}{V_T}} + 1\right] = I_{EE}$$

$$I_{C2} = \frac{I_{EE}}{1+e^{\left(\frac{V_{in1}-V_{in2}}{V_T}\right)}} \quad \text{...(vii)}$$

$$I_{C1} = I_{EE} - I_{C2} = I_{EE}\left[1 - \frac{1}{1+e^{\left(\frac{V_{in1}-V_{in2}}{V_T}\right)}}\right]$$

$$I_{C1} = \frac{I_{EE}e^{\left(\frac{V_{in1}-V_{in2}}{V_T}\right)}}{1+e^{\left(\frac{V_{in1}-V_{in2}}{V_T}\right)}} \quad \text{...(viii)}$$

From equation (iii)

So,

$$V_{out} = -R_C(I_{C1} - I_{C2})$$

$$V_{out} = -R_C\left[\frac{I_{EE}e^{\left(\frac{V_{in1}-V_{in2}}{V_T}\right)}}{1+e^{\frac{V_{in1}-V_{in2}}{V_T}}} - \frac{I_{EE}}{1+e^{\left(\frac{V_{in1}-V_{in2}}{V_T}\right)}}\right]$$

$$V_{out} = -I_{EE}R_C\left[\frac{e^{\frac{V_{in1}-V_{in2}}{V_T}} - 1}{1+e^{\frac{V_{in1}-V_{in2}}{V_T}}}\right]$$

$$= -I_{EE}R_C\frac{e^{\frac{V_{in1}-V_{in2}}{2V_T}}}{e^{\frac{V_{in1}-V_{in2}}{2V_T}}}\left[\frac{e^{\frac{V_{in1}-V_{in2}}{2V_T}} - e^{-\left(\frac{V_{in1}-V_{in2}}{2V_T}\right)}}{e^{\left(\frac{V_{in1}-V_{in2}}{2V_T}\right)} + e^{-\frac{V_{in1}-V_{in2}}{2V_T}}}\right]$$

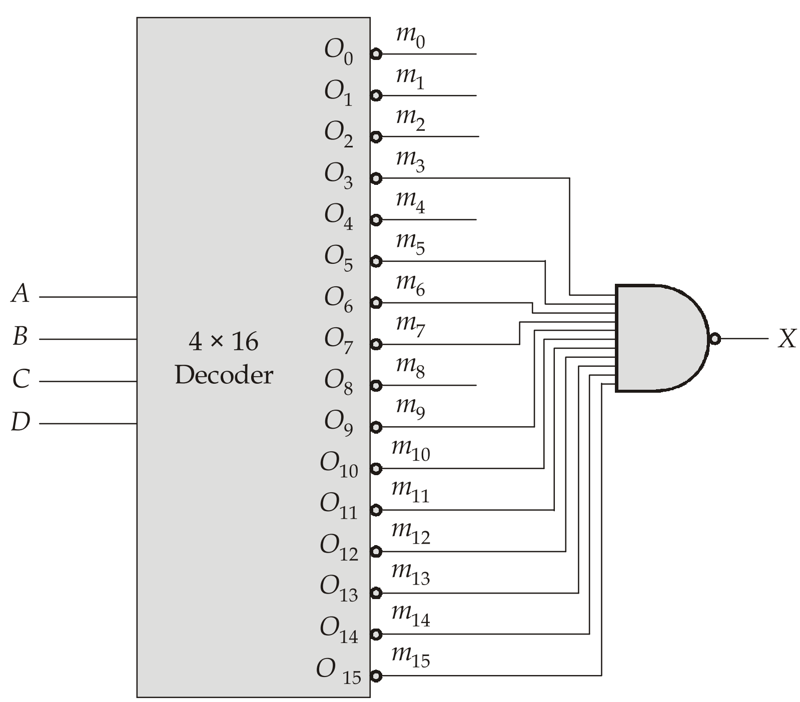
$$V_{out} = -I_{EE}R_C\tanh\left(\frac{V_{in1}-V_{in2}}{2V_T}\right) \qquad \left[\because \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}\right]$$

Q.3 (b) Solution:

(i) The truth table for the given circuit,

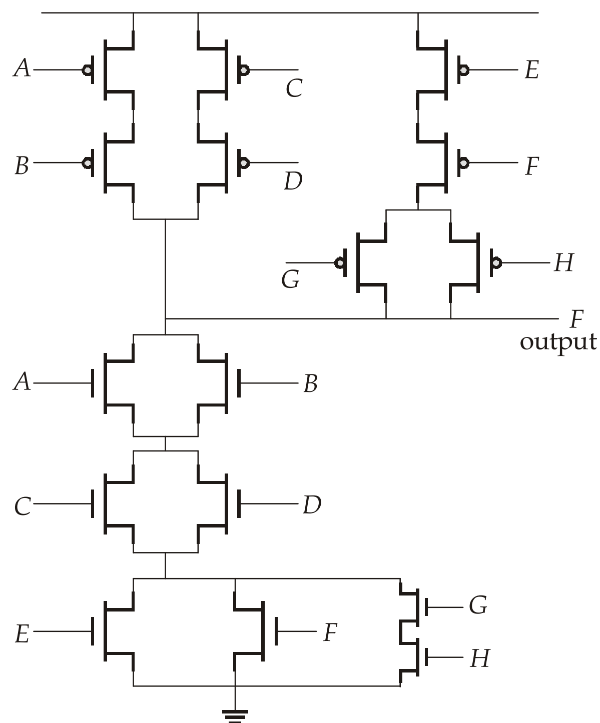
A	*B*	*C*	*D*	*X*
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

(or) $X(A, B, C, D) = \Sigma m[m_3, m_5, m_6, m_7, m_9, m_{10}, m_{11}, m_{12}, m_{13}, m_{14}, m_{15}]$



(ii) Given output logic function,

$$F = \overline{(A+B)(C+D)(E+F+GH)}$$



(iii) 1. Given,

$$0.1101_{(2)} = (2^{-1} \times 1) + (2^{-2} \times 1) + (2^{-3} \times 0) + (2^{-4} \times 1)$$

$$= \left(\frac{1}{2}\times 1\right)+\left(\frac{1}{4}\times 1\right)+\left(\frac{1}{8}\times 0\right)+\left(\frac{1}{16}\times 1\right)$$

$$= 0.5 + 0.25 + 0 + 0.0625$$

$$\therefore \quad 0.1101_{(2)} = 0.8125_{(10)}$$

2.

$$0.B2_{(16)} = (16^{-1} \times B) + (16^{-2} \times 2)$$

$$= \frac{11}{16} + \frac{2}{256}$$

$$= (0.0625 \times 11) + (2 \times 0.00390625)$$

$$0.B2_{(16)} = 0.6953125_{(10)}$$

3.

$$0.144_{(8)} = (8^{-1} \times 1) + (8^{-2} \times 4) + (8^{-3} \times 4)$$

$$= \frac{1}{8} + \frac{4}{64} + \frac{4}{512}$$

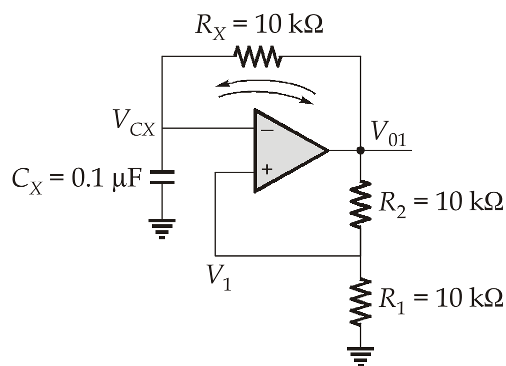
$$= \frac{1}{8} + \frac{1}{16} + \frac{1}{128}$$

$$= 0.125 + 0.0625 + 0.0078125$$

$$0.144_{(8)} = 0.1953125_{(10)}$$

Q.3 (c) Solution:

Given circuit is



$$V_{sat} = \pm 10 \text{ V}$$

The given circuit is Astable multivibrator with the output oscillating between the high state and the low state.

R_1 and R_2 forms voltage divider and it result in the feedback action.

$$V_1 = \frac{V_{01}R_1}{R_1 + R_2} = \beta V_{01} \quad ; \quad \beta = \frac{R_1}{R_1 + R_2} \text{ feedback factor}$$

At the input of comparator, $V_d = V_1 - V_{CX}$

Thus, the voltage across capacitor is compared with V_1 and we have,

$$V_{01} = \pm V_{sat}; \quad V_1 = \pm\beta V_{sat}$$

The upper and lower threshold voltage at which the output switch is given by

$$V_{UT} = \beta V_{sat}$$

$$V_{LT} = -\beta V_{sat}$$

Case-I: Let $V_{01} = +V_{sat}$

$$V_1 = +\beta V_{sat}$$

V_{CX} is compared with βV_{sat}

$$V_d = \beta V_{sat} - V_{CX}$$

Capacitor C_X charges through resistor R_X (opamp provides charging current to capacitor)

Hence, capacitor voltage; V_{CX} increases exponentially upto $+\beta V_{sat}$.

When V_{CX} becomes slightly greater than βV_{sat}, then difference input V_d becomes –ve. Hence output changes from $+V_{sat}$ to $-V_{sat}$.

Case-2 Let $V_{01} = -V_{sat}$

$$V_{01} = -\beta V_{sat}$$

$$V_1 = -\beta V_{sat} = V_{LT}$$

V_{CX} is compared with $-\beta V_{sat}$.

$$V_d = -\beta V_{sat} - V_{CX}$$

Capacitor starts discharging through resistor R_X (discharging current flows into OPAMP).

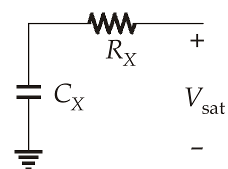
Capacitor voltage decreases exponentially upto $-\beta V_{sat}$.

When V_{CX} becomes slightly less than $-\beta V_{sat}$, then difference input V_d becomes +ve. Hence, output changes from $-V_{sat}$ to $+V_{sat}$.

Given that at $t = 0$, output V_{01} switches from its low state to its high state. Therefore,

At $t = 0$, $\quad V_{CX} = -\beta V_{sat} = V_{initial}$

For $t > 0$; C_X starts charging in the direction of $+V_{sat}$.



$$V_{Final} = V_{sat}$$

$$V_{CX} = V_{final} + (V_{initial} - V_{final})e^{-\frac{t}{R_X C_X}}$$

$$= V_{sat} + (-\beta V_{sat} - V_{sat})e^{-\frac{t}{R_X C_X}}$$

$$V_{CX} = V_{sat}\left[1 - (\beta + 1)e^{-\frac{t}{R_X C_X}}\right]$$

At $t = \frac{T_{01}}{2}$, $V_{CX} = +\beta V_{sat}$ and the output switches from high state to low state.

So,
$$\beta V_{sat} = V_{sat}\left\{1 - (1+\beta)e^{-\frac{T_{01}}{2R_X C_X}}\right\}$$

$$\beta = 1 - (1+\beta)e^{\frac{-T_{01}}{2R_X C_X}}$$

$$(1+\beta)e^{-\frac{T_{01}}{2R_X C_X}} = (1 - \beta)$$

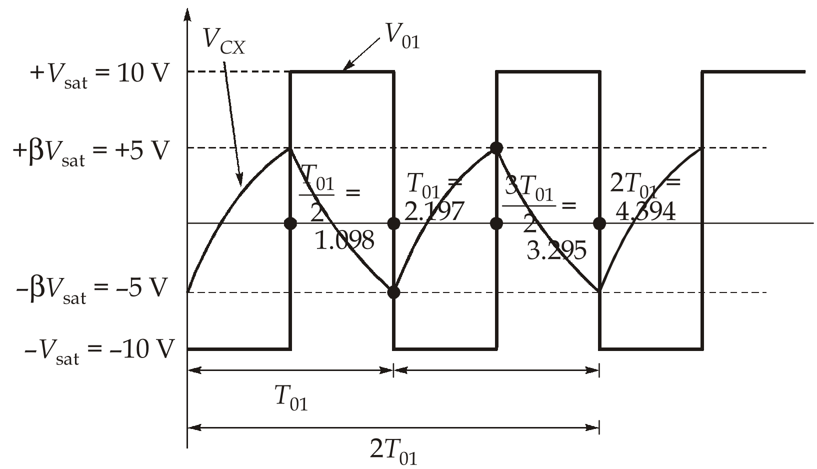
$$e^{\frac{T_{01}}{2R_X C_X}} = \frac{1+\beta}{1-\beta}$$

$$\frac{T_{01}}{2R_X C_X} = \ln\left(\frac{1+\beta}{1-\beta}\right)$$

$$T_{01} = 2R_X C_X \ln\left(\frac{1+\beta}{1-\beta}\right); \ \beta = \frac{R_1}{R_1 + R_2} = \frac{10}{20} = 0.5$$

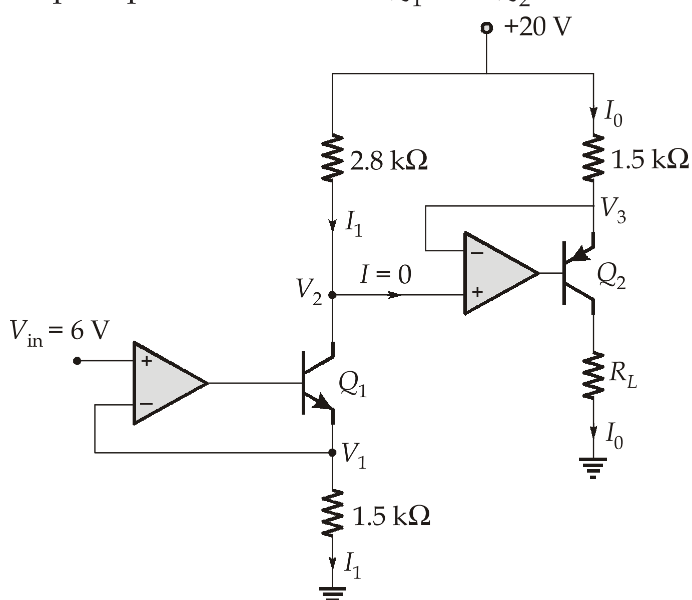
$$T_{01} = 2\times10\times10^{3}\times0.1\times10^{-6}\ln\left(\frac{1.5}{0.5}\right)$$

$$T_{01} = 2.197 \text{ ms}$$



Q.4 (a) Solution:

(i) Assuming: both op-amps to be ideal and Q_1 and Q_2 to be in saturation.



Here, $\quad V_1 = V_{in} = 6$ V (voltage follower)

So, $\quad I_1 = \dfrac{V_1}{1.5} = \dfrac{6}{1.5} = 4$ mA

By KVL, $\quad V_2 = V_{CC} - (I_1)2.8$

$$= 20 - 4 \times 2.8 = 8.8 \text{ V}$$

Using the virtual-short concept,

$$V_3 = V_2 = 8.8 \text{ V}$$

So, $$I_0 = \frac{V_{CC} - V_3}{1.5} = \frac{20 - 8.8}{1.5} = 7.467 \text{ mA}$$

So, output current, $$I_0 = 7.467 \text{ mA}$$

For maximum value of R_L, the transistor Q_2 should be at boundary of active and saturation.

So, $$(V_{EC})_{sat} = 0.2 \text{ V}$$

Applying KVL in transistor Q_2,

$$V_{CC} = I_0(1.5 + R_L) + (V_{EC})_{sat}$$

$$20 = 7.467(1.5 + R_L) + 0.2$$

$$R_L + 1.5 = \frac{20 - 0.2}{7.467}$$

$$R_L = \frac{19.8}{7.467} - 1.5 = 1.151 \text{ k}\Omega$$

$$R_L = 1.151 \text{ k}\Omega$$

So, maximum value of $R_L = 1.15$ kΩ

(ii) Given data: $V_p = 160$ V; $f = 60$ Hz; $R = 20$ kΩ

V_r (Peak to peak ripple) = 2.5 V

We know that, for a half wave peak rectifier with capacitor filter,

$$C = \frac{V_p}{V_r f R} = \frac{160}{2.5 \times 60 \times 20 \times 10^3}$$

$$= 5.34 \times 10^{-5} \text{ F}$$

$$= 53.4 \ \mu\text{F}$$

The conduction angle $\omega\Delta t$ is found as

$$\omega\Delta t = \sqrt{\frac{2V_r}{V_p}} = \sqrt{\frac{2 \times 2.5}{160}} = 0.176 \text{ rad}$$

The diode conducts for $\frac{0.176}{2\pi} \times 100$

$= 2.8\%$ of the cycle.

The average diode current is

$$\text{Load Current, } I_L = \frac{160}{20} = 8 \text{ mA}$$

$$i_{D\,avg} = I_L\left(1+\pi\sqrt{\frac{2V_p}{V_r}}\right)$$

$$= 8\left(1+\pi\sqrt{\frac{2\times 160}{2.5}}\right) = 292.34 \text{ mA}$$

The peak diode current is found

$$i_{D\,max} = I_L\left(1+2\pi\sqrt{\frac{2V_p}{V_r}}\right)$$

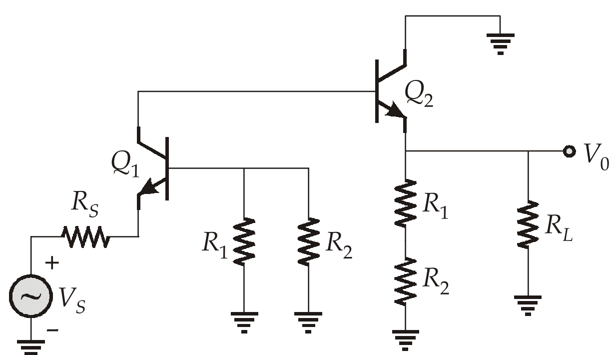
$$= 8\left(1+2\pi\sqrt{\frac{2\times 160}{2.5}}\right)$$

$$i_{D\,max} = 576.689 \text{ mA}$$

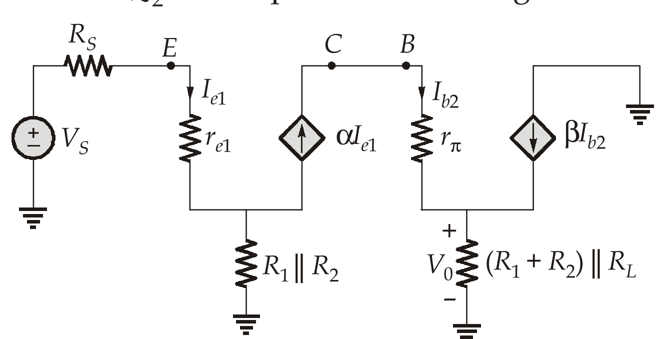
Q.4 (b) Solution:

Given circuit has voltage-series feedback and acts as a voltage amplifier. The feedback is created by R_1 and R_2 voltage divider.

Considering circuit without feedback,



Replacing the common base transistor Q_1 with equivalent small signal T model and common collector transistor Q_2 with equivalent small signal π-model. We get,



From the figure,

$$V_0 = (1 + \beta)[(R_1 + R_2) \parallel R_L]I_{b2} \qquad [\because I_{b2} = \alpha I_{e1}]$$

$$= (1 + \beta)[(R_1 + R_2) \parallel R_L]\alpha I_{e1} \qquad ...(1)$$

Applying KVL in input side of amplifier,

$$V_s = [R_s + r_{e1} + (1 - \alpha)(R_1 \parallel R_2)]I_{e1}$$

$$1 + \beta = \frac{1}{1-\alpha}$$

So,
$$V_s = \left[R_s + r_{e1} + \left(\frac{R_1 \parallel R_2}{1+\beta}\right)\right]I_{e1} \qquad ...(2)$$

From equation (1) and (2), we get,

$$\frac{V_0}{V_s} = \frac{(1+\beta)[(R_1+R_2)\parallel R_L]\alpha I_{e1}}{[R_s + r_{e1} + (1-\alpha)(R_1 \parallel R_2)]I_{e1}}$$

$$A_V = \frac{V_0}{V_s} = \frac{\beta[(R_1+R_2)\parallel R_L]}{\left[R_s + r_{e1} + \left(\frac{R_1 \parallel R_2}{1+\beta}\right)\right]} \qquad ...(3)$$

Input resistance, from equation (2) is obtained as

$$R_i = R_s + r_{e1} + \frac{R_1 \parallel R_2}{1+\beta} \qquad ...(4)$$

Output resistance, $R_0 = R_L \parallel (R_1 + R_2)$ ...(5)

Now, the feedback factor,

$$\beta = \frac{V_f}{V_0} = \frac{R_1}{R_1 + R_2}$$

$$\beta = \frac{2}{2+12} = \frac{1}{7}$$

On putting respective values in equation (3),

$$A_V = \frac{100[14 \parallel 1.5]}{0.15 + 0.305 + \frac{2\times 12}{14\times 101}} = 287.058 \text{ V/V}$$

From equation (4)

$$R_i = 0.15 + 0.305 + \frac{2\times 12}{14 \times 101} = 0.4719 \text{ k}\Omega$$

From equation (5),

$$R_0 = R_L \parallel (R_1 + R_2) = 1.5 \parallel 14 = \frac{14 \times 1.5}{15.5} = 1.354 \text{ k}\Omega$$

As it is voltage amplifier the feedback reduces the overall gain, increases the input resistance and decreases the output resistance by a factor of $(1 + A_v\beta)$. Thus,

$$A_{vf} = \frac{A_V}{1 + A_V\beta} = \frac{287.058 \text{ V/V}}{1 + 287.058 \times \frac{1}{7}} = 6.833 \text{ V/V}$$

$$R_{if} = R_i(1 + A_V\beta)$$

$$= 0.4719\left(1 + 287.058 \times \frac{1}{7}\right)$$

$$= 19.8237 \text{ k}\Omega$$

$$R_{of} = \frac{R_0}{1 + A_V\beta} = \frac{1.354}{1 + 287.058 \times \frac{1}{7}} = 0.0322 \text{ k}\Omega$$

Q.4 (c) Solution:

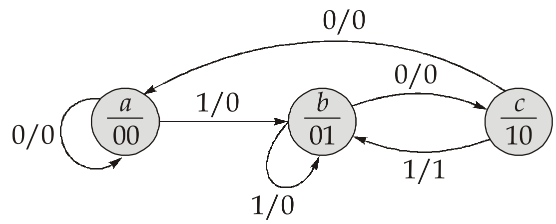
(i) State table for the given state diagram

Present state	Next state		Output	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
a	*a*	*b*	0	0
b	*c*	*b*	0	0
c	*a*	*b*	0	1
d	*a*	*b*	0	1

Thus, '*c*' and '*d*' are redundant states. The reduced state table is thus obtained as below:

Present state	Next state		Output	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
a	*a*	*b*	0	0
b	*c*	*b*	0	0
c	*a*	*b*	0	1

Reduced state diagram,



(ii) Excitation table for *JK* FF:

Q_n	Q_{n+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Since there are 4-states in the given state diagram, i.e., $N = 4$

The number of flip-flops,

$$2^n \geq N$$

$$2^n \geq 4$$

$$n \geq 2$$

Therefore, 2 JK flip flops are required to implement the sequential circuit.

Input	Present State		Next State		FF-inputs				Output
X	A	B	A^+	B^+	J_A	K_A	J_B	K_B	Y
0	0	0	0	0	0	X	0	X	0
1	0	0	0	1	0	X	1	X	0
0	0	1	1	0	1	X	X	1	0
1	0	1	0	1	0	X	X	0	0
0	1	0	0	0	X	1	0	X	0
1	1	0	0	1	X	1	1	X	1
0	1	1	X	X	X	X	X	X	X
1	1	1	X	X	X	X	X	X	X

For J_A:

X \ AB	00	01	11	10
0	0	1	X	X
1	0	0	X	X

$\therefore \quad J_A = X'B$

For K_A:

X \ AB	00	01	11	10
0	X	X	X	1
1	X	X	X	1

$\therefore \quad K_A = 1$

For J_B:

X \ AB	00	01	11	10
0		X	X	
1	1	X	X	1

$\therefore \quad J_B = X$

For K_B:

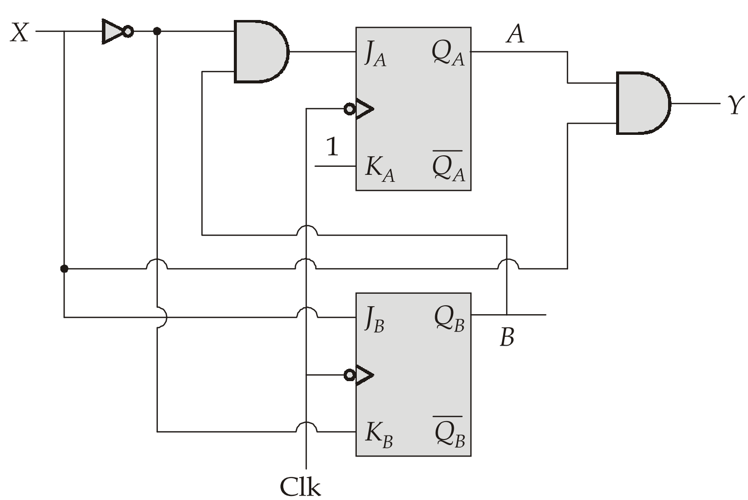
X \ AB	00	01	11	10
0	X	1	X	X
1	X		X	X

$\therefore \quad K_B = X'$

For output Y:

X \ AB	00	01	11	10
0			X	
1			X	1

$\therefore \quad Y = XA$



Section B : Advanced Electronics-1 + Electronic Measurements and Instrumentation-1 Electromagnetics-2 + Basic Electrical Engineering-2

Q.5 (a) Solution:

For the sake of ease in calculations, the observations are tabulated and manipulated as under:

x	d $(x-\bar{x})$	d^2 $(x-\bar{x})^2$
41.7	–0.27	0.0729
42	0.03	0.0009
41.8	–0.17	0.0289
42	0.03	0.0009
42.1	0.13	0.0169
41.9	–0.07	0.0049
42	0.03	0.0009
41.9	–0.07	0.0049
42.5	0.53	0.2809
41.8	–0.17	0.0289
$\Sigma x = 419.7$		$\Sigma d^2 = 0.441$

(i) Mean $\bar{x} = \frac{\Sigma x}{n} = \frac{419.7}{n} = \frac{419.7}{10} = 41.97$

(ii) The value of standard deviation is,

$$\sigma = \sqrt{\frac{\Sigma d^2}{n-1}} = \sqrt{\frac{0.441}{(10-1)}} = 0.22$$

(iii) Probable error of one reading

$$r_1 = \pm 0.6745\sigma$$

$$r_1 = \pm 0.6745 \times 0.22$$

$$r_1 = \pm 0.15 \text{ volt}$$

(iv) Probable error of the mean,

$$r_m = \frac{r_1}{\sqrt{n-1}} = \frac{\pm 0.15}{\sqrt{10-1}} = \pm 0.05 \text{ volt}$$

(v) Range = 42.5 – 41.7 = 0.8 volt

Q.5 (b) Solution:

Latch up refers to short circuit/low impedance path formed between power and ground rails of a MOSFET circuit leading to high current and damage to the IC. It occurs due to interaction between parasitic pnp and npn transistors. The structure formed by these resembles a silicon controlled rectifier (SCR). These forms a positive feedback loop, by short circuiting the power rail and ground rail, which eventually causes excessive current, and can even permanently damage the device.

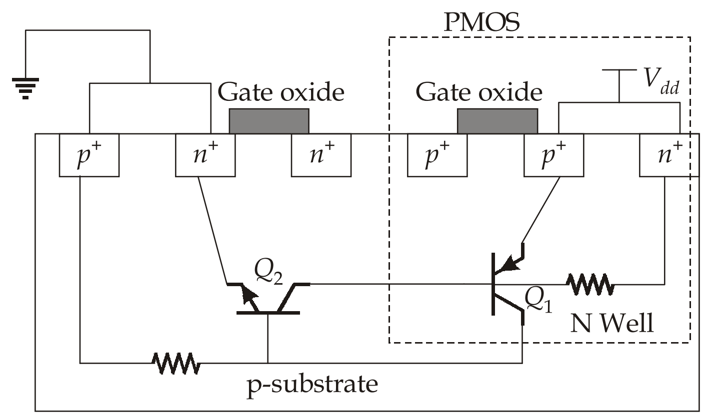


Fig. Latch up formation in a CMOS Device

Latchup formation

Figure above is a CMOS transistor consisting of an NMOS and a PMOS device. Q_1 and Q_2 are parasitic transistor elements residing inside it. Q_1 is double emitter pnp transistor whose base is formed by n well substrate of PMOS, two emitters are formed by source and drain terminal of PMOS and collector is formed by substrate (p type) of NMOS. The reverse is true for Q_2. The two parasitic transistors form a positive feedback loop and is equivalent to an SCR.

Analysis of latch formation:

Unless SCR is triggered by an external disturbance, the collector current of both transistors consists of reverse leakage current. But if collector current of one of BJT is temporarily increased by disturbance, resulting positive feedback loop causes current perturbation to be multiplied by $\beta_1\beta_2$ as explained below.

The disturbance may be a spike of input voltage on an input or output pin, leading to junction breakdown, or ionizing radiations.

Because collector current of one transistor Q_1 is fed as input base current to another transistor Q_2, the collector current of Q_2, $I_{c2} = \beta_2 \times I_{b2}$ and this collector current I_{c2} is fed as input base current I_{b1} to another transistor Q_1. In this way, both transistors feedback each other and the collector of each goes on multiplying.

Net gain of SCR device = $\beta_1 \times \beta_2$

Total current in one loop = Current perturbation × Gain

If $\beta_1 \times \beta_2 \geq 1$, both transistors will conduct a high saturation current even after the triggering perturbation is no longer available. This current will eventually becomes so large that it may damage the device.

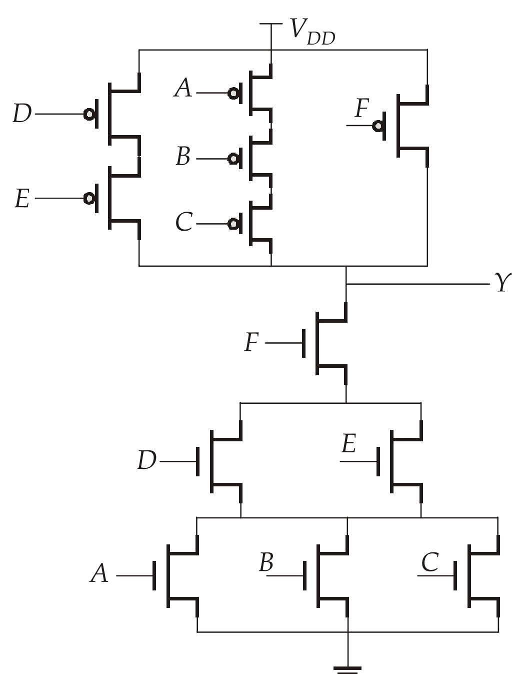
Latch up prevention techniques:

Latch up prevention includes putting a high resistance in the path so as to limit the current through supply and make $\beta_1 * \beta_2 < 1$. This can be done with the help of following techniques:

1. Surrounding PMOS and NMOS transistors with an insulating oxide layer (trench) This breaks parasitic SCR structure.
2. Latch protection technology circuitry which shuts off the device when latch up is detected.
3. Use of p^+ and n^+ guard rings around nMOS and pMOS connected to the ground and V_{DD} respectively.
4. Keeping sufficient spacing between nMOS and pMOS transistors.

Q.5 (c) Solution:

(i) The CMOS logic circuit design for $Y = \overline{(A+B+C)(D+E)F}$ is



The equivalent $\frac{W}{L}$ ratios of the nMOS and pMOS network are determined by using the series parallel equivalent rules.

For a number of MOSFETs connected in series,

$$\frac{1}{(W/L)_{eq}} = \frac{1}{(W/L)_1} + \frac{1}{(W/L)_2} + \ldots$$

For a number of MOSFETs connected in parallel,

$$(W/L)_{eq} = (W/L)_1 + (W/L)_2 + \ldots\ldots$$

So,

$$\left(\frac{W}{L}\right)_{n,A,B,C} = \left(\frac{W}{L}\right)_A + \left(\frac{W}{L}\right)_B + \left(\frac{W}{L}\right)_C$$

$$= 2 + 2 + 2 = 6$$

$$\left(\frac{W}{L}\right)_{n,D,E} = \left(\frac{W}{L}\right)_D + \left(\frac{W}{L}\right)_E$$

$$= 2 + 2 = 4$$

$$\left(\frac{W}{L}\right)_{n,\,eq} = \frac{1}{\frac{1}{\left(\frac{W}{L}\right)_{n,A,B,C}} + \frac{1}{\left(\frac{W}{L}\right)_{n,D,E}} + \frac{1}{\left(\frac{W}{L}\right)_{n,F}}}$$

$$= \frac{1}{\frac{1}{6} + \frac{1}{4} + \frac{1}{2}}$$

$$= \frac{1}{\frac{2+3+6}{12}} = \frac{12}{11}$$

$$\left(\frac{W}{L}\right)_{p,A,B,C} = \frac{1}{\frac{1}{\left(\frac{W}{L}\right)_{p,A}} + \frac{1}{\left(\frac{W}{L}\right)_{p,B}} + \frac{1}{\left(\frac{W}{L}\right)_{p,C}}}$$

$$= \frac{1}{\frac{1}{5} + \frac{1}{5} + \frac{1}{5}} = \frac{5}{3}$$

$$\left(\frac{W}{L}\right)_{p,D,E} = \frac{1}{\frac{1}{\left(\frac{W}{L}\right)_{p,D}} + \frac{1}{\left(\frac{W}{L}\right)_{p,E}}} = \frac{1}{\frac{1}{5} + \frac{1}{5}} = \frac{5}{2}$$

$$\left(\frac{W}{L}\right)_{p,eq} = \left(\frac{W}{L}\right)_{p,A,B,C} + \left(\frac{W}{L}\right)_{p,D,E} + \left(\frac{W}{L}\right)_{p,F}$$

$$\left(\frac{W}{L}\right)_{p,eq} = \frac{5}{3} + \frac{5}{2} + 5 = \frac{10+15+30}{6} = \frac{55}{6}$$

(ii)

Feature	Dry Oxidation	Wet Oxidation
Definition	Oxidation process in an oxygen rich environment without the presence of water vapour or steam.	Oxidation process in the presence of water vapour or steam.
Oxidizing Agent	Oxygen gas	Oxygen gas and water vapour or steam.
Temperature	High temperatures usually required.	Lower temperatures usually required.
Mechanism	Surface reaction between silicon and oxygen molecules.	Combination of surface reaction and hydrolysis of silicon dioxide.
Growth rate	Slower growth rate.	Faster growth rate.
Kinetics	Typically follows a parabolic rate law.	Kinetics influenced by the concentration of water vapour and temperature.
Oxide quality	High quality oxide layers are produced, less prone to defects.	Oxide layer may contain impurities and defects due to the presence of water vapour.
Stress	Lower stress due to slower oxidation rate.	Higher stress due to faster oxidation rate and incorporation of water molecules.
Uniformity	Less uniform oxide thickness.	More uniform oxide thickness.
Thickness range	Typically thinner oxide layers.	Can produce thicker oxide layers.
Dopant Diffusion	Less dopant diffusion due to slower oxidation rate	Increased dopant diffusion due to higher oxidation rate and presence of water vapour.

Q.5 (d) Solution:

Given:

$$\text{Angular frequency, } \omega = 2\pi \times 2 \times 10^9$$

$$\omega = 4\pi \times 10^9 \text{ rad/sec}$$

The voltage of the forward travelling wave is given as

$$V_f(t) = |V^+|e^{-\alpha x}\cos(\phi^+ + \omega t - \beta x)$$

given, $\phi^+ = 0$ and $V_f(t) = 2$ V at $x = 0$, $t = 0$

Therefore, we get, $|V^+| = 2$ V

Similarly, for the backward travelling wave

$$V_b(t) = |V^-| e^{\alpha x} \cos(\phi^- + \omega t + \beta x)$$

Given: $\phi^- = \frac{\pi}{3}$ and $V_b(t) = 0.5$ V at $x = 0$ and $t = 0$. Therefore we have

$$0.5 = |V^-| \cos\left(\frac{\pi}{3}\right)$$

$$|V^-| = 1 \text{ V}$$

Now, the propagation constant of the line is

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{(0.1 + j \times 2 \times \pi \times 2 \times 10^9 \times 0.01 \times 10^{-6}) \times (0.01 + j4\pi \times 10^9 \times 100 \times 10^{-12})}$$

$$\gamma = [157.91\angle 179.498°]^{\frac{1}{2}}$$

$$\gamma = 12.56\ \angle 89.749°$$

$$\gamma = 0.05 + 12.56j \text{ per meter} = \alpha + j\beta$$

We get, $\alpha = 0.055$ nepers/m

$\beta = 12.56$ rad/m

Voltage on the line is superposition of the forward and backward wave voltages giving

$$V(t) = \text{Re}\{|V^+| e^{j\phi+} e^{-\alpha x} e^{j(\omega t - \beta x)} + |V^-| e^{j\phi-} e^{\alpha x} e^{j(\omega t + \beta x)}\}$$

$$V(t) = \text{Re}\left\{2e^{-0.055x} e^{j(\omega t - \beta x)} + e^{j\frac{\pi}{3}} e^{0.055x} \cdot e^{j(\omega t + \beta x)}\right\}$$

The current on the line is

$$i(t) = \text{Re}\left\{\frac{|V^+| e^{j\phi+}}{Z_0} e^{-\alpha x} e^{j(\omega t - \beta x)} - \frac{|V^-| e^{j\phi-}}{Z_0} e^{\alpha x} e^{j(\omega t + \beta x)}\right\}$$

$$i(t) = \text{Re}\left\{\frac{2}{Z_0} e^{-0.055x} e^{j(\omega t - \beta x)} - \frac{e^{j\pi/3}}{Z_0} e^{0.055x} e^{j(\omega t + \beta x)}\right\}$$

where $$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{0.1 + j125.66}{0.01 + j1.2566}} \approx 10\ \Omega$$

Therefore at $x = 50$ cm $= 0.5$ m and $t = 1$ nsec $= 10^{-9}$ sec

We get, $v(t) = 2.42$ V

$i(t) = 0.148$ A

Q.5 (e) Solution:

We have,

$$\text{Stator copper losses, } P_s = 2 \text{ kW}$$
$$\text{Rotor copper losses, } P_R = 700 \text{ W}$$
$$\text{Friction and windage losses, } P_W = 600 \text{ W}$$
$$\text{Core losses, } P_c = 1800 \text{ W}$$

(i) We know that,

$$\text{input power, } P_{in} = \sqrt{3} V_L I_L \cos\theta$$
$$P_{in} = \sqrt{3} \times 480 \times 60 \times 0.85$$
$$P_{in} = 42.4 \text{ kW}$$

The air gap power, $P_{AG} = P_{in} - P_B$ (Stator Copper loss + Stator Core loss)

$$P_{AG} = P_{in} - P_s - P_c$$
$$= 42.4 - 2 - 1.8$$
$$= 38.6 \text{ kW}$$

(ii) The power converted from electrical to mechanical form is,

P_{con} = The air gap power (P_{AG}) – Rotor copper loss (P_R)

$$P_{con} = P_{AG} - P_R = 38.6 \text{ kW} - 0.7 \text{ kW}$$
$$P_{con} = 37.9 \text{ kW}$$

(iii) The output power, P_{out} = Power converted from electrical to mechanical form (P_{con}) – Friction and Windage loss (P_W)

$$= 37.9 \text{ kW} - 0.6 \text{ kW}$$
$$= 37.3 \text{ kW}$$

(iv)

$$\text{Efficiency, } \eta\% = \frac{P_{out}}{P_{in}} \times 100$$
$$\eta\% = \frac{37.3}{42.4} \times 100 = 87.97\%$$

Q.6 (a) Solution:

An error may be defined as the difference between the measured and actual values. During the measurement, errors may arise from different sources and are usually classified as under:

1. Gross Errors **2.** Systematic Errors **3.** Random Errors

1. Gross Errors:

This class of errors mainly covers human mistakes in reading instruments, recording and calculating measurement results. The responsibility of the mistake normally lies with the experimenter. The experimenter may grossly misread the scale. For example, he may, due to an oversight, read the temperature as 31.5°C while the actual reading may be 21.5°C. Gross errors may be of any amount and therefore their mathematical analysis is impossible. However, they can be avoided by adopting two means:

- Great care should be taken in reading and recording the data.
- Two, three or even more readings should be taken for the quantity under measurement. These readings should be taken preferably by different experimenters and the readings should be taken at a different reading points to avoid re-reading with the same error.

2. Systematic Errors:

These types of errors are divided into three categories:

(i) Instrumental Errors **(ii)** Environmental Errors **(iii)** Observational Errors

(i) Instrumental Errors:

The instrument error is generated due to instrument itself. These errors arise due to three main reasons:

- Due to inherent short comings in the instruments:

  These errors are inherent in instruments because of their mechanical structure. They may be due to construction, calibration or operation of the instruments or measuring devices.

- Due to misuse of the instruments:

  Too often, the errors caused in measurements are due to the fault of the operator than that of the instrument. A good instrument used in an unintelligent way may give erroneous results.

- Due to loading effects of instruments:

  One of the most common errors is use due to the loading of the instrument. For example, a well caliberated voltmeter may give a misleading voltage reading when connected across a high resistance circuit.

(ii) Environmental Errors:

These errors are due to conditions external to the measuring device including conditions in the area surrounding the instrument. These may be due to the variations in temperature, pressure, humidity or effect of dust, vibrations or of external magnetic or electrostatic fields.

The corrective measures which can be employed to eliminate or to reduce these undesirable effects are:

- Arrangements should be made to keep the conditions as nearly constant as possible.
- Using equipment which is immune to these effects, like resistance materials with a low resistance temperature co-efficient to minimize variations in resistance with temperature.
- Employing tehcniques which eliminate the effects of these disturbances, such as hermetically sealing equipment to eliminate humidity and dust.
- By applying computed corrections, however efforts are normally made to avoid the use of application of computed corrections, but where these corrections are needed and are necessary, they are incorporated for the computations of the results.
- Providing magnetic or electrostatic shields if external fields may affect instrument readings.

(iii) Observational Errors:

Observational errors occurs due to improper observational methodology. As an example, the pointer of a voltmeter rests slightly above the surface of the scale. Thus, a parallax error will be incurrerd unless the line of vision of the observer is exactly above the pointer.

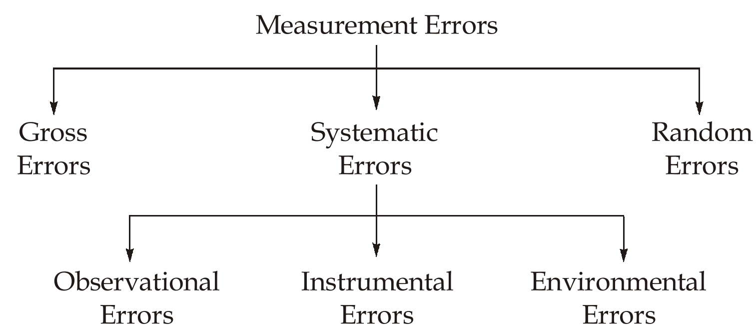
To minimize the observational errors:

- Operators must be given training, on how to use the instruments. The skillful operator with specilized knowledge can minimise the observational errors.
- Instruments with digital display can help largely eliminate observational errors.

3. Random Errors:

It has been consistently found that experimental results show variation from one reading to another, even after all systematic errors have been accounted for. These errors are due to a multitude of small factors which change or fluctuate from one measurements to another. The quantity being measured is affected by many happenings throughout the universe. We are aware of the account for some of the factors influencing the measurements, but about the rest we are unaware. The happenings or disturbances about which we are unaware are lumped together and called "Random" or "Residual". Hence the errors caused by these happenings are called Random Errors.

The measurement errors are represented in flow chart form with their types below:



Q.6 (b) Solution:

For the waveguide $a = 0.04$ m and $b = 0.03$ m

Phase constant,
$$\beta = \sqrt{\omega^2\mu\varepsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

For the dominant mode TE_{10}, we have

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \left(\frac{\pi}{0.04}\right)^2$$

$$\therefore \quad \beta = \sqrt{(2\pi\times6\times10^9)^2\times\frac{4\pi\times10^{-7}}{36\pi\times10^{+9}} - \left(\frac{\pi}{0.04}\right)^2}$$

$$\beta = 98.096 \text{ rad/m}$$

The fields for TE_{10} mode are given by

$$H_z = H_0\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} \quad \text{...(i)}$$

$$E_x = \frac{j\omega\mu}{h^2}H_0\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} = 0 \quad \text{...(ii)}$$

where,
$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$E_y = \frac{-j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right)H_0\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$E_y = \frac{-j\omega\mu}{\left(\frac{\pi}{a}\right)^2} \times \left(\frac{\pi}{a}\right) H_0 \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} \quad \text{...(iii)}$$

$$H_x = \frac{j\beta}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$$H_x = \frac{j\beta}{\left(\frac{\pi}{a}\right)^2}\left(\frac{\pi}{a}\right) H_0 \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} \quad \text{...(v)}$$

$$H_y = \frac{j\beta}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z} \quad \text{...(vi)}$$

$$H_y = 0$$

Now it is given that $$|E| = |E_y| = \frac{\omega\mu H_0}{\left(\frac{\pi}{a}\right)} = 50 \text{ V/m}$$

$\therefore$ $$H_0 = \frac{50\left(\frac{\pi}{a}\right)}{\omega\mu}$$

$$H_0 = \frac{50\left(\frac{\pi}{0.04}\right)}{2\pi\times6\times10^9\times4\pi\times10^{-7}} = 82.89\times10^{-3} \text{ A/m}$$

Substituting for H_0, we get the fields inside the waveguide as

$$E_y = -j50\sin\left(\frac{\pi x}{0.04}\right) e^{-j\beta z} \text{ V/m}$$

$$H_x = j\frac{50\beta}{\omega\mu}\sin\left(\frac{\pi x}{0.04}\right) e^{-j\beta z} \text{A/m}$$

$$H_x = j0.1035\sin\left(\frac{\pi x}{0.04}\right) e^{-j\beta z} \text{A/m}$$

$$H_z = 82.89\times10^{-3}\cos\left(\frac{\pi x}{0.04}\right) e^{-j\beta z} \text{A/m}$$

Now the power density of the propagating mode is

$$P = \frac{1}{2}\text{Re}\{E\times H^*\} = \frac{1}{2}E_y H_x^* \hat{a}_z$$

$$P = \frac{1}{2}\times50\times0.1035\times\sin^2\left(\frac{\pi x}{a}\right)$$

The total power carried by the mode can be obtained by integrating P over the waveguide cross-section.

The total power carried by the waveguide in the propagating mode is

$$W = \int_{x=0}^{a}\int_{y=0}^{b} P\,dx\,dy$$

$$W = 2.5875b\int_{0}^{a}\sin^2\left(\frac{\pi x}{a}\right)dx$$

$$W = 2.5875b\times\frac{a}{2} = 1.55\times10^{-3} \text{ Watts}$$

$$W = 1.55 \text{ mW}$$

Q.6 (c) Solution:

We have,

Rotor resistance per phase, $R_2 = 0.2\ \Omega$

Maximum torque, $T_{max} = 12$ N-m

Rotor speed at maximum torque,

$$(N_r)_{max} = 825 \text{ rpm}$$

(i) We know that, slip $(s)_{T\max} = \dfrac{N_s - (N_r)_{max}}{N_s}$

where, N_s = Synchronous speed = $\dfrac{120\times f}{P}$

$$= \frac{120\times 50}{6} = 1000 \text{ rpm}$$

$$s = \frac{1000-825}{1000}$$

$$(s)_{T\max} = 0.175$$

At maximum torque,

$$(s)_{T\max} = \frac{R_2}{X_2}$$

$$X_2 = \frac{R_2}{(s)_{T\max}}$$

$$X_2 = \frac{0.2}{0.175}$$

$$X_2 = 1.14\ \Omega/\text{phase}$$

Since,

At $s = (s)_{T\max} \Rightarrow T = T_{\max}$

$$T_{\max} = \frac{3\times60}{2\pi N_s}\times\frac{V_2^2}{2X_2} \quad ...(1)$$

As $T_{\max}$ = 12 N-m; N_s = 1000 rpm and X_2 = 1.14 Ω

We have,

$$12 = \frac{3\times60}{2\pi\times1000}\times\frac{V_2^2}{2\times1.14}$$

$$V_2^2 = \frac{12\times2\pi\times1000\times2\times1.14}{3\times60}$$

$$V_2 \approx 31 \text{ Volt}$$

When slip s = 0.05

$$T = \frac{3\times60}{2\pi N_s}\times\frac{sV_2^2R_2}{R_2^2+(sX_2)^2}$$

$$T = \frac{3\times60}{2\pi\times1000}\times\frac{0.05(31)^2\times0.2}{(0.2)^2+(0.05\times1.14)^2}$$

$$T = 6.37 \text{ N-m}$$

(ii) Let R_{ext} be resistance added externally,

$$T_{start} = \frac{3\times60}{2\pi N_s}\times\frac{V_2^2(R_2+R_{ext})}{(R_2+R_{ext})^2+X_2^2} \quad \text{[At starting, } s = 1]$$

$$T_{\max} = \frac{3\times60}{2\pi N_s}\times\frac{V_2^2}{2(X_2)}$$

$$\frac{T_{start}}{T_{\max}} = \frac{(R_2+R_{ext})}{\left[(R_2+R_{ext})^2+X_2^2\right]}\cdot2(X_2)$$

Let, $R_2 + R_{ext} = R_t$

$$\frac{55}{100} = \left[\frac{R_t}{R_t^2+X_2^2}\right]\cdot2(X_2)$$

$$0.55 = \frac{R_t}{R_t^2+(1.2996)}\times2(1.14)$$

$$0.55R_t^2+0.71478 = 2.28\,R_t$$

$$0.55R_t^2-2.28R_t+0.71478 = 0$$

$$R_t = 3.80\ \Omega,\ 0.34\ \Omega$$

or $$R_2 + R_{ext} = 3.80\ \Omega; \qquad R_2 + R_{ext} = 0.34\ \Omega$$

$$R_{ext} = 3.6\ \Omega; \qquad R_{ext} = 0.14\ \Omega$$

It may be noted that, $R_{ext} = 3.6\ \Omega$ will correspond to T_{max} lying in the region, $s = (R_2 + R_{ext})/X_2 > 1$. Therefore, external resistance added is 0.14 Ω to obtain 55% of the maximum torque at starting.

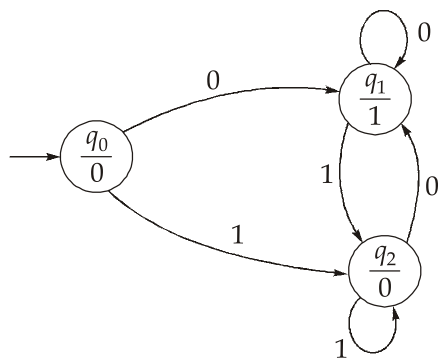
Q.7 (a) Solution:

(i) The challenges imposed by inter connect lines in deep sub micron MOS (Metal oxide semiconductor) devices are similar to those in other deep sub micron devices but with some specific considerations due to the nature of MOS technology. These challenges include:

1. **Scaling:** As MOS devices scale down to deep sub micron dimensions, the aspect ratio of interconnect lines increases, leading to increased resistance and capacitance, which can degrade performance and increase power consumption.
2. **Interconnect delay:** The RC delay of interconnect lines becomes a significant factor limiting the speed of signal propagation in MOS devices, impacting overall device performance.
3. **Signal Integrity:** Shrinked dimensions and increasing integration density can lead to signal integrity issues such as cross-talk, noise coupling and electromagnetic interference, affecting the reliability and performance of MOS device.
4. **Electromigration:** With smaller feature sizes, the risk of electromigration induced failures in interconnect lines increases, as metal atoms migrate under high current densities, leading to line degradation and eventual failure.
5. **Reliability:** MOS devices face reliability challenges such as time dependent dielectric breakdown (TDDB), hot carrier injection (HCI), and negative bias temperature instability (NBTI), which can degrade the performance and lifespan of interconnect lines.
6. **Manufacturing variability:** Variations in fabrication processes become more critical at deep sub micron scales, leading to increased variability in interconnect line dimensions, electrical properties, and performance, which can affect device yield and reliability.

(ii) To generate 1's complement of a given binary number, the simple logic is that if the input is 0 then the output will be 1 and if the input is 1 then the output will be 0. That means there are three states. One state is start state. The second state is for taking 0's as input and producing output as 1. The third state is for taking 1's as input and producing output as 0.

Hence, the Moore machine will be



For instance, take one binary number 1011 then

Input		1	0	1	1
State	q_0	q_2	q_1	q_2	q_2
Output	0	0	1	0	0

Thus, we get 00100 as 1's complement of 1011, we can neglect the initial 0 and the output which we get is 0100 which is 1's complement of 1011. The transaction table is as follows:

Current state	**Next state**		**Output**
	0	1	
→ q_0	q_1	q_2	0
q_1	q_1	q_2	1
q_2	q_1	q_2	0

Q.7 (b) Solution:

(i) For the array, in general

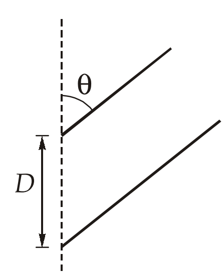


Fig.: Two element array (example)

Phase difference, $\psi = \frac{2\pi}{\lambda} d\cos\theta + \delta$, where δ is the intrinsic phase difference between the two sources.

The total field is given by

$$E_T = E_0 e^{-j\psi/2} + E_0 e^{j\psi/2} = 2E_0 \cos(\psi/2)$$

The normalized radiation intensity is given by

$$U(\theta) = [AF]^2 = \cos^2(\psi/2)$$

and the radiation pattern is described by

Array Factor, $$AF = \left|\frac{E_T}{E_{T\max}}\right| = \cos\left(\frac{\psi}{2}\right)$$

Directivity of the array $$D = \frac{U_{max}}{U_{avg}} = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} |E|^2 \sin\theta d\theta d\phi}$$

Broadside array: For Broadside array, $\delta = 0$. Thus, we get

$$\psi = \frac{2\pi}{\lambda} d\cos\theta$$

$$D = \frac{4\pi}{2\pi\int_0^{\pi} \cos^2\left(\frac{\pi d}{\lambda}\cos\theta\right)\sin\theta d\theta}$$

Integral can be solved by putting $\pi\frac{d}{\lambda}\cos\theta = t$

$$D = \frac{4\pi}{2\pi\int_{\frac{\pi d}{\lambda}}^{\frac{-\pi d}{\lambda}} \cos^2 t\left(\frac{-\lambda}{\pi d}\right)dt}$$

$$D = \frac{\frac{-2\pi d}{\lambda}}{\int_{\frac{\pi d}{\lambda}}^{\frac{-\pi d}{\lambda}} \left(\frac{1+\cos 2t}{2}\right)dt}$$

$$D = \frac{-2\pi d/\lambda}{\frac{1}{2}\left[t + \frac{\sin 2t}{2}\right]_{\frac{\pi d}{\lambda}}^{-\frac{\pi d}{\lambda}}}$$

$$D = \frac{-2\pi d/\lambda}{\frac{1}{2}\left[\frac{-\pi d}{\lambda} + \frac{1}{2}\sin\left(\frac{-2\pi d}{\lambda}\right) - \frac{\pi d}{\lambda} - \frac{1}{2}\sin\left(\frac{2\pi d}{\lambda}\right)\right]}$$

$$D = \frac{2\pi d/\lambda}{\frac{1}{2}\left[\frac{2\pi d}{\lambda} + \sin\left(\frac{2\pi d}{\lambda}\right)\right]}$$

$$D = \frac{2}{\left[1 + \frac{\sin\left(\frac{2\pi d}{\lambda}\right)}{\frac{2\pi d}{\lambda}}\right]} = \frac{2}{1 + \frac{\sin(\beta d))}{\beta d}}$$

End-fire array:

In this case,
$$\delta = -\beta d = \frac{-2\pi d}{\lambda}$$

$\Rightarrow$
$$\psi = \frac{2\pi d}{\lambda}(\cos\theta - 1)$$

The directivity of end-fire array can be calculated as:

$$D = \frac{4\pi}{2\pi\int_0^{\pi}\cos^2\left\{\frac{\pi d}{\lambda}(\cos\theta - 1)\right\}\sin\theta d\theta}$$

Let $\frac{\pi d}{\lambda}(\cos\theta - 1) = t$, we get,

$$D = \frac{4\pi}{2\pi\int_0^{\frac{-2\pi d}{\lambda}}\cos^2 t\left(\frac{-\lambda}{\pi d}\right)dt}$$

$$D = \frac{\frac{-2\pi d}{\lambda}}{\int_0^{\frac{-2\pi d}{\lambda}}\left(\frac{1+\cos 2t}{2}\right)dt}$$

$$D = \frac{\frac{-2\pi d}{\lambda}}{\frac{1}{2}\left[t + \frac{\sin 2t}{2}\right]_0^{\frac{-2\pi d}{\lambda}}}$$

$$D = \frac{\frac{-2\pi d}{\lambda}}{\frac{1}{2}\left[\frac{-2\pi d}{\lambda} - \frac{1}{2}\sin\left(\frac{4\pi d}{\lambda}\right)\right]}$$

$$D = \frac{2}{1 + \frac{\sin\left(\frac{4\pi d}{\lambda}\right)}{\left(\frac{4\pi d}{\lambda}\right)}} = \frac{2}{1 + \frac{\sin(2\beta d)}{2\beta d}}$$

when $d = \frac{\lambda}{4},\ \beta d = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$

Directivity of the broadside array

$$D_{BS} = \frac{2}{1 + \sin\left(\frac{\pi}{2}\right) \Big/ \left(\frac{\pi}{2}\right)} = 1.222$$

Directivity of the end fire array

$$D_{EF} = \frac{2}{1 + \sin\frac{(\pi)}{\pi}} = 2$$

(ii) Let I be the amplitude of the current in the wire dipole having a loss resistance R_l. Then ohmic power loss is

$$P_l = \frac{1}{2} I^2 R_l \qquad ...(1)$$

In terms of radiation resistance R_r, the radiated power is

$$P_r = \frac{1}{2} I^2 R_r \qquad ...(2)$$

From (1) and (2), we get

$$\eta_r = \frac{P_r}{P_r + P_l} = \frac{R_r}{R_r + R_l} \qquad ...(3)$$

$$= \frac{1}{1 + \left(\frac{R_l}{R_r}\right)}$$

where R_r is given as

$$R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

The loss resistance R_l of the metal wire can be expressed in terms of the surface resistance R_s.

$$R_l = R_s\left(\frac{dl}{2\pi a}\right)$$

$$R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}}$$

$$\therefore \qquad \eta = \frac{1}{1+\frac{R_s}{160\pi^3}\left(\frac{\lambda}{a}\right)\left(\frac{\lambda}{dl}\right)}$$

Q.7 (c) Solution:

(i) In figure, at the junction we have two impedances Z_1 and Z_2 in parallel. Since the cable has been terminated in its characteristic impedance, Z_1 will be same as the characteristic impedance 50 Ω. Z_2 however will be transformed version of 75 Ω impedance calculated as

$$Z_2 = Z(l_1) = Z_0\left[\frac{75\cos\beta l_1 + j50\sin\beta l_1}{50\cos\beta l_1 + j75\sin\beta l_1}\right]$$

and $$\beta l_1 = \frac{2\pi}{\lambda}(0.3\lambda) = 0.6\pi = 108°,$$

giving, $$Z_2 = Z(l_1) = 50\left[\frac{75\cos 108° + j50\sin 108°}{50\cos 108° + j75\sin 108°}\right]$$

$$Z_2 = 35.200 + 8.621j\ \Omega$$

Since, at the junction, the two impedances are connected in parallel, the impedance Z is

$$Z = \frac{Z_1 Z_2}{Z_1+Z_2} = \frac{50[35.200+8.621j)]}{50+35.200+8.621j}$$

$$Z = 20.9549 + j2.9389\ \Omega$$

The impedance at a distance of l_2 from the junction is

$$Z(l_2) = Z_0\left[\frac{Z\cos\beta l_2 + j50\sin\beta l_2}{50\cos\beta l_2 + jZ\sin\beta l_2}\right]$$

and $$\beta l_2 = \frac{2\pi}{\lambda}\times(0.2\lambda) = 0.4\pi = 72°,$$

We get,

$$Z(l_2) = \frac{50[(20.9549 + 2.9389j)\cos 72° + j50\sin 72°]}{50\cos 72° + j(20.9549 + 2.9389j)\sin 72°}$$

$$Z(l_2) = 94 + j43.47\ \Omega$$

The magnitude of the reflection coefficient on the line is

$$|\Gamma| = \left|\frac{Z - Z_0}{Z + Z_0}\right| = \left|\frac{20.9549 + 2.9389j - 50}{20.9549 + 2.9389j + 50}\right|$$

$$= 0.41$$

$\Rightarrow$ VSWR on the line,

$$\rho = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.41}{1-0.41}$$

$$\rho = 2.3898$$

(ii) The antenna has nulls when

$$F(\theta) = 0$$

$$\sin(10\cos\theta_{null}) = 0$$

provided $\sin(2\cos\theta_{null}) \neq 0$

$$10\cos\theta_{null} = \pm m\pi \qquad m = 1, 2, 3, ...$$

$$\Rightarrow \qquad \cos\theta_{null} = \pm\frac{m\pi}{10}$$

$|\cos\theta_{null}|$ has to be ≤ 1. We therefore get,

$$\theta_{null} = \cos^{-1}\left(\pm\frac{m\pi}{10}\right) \text{ and } \frac{m\pi}{10} \le 1 \text{ giving } m \le \frac{10}{\pi}$$

Since, m is an integer, we get, $m = 1, 2, 3$.

The directions of nulls therefore are

$$\theta_{null} = \cos^{-1}\left(\pm\frac{\pi}{10}\right), \cos^{-1}\left(\pm\frac{\pi}{5}\right), \cos^{-1}\left(\pm\frac{3\pi}{10}\right)$$

Since, $0 \le \theta \le \pi$, the nulls are at $\theta = \frac{\pi}{2}$, the denominator of $F(\theta)$ is zero making $\theta = \frac{\pi}{2}$ the direction of maximum radiation.

To obtain the HPBW of the antenna, we have to find two angles θ_1 and θ_2 around the direction of maximum radiation, $\theta = \frac{\pi}{2}$, along which the field is $\frac{1}{\sqrt{2}}$ of its maximum value.

The maximum value of $F(\theta)$ is along $\theta = \frac{\pi}{2}$ and can be obtained as

$$|F(\theta)|_{max} = \left|\frac{\sin(10\cos\theta)}{\sin(2\cos\theta)}\right|_{at\ \theta=\frac{\pi}{2}}$$

Since, at $\theta = \frac{\pi}{2}$, $F(\theta)$ has $\frac{0}{0}$ form, we evaluate it in the form of a limit.

$$|F(\theta)|_{max} = \underset{\theta\to\frac{\pi}{2}}{lt}\left|\frac{\sin(10\cos\theta)}{\sin(2\cos\theta)}\right|$$

$$= \underset{\theta\to\frac{\pi}{2}}{lt}\frac{\left\{10\cos\theta - \frac{(10\cos\theta)^3}{3!} + \ldots\right\}}{2\cos\theta - \frac{(2\cos\theta)^3}{3!} + \ldots} = 5$$

For the half power points, we have,

$$F(\theta) = \frac{5}{\sqrt{2}}$$

$$\frac{\sin(10\cos\theta)}{\sin(2\cos\theta)} = \frac{5}{\sqrt{2}}$$

Solving numerically, we get HP angles θ_1 and θ_2 as

$\theta_1 = 82.08$ and $\theta_2 = 97.93°$

The HPBW = $\theta_2 - \theta_1 \simeq 16°$

Q.8 (a) Solution:

(i) (a) For lap winding,

$$A = P = 4$$

$$\text{Number of turns/path} = \frac{120}{4} = 30 \text{ turns/path}$$

EMF generated per path,

$$E_{path} \text{ or } E_a = \frac{EMF}{turn} \times \frac{\text{number of turns}}{\text{path}}$$

$$= 10 \times 30 = 300 \text{ V}$$

$$\text{Generator current, } I_a = I_{path} \times A$$
$$= 15 \times 4 = 60 \text{ A}$$
$$\text{Generated power, } P_a = E_a I_a$$
$$P_a = 300 \times 60$$
$$P_a = 18 \text{ kW}$$

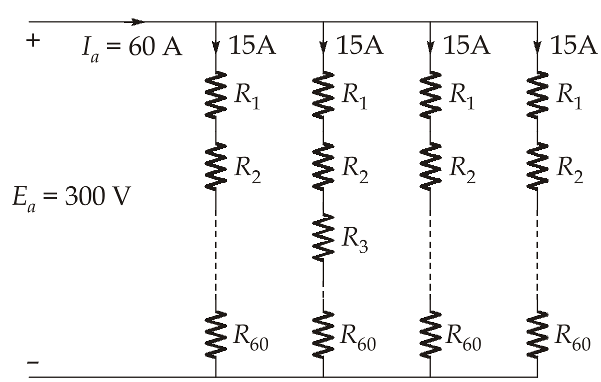
(b) For wave winding,

$$A = 2$$
$$\frac{\text{Number of turns}}{\text{Path}} = \frac{120}{2} = 60$$
$$\text{Emf generated per path} = 10 \times 60 = 600 \text{ V}$$
$$\text{Generator current, } I_a = I_{path} \times A$$
$$= 15 \times 2 = 30 \text{ A}$$
$$\text{Generated power, } P_a = I_a E_a$$
$$= 30 \times 600$$
$$= 18 \text{ kW}$$

(ii) As Generator has 120 turns; and as we know, one turn has two conductors, hence total number of conductors is 120 × 2 = 240 conductors.

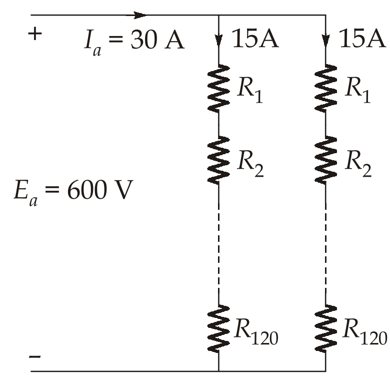
For lap connection,

$$\text{Number of conductors per path} = \frac{240}{A} = \frac{240}{4} = 60 \text{ conductors/path}$$



For wave connection,

$$\text{Number of conductors per path} = \frac{240}{A} = \frac{240}{2} = 120 \text{ conductors/path}$$



Q.8 (b) Solution:

(i) We have, $P_1 = 7500$ Watt and $P_2 = -1500$ W

We know that,
$$\phi = \tan^{-1}\left[\frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2}\right]$$

$$\phi = \tan^{-1}\left[\frac{\sqrt{3}(7500-(-1500)}{7500-1500}\right]$$

$$\phi = 69°$$

Power factor $\cos\phi = \cos 69°$
$$= 0.36$$

(ii) Power consumed by each phase $= \dfrac{P_1 + P_2}{3}$
$$= \frac{7500+(-1500)}{3} = 2000 \text{ W}$$

For star connection, Voltage across each phase,
$$V_{ph} = \left(\frac{400}{\sqrt{3}}\right) \approx 231 \text{ V}$$

Current in each phase,
$$I_{ph} = \frac{P}{V_{ph}\cos\phi} = \frac{2000}{231\times 0.36} = 24.05 \text{ A}$$

Impedance of each phase,
$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{231}{24.05} = 9.60\ \Omega$$

Resistance of each phase,
$$R_{ph} = \frac{2000}{(24.05)^2} = 3.46\ \Omega$$

Reactance of each phase,

$$X_{ph} = \sqrt{(9.60)^2 - (3.46)^2} = 8.95\ \Omega$$

In order that one of the wattmeter should read zero, the power factor should be 0.5.

$$\therefore \qquad \cos\phi = 0.5 \text{ and } \tan\phi = 1.73$$

Now,
$$\tan\phi = \frac{X'_{ph}}{R_{ph}}$$

∴ Reactance of circuit,

$$X'_{ph} = R_{ph}\tan\phi$$
$$= 3.44 \times 1.73 = 5.95\ \Omega$$

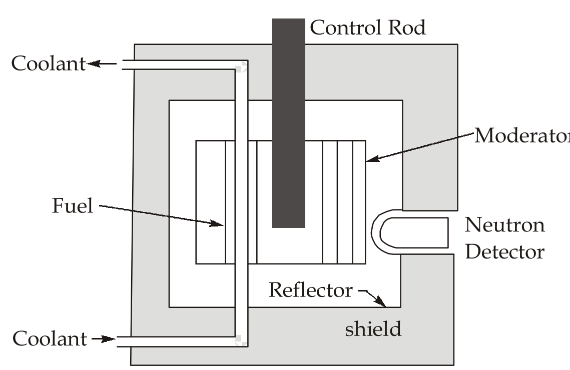
∴ Capacitive reactance required to be introduced,

$$X_C = 8.95 - 5.95 = 3\ \Omega$$

Hence,
$$\text{capacitance, } C = \frac{1}{2\pi\times 50\times 3}\text{ F}$$
$$C = 1060\ \mu\text{F}$$

Q.8 (c) Solution:

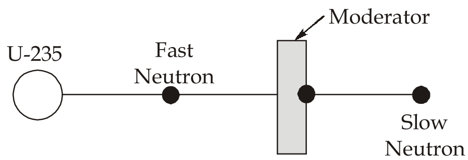
(i) Reactor is that part of nuclear power plant where nuclear fuel is subjected to nuclear fission and the energy released in the process is utilized to heat the coolant which may in turn generate steam or be used in a gas turbine. A nuclear reactor consists of the following basic components:



Basic Components of a Nuclear Reactor

1. **Reactor Core:** It contains a number of fuel rods made of fissile material. They may be diluted with non – fissionable material for better control of the reaction or to reduce the damage from fission product poisoning. The size of core, just sufficient to maintain a chain reaction is the **critical size**. It can be brought **down** by using enriched uranium as fuel.

2. **Moderator:** Neutrons produced by the fission process are ejected from the nucleus at a very high velocity of about 1.5 x 10^7 m/s and therefore, have a very large kinetic energy and are termed as **fast neutrons**. The elements which can undergo a fission reaction with fast neutrons are U-233, U-235 and Pu-239. Natural uranium contains only 0.7 % U-235. Fast neutrons are slowed down by **elastic scattering process** and chain reaction can still occur.



Moderator Slows Down a Fast Neutron

But during this process, there is a possibility of their getting absorbed by U-238 and the chain reaction may not be maintained. If the proportion of U-235 in the metal is increased to more than 10%, the above absorption effect can be overcome and a chain reaction is possible. This occurs in **fast reactors** but the **enriching process** is expensive.

For more effective use in nuclear reactor, it is desirable to slow down the fast neutrons to speeds corresponding to the speed of molecules in a gas at NTP (i.e. to a speed of about 2.2 x 10^3, m/s). Such neutrons are known as slow or thermal neutrons. The absorption properties of U-238 are very much reduced with thermal neutrons. Thus, if natural uranium is bombarded by thermal or slow neutrons, the chain reaction can be maintained. This is accomplished with the help of **'moderator'** which is mixed with the fissile material in a suitable manner.

Thus the purpose of moderator material in the reactor core is to **moderate**, or **reduce** the neutron speeds to a value that increases the probability of fission occurrence. The fast neutrons collide with the nuclei of moderator material, loose their energy and get slowed down.

Heavy water is an **ideal moderating material** and is used in many reactors inspite of its heavy cost. Ordinary water is cheap but it has high neutron absorption and can be used only with enriched uranium.

The moderator and the fuel can either be intimately mixed or the fuel may be scattered through out the moderator in discrete lumps. These two arrangements are called homogenous and heterogenous arrangements respectively.

3. **Control Rods:** Control rods are meant for controlling the rate of fission of U-235. These are made of **boron-10, cadmium** or **hafnium**, that absorb some of the slowed neutrons. In a reactor, nuclear chain reaction has to be initiated when

started from cold, and the chain reaction is to be maintained at a steady value during the operation of reactor. Also the reactor must be able to shut-down automatically under emergency conditions. All this requires a control of reactor so as to prevent the melting of fuel roads, disintegration of coolant and destruction of reactor as the amount of energy released is enormous.

The control rods are inserted into the reactor core from the top of the reactor vessel. These rods regulate the fissioning in the reactor by absorbing the excess neutrons. These rods can be moved in and out of the holes in the reactor core assembly. If the fissioning rate of the chain reaction is to be increased, the control rods are moved out slightly so that they absorb less number of neutrons and vice-versa.

4. **Coolant:** It is a medium through which the heat generated in the reactor is transferred to the heat exchanger for further utilisation in power generation. Sometimes when water, is used as a coolant it takes up heat and gets converted into steam in the reactor which is directly used for driving steam turbines. Coolant flows through and around the reactor core. It performs the additional function of keeping the interior of reactor at the desired temperature.

   A good coolant should not absorb neutrons, should be non-oxidising, non-toxic and non-corrosive and have high chemical and radiation stability and good heat transfer capability. **Air, helium, hydrogen and CO_2 amongst the gases, light and heavy water amongst the liquids, and the molten sodium and lithium amongst the metals are the materials used as coolants.**

5. **Reflector:** This completely surrounds the reactor core within the thermal shielding arrangement and bounces back most of the neutrons that escape from the fuel core. This conserves the nuclear fuel, as the low speed neutrons thus returned are useful in continuing the chain reaction. The reflector gets heated due to collision of neutrons with its atom, therefore, its cooling is essential. The reflector should have good neutron scattering proper ties and preferably a small tendency to absorb neutrons. **It is often a moderating material and sometimes the same material is used both for moderator and reflector.**

6. **Thermal Shielding:** The shielding is usually constructed from iron and help in giving protection from deadly α and β **particle** radiations and **γ-rays** as well as neutrons given off by the process of fission with in the reactor. In this manner it gets heated and prevents the reactor wall from getting heated. Coolant flows over the shielding to take away the heat.

7. **Reactor Vessel:** The reactor core, reflector and thermal shielding are all enclosed in the main body of the reactor and is called the **reactor vessel or tank**. It is a **strong walled container** and provides the entrance and exit for the coolant and also the passage for its flow through and around the reactor core. There are holes at the top to allow the control rods to pass through them. The reactor, core (fuel and moderator assembly) is usually placed at the bottom of the vessel. The reactor vessel has to withstand **high pressures (upto 21 MPa)**.
8. **Biological Shield:** The whole of the reactor is enclosed in a biological shield to prevent the escape or leak away of the fast neutrons, slow neutrons, β-particles and γ-rays as these radiations are very harmful for living organisms. **Lead iron** or **dense concrete shields**, are used for this purpose.

(ii) Advantages and Disadvantages of Nuclear Power Plant

Advantages:

1. The amount of fuel required is quite small, therefore, there is no problem of transportation, storage etc.
2. The demand for coal, oil and gas is reduced which are tending to rise in cost as the stocks are becoming depleted.
3. These plants need less area as compared to any other plant of the same size. A 2,000 MW nuclear power plant needs about 80 acres whereas the coal-fired steam power plant of the same capacity needs about 250 acres of land.
4. In addition to producing large amounts of power, the nuclear power plant can produce valuable fissile material, which is extracted when the fuel has to be renewed.
5. These plants, because of the negligible cost of transportation of fuel, can be located near the load centres, therefore, primary distribution cost is reduced.
6. These plants are most economical in large capacity (100 MVA and more).
7. The output control is extremely flexible i.e. the output can be instantaneously adjusted from zero to an upper limit. The limit is set by the capacity of the heat – removal system to prevent over heating of the pile.
8. There are large deposits of nuclear fuel available all over the world. Therefore such plants can ensure continued supply of electrical energy for thousands of years.
9. A coal-fired steam power plant needs thousands of tonnes of coal per day and usually, looks like a mad house of trains, trucks, coal and ashes, whereas a nuclear power plant will be very neat and clean and is hospital quiet.

10. The operating cost is quite low and once the installation is completed, the loading of the power plant will have no effect on the generation cost. Therefore, a nuclear power plant is always operated as a base load plant. The nuclear power plant are usually not operated at a load factor less than 0.8.

Disadvantages:

1. The initial capital cost is very high as compared to other types of power plants.
2. The erection and commissioning of the plant requires greater technical know-how.
3. The fission by-products are generally radio-active and may cause a dangerous amount of radio-active pollution.
4. Nuclear power plants are not well suited for varying loads since the reactor does not respond to the fluctuations of load efficiently.
5. The fuel used is expensive and is difficult to recover.
6. Maintenance charges are high owing to lack of standardization. Salary bill of the maintenance staff is also high as specially trained personnels are required to handle the plant.
7. The disposal of the products, which are radio-active, is a big problem. They have either to be disposed off in a deep trench or in a sea away from sea-shore.
8. The cooling water requirements of a nuclear power plant are very heavy (more than twice the water required for the same size coal-fired steam power plant) Hence, cooling towers required for nuclear power plants are larger and costlier than those for conventional steam power plants.

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