MADE EASY

India's Best Institute for IES, GATE & PSUs

Detailed Solutions

ESE-2024 Mains Test Series

Electrical Engineering Test No : 9

Section A : Electromagnetic Theory + Digital Electronics + Communication Systems

Q.1 (a) Solution:

(i) We can simplify the given expression as follows:

$$\overline{\overline{A\overline{B}+ABC}+A(B+A\overline{B})} = \overline{\overline{A(\overline{B}+BC)}+A(B+A)}$$

$$= \overline{\overline{A(\overline{B}+C)}+AB+A\cdot A}$$

$$= \overline{\overline{A\overline{B}+AC}+AB+A}$$

$$= \overline{\overline{A\overline{B}+AC}+A(B+1)}$$

$$= \overline{\overline{A\overline{B}+AC}+A\cdot 1}$$

$$= \overline{\overline{A\overline{B}+AC}+A}$$

$$= \overline{(\overline{A\overline{B}})\cdot\overline{AC}+A}$$

$$= \overline{(\overline{A}+B)(\overline{A}+\overline{C})+A}$$

$$= \overline{\overline{A}+B\overline{C}+A} \qquad \text{... } [\because (\overline{A}+B)(\overline{A}+\overline{C}) = \overline{A}+B\overline{C}]$$

$$= \overline{1+B\overline{C}} \qquad \text{.... } [\overline{A}+A=1]$$

$$= \overline{1} \qquad \text{.... } [1+A=1]$$

$$= 0$$

(ii) L.H.S. of given expression is $AB+\bar{C}(\bar{A}+\bar{D})+0$

$$= AB+\bar{C}(\bar{A}+\bar{D})+\bar{A}B+C\bar{D} \quad (\text{Given that } \bar{A}B+C\bar{D}=0)$$

$$= AB+\bar{A}\bar{C}+\bar{C}\bar{D}+\bar{A}B+C\bar{D}$$

$$= B(A+\bar{A})+\bar{D}(C+\bar{C})+\bar{A}\bar{C}$$

$$= B+\bar{D}+\bar{A}\bar{C}$$

R.H.S. of given expression is $AB+BD+\bar{B}\bar{D}+\bar{A}\bar{C}D+0$

$$= AB+BD+\bar{B}\bar{D}+\bar{A}\bar{C}D+\bar{A}B+C\bar{D}$$

(Given that $\bar{A}B+C\bar{D}=0$)

$$= B(A+\bar{A})+BD+\bar{B}\bar{D}+\bar{A}\bar{C}D+C\bar{D}$$

$$= B(1+D)+\bar{B}\bar{D}+\bar{A}\bar{C}D+C\bar{D}$$

$$= B+B\bar{D}+\bar{A}\bar{C}D+C\bar{D}$$

$$= B+\bar{D}+\bar{A}\bar{C}D+C\bar{D}$$

$$= B+\bar{D}+\bar{A}\bar{C}D+C\bar{D} \quad (\because A+\bar{A}B=A+B)$$

$$= B+\bar{D}+D\bar{A}\bar{C}$$

$$= B+\bar{D}+\bar{A}\bar{C} \quad (\because A+\bar{A}B=A+B)$$

Hence, L.H.S. = R.H.S.

Q.1 (b) Solution:

(i)

$$S = x^2y+xyz$$

$$\nabla^2 S = \frac{\partial^2 S}{\partial x^2}+\frac{\partial^2 S}{\partial y^2}+\frac{\partial^2 S}{\partial z^2}$$

$$= \frac{\partial}{\partial x}(2xy+yz)+\frac{\partial}{\partial y}(x^2+xz)+\frac{\partial}{\partial z}(xy) = 2y$$

(ii)

$$S = r^2 z(\cos\phi+\sin\phi)$$

$$\nabla^2 s = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left[r^2 z(\cos\phi-\sin\phi)\right]+\frac{\partial}{\partial z}\left[r^2(\cos\phi+\sin\phi)\right]$$

$$= 4z(\cos\phi+\sin\phi) - z(\cos\phi+\sin\phi)$$

$$= 3z(\cos\phi+\sin\phi)$$

(iii)

$$\nabla^2 s = \frac{1}{\rho^2}\frac{\partial}{\partial\rho}\left(\rho^2\frac{\partial S}{\partial\rho}\right)+\frac{1}{\rho^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial S}{\partial\theta}\right)+\frac{1}{\rho^2\sin^2\theta}\frac{\partial^2 S}{\partial\phi^2}$$

$$\therefore \quad \nabla^2 s = \frac{1}{\rho^2}\frac{\partial}{\partial \rho}\left[\rho\cos\theta\sin\phi + 2\rho^3\phi\right] - \frac{1}{\rho^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin^2\theta\sin\phi\ln\rho\right]$$

$$+\frac{1}{\rho^2\sin^2\theta}\frac{\partial}{\partial\phi}\left[\cos\theta\cos\phi\ln\rho + \rho^2\right]$$

$$= \frac{1}{\rho^2}\left[\cos\theta\sin\phi + 6\rho^2\phi\right] - \frac{1}{\rho^2\sin\theta}\left[2\sin\theta\cos\theta\sin\phi\ln\rho\right]$$

$$-\frac{1}{\rho^2\sin^2\theta}\left[\cos\theta\sin\phi\ln\rho\right]$$

$$= \frac{\cos\theta\sin\phi}{\rho^2}(1 - 2\ln\rho - \text{cosec}^2\theta\ln\rho) + 6\phi$$

Q.1 (c) Solution:

$$x(t) = A_c\cos(\omega_c t) + m(t)\cos(\omega_c t) + \hat{m}(t)\sin(2\pi f_c t)$$

As, $m(t) = \dfrac{1}{1+t^2}$,

We have $\hat{m}(t) = \dfrac{t}{1+t^2}$

Now envelope, $$[x(t)]_{\text{env}} = \left[\left(A_c + \frac{1}{1+t^2}\right)^2 + \left(\frac{t}{1+t^2}\right)^2\right]^{1/2}$$

$$= \left[A_c^2 + \left(\frac{1}{1+t^2}\right)^2 + \frac{2A_c}{1+t^2} + \frac{t^2}{(1+t^2)^2}\right]^{1/2}$$

$$= \left[A_c^2 + \frac{1}{1+t^2} + \frac{2A_c}{1+t^2}\right]^{1/2}$$

$$= A_c\left[1 + \frac{2}{A_c(1+t^2)} + \underbrace{\frac{1}{A_c^2(1+t^2)}}_{\text{Neglect}}\right]^{1/2}$$

$$[x(t)]_{\text{env}} = A_c\left[1 + \frac{2}{A_c(1+t^2)}\right]^{1/2}$$

Using exp. $$[x(t)]_{\text{env}} = A_c\left[1 + \frac{1}{2}\frac{2}{A_c(1+t^2)}\right] = A_c + \frac{1}{1+t^2}$$

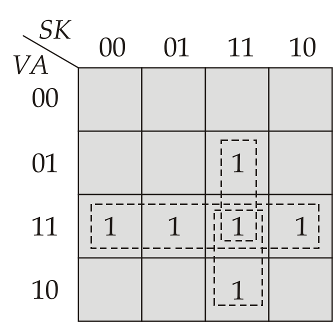
Q.1 (d) Solution:

Let 1 represent a vote for burger and 0 represent a vote for chicken then based on above conditions truth table can be drawn as:

Truth table for Voters

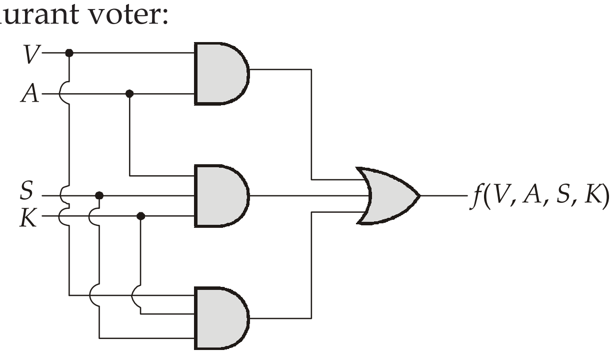
Vikas (V)	Archana (A)	Saurabh (S)	Kunal (K)	f (restaurant serving only burger)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

minimizing above using k-map:



$$f = VA + VSK + ASK$$

Logic circuit for restaurant voter:



thus, if $f(V, A, S, K) = 1$, then the family will go to the restaurant serving burger only and if $f(V, A, S, K) = 0$, then the family will go to the restaurant serving chicken only.

Q.1 (e) Solution:

$$\vec{A} = x^2y\,\hat{a}_x + y^2x\,\hat{a}_y - 4xyz\,\hat{a}_z \text{ Wb/m}$$

$$\vec{\nabla}\times\vec{A} = \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2y & y^2x & -4xyz \end{bmatrix}$$

$$\vec{\nabla}\times\vec{A} = (-4xz)\hat{a}_x - \hat{a}_y(-4yz) + \hat{a}_z[y^2 - x^2]$$

$$\Rightarrow \quad \vec{\nabla}\times\vec{A} = -4xz\hat{a}_x + 4yz\hat{a}_y + [y^2 - x^2]\hat{a}_z = \vec{B}$$

(i) $\vec{B}$ at (–1, 2, 5)

$$\vec{B} = (20\hat{a}_x + 40\hat{a}_y + 3\hat{a}_z) \text{ Wb/m}^2$$

(ii) Flux through the surface:

$$\phi = \oint\!\!\oint_s \vec{B}\cdot dS \qquad \Rightarrow \phi = \int_{-1}^{4}\int_{0}^{1}(y^2 - x^2)\,dx\,dy$$

$$\phi = \int_{-1}^{4}\left[xy^2 - \frac{x^3}{3}\right]_0^1 dy \qquad \Rightarrow \phi = \int_{-1}^{4}\left[y^2 - \frac{1}{3}\right]dy$$

$$\phi = \left[\frac{y^3}{3} - \frac{y}{3}\right]_{-1}^{4} \qquad \Rightarrow \phi = \left(\frac{60}{3} - \left[\frac{-1}{3} + \frac{1}{3}\right]\right)W_b$$

$$\Rightarrow \quad \phi = 20 \text{ Wb}$$

Q.2 (a) (i) Solution:

$$V = \frac{\vec{P}_1.\vec{R}_1}{4\pi\epsilon_0\,|R_1|^3} + \frac{\vec{P}_2.\vec{R}_2}{4\pi\epsilon_0\,|R_2|^3} = \frac{1}{4\pi\epsilon_0}\left[\frac{\vec{P}_1\cdot\vec{R}_1}{(R_1)^3} + \frac{\vec{P}_2\cdot\vec{R}_2}{(R_2)^3}\right]$$

Where

$$\vec{P}_1 = -5\hat{a}_z,$$

$$\vec{P}_2 = 9\hat{a}_z$$

$$\vec{R}_1 = (0, 0, 0) - (0, 0, -2)$$

$$= 2\hat{a}_z, R_1 = 2$$

$$\vec{R}_2 = (0, 0, 0) - (0, 0, 3)$$
$$= -3\hat{a}_z, R_2 = 3$$

Therefore
$$V = \frac{1}{4\pi \times \frac{10^{-9}}{36\pi}}\left[\frac{-5\times 2}{2^3} - \frac{27}{3^3}\right].10^{-9}$$
$$V = -20.25 \text{ V}$$

Q.2 (a) (ii) Solution:

$$\vec{J} = 2x^2\hat{a}_x + 2xy^3\hat{a}_y + 2xy\,\hat{a}_z$$
$$\vec{\nabla}\cdot\vec{J} = 4x + 6xy^2 + 0 = (4 + 6y^2)x$$

Current,
$$I = \oiint_s \vec{J}\cdot\vec{dS} = \iiint_V \vec{\nabla}\cdot\vec{J}\,dV \quad \text{(from divergence theorem)}$$

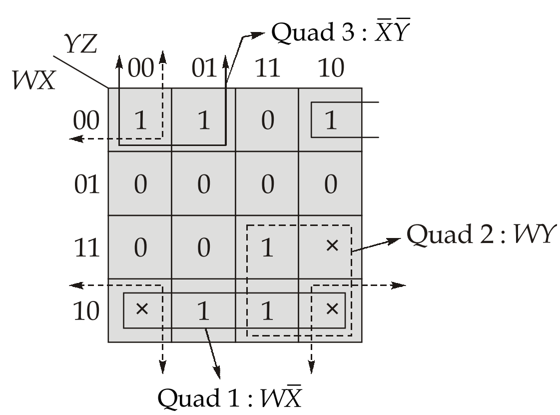
$$\Rightarrow \quad I = \int_0^1\int_0^1\int_0^1 (4+6y^2)x\,dx\,dy\,dz = \int_0^1 dz\int_0^1 (4+6y^2)dy\int_0^1 x\,dx$$

$$\Rightarrow \quad I = 1\times\left[4y+2y^3\right]_0^1 \times \left[\frac{x^2}{2}\right]_0^1 = 1\times 6\times\frac{1}{2}$$

$$\Rightarrow \quad I = 3 \text{ A}$$

Q.2 (b) Solution:

K-map and simplification



Hence the simplified expression is as follows:

$$F = w\bar{x} + wy + \bar{x}\,\bar{y} + \bar{x}\,\bar{z}$$

Conversion into NOR:

$$F = w\overline{x} + wy + \overline{x}\,\overline{y} + \overline{x}\,\overline{z}$$

Take double inversion to get

$$F = \overline{\overline{w\overline{x} + wy + \overline{x}\,\overline{y} + x\overline{z}}}$$

Using De-Morgan's theorem

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

We get, $$F = \overline{\overline{(w\overline{x})} \cdot \overline{(wy)} \cdot \left(\overline{(\overline{x}\,\overline{y})}\right) \cdot \left(\overline{(\overline{x}\,\overline{z})}\right)}$$

Using De-Morgan's theorem

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

We get $$F = \overline{(\overline{w} + x) \cdot (\overline{w} + \overline{y}) \cdot (x + y) \cdot (x + z)}$$

Using De-Morgan's theorem

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

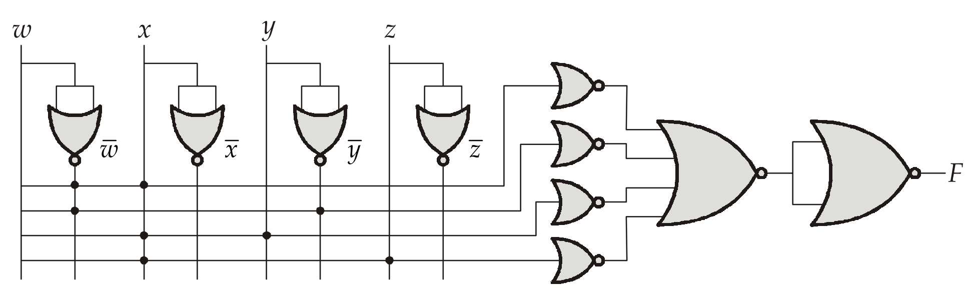
We get $$F = (\overline{\overline{w} + x}) + (\overline{\overline{w} + \overline{y}}) + (\overline{x + y}) + (\overline{x + z})$$

Taking double inversion,

We get $$F = \overline{\overline{(\overline{\overline{w} + x}) + (\overline{\overline{w} + \overline{y}}) + (\overline{x + y} + (\overline{x + z}))}}$$

Implementation:

The above expression is implemented



Q.2 (c) Solution:

Given: $(\text{SNR})_0 = 40\text{ dB} = 10^4$;

Message signal bandwidth,

$$f_m = 10\text{ kHz};$$

$$\text{Channel loss} = 40\text{ dB}$$

Power spectrial density of white noise,

$$\frac{\eta}{2} = 10^{-9}\text{ W/Hz}$$

(i) DSB: Transmission bandwidth,

$$B_T = 2f_m = 20 \text{ kHz}$$

For DSB, FOM = 1

$$\Rightarrow \quad \frac{(SNR)_0}{(SNR)_r} = 1$$ where, $(SNR)_r$ is the signal to noise ratio at detector input

$$\Rightarrow \quad (SNR)_r = (SNR)_0 = 40 \text{ dB}$$

$$\Rightarrow \quad \frac{S_r}{N_r} = 10^4,$$ where N_r is the noise power affecting message signal

$$\Rightarrow \quad S_r = 10^4 N_r = 10^4 \times \left(\frac{\eta}{2} \times 2f_m\right) = 10^4 \times 10^{-9} \times 2 \times 10 \times 10^3$$

$$\Rightarrow \quad S_r = 0.2 \text{ W}$$

Considering channel loss = 40 dB = 10^4, we get transmission power as

$$S_T = 10^4 \times S_r = 10^4 \times 0.2 = 2 \text{ kW}$$

(ii) SSB:

Transmission bandwidth,

$$B_T = f_m = 10 \text{ kHz}$$

For SSB, FOM = 1. Hence, we get,

$$S_T = 2 \text{ kW}$$

(iii) AM:

Transmission bandwidth,

$$B_T = 2f_m = 20 \text{ kHz}$$

For envelope detection,

$$\text{FOM} = \frac{\mu^2}{2+\mu^2} = \frac{0.5}{1+0.5}$$

$$\text{FOM} = \frac{(SNR)_0}{(SNR)_r} = \frac{1}{3}$$

$$\Rightarrow \quad (SNR)_r = \frac{S_r}{N_r} = 10^4 \times 3$$

$$\Rightarrow \quad S_r = 3 \times 10^4 \times \left(\frac{\eta}{2} \times 2f_m\right) = 3 \times 10^4 \times 10^{-9} \times 2 \times 10 \times 10^3 = 0.6 \text{ W}$$

Considering channel loss= 40 dB = 10^4,

we get transmission power as

$$S_T = 10^4 S_r = 10^4 \times 0.6 \text{ W} = 6 \text{ kW}$$

Q.3 (a) Solution:

(i) Total current is given as, $I = \int \vec{J} \cdot d\vec{s}$

Here, $d\vec{s} = r^2 \sin\theta \, d\phi \, d\theta \, \hat{a}_r$

Total current passing through a hemispherical shell of radius 20 cm is

$$I = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \frac{1}{r^3}(2\cos\theta \, \hat{a}_r + \sin\theta \hat{a}_\phi) \cdot (r^2 \sin\theta \, d\phi \, d\theta \, \hat{a}_r)\Big|_{r=0.2}$$

$$= \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \frac{1}{r^3} 2\cos\theta r^2 \sin\theta \, d\phi \, d\theta\Big|_{r=0.2}$$

$$= \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \frac{1}{r^3} 2\cos\theta \, r^2 \sin\theta \, d\phi \, d\theta\Big|_{r=0.2}$$

$$= \frac{4\pi}{0.2}\left[\frac{\sin^2\theta}{2}\right]_0^{\pi/2} = 10\pi = 31.42 \text{ A}$$

(ii) Total current passing through a spherical shell of radius 10 cm is

$$I = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{r^2}(2\cos\theta \, \hat{a}_r + \sin\theta \hat{a}_\phi) \cdot (r^2 \sin\theta \, d\phi \, d\theta \, \hat{a}_r)\Big|_{r=0.1}$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{r^3} 2\cos\theta r^2 \sin\theta \, d\phi \, d\theta\Big|_{r=0.1}$$

$$= 2\pi \times \frac{2}{r} \int_{\theta=0}^{\pi} \sin\theta \, d(\sin\theta)\Big|_{r=0.1} = \frac{4\pi}{0.1}\left[\frac{\sin^2\theta}{2}\right]_0^{\pi} = 0$$

Q.3 (b) Solution:

Given, $\vec{B}_2 = 5\hat{a}_x + 8\hat{a}_z$ Wb/m²

$$\vec{H}_2 = \frac{\vec{B}_2}{\mu_2} = \frac{1}{4\mu_0}(5\hat{a}_x + 8\hat{a}_z) \text{ A/m}$$

and $$\vec{H}_1 = \frac{\vec{B}_1}{\mu_1} = \frac{1}{6\mu_0}\left(B_{1x}\hat{a}_x + B_{1y}\hat{a}_y + B_{1z}\hat{a}_z\right) \text{ A/m}$$

From boundary conditions (normal components)

$$\vec{B}_{1n} = \vec{B}_{2n}$$

$$\Rightarrow \qquad B_{1z} = 8$$

To find tangential components, we use

$$\left(\vec{H}_1 - \vec{H}_2\right)\times \hat{a}_{n12} = \vec{K}$$

or, $$\hat{H}_1 \times \hat{a}_z = \hat{H}_2 \times \hat{a}_z + \vec{K}$$

on substitution, we have

$$\frac{1}{6\mu_0}\left(B_{1x}\hat{a}_x + B_{1y}\hat{a}_y + B_{1z}\hat{a}_z\right)\times \hat{a}_z = \frac{1}{4\mu_0}(5\hat{a}_x + 8\hat{a}_z)\times \hat{a}_z + \frac{1}{\mu_0}\hat{a}_y$$

$$\frac{B_{1x}}{6}(-\hat{a}_y) + \frac{B_{1y}}{6}\hat{a}_x = \frac{5}{4}(-\hat{a}_y) + \hat{a}_y$$

equating components results,

$$B_{1y} = 0,$$

$$\frac{-B_{1x}}{6} = \frac{-5}{4} + 1$$

$$\Rightarrow \qquad B_{1x} = 1.5$$

Finally substituting $\vec{B}_{1x}$ and $\vec{B}_{1y}$ in $\vec{B}_1$, we get

$$\vec{B}_1 = 1.5\hat{a}_x + 8\hat{a}_z \text{ Wb/m}^2$$

$$\vec{H}_1 = \frac{\vec{B}_1}{\mu_1} = \frac{1}{\mu_0}\left(\frac{1.5}{6}\hat{a}_x + \frac{8}{6}\hat{a}_z\right) \text{ A/m}$$

or, $$\vec{H}_1 = \frac{1}{\mu_0}(0.25\,\hat{a}_x + 1.33\,\hat{a}_z) \text{ A/m}$$

Q.3 (c) Solution:

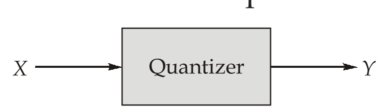
The signal-to-quantization noise ratio can be given as,

$$\left(\frac{S}{N}\right)_Q = \frac{S}{N_Q}$$

Here, S is the signal power and N_Q is the quantization noise power.

Finding quantization noise power:

Let us assume that, the samples at the quantizer input are denoted as X, the samples at the quantizer output are denoted as Y and the quantization error as Q.



For the analytical purpose, we can treat the quantization error as additive noise.

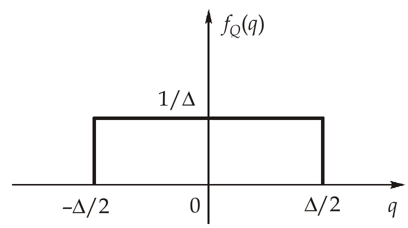
So, $$Q = X - Y$$

For a uniform quantizer, the range of quantization error resulted while quantizing each sample can be given as,

$$|q| \leq \frac{\Delta}{2}; \ \Delta = \text{Step size of the quantizer}$$

If the step-size is sufficiently small, we can assume that, the quantization error "Q" is a random variable uniformly distributed in the range $\left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$.

So, the probability density function of Q can be given as,



The quantization noise can be derived by calculating the mean square value of the random variable "Q".

So,
$$N_Q = E[Q^2] = \int_{-\infty}^{\infty} q^2 \, f_Q(q)\, dq$$

$$= \int_{-\Delta/2}^{\Delta/2} \frac{1}{\Delta}(q^2)\, dq = \frac{1}{\Delta}\left[\frac{q^3}{3}\right]_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{3\Delta}\left[\left(\frac{\Delta}{2}\right)^3 - \left(-\frac{\Delta}{2}\right)^3\right] = \frac{2\Delta^3}{24\Delta} = \frac{\Delta^2}{12} \quad \text{...(i)}$$

When the input samples are distributed in the range $[-a, a]$, the step-size of an uniform quantizer can be given as,

$$\Delta = \frac{a-(-a)}{L} = \frac{2a}{2^n} \quad \text{...(ii)}$$

where, L = Number of quantization levels and n = Number of bits/sample.

By substituting equation (ii) in equation (i), we get,

$$N_Q = \frac{(2a/2^n)^2}{12} = \frac{4a^2}{(12)(2^{2n})} = \frac{a^2}{(3)(2^{2n})} \quad \text{...(iii)}$$

Finding signal power (S):

$$S = E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

It is given that amplitude of samples of $X(t)$ is uniformly distributed in the range $[-a, a]$.

So,
$$S = \int_{-a}^{a} \frac{1}{2a} x^2 dx = \frac{1}{2a}\left(\frac{x^3}{3}\right)_{-a}^{a}$$

$$= \frac{1}{6a}(2a^3) = \frac{a^2}{3} \quad \text{...(iv)}$$

From equations (iii) and (iv), we can write $\left(\frac{S}{N}\right)_Q$ as,

$$\left(\frac{S}{N}\right)_Q = \frac{S}{N_Q} = \frac{a^2/3}{a^2/3(2^{2n})} = 2^{2n} \quad \text{...(v)}$$

In decilogs,
$$\left[\frac{S}{N}\right]_Q = 10\log_{10}\left(\frac{S}{N}\right)_Q = 10\log_{10} 2^{2n}$$

$$= 2n[10\log_{10} 2] = 6.02n \text{ dB} \quad \text{...(vi)}$$

Given that, $n = 8$ bits/sample

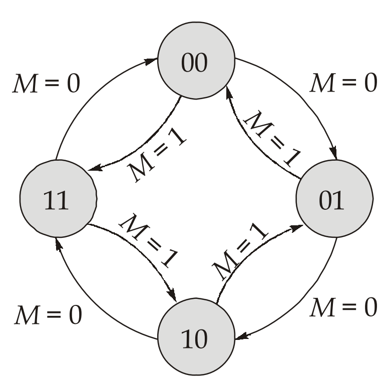
From equations (v) and (vi), we get,

$$\left(\frac{S}{N}\right)_Q = 2^{2n} = 2^{2(8)} = 2^{16} = 65536$$

In decilogs,
$$\left[\frac{S}{N}\right]_Q = 6.02n \text{ dB} = 48.16 \text{ dB}$$

Q.4 (a) Solution:

For a 2-bit counter total 4 states are needed and state diagram for up/down counter can be drawn as



State table with D flip flop excitation for given state diagram.

Mode	Present	State	Next	State	D Flip-Flop Input	
M	Q_A	Q_B	Q_A^+	Q_B^+	D_A	D_B
0	0	0	0	1	0	1
0	0	1	1	0	1	0
0	1	0	1	1	1	1
0	1	1	0	0	0	0
1	0	0	1	1	1	1
1	0	1	0	0	0	0
1	1	0	0	1	0	1
1	1	1	1	0	1	0

***K* Map of D_A**

$M \backslash Q_AQ_B$	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$D_A = \bar{M}\bar{Q}_AQ_B + \bar{M}Q_A\bar{Q}_B + M\bar{Q}_A\bar{Q}_B + MQ_AQ_B$$

***K* Map for D_B**

$M \backslash Q_AQ_B$	00	01	11	10
0	1	0	0	1
1	1	0	0	1

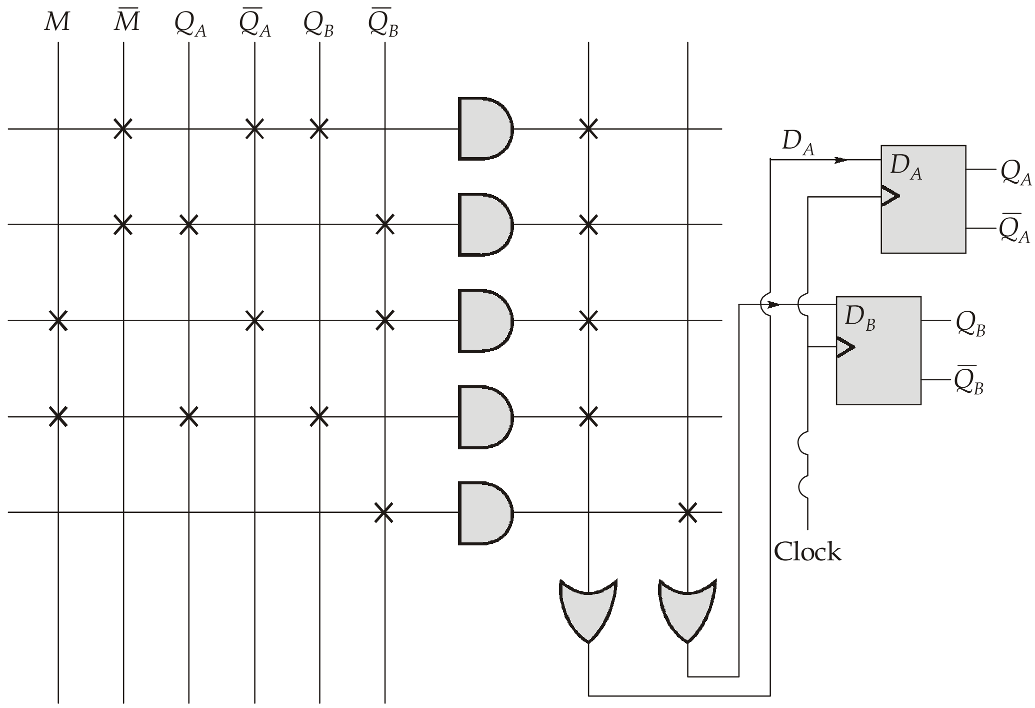
$$D_B = \bar{Q}_B$$

For PLA based design number of product terms are five.
So, A 3-input; 2 output and 5 AND gate based PLA is needed for design.

PLA table

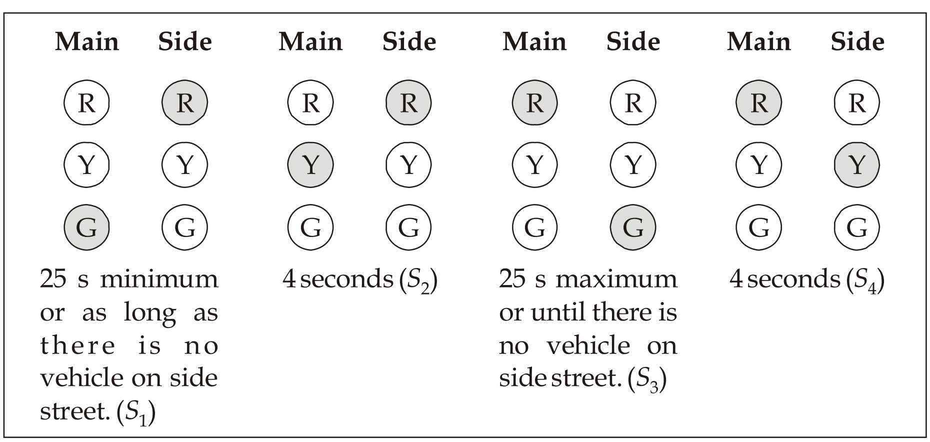
M	Q_A	Q_B	D_A	D_B
–	–	0	0	1
0	0	1	1	0
0	1	0	1	0
1	0	0	1	0
1	1	1	1	0

Circuit Block Diagram:



Q.4 (b) Solution:

Total four states are required as defined below:



Defining the variables:

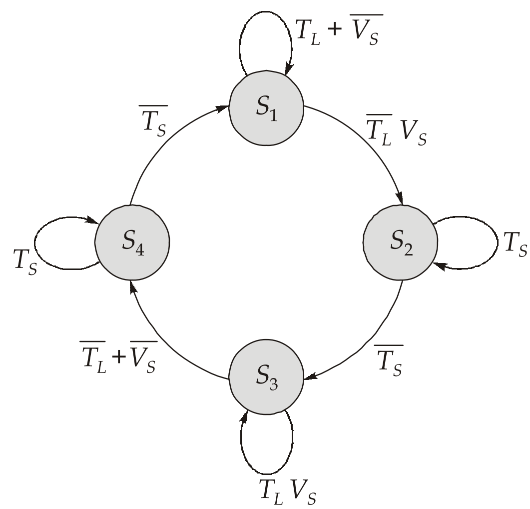
The variables that determine how the system sequences through the various states are defined as follows:

V_S : A vehicle is present on the side street.

T_L : The 25 sec timer is ON.

T_S : The 4 sec timer is ON.

State diagram:



First State: In this state (S_1), the light is green on the main street and red on the side street for 25 s when the long timer is ON or till there is no vehicle on the side street. This condition is expressed as $T_L + \overline{V}_S$. The system transitions to the next state when the long timer goes OFF and there is a vehicle on the side street. This condition is expressed as $\overline{T}_L V_S$.

Second State: In this state (S_2), the light is yellow on the main street and red on the side street. The system remains in this state for 4 s when the short timer is ON. This condition is expressed as T_S. The system transitions to the next state when the short timer goes OFF. This condition is expressed as $\overline{T}_S$.

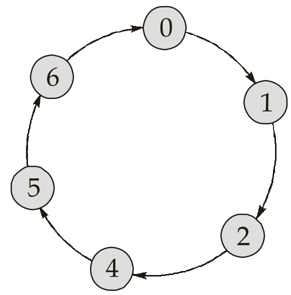
Third State: In this state (S_3), the light is red on the main street and green on the side street for 25 s when the long timer is ON and as long as there is a vehicle on the side street. This condition is expressed as $T_L V_S$. The system transitions to the next state when the long timer goes OFF or when there is no vehicle on the side street. This condition is expressed as $\overline{T}_L + \overline{V}_S$.

Fourth State: In this state (S_4), the light is red on the main street and yellow on the side street. The system remains in this state for 4 s when the short timer is ON. This condition is expressed as T_S. The system transitions back to the first state when the short timer goes OFF. This condition is expressed as $\overline{T}_S$.

Q.4 (c) Solution:

Number of valid states = 6. So, minimum number of FFs required = 3.

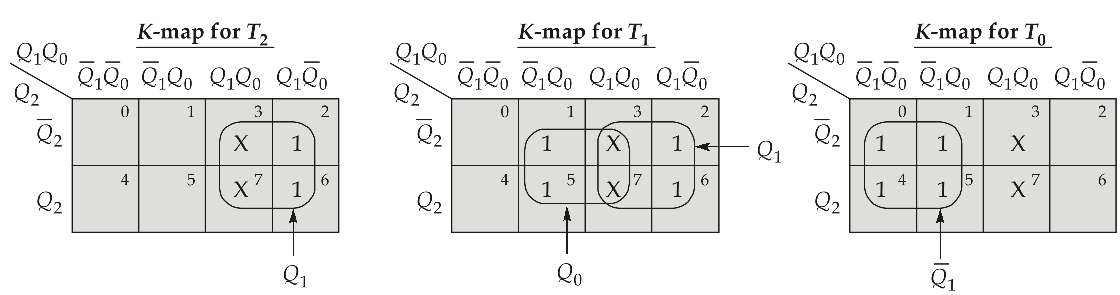
Sequence diagram:



Excitation table:

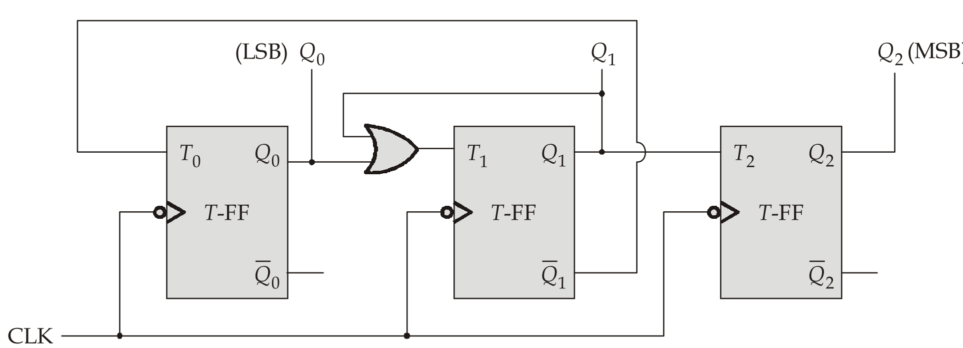
Present state			Next state			Required excitations		
Q_2	Q_1	Q_0	Q_2	Q_1	Q_0	T_2	T_1	T_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	1	0	0	1	1	0
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	0	0	0	1	1	0

Minimization:



$T_2 = Q_1$ $\quad$ $T_1 = Q_0 + Q_1$ $\quad$ $T_0 = \overline{Q}_1$

Logic circuit:



Check for self-starting:

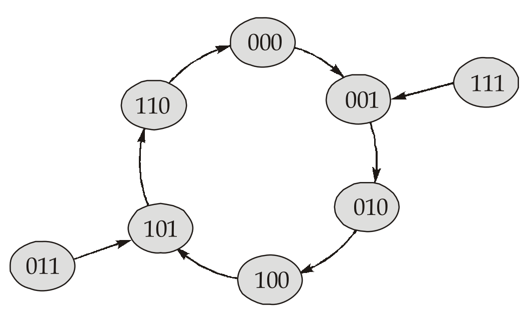
- To check for a given counter is self-starting or not, take each invalid or unused state as the counters initial state and observe the output states of the counter for some finite subsequent clock pulses.
- If the counter is entering into a valid state after some clock pulses from each unused state, the counter is self-starting (or) self-correcting counter. Otherwise the counter is not a self-starting counter.
- In the given problem, 3 and 7 are invalid states.

$T_2 = Q_1$
$T_1 = Q_0 + Q_1$
$T_0 = \overline{Q_1}$

Present state			T_2	T_1	T_0	Next state		
Q_2	Q_1	Q_0				Q_2	Q_1	Q_0
0	1	1	1	1	0	1	0	1
1	1	1	1	1	0	0	0	1

- When the counter finds the state 011 (3) at the output, in the next clock pulse it is entering into the state 101 (5) which is a valid state. Similarly from another unused state 111 (7) the counter entering into the state 001 (1) which is also a valid state. Hence the given counter is a "self-starting" or "self-correcting counter".

Complete sequence diagram:

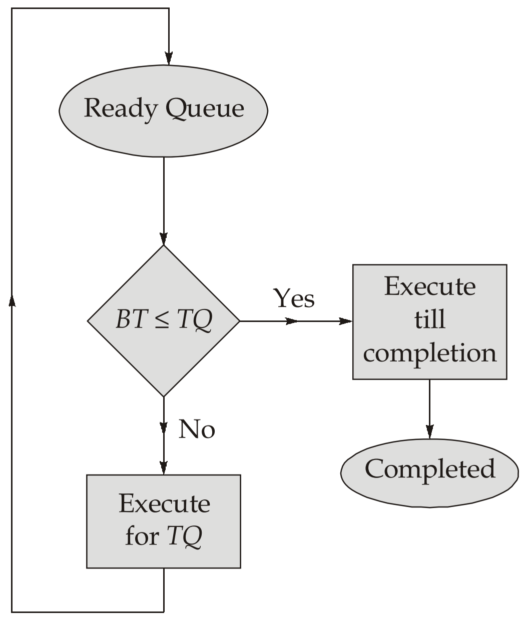


Section B : Computer Fundamentals - 2 + Electrical and Electronic Measurements - 2

Q.5 (a) Solution:

Round Robin Scheduling Algorithm: Round Robin scheduling algorithm is one of the most popular scheduling algorithm mainly used for multitasking and can be implemented in most of the operating systems.

This is the preemptive version of first come first serve scheduling. The algorithm focuses on time sharing. In this algorithm, every process gets executed in a cyclic way. A certain time slice is defined in the system which is called time quantum. Each process present in the ready queue is assigned the CPU for that time quantum, if the execution of the process is completed during that time then the process will terminate else the process will go back to the ready queue and waits for the next turn to complete the execution.



Here AT → Arrival Time, BT → Burst Time (remaining), TQ → Time Quantum

P_{id}	AT	BT	CT	TAT = CT – AT	WT = TAT – BT
P_1	0	8	25	25	17
P_2	5	2	14	9	7
P_3	1	7	26	25	18
P_4	6	3	17	11	8
P_5	8	5	28	20	15
P_6	2	3	9	7	4

Ready Queue: $P_1\ P_3\ P_6\ P_1\ P_2\ P_4\ P_3\ P_5\ P_1\ P_3\ P_5$

Gantt Chart:

P_1	P_3	P_6	P_1	P_2	P_4	P_3	P_5	P_1	P_3	P_5

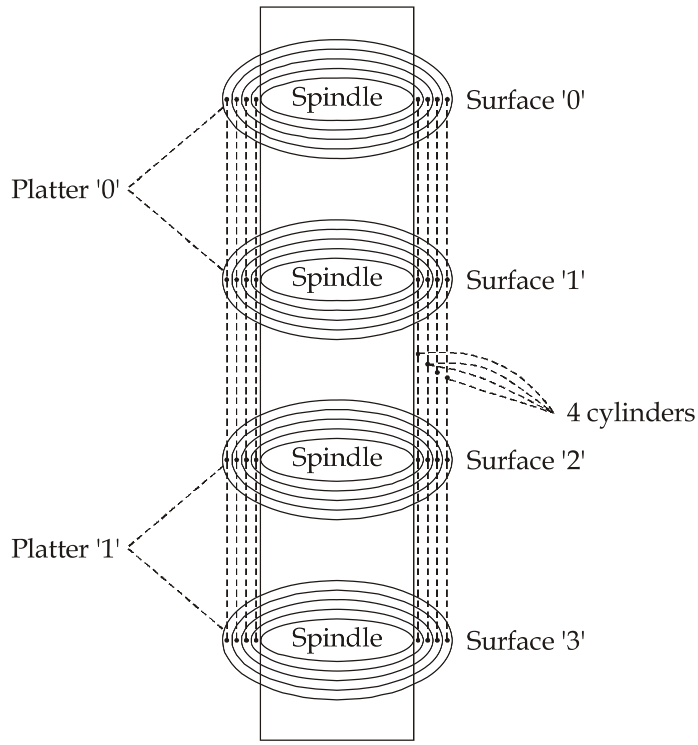
0 3 6 9 12 14 17 20 23 25 26 28

$$\therefore \quad \text{Average waiting time, } T_{avg} = \frac{17+7+18+8+15+4}{6} = \frac{69}{6}$$

$$T_{avg} = 11.5 \text{ nsec}$$

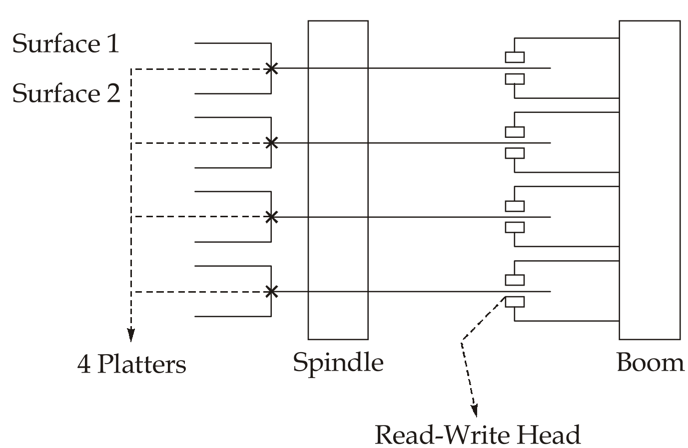
Q.5 (b) Solution:

In a disc, there are few platters, each with two recording surface, each of which contains several tracks. Each track is divided into sectors. There is one read/write head for every surface of the disc. The same track on all surfaces collectively known as a cylinder.

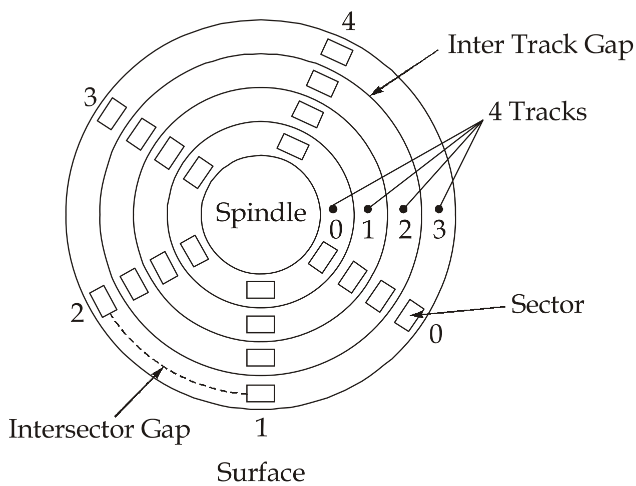


To read from a disc, one must specify :

- cylinder
- surface
- sector
- transfer size
- memory address



Hard Disk with 4 Platters



One disc access is the sum of seek time, rotational latency and transfer time.

Seek Time : Time required to position the arm to desired track.

Rotational Latency : Time required to rotate the desired sector under read/write head. Average latency is half of one rotation time.

Transfer Time : Time required to transfer 1 sector data.

$$\text{1 sector transfer time} = \frac{\text{1 rotation time}}{\text{Number of sectors per track}}$$

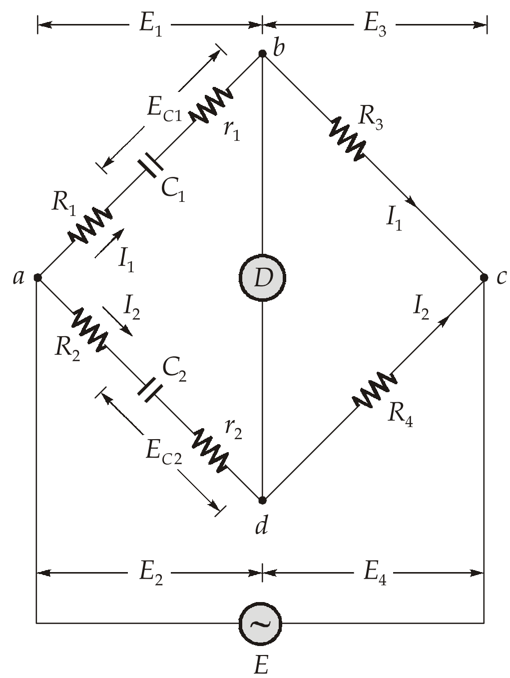
Q.5 (c) Solution:

Computer architecture is a set of rules and method that describe the functionality organization and implementation of computer system. It is description of capabilities and programming model of a computer but not a particular implementation.

Reduced Instruction Set Computer architecture (RISC) has following properties:

1. RISC has simpler instruction, hence simpler instruction decoding is involved in process.
2. Instructions size is less than size of one word.
3. Instruction take single clock cycle to be executed.
4. RISC has more number of general purpose registers.
5. RISC has simple addressing models.
6. RISC has less data type involved.
7. It has hardwired unit of programming.
8. RISC processors are highly pipelined.
9. Execution time in RISC is very less.
10. RISC has no requirement of external memory for calculations.
11. RISC has applications in video processing telecommunication and image processing.

Q.5 (d) Solution:



Let

$$C_1 = \text{Capacitor whose capacitance is to be measured,}$$

$$C_2 = \text{A standard capacitor,}$$

$$R_3, R_4 = \text{non-inductive resistors}$$

Resistance R_1 and R_2 are connected in series with C_1 and C_2 respectively, r_1 and r_2 are resistances representing the loss component of the two capacitors.

At balance,

$$\left(R_1 + r_1 + \frac{1}{j\omega C_1}\right)R_4 = \left(R_2 + r_2 + \frac{1}{j\omega C_2}\right)R_3$$

Comparing imaginary and real parts on both sides

$$\frac{R_4}{\omega C_1} = \frac{R_3}{\omega C_2}$$

$$C_1 = \frac{C_2 R_4}{R_3}$$

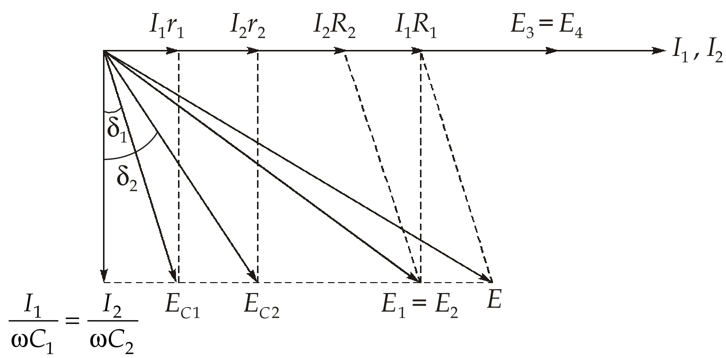
and

$$(R_1 + r_1)R_4 = (R_2 + r_2)R_3$$

$$R_1 + r_1 = (R_2 + r_2)\frac{R_3}{R_4}$$

$$r_1 = (R_2 + r_2)\frac{R_3}{R_4} - R_1$$

The phasor diagram of the bridge under balance condition



Q.5 (e) Solution:

To write an interactive program in *C* language we have

```
# include stdio.h ( ) ; for an input/output
# include conio.h ( ) ; for output display on screen
# define pi = 3.14159
      void main ( )
{
      Float r, perimeter ; radius and perimeter are floating type.
      printf (“enter the radius of circle”);
      scanf (“ %f” , &r) ; getting the value of radius from user
      printmeter = 2 * pi * r ; calculate the value of perimeter
      printf (“ %f”, perimeter) ; display the output value of perimeter
      getch ( ) ;
}
```

Q.6 (a) Solution:

(i) Total secondary circuit resistance

$$= 1.0 + 0.4 = 1.4\ \Omega$$

Total secondary circuit reactance

$$= 0.4 + 0.4 = 0.8\ \Omega$$

Secondary circuit phase angle

$$\delta = \tan^{-1}\frac{0.8}{1.4} = 29°42'$$

or $\cos\delta = 0.8686$ and $\sin\delta = 0.4955$

Primary winding turns, $N_p = 1$

Secondary winding turns,

$$N_s = 200$$

Magnetizing current, $$I_m = \frac{\text{Magnetizing mmf}}{\text{Primary turns}} = \frac{120}{1} = 120\text{ A}$$

Loss component, $$I_e = \frac{\text{mmf equivalent to iron loss}}{\text{Primary winding turns}}$$

$$= \frac{60}{1} = 60\text{ A}$$

∴ Actual ratio, $$R = n + \frac{I_e\cos\delta + I_m\sin\delta}{I_s}$$

$$= 200 + \frac{60\times0.8686 + 120\times0.4955}{5} = 222.3152$$

Primary current, I_p = Actual transformation ratio × Secondary current

= 1111.576 A

$$\text{Ratio error} = \frac{K_n - R}{R}\times100 = \frac{222.3152 - 200}{200}\times100 = 11.16\%$$

(ii) In order to eliminate the ratio error, we must reduce the secondary winding turns or in other words we must reduce the turns ratio.

The nominal ratio is 200 and therefore for zero ratio error, the actual transformation ratio should be equal to the nominal ratio.

Nominal ratio, $K_n = 200$

Actual ratio, $$R = n + \frac{I_e\cos\delta + I_m\sin\delta}{I_s}$$

∴ For zero ratio error, $K_n = R$

$$200 = n + \frac{60\times0.8686 + 120\times0.4955}{5} = n + 22.3152$$

Turns ratio, $n = 177.6848$

Hence, secondary winding turns,

$$N_s = nN_p = 177.6848$$

Reduction in secondary winding turns

$$= 200 - 177.6848 \approx 23$$

Q.6 (b) Solution:

(i) **1.** $R_1(ABC)$ has functional dependency

$$AB \to C$$
$$AC \to B$$
$$BC \to A$$

Key $\{AB, AC, BC\}$

So, relation R_1 is in BCNF.

2. R_2 $(ABCD)$ has FDs:

$$AB \to C$$
$$AC \to B$$
$$BC \to A$$
$$B \to D$$

Key $\{AB, AC, BC\}$

Non-prime $\Rightarrow \{D\}$

So, relation $R_2(ABCD)$ is in 1 NF because partial dependency $B \to D$ exist.

3. R_3 $(ABCEF)$ has FDs:

$$AB \to C$$
$$AC \to B$$
$$BC \to A$$
$$E \to F$$

Key = $\{ABE, ACE, BCE\}$

Non-prime $\Rightarrow \{F\}$

So, R_3 is in 1 NF because partial dependency exist $E \to F$.

(ii) **File system:** The file system is basically a way of arranging the files in a storage medium like a hard disk.

Database Management System: Database Management system is basically a software that manages the collection of related data. It is used for storing data and retrieving the data effectively when it is needed.

File System	Database Management System
1. File system is a collection of data. In this system, the user has to write the procedures for managing the database.	1. Database management system is a collection of data in which, the user is not required to write the procedures.
2. File system provides the detail of the data representation and storage of data.	2. DBMS gives an abstract view of data that hides the details.
3. It is very difficult to protect a file under the file system.	3. DBMS provides a good protection mechanism.
4. File system can't efficiently store and retrieve the data.	4. DBMS contains a wide variety of sophisticated techniques to store and retrieve the data.
5. Redundancy is not controlled in file system.	5. Redundancy is controlled in DBMS.
6. In file system, concurrent access has many problems like redirecting the file while other deleting or updating some information.	6. DBMS takes care of concurrent access of data using some form of locking.
7. Unauthorized access is not restricted in file system.	7. Unauthorized access is restricted in DBMS.
8. Data lost in file system and can't be recovered.	8. DBMS provides back-up and recovery.
9. Data is isolated in file system.	9. DBMS provide multiple user interface.
10. A file-processing system is usually designed to allow predetermined access to data (i.e., compiled programs)	10. A database management system is designed to allow flexible access to data (i.e., queries)
11. A file-processing system coordinates only the physical access.	11. A DBMS coordinates both the physical and the logical access to the data.
12. A file processing system is usually designed to allow one or more programs to access different data files at the same time.	12. A DBMS system is designed to coordinate multiple users accessing the same data at the same time.

Q.6 (c) Solution:

(i) Digital Voltmeter (DVM):

- A digital voltmeter (DVM) displays the value of AC or DC voltage being measured directly as discrete numerals in the decimal number system.
- Numerical readout is advantageous in many applications because it reduces human reading and interpolations errors and eliminates parallax errors.
- The use of digital voltmeters increase the speed with which reading can be taken, also the output of digital voltmeter can be fed to memory devices for storage and future computations.
- A digital voltmeter is a versatile and accurate voltmeter which has many laboratory applications.

Types of DVMs:

(i) Ramp type DVM. (ii) Integrating type DVM. (iii) Potentiometric type DVM. (iv) Successive approximation type DVM. (v) Continuous type DVM.

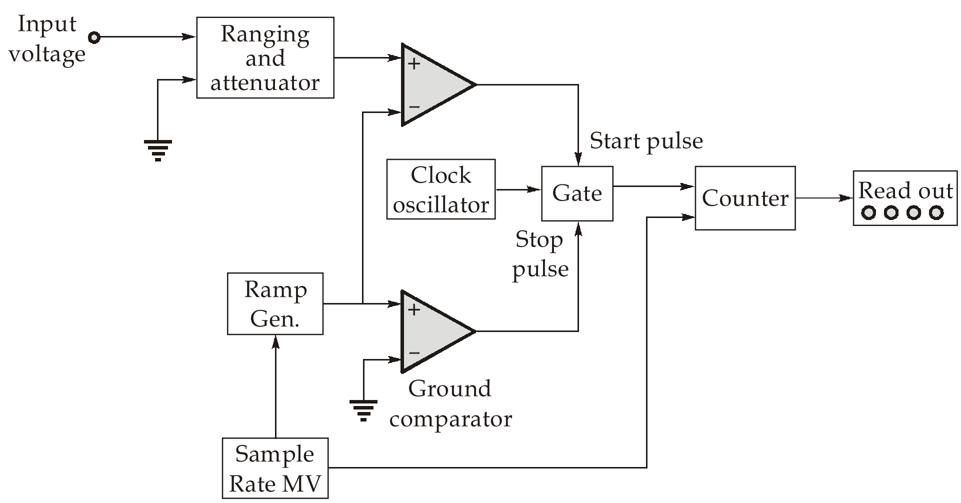
Basic Function:

In every case the basic function that is performed is an analog to digital (A/D) conversion.

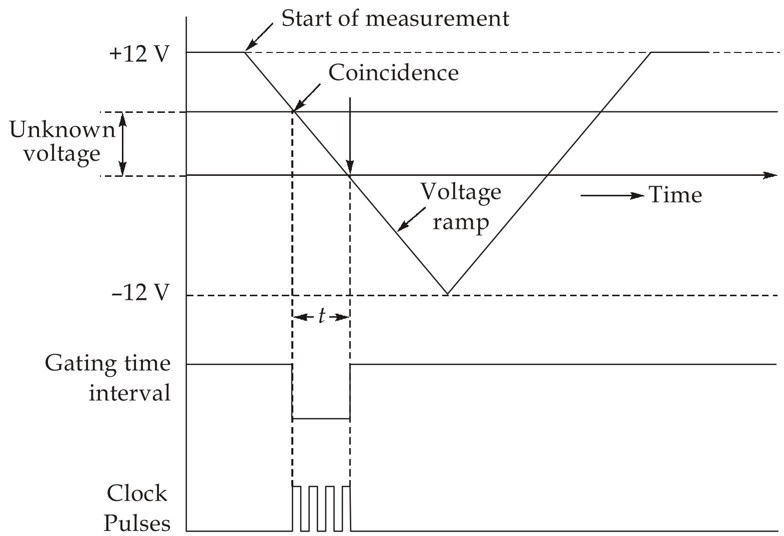
For example a voltage value may be changed to a proportional time interval, which starts and stops a clock oscillator. In turn the oscillator output is applied to an electronic counter which is provided with a read out in terms of voltage values.

Ramp Type Digital Voltmeter:

- When an analog voltage of ramp type is applied to the ramp type digital voltmeter it measure the time interval with an electronic time interval counter and count is displayed as a number of digits on electronic indicating tubes of the output read out of the voltmeter.
- Block diagram of ramp type DVM is shown on figure below:



- The conversion of a voltage value of a time interval is as shown in figure below:



(Timing diagram showing voltage to time conversion)

- The decimal number as indicated by the read out is a measure of the value of input voltage.
- The sample rate multivibrator determines the rate at which the measurement cycles are indicated.
- The sample rate circuit provides an indicating pulse for the ramp generator to start its next ramp voltage.
- At the same time it sends a pulse to the counter which sets all of them to 0. This momentarily removes the digital display of the readout.

(ii) Controlling torque at full scale deflection

$$T_c = 240 \times 10^{-6} \text{ N-m}$$

Deflecting torque at full scale deflection

$$T_d = NBldI$$

$$= 100 \times 1 \times 40 \times 10^{-3} \times 30 \times 10^{-3} I$$

$$= 120 \times 10^{-3} I \text{ N-m}$$

At final steady state position,

$$T_d = T_c$$

or $$120 \times 10^{-3} I = 240 \times 10^{-6}$$

∴ Current at full scale deflection, I

$$= 2 \times 10^{-3} \text{ A} = 2 \text{ mA}$$

Let the resistance of the voltmeter circuit be R

∴ Voltage across the instrument = $2 \times 10^{-3} R$

This produces a deflection of 100 division

$\therefore$ Volts per division $= 2 \times 10^{-3} R/100$

This value should be equal to 1 in order to get 1 volt per division

$\therefore$ $2 \times 10^{-3} R/100 = 1$

or $R = 50000\ \Omega = 50\ \text{k}\Omega$

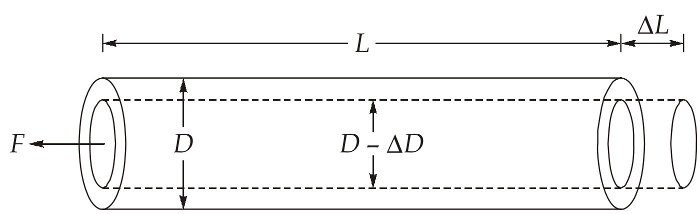
Q.7 (a) Solution:

(i) Let us consider a strain gauge made of circular wire.

Resistance of unstrained gauge,

$$R = \frac{\rho L}{A} \quad \text{...(i)}$$

where ρ is the resistivity of circular crosssection wire.



Let a tensile stress S is applied to the wire. This produces increase in length and decrease in area of the wire.

Differentiating R w.r.t. S,

$$\frac{dR}{dS} = \frac{\rho}{A}\frac{\partial L}{\partial S} - \frac{\rho L}{A^2}\frac{\partial A}{\partial S} + \frac{L}{A}\frac{\partial \rho}{\partial S}$$

or
$$\frac{1}{R}\frac{dR}{dS} = \frac{1}{L}\frac{\partial L}{\partial S} - \frac{1}{A}\frac{\partial A}{\partial S} + \frac{1}{\rho}\frac{\partial \rho}{\partial S} \quad \text{...(ii)}$$

Area, $A = \frac{\pi}{4}D^2$, D = Diameter of wire

$$\therefore \quad \frac{\partial A}{\partial S} = 2 \times \frac{\pi}{4} \times D \times \frac{\partial D}{\partial S}$$

or
$$\frac{1}{A}\frac{\partial A}{\partial S} = \frac{\left(\frac{2\pi}{4}\right)D}{\left(\frac{\pi}{4}\right)D^2}\cdot\frac{\partial D}{\partial S} = \frac{2}{D}\frac{\partial D}{\partial S}$$

Now, equation (ii) can be written as

$$\frac{1}{R}\frac{dR}{dS} = \frac{1}{L}\frac{\partial L}{\partial S} - \frac{2}{D}\frac{\partial D}{\partial S} + \frac{1}{\rho}\frac{\partial \rho}{\partial S}$$

Now, Poisson's ratio, $\nu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{-\partial D/D}{\partial L/L}$

or $\frac{-\partial D}{D} = \frac{-\nu \partial L}{L}$

$\therefore \quad \frac{1}{R}\frac{dR}{dS} = \frac{1}{L}\frac{\partial L}{\partial S} + \nu\frac{2}{L}\frac{\partial L}{\partial S} + \frac{1}{\rho}\frac{\partial \rho}{\partial S}$

For small variations, the above relationship can be written as

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2\nu\frac{\Delta L}{L} + \frac{\Delta\rho}{\rho} \quad \text{...(iii)}$$

The gauge factor is defined as the ratio of per unit change in resistance to per unit change in length, i.e.,

$$\text{Gauge factor } (G_f) = \frac{\Delta R/R}{\Delta L/L}$$

From equation (iii),

where, $\varepsilon = \text{Strain} = \frac{\Delta L}{L}$

If the change in the value of resistivity of material when strained is neglected, the gauge factor is

$$G_f = 1 + 2\nu$$

(ii) Given : $G_F = 2$

Stress, $P = 2100 \text{ kg/cm}^2$

Modulus of elasticity, $E = 2.1 \times 10^6 \text{ kg/cm}^2$

We know that, Strain, $\frac{\Delta L}{L} = \frac{P}{E} = \frac{2100}{2.1\times 10^6} = 10^{-3}$

Now, $\frac{\Delta R}{R} = \frac{\Delta L}{L} G_F = 10^{-3} \times 2 = 2 \times 10^{-3}$

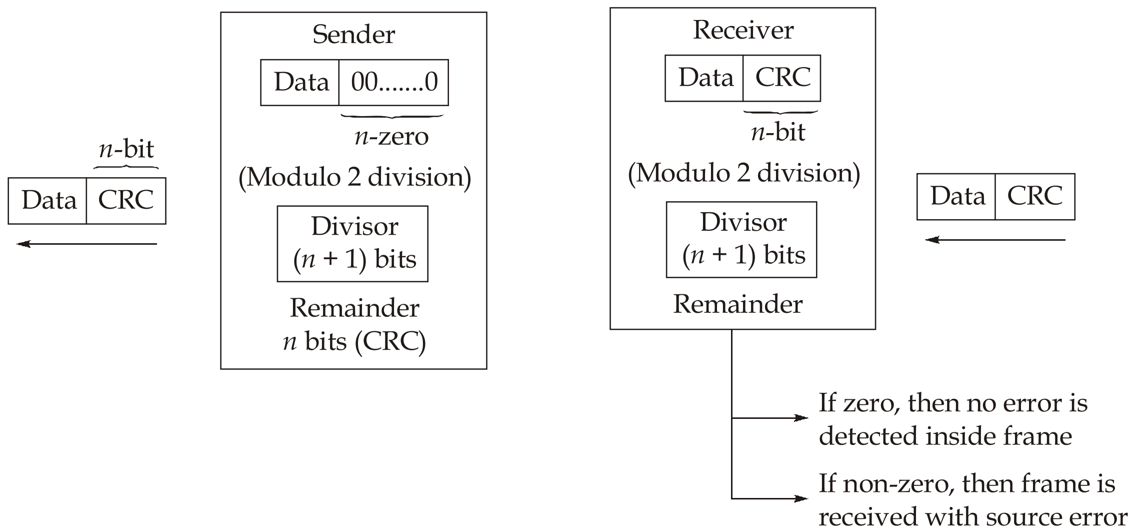
Change in resistance, $\frac{\Delta R}{R} = 2 \times 10^{-3}$

Q.7 (b) Solution:

(i)
- Both sender and receiver agree on same generator polynomial $G(X)$.
- Generator polynomial is a polynomial function where coefficient are either zero or one.

  i.e., $G(X) = X^3 + X + 1 = 1.X^3 + 0.X^2 + 1.X^1 + 1.X^0$
- Divisor is $(n + 1)$ bits long where n is degree of generator polynomial.

Divisor ⇒ 1011 (in e.g.)



(ii)

$$\text{Transmission time} = \frac{\text{Packet Size}}{\text{Data Transfer Size}}$$

$$t_X = \frac{1000 \text{ bytes}}{1 \text{ Mbps}} = \frac{8000 \text{ bits}}{10^6 \text{ bits/sec}} = 8 \text{ msec}$$

$$\text{Propagation delay} = \frac{\text{Distance}}{\text{Signal Speed}}$$

$$t_P = \frac{2000 \text{ km}}{2 \times 10^8 \text{ m/sec}} = \frac{2 \times 10^6 \text{ m}}{2 \times 10^8 \text{ m/sec}} = 10 \text{ msec}$$

$$\text{Rount Trip Time (RTT)} = t_X + 2t_P = (8 + 2 \times 10) = 28 \text{ msec}$$

$$\text{Efficiency, } \eta = \frac{t_X}{\text{RTT}} = \frac{8 \text{ msec}}{28 \text{ msec}} = 0.2857$$

Therefore, $\text{Throughput} = \text{Efficiency of Data Transfer Rate}$

$$= \left(\frac{8}{28}\right) \times 1 \text{ Mbps} = 0.2857 \text{ Mbps}$$

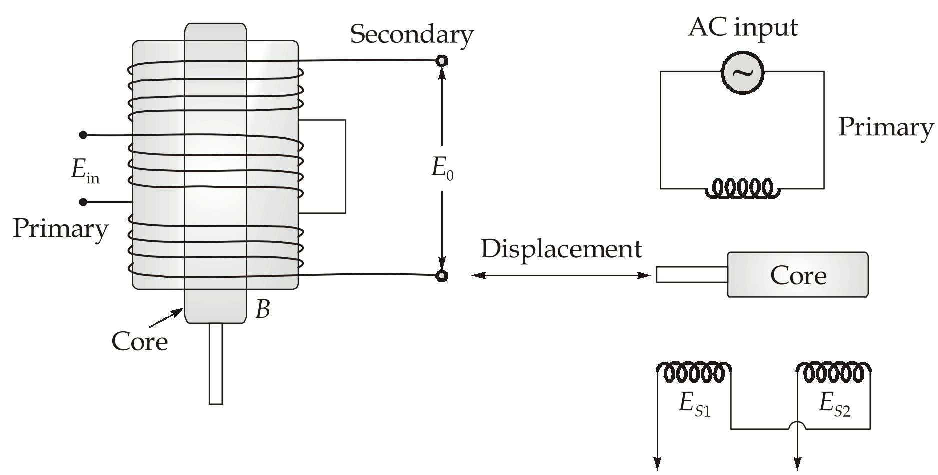
Q.7 (c) Solution:

LVDT consists of a transformer consisting of a single primary winding P_1 and two secondary windings S_1 and S_2 wound on a hollow cylindrical former. The secondary windings have an equal number of turns and are identically placed on either side of the primary windings. The primary winding is connected to an ac source.

A movable soft iron core slides within the hollow former and therefore affects the magnetic coupling between the primary and two secondaries.

The displacement to be measured is applied to an arm attached to the soft iron core.

When the core is in its normal (null) position, equal voltages are induced in the two secondary windings. The frequency of the ac applied to the primary winding varies from 50 Hz to 20 kHz.



The output voltage of the secondary windings S_1 is E_{S1} and that of secondary winding S_2 is E_{S2}.

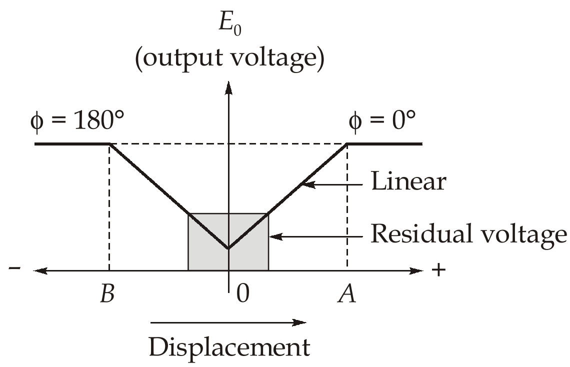
In order to convert the output from S_1 to S_2 into a single voltage signal, the two secondaries S_1 and S_2 are connected in series opposition. Hence the output voltage of the transducer is the difference of the two voltages. Therefore the differential output voltage is $E_0 = E_{S1} \sim E_{S2}$.

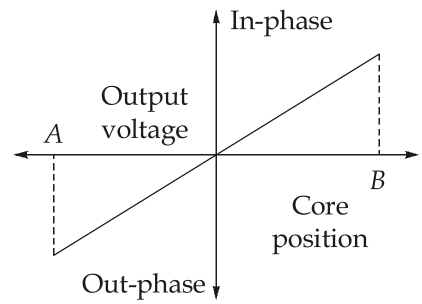
When the core is at its normal position, the flux linking with both secondary windings is equal, and hence equal emf's are induced in them. Hence at null position, $E_{S1} = E_{S2}$. Since the output voltage of the transducer is the difference of the two voltages, the output voltage E_0 is zero at null position.

Now, if the core is moved to the left of the null position, more flux links with winding S_1 and less with winding S_2. Hence output voltage E_{S1} of the secondary winding S_1 is greater than E_{S2}. The magnitude of the output voltage of secondary is then $E_{S1} - E_{S2}$, in phase with E_{S1} (the output voltage of secondary winding S_1).

Similarly if the core is moved to the right of the null position, the flux linking with winding S_2 becomes greater than that linked with winding S_1. The output voltage in this case is $E_0 = E_{S2} - E_{S1}$ and is in phase with E_{S2}.

The output voltage of an LVDT is a linear function of the core displacement within a limited range of motion (say 5 mm from the null position).





Advantages of LVDT:

- ***Linearity***: The output voltage of this transducer is practically linear for displacement upto 5 mm.
- ***Infinite resolution***: The change in output voltage is stepless. The effective resolution depends more on the test equipment than on the transducer.
- ***High output***: It gives a high output (therefore there is no need for intermediate amplification devices).
- ***High sensitivity***: The transducer possesses a sensitivity as high as 40 V/mm.
- ***Ruggedness***: These transducers can usually tolerate a high degree of vibration and shock.
- ***Less friction***: There are no sliding contacts.
- ***Low hysteresis*** **:** This transducer has low hysteresis, hence repeatability is excellent under all conditions.
- ***Low power consumption***: Most LVDTs consume less than 1 W of power.

Q.8 (a) Solution:

(i) **1.** The voltage across the resistance R_b without either meter connected, is calculated using the voltage divider formula.

Therefore,
$$V_{R_b} = \frac{5K}{25K+5K} \times 30 = \frac{150K}{30K} = 5\text{ V}$$

2. Starting with meter 1, having sensitivity $S = 1\text{ k}\Omega/\text{V}$. Therefore, the total resistance it presents to the circuit.

$$R_{m1} = S \times \text{Range}$$
$$= 1\text{ k}\Omega/\text{V} \times 10$$
$$= 10\text{ k}\Omega$$

The total resistance across R_b is, R_b in parallel with meter resistance R_{m1}.

$$R_{eq} = \frac{R_b \times R_{m1}}{R_b + R_{m1}} = \frac{5K \times 10K}{5K + 10K} = 3.33 \text{ k}\Omega$$

Therefore, the voltage reading obtained with meter 1 using the voltage divider equation is

$$V_{R_b} = \frac{R_{eq}}{R_{eq} + R_a} \times V = \frac{3.33K}{3.33K + 25K} \times 30$$

$$= 3.53 \text{ V}$$

3. The total resistance that meter 2 presents to the circuit is

$$R_{m2} = S \times \text{range} = 20 \text{ k}\Omega/\text{V} \times 10 \text{ V} = 200 \text{ k}\Omega$$

The parallel combination of R_b and meter 2 gives

$$R_{eq} = \frac{R_b \times R_{m2}}{R_b + R_{m2}} = \frac{5K \times 200K}{5K + 200K} = 4.88 \text{ k}\Omega$$

Therefore, the voltage reading obtained with meter 2, using the voltage divider equation is

$$V_{R_b} = \frac{4.88K}{2.5K + 4.88K} \times 30 = 4.9 \text{ V}$$

4. The error in the reading of the voltmeter is given as :

$$\% \text{ Error} = \frac{\text{Voltage reading observed in meter} - \text{Actual Voltage}}{\text{Actual voltage}} \times 100\%$$

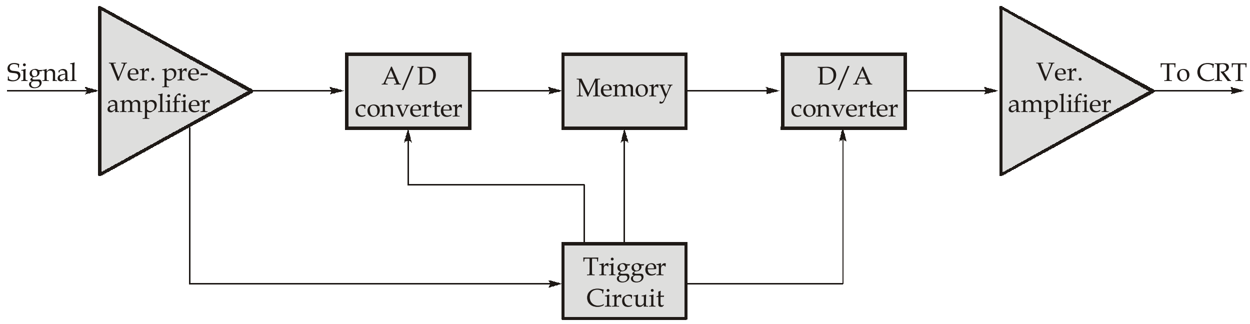
$$\therefore \quad \text{Voltmeter 1 error} = \frac{3.33 \text{ V} - 5 \text{ V}}{5 \text{ V}} \times 100\%$$

$$= -33.4\%$$

Similarly,

$$\text{Voltmeter 2 error} = \frac{4.9 \text{ V} - 5 \text{ V}}{5 \text{ V}} \times 100\% = -2\%$$

(ii) Digital Storage Oscilloscope : Digital Storage Oscilloscope is an instrument that analyze the signal digitally and store the data in the electronic digital memory. By examining the stored traces in memory, it can display visual as well as numerical values. It digitizes the input signal in order to have subsequent digital signals. The input is stored in digital memory in the form of 0 and 1. This stored digitized signal is then viewed on the CRT screen after the signal reconstruction in analog form. A schematic block diagram of a digital storage oscilloscope is shown in the figure below.



It uses an A/D converter to digitize the input signal. That means input signal waveform is sampled at many points and then the instantaneous amplitude at each point is converted into a binary number proportional to the amplitude. The binary numbers are then stored in the memory. A D/A converter, at the output of the memory, reconverts the binary words into analog voltages.

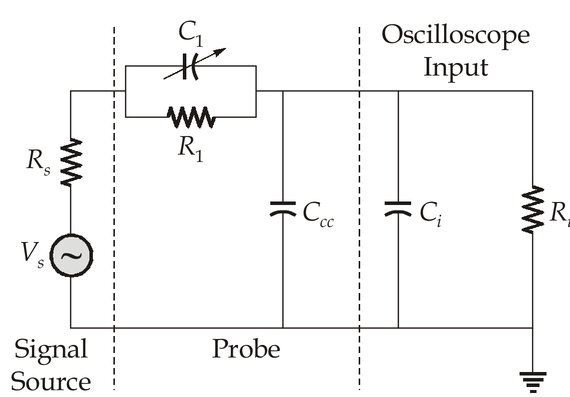
The vertical amplifier amplifies the analog output voltage to value capable of driving the CRT vertical deflection system.

The digital technique is used in some high frequency oscilloscope designed to display transient phenomena. However, the technique is very expensive for high frequency range. This is because of the fact that the sampling rule requires a sampling rate for any given waveform of twice the highest frequency Fourier series component of the waveform. This is required for satisfactory reproduction of the original waveform. For this a lot of memory is required. The memory is expensive.

It also requires fast tracking ADC and DAC which also increase the cost. However, a digital storage oscilloscope finds uses in medical and physiological oscilloscopes.

Q.8 (b) Solution:

(i) Circuit diagram of 10 : 1 probe.



$$C_2 = C_{cc} + C_i$$

C_2 is the sum of input capacitance and co-axial cable capacitance.

$$C_2 = 100 + 30 = 130 \text{ pF}$$

Also,
$$C_1 = C_2 \times \frac{R_i}{R_1}$$

Therefore,
$$C_1 = (130 \times 10^{-12}) \times \frac{1 \times 10^6}{9 \times 10^6} = 14.4 \text{ pF}$$

Probe input capacitance

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_T} = \frac{1}{14.4} + \frac{1}{130}$$

$$C_T = 13 \text{ pF}$$

(ii) Gantt Chart :

	P_4	P_5	P_3	P_2	P_4	P_1	P_6	P_3	P_2	P_4	P_1	P_3
0–1	1–4	4–6	6–9	9–12	12–15	15–18	18–21	21–24	24–27	27–30	30–32	32–33

Now, we know

Turn around time = Exit time – Arrival time

Waiting time = Turn around time – Burst time

Processes Id	Exit Time	Turn Around Time	Waiting Time
P_1	32	32 – 5 = 27	27 – 5 = 22
P_2	27	27 – 4 = 23	23 – 6 = 17
P_3	33	33 – 3 = 30	30 – 7 = 23
P_4	30	30 – 1 = 29	29 – 9 = 20
P_5	6	6 – 2 = 4	4 – 2 = 2
P_6	21	21 – 6 = 15	15 – 3 = 12

$$\text{Average TAT} = \frac{27+23+30+29+4+15}{6} = \frac{128}{6} = 21.33 \text{ units}$$

$$\text{Average waiting time} = \frac{22+17+23+20+2+12}{6} = \frac{96}{6} = 16 \text{ units}$$

Q.8 (c) Solution:

Given,

$$R_1 = 3.1\text{ k}\Omega, \quad R_3 = 25\text{ k}\Omega, \quad R_4 = 100\text{ k}\Omega,$$

$$C_1 = 5.2\ \mu\text{F}, \quad f = 2.5\text{ kHz}$$

$$Z_1 = R_1 - \frac{j}{\omega C_1}$$

$$Y_3 = \frac{1}{R_3} + j\omega C_3$$

Using bridge balance equation,

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_2 = Z_1 Z_4 Y_3$$

$$R_2 = R_4\left(R_1 - \frac{j}{\omega C_1}\right)\left(\frac{1}{R_3} + j\omega C_3\right)$$

$$R_2 = \frac{R_1 R_4}{R_3} - \frac{jR_4}{\omega C_1 R_3} + j\omega R_1 C_3 R_4 + \frac{C_3}{C_1}R_4$$

$$R_2 = \left(\frac{R_1 R_4}{R_3} + \frac{C_3}{C_1}R_4\right) - j\left(\frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4\right)$$

In equation imaginary part equal to zero, we get

$$\frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4 = 0$$

$$\therefore \quad \omega = \frac{1}{\sqrt{C_1 R_1 C_3 R_3}}$$

$$\therefore \quad f = \frac{1}{2\pi\sqrt{C_1 R_1 C_3 R_3}} \quad \text{...(i)}$$

and

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{C_3}{C_1}R_4 \quad \text{...(ii)}$$

Putting values in equation (i),

$$2500 = \frac{1}{2\pi\sqrt{(5.2\times 10^{-6})(3100)\times C_3 \times (25000)}}$$

Squaring both sides and computing C_3

$$C_3 = \frac{1}{4\pi^2 \times 6250000 \times 5.2\times 10^{-6} \times 3100 \times 2500} = 0.01\text{ nF}$$

Putting C_3 in equation (ii), we get

$$R_2 = \frac{3100 \times 100000}{25000} + \frac{0.01 \times 10^{-9}}{5.2 \times 10^{-6}} \times 10^5 = 12400 + 0.19$$

$$R_2 = 12.419 \text{ k}\Omega$$

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